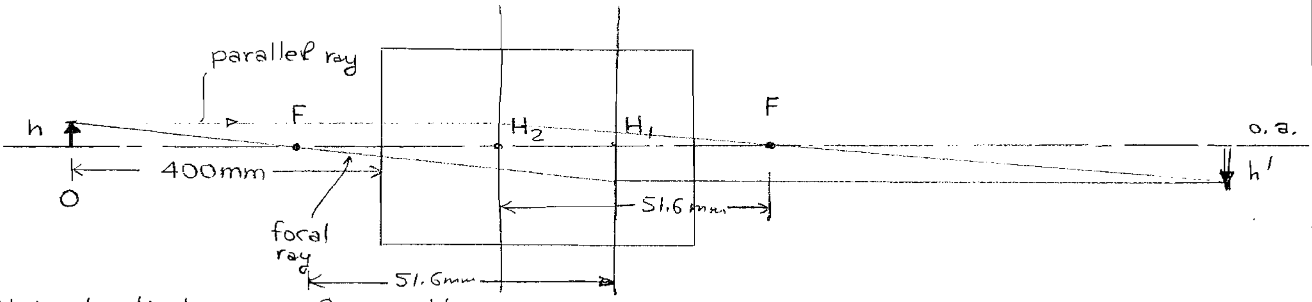
ΟΠΤΙΚΗ ΕΠΙΣΤΗΜΗ & ΤΕΧΝΟΛΟΓΙΑ
ΣΕΙΡΑ ΑΣΚΗΣΕΩΝ No. 3

ΑΣΚΗΣΗ 1:

1. Nikkor-H Imaging Lens:



Object distance from H_1:

$$s = 400mm + (51.6mm - 23.7mm) = 427.9\,mm$$

The object is a real object for the lens ~ $s = 427.9$ mm

$f = 51.6$ mm (positive since this is a convergent lens).

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s'} = \frac{1}{-427.9} + \frac{1}{51.6} \Rightarrow s' = 58.68\,mm$$

1) Image distance from the back surface of the rear lens =

$$= 58.68mm - (57.8mm - 44.3mm) = \underline{45.18\,mm}$$

2) Lets compute the transverse magnification first.

$$m = -\frac{s'}{s} = -\frac{58.68mm}{427.9mm} = -0.137 = \frac{h'}{h}$$

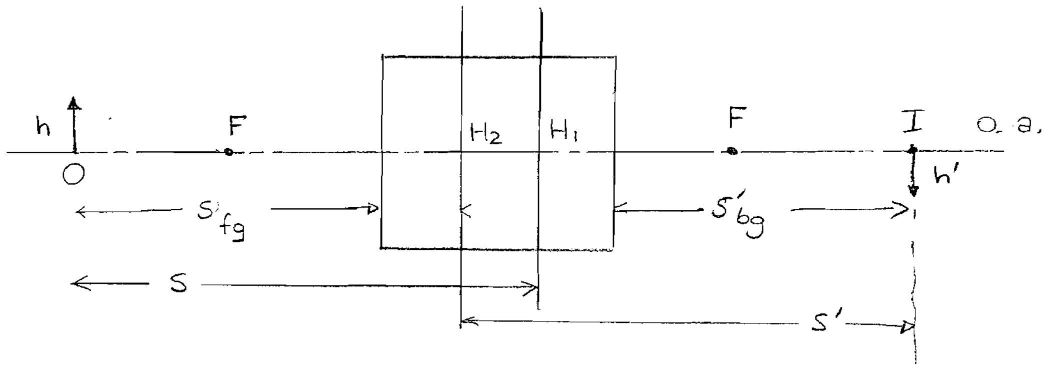
$$h' = mh = -0.137 \cdot 10mm = \underline{-1.37\,mm} \quad \text{(below o.a.)}$$

3) $m = -0.137$

4) Image is real since $s' = 58.68mm > 0$

5) Look at the figure (qualitatively only).

ΑΣΚΗΣΗ 2:



(a)

The needed magnification (transverse) is $\frac{1}{10} = 0.1$ and since the image should be real $m = -0.1 = \frac{s'}{s} = \frac{h'}{h}$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} + \frac{1}{-ms} = \frac{1}{f} \Rightarrow \frac{1}{s}\left(1 - \frac{1}{m}\right) = \frac{1}{f} \Rightarrow$$

$$s = f\left(1 - \frac{1}{m}\right) \rightarrow s = 51.6\,mm\,(1-(-0.1)) = 567.6\,mm$$

Thus in magnitude the distance of the items from H_1 is 567.6 mm

The distance from the front glass, o_{fg}, is:

$$o_{fg} = 567.6\,mm - (51.6\,mm - 23.7\,mm) = 539.7\,mm$$

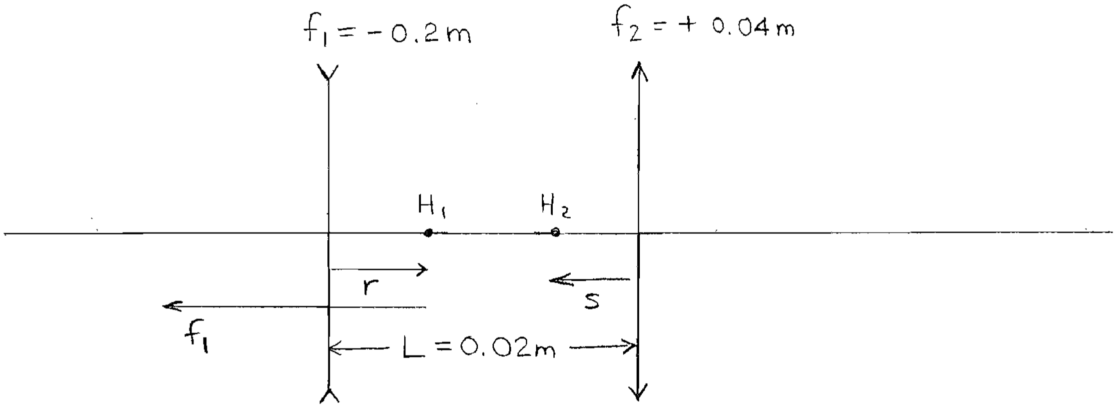
(b) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{-\frac{s'}{m}} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s'}(-m+1) = \frac{1}{f} \Rightarrow s' = f(1-m) \Rightarrow$

$$s' = 51.6\,mm\,(1 + 0.1) = 56.76\,mm$$

Distance of photocathode face from the back glass, i_{bg}, is:

$$i_{bg} = 56.76\,mm - (57.8\,mm - 44.3\,mm) = 43.26\,mm$$

ΑΣΚΗΣΗ 3:



Matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \tilde{M} = \tilde{M}_2 \tilde{M}_T \tilde{M}_1$ where

$\tilde{M}_2 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix}$ $M_T = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$ $\tilde{M}_1 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix}$

$$\tilde{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{0.04} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0.02 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{-0.2} & 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \tilde{M} = \begin{bmatrix} 1.1 & 0.02 \\ -22.5 & 0.5 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

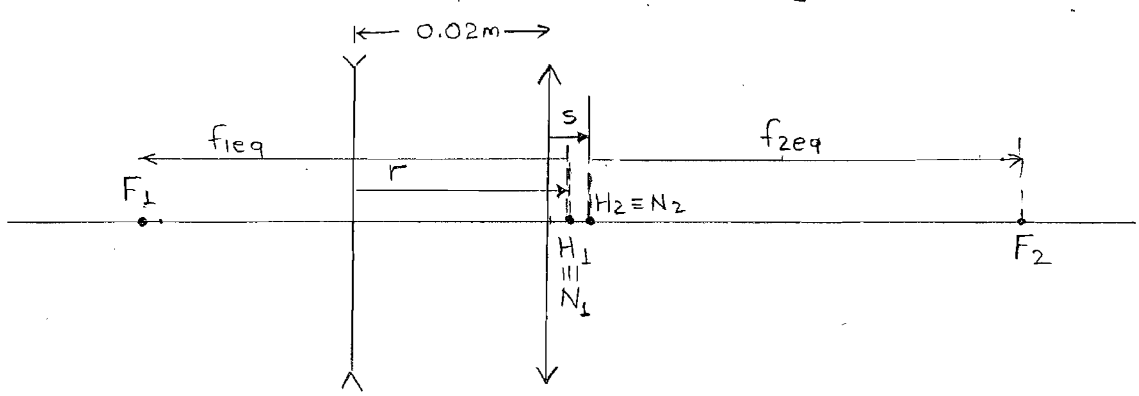
Thus, $A = 1.1$, $B = 0.02$, $C = -22.5$, and $D = 0.5$.

$|f_{1eq}| = |f_{2eq}| = 1/|C| = 0.04444$ m

$r = \frac{D-1}{C} = 0.02222$ m (specifies H_1)

$s = \frac{1-A}{C} = 0.004444$ m (specifies H_2).

Thus, the cardinal points of the system are:



Now we can use the equivalent thin lens equation for the system:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f_{eq}} \qquad f_{eq} = 0,04444 \text{ m}$$

s = distance of the object from H_1 = 0.12 + r = 0,142222 m

$$\frac{1}{0,142222\text{m}} + \frac{1}{s'} = \frac{1}{0,04444 \text{ m}} \sim s' = 0,064646 \text{ m}$$

s' is the distance of the image from H_2. Thus the distance of the image from the second lens is s' + 0,004444m =

= 0.0690909 m ≃ 6.91 cm.

<u>$A\Sigma KH\Sigma H$ 4 : (Spherical Aberration)</u>

Pedrotti's equation A [originally from Jenkins and White, "Fundamentals of Optics", 4th Ed., Eq. (9e)] can be applied immediately to calculate $s'(h)$. The paraxial focal distance $s' = s'(0)$ can be easily calculated from the thin lens equation $1/s + 1/s' = 1/f$. Since the object is assumed to be a large distance away, this corresponds to $s' \simeq f$ (this implies that the incident rays are parallel to the optical axis). Therefore, the factor $p = (s'-s)/(s'+s) = -1$. The shape factor σ varies from $[-2, +2]$. The axial difference between the paraxial and the 3rd order correction of the spherical aberration is $s'(0) - s'(h) = f - s'(h)$ where $s'(h)$ is determined from equation A as follows

$$\frac{1}{s'(h)} - \frac{1}{f} = \frac{h^2}{8f^3}\frac{1}{n_2(n_2-1)}\left[\frac{n_2+2}{n_2-1}\sigma^2 + 4(n_2+1)p\sigma + (3n_2+2)(n_2-1)p^2 + \frac{n_2^3}{n_2-1}\right]. \quad (1)$$

In order to work with equation B requires a bit more thought since it is given for a single refractive interface. First, it is needed to determine how to specify the radii of curvature R_1 and R_2 in order to retain the focal length f constant. From the definition of the shape factor and from the thin lens focal length equation it is straightforward to show

$$\sigma = \frac{R_2+R_1}{R_2-R_1} \implies R_2 = \frac{\sigma+1}{\sigma-1}R_1 = kR_1, \quad (2)$$

$$\frac{1}{f} = \frac{n_2-n_1}{n_1}\Big(\frac{1}{R_1} - \frac{1}{R_2}\Big) \implies R_1 = \frac{k-1}{k}\frac{n_2-n_1}{n_1}f, \qquad R_2 = (k-1)\frac{n_2-n_1}{n_1}f. \quad (3)$$

Now that the radii of curvature R_1 and R_2 have been determined in order to retain the focal length as the shape factor is changing we need to apply equation B (Jenkins and White) twice for each spherical refractive boundary of the thin lens.

$$\frac{n_1}{s} + \frac{n_2}{s''} = \frac{n_2-n_1}{R_1} + \frac{h^2}{2}\frac{R_1}{f'}\frac{n_1^2}{n_2}\left(\frac{1}{s} + \frac{1}{R_1}\right)^2\left(\frac{1}{R_1} + \frac{n_2+n_1}{n_1 s}\right), \quad (4)$$

$$f' = \frac{n_2}{n_2-n_1}R_1, \quad (5)$$

$$\frac{n_2}{-s''} + \frac{n_1}{s'(h)} = \frac{n_1-n_2}{R_2} + \frac{h^2}{2}\frac{R_2}{f''}\frac{n_2^2}{n_1}\left(\frac{1}{-s''} + \frac{1}{R_1}\right)^2\left(\frac{1}{R_1} + \frac{n_2+n_1}{n_2(-s'')}\right), \quad (6)$$

$$f'' = \frac{n_1}{n_1-n_2}R_2. \quad (7)$$

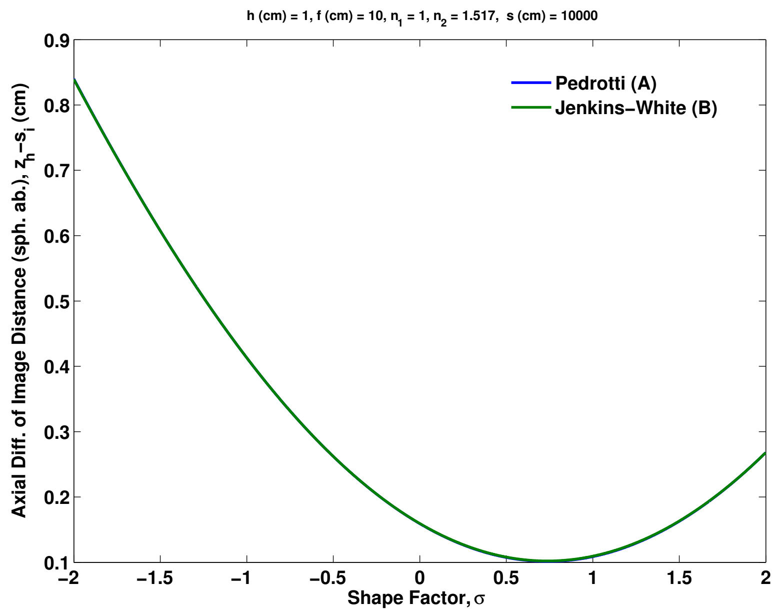
Solving from Eq. (4)(first refractive interface of radius of curvature R_1) for s'' we insert it into Eq.(6) (second refractive interface of radius of curvature R_2) with the "$-$" sign since it would be a virtual object for the second interface. Then from Eq.(6) $s'(h)$ can be determined. Plotting the term $f - s'(h)$ as a function of the shape factor the plots shown below are obtained (using MatLab).

From Eq.(1) the optimum shape factor can be easily determined as follows:

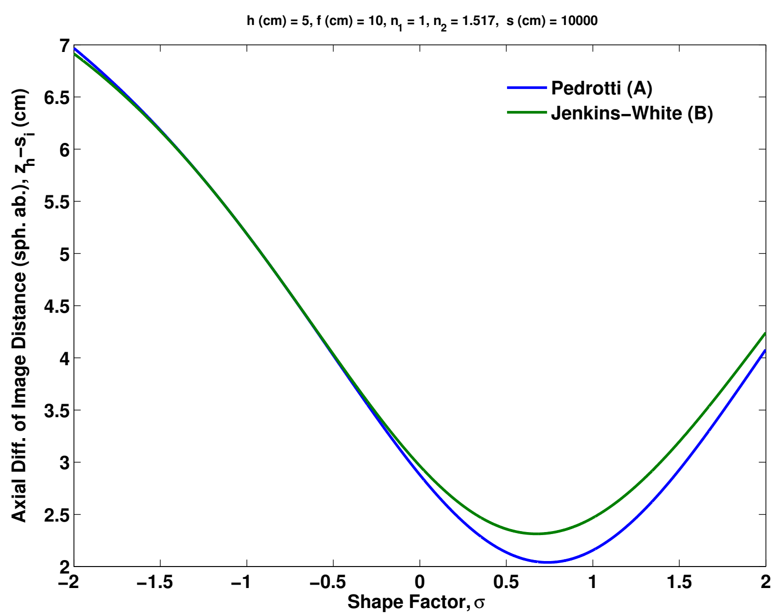
$$\begin{aligned}
\frac{1}{s'(h)} - \frac{1}{f} &= \frac{h^2}{8f^3}\frac{1}{n_2(n_2-1)}\left[\frac{n_2+2}{n_2-1}\sigma^2 + 4(n_2+1)p\sigma + (3n_2+2)(n_2-1)p^2 + \frac{n_2^3}{n_2-1}\right] \Longrightarrow \\
\frac{1}{s'(h)} - \frac{1}{f} &= \frac{h^2}{8f^3}\frac{1}{n_2(n_2-1)}g(\sigma) = ag(\sigma) \Longrightarrow \\
s'(h) &= \frac{f}{1+afg(\sigma)} \Longrightarrow \\
\frac{d(f-s'(h))}{d\sigma} &= 0 \Longrightarrow -\frac{af^2}{[1+afg(\sigma)]^2}\frac{dg}{d\sigma} = 0 \Longrightarrow \\
\sigma_{opt} &= -2\,\frac{n_2^2-1}{n_2+2}\,p. \qquad (8)
\end{aligned}$$

Using $p = -1$ and $n_2 = 1.517$ the optimum shape factor is determined to be $\sigma_{opt} = 0.74$ that agrees well with the curves shown in the figures. The optimum shape corresponds to $R_1 = 5.9425\,\text{cm}$ and $R_2 = -39.7691\,\text{cm}$ which resembles a planoconvex lens with the convex side at the left and the almost planar side at the right of the lens. For $h = 1\,\text{cm}$ the solutions from both equations coincide. For $h = 5\,\text{cm}$ and $h = 10\,\text{cm}$ the two solutions differ since equation A has more approximations than equation B. It can also be observed from the two figures that as the height from the optical axis increases so the spherical aberration increases too.

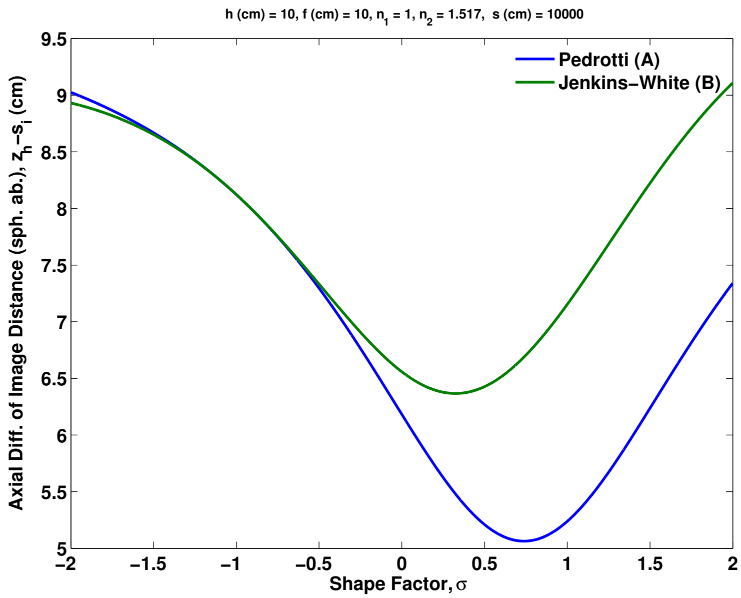
Height from the optical axis, $h = 1\,\text{cm}$:



Height from the optical axis, $h = 5\,\text{cm}$:



Height from the optical axis, $h = 10\,\text{cm}$:



$\underline{A\Sigma K H\Sigma H\ \ 5 : \text{(Spherical Aberration)}}$

(α)

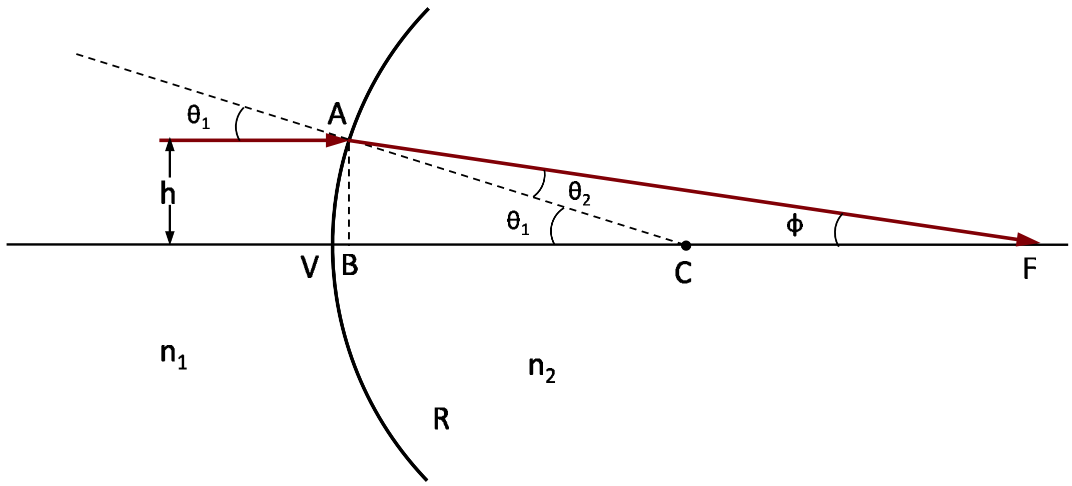


Figure 1: The geometry of the convex spherical refractive interface.

Initially the angles θ_1 and θ_2 can be determined (see figure) from the orthogonal triangle ABC, and the Snell's law respectively,

$$\theta_1 = \sin^{-1}\left\{\frac{h}{R}\right\}, \tag{1}$$

$$\theta_2 = \sin^{-1}\left\{\frac{n_1}{n_2}\frac{h}{R}\right\}. \tag{2}$$

From ACF and ABF triangles it is straightforward to find

$$\phi = \theta_1 - \theta_2 = \sin^{-1}\left\{\frac{h}{R}\right\} - \sin^{-1}\left\{\frac{n_1}{n_2}\frac{h}{R}\right\}, \tag{3}$$

$$FB = \frac{h}{\tan\phi}. \tag{4}$$

From ABC triangle the VB distance can be determined and from that the final focal distance VF can be calculated as follows:

$$VB = R - \frac{h}{\tan\theta_1}, \tag{5}$$

$$VF = R - \frac{h}{\tan\theta_1} + \frac{h}{\tan\phi}. \tag{6}$$

The same focal distance VF can be determined approximately using the paraxial wave approximation

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Longrightarrow s' = VF = \frac{n_2}{n_2 - n_1} R, \tag{7}$$

where for the parallel incident ray $s = \infty$.

(β) Using $R = 100\,\text{mm}$, $n_1 = 1.0$, and $n_2 = 1.5$ and the above derived equations, the VF can be determined both in exact form and in its paraxial approximation. The maximum height of course that can be used is equal to the radius of curvature of the spherical interface. I.e. $h_{max} = R$. The 3-order corrected f_2 can be calculated from Jenkins and White equation (for $s = \infty$) and is given by

$$f_2 \text{ (3rd-order)} = \frac{n_2}{\left(\frac{n_2 - n_1}{R}\right) + \frac{h^2}{2}\left(\frac{n_2 - n_1}{R^3}\right)\frac{n_1^2}{n_2^2}} \tag{8}$$

The resutls for the asked values of h are shown in Table 1, and the graph is shown in Fig. 2.

Table 1: Tabulated Focal Distance (all distances in mm)

Height h	Focal Distance f_2	Paraxial f_2	3rd-order f_2
0.5	299.998	300	299.998
1.0	299.993	300	299.993
5.0	299.833	300	299.833
10.0	299.332	300	299.335
25.0	295.781	300	295.890
40.0	288.976	300	289.700
50.0	282.419	300	284.211
75.0	256.838	300	266.667
95.0	217.845	300	249.884

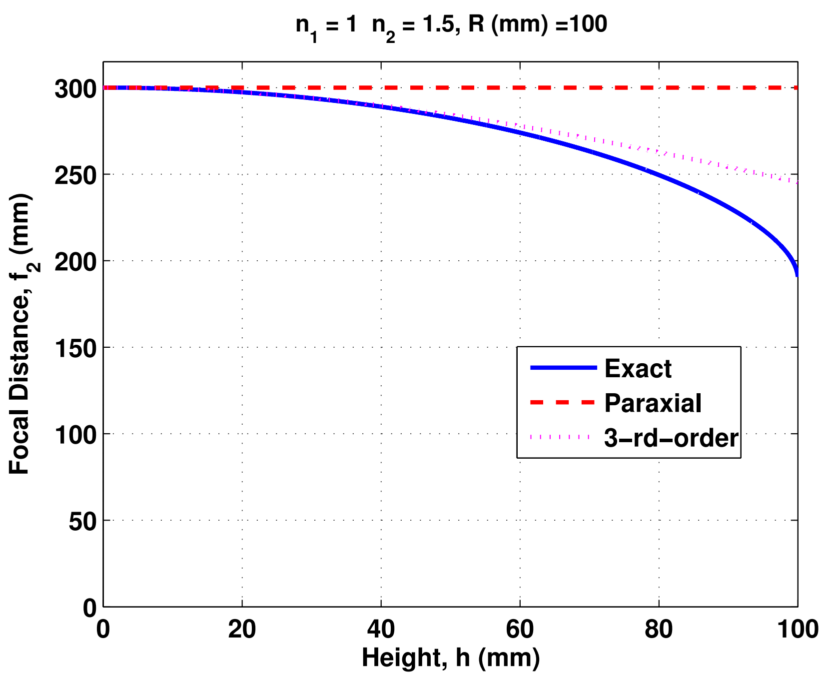


Figure 2: The focal distance $f_2 = VF$ as a function of height.

(γ) In this question, the refractive index of region 2 (corresponds to fused silica) and has a refractive index that varies with wavelength according to Sellmeier formula

$$n_2^2(\lambda) = A + \frac{G_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{G_2\lambda^2}{\lambda^2 - \lambda_2^2} + \frac{G_3\lambda^2}{\lambda^2 - \lambda_3^2}, \tag{9}$$

where the parameters are given in the problem statement.

Table 2: Tabulated Focal Distance for $h = 20\,\text{mm}$ as function of freespace wavelength

Wavelength λ (μm)	Focal Distance f_2 (mm)	Paraxial f_2 (mm)
0.4	309.796	312.713
0.5	313.315	316.297
0.6	315.304	318.322
0.7	316.596	319.639

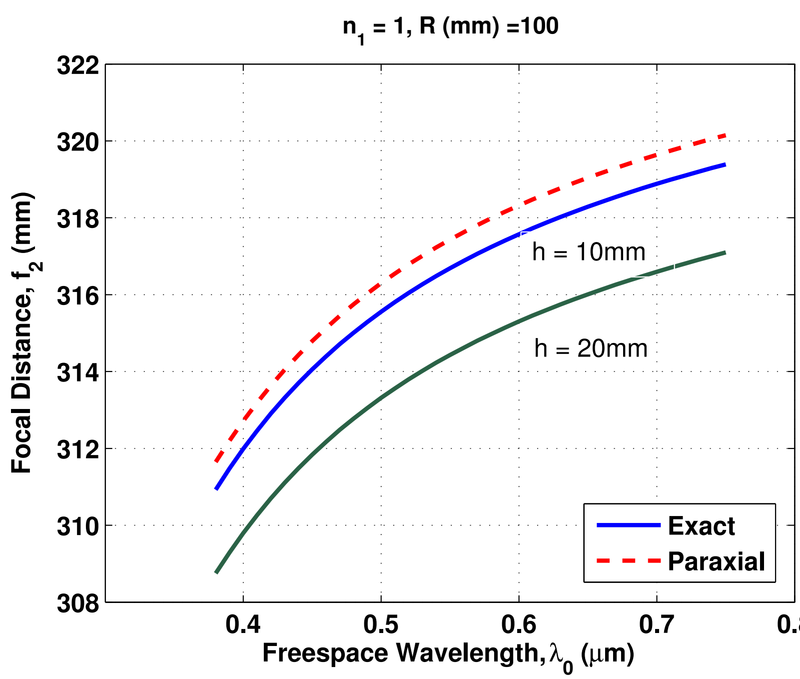


Figure 3: The focal distance VF as a function of freespace wavelength for $h = 10$ and $20\,\text{mm}$.

Problem 4: (Two Hemispherical Lenses System)

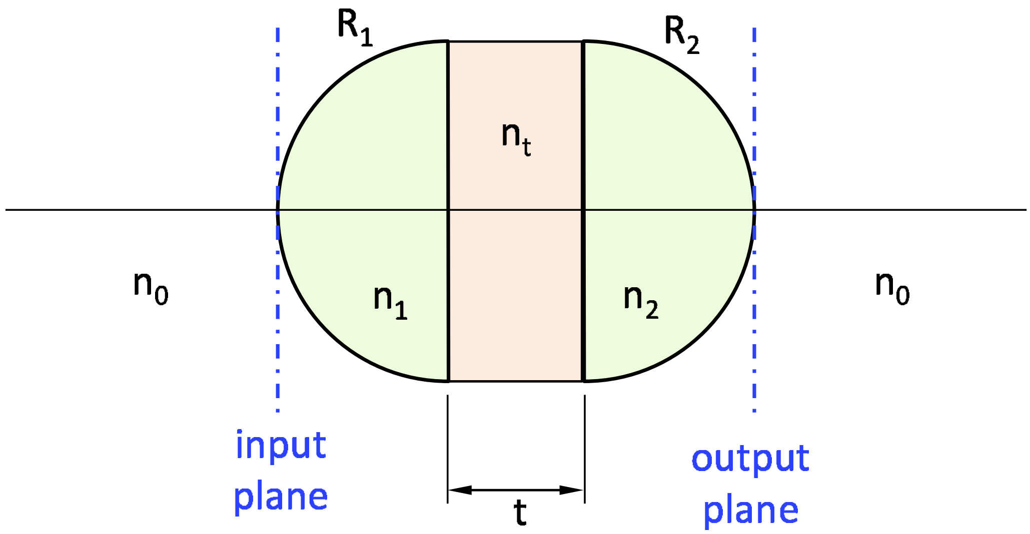


Figure 1: The two hemi-spherical lenses system. The lenses are separated by a slab of thickness t and refractive index n_t.

(α) We are going to follow the matrix method (paraxial approximation or Gaussian optics). The system between the input and the output planes can be described by a product of seven ABCD matrices. These are the following (considering light propagation from left to right):

1. First (left) Spherical Refractive Interface:

$$\tilde{M}_1 = \begin{bmatrix} 1 & 0 \\ \dfrac{n_0 - n_1}{n_1 R_1} & \dfrac{n_0}{n_1} \end{bmatrix}$$

2. Propagation Inside the First Lens:

$$\tilde{M}_2 = \begin{bmatrix} 1 & R_1 \\ 0 & 1 \end{bmatrix}$$

3. First Planar Refractive Interface:

$$\tilde{M}_3 = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{n_1}{n_t} \end{bmatrix}$$

4. Propagation between Lenses:

$$\tilde{M}_4 = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}$$

5. <u>Second Planar Refractive Interface:</u>

$$\tilde{M}_5 = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{n_t}{n_2} \end{bmatrix}$$

6. <u>Propagation Inside the Second Lens:</u>

$$\tilde{M}_6 = \begin{bmatrix} 1 & R_2 \\ 0 & 1 \end{bmatrix}$$

7. <u>Second (right) Spherical Refractive Interface:</u>

$$\tilde{M}_7 = \begin{bmatrix} 1 & 0 \\ \dfrac{n_2 - n_0}{n_0(-R_2)} & \dfrac{n_2}{n_0} \end{bmatrix}$$

In the $\tilde{M}_7$ matrix the sign of R_2 was set negative to conform with the convention used in the ABCD matrices. The final ABCD matrix is given by the product

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \tilde{M}_7\tilde{M}_6\tilde{M}_5\tilde{M}_4\tilde{M}_3\tilde{M}_2\tilde{M}_1$$

where the A, B, C, and D elements are given by

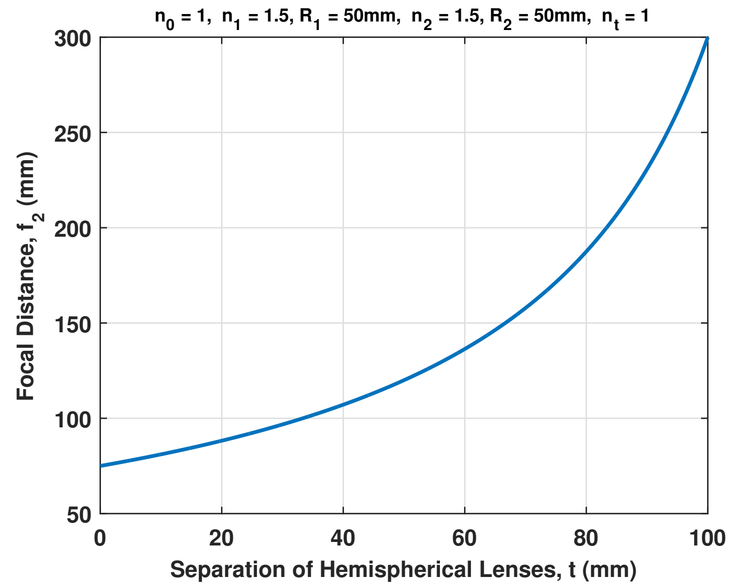
$$A = \frac{n_0}{n_1} + \frac{(n_0 - n_1)}{R_1}\left(\frac{t}{n_t} + \frac{R_2}{n_2}\right)$$

$$B = n_0\left(\frac{R_1}{n_1} + \frac{R_2}{n_2} + \frac{t}{n_t}\right)$$

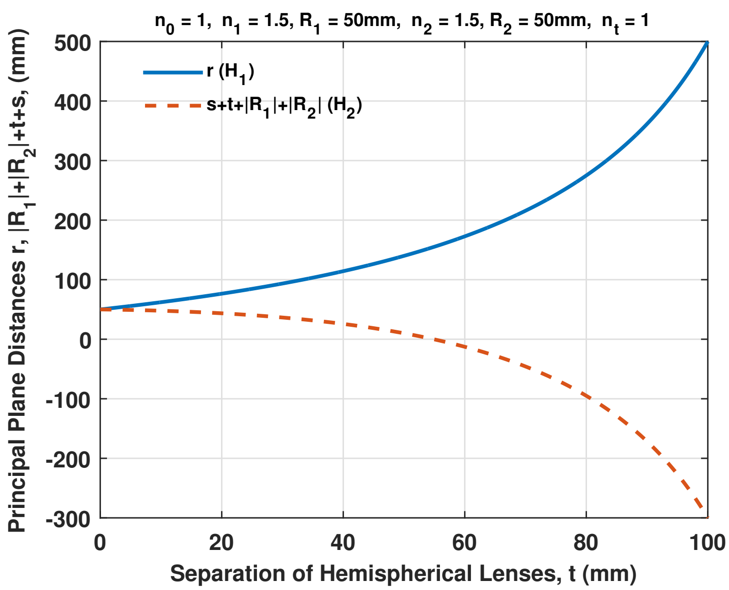
$$C = -\frac{n_2 - n_0}{n_0 R_2} - \frac{n_1 - n_0}{n_1 R_1}\left(\frac{n_0 - n_2}{n_0}\frac{R_1}{R_2} + \frac{n_1}{n_2} + \frac{n_1(n_0 - n_2)}{n_0 n_t}\frac{t}{R_2}\right)$$

$$D = \frac{t}{R_2 n_t}(n_0 - n_2) - \frac{R_1}{R_2}\frac{n_2 - n_0}{n_1} + \frac{n_0}{n_2}$$

(b) For the numerical computation the parameters take the following values: $R_1 = R_2 = 50\,\text{mm}$, $n_1 = n_2 = 1.50$, $n_0 = n_t = 1.0$, and $0 \leq t \leq 100\,\text{mm}$.



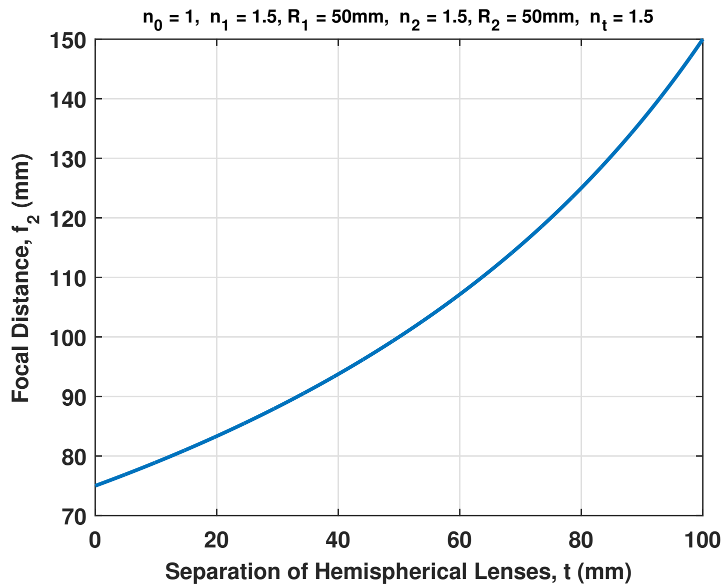
(a)



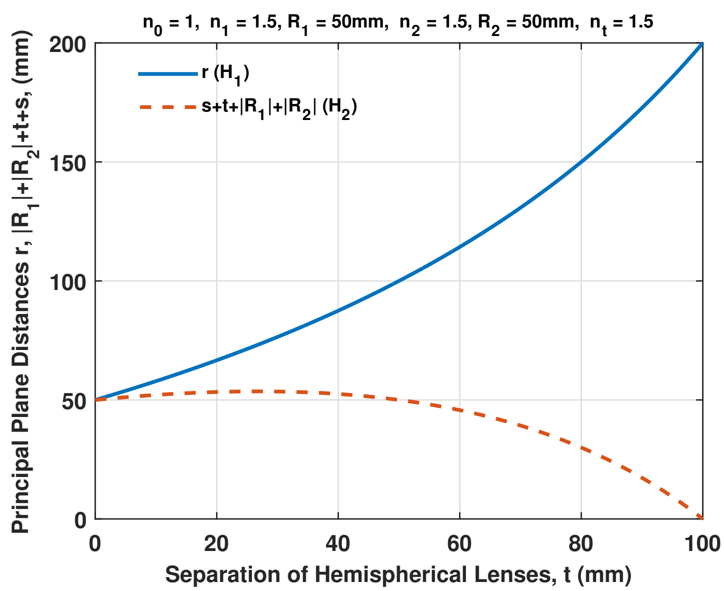
(b)

Figure 2: (a) The focal distance f_2 as a function of t for $n_t = 1$. (b) The distances r (of principal point H_1) and $R_1 + R_2 + t + s$ (for principal pont H_2) as they measured from the input plane.

(c) For the numerical computation the parameters take the following values: $R_1 = R_2 = 50\,\text{mm}$, $n_1 = n_2 = 1.50$, $n_0 = 1.0$, but now $n_t = 1.50$, and $0 \le t \le 100\,\text{mm}$.



(a)



(b)

Figure 3: (a) The focal distance f_2 as a function of t for $n_t = 1.50$. (b) The distances r (of principal point H_1) and $R_1 + R_2 + t + s$ (for principal pont H_2) as they measured from the input plane.