Problem No. 0: (deBroglie Wavelength)

$m_o = 9.10938291 \cdot 10^{-31}\,kg$, $c = 2.99792458 \cdot 10^8\,m/s$

$eV = 1.602176565 \cdot 10^{-19}\,J$, $h = 6.62606957 \cdot 10^{-34}\,J \cdot s$

(α) $\upsilon = 1 \cdot 10^5\,m/s$ $\quad \upsilon/c = \dfrac{1 \cdot 10^5}{2.99792458 \cdot 10^8} = 3{,}334 \cdot 10^{-4} \ll 1$

(μη σχετικιστική ταχύτητα)

$KE = \frac{1}{2} m_o \upsilon^2 =$

$= 4.5547 \cdot 10^{-21}\,J = 0{,}0284\,eV$

$p = \sqrt{2(KE)\,m_o} = 9.1094 \cdot 10^{-26}\,kg\,m/s$

$\lambda = \dfrac{h}{p} = 7.2739 \cdot 10^{-9}\,m = 7.2739\,nm$

(β) $\upsilon = 0{,}99c \rightarrow \upsilon/c = 0{,}99$ (σχετικιστική ταχύτητα).

$$\left.\begin{array}{l} KE = (m-m_o)c^2 \\ m - m_o = m_o\left(\dfrac{1}{\sqrt{1-(\upsilon/c)^2}} - 1\right) \end{array}\right\} \Rightarrow KE = 4.9850 \cdot 10^{-13}\,J =$$

$= 3{,}1114 \cdot 10^6\,eV =$

$= 3{,}1114\,MeV$

$p = \left[KE^2 + 2KE\,m_o c^2\right]^{1/2} / c = 1{,}9165 \cdot 10^{-21}\,kg\,m/s$

$\lambda = h/p = 3{,}4573 \cdot 10^{-13}\,m = 0{,}34573\,pm$!

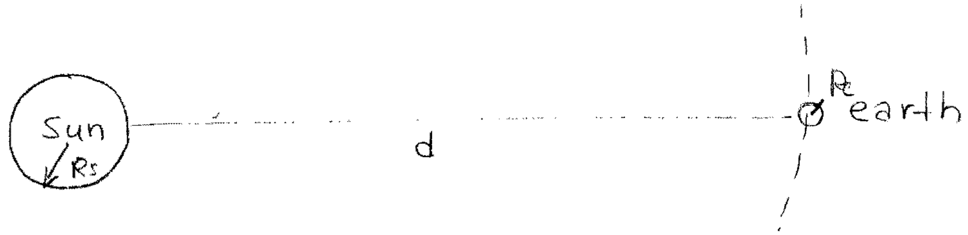
Problem No. 2: (Blackbody Radiation)

(α) The total power per unit area emitted by the sun is $M_s = \sigma T_s^4$ (Stefan's Law).

From Wien's Law: ($\lambda_m = 501.4\,nm$)

$$\lambda_m T_s = 2898\ \mu m\ {}^\circ K \Rightarrow T_s = \frac{2898}{0.5014}\ {}^\circ K \Rightarrow$$

$$T_s = 5779.8\ {}^\circ K$$



The total power is approximately constant.

$(4\pi R_s^2) M_s = (4\pi d^2) M_s^e$ where M_s^e is the electromagnetic power density arriving at earth.

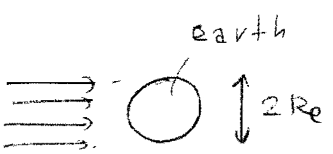
$$M_s^e = M_s \left(\frac{R_s}{d}\right)^2 = (\sigma T_s^4)\left(\frac{R_s}{d}\right)^2$$

for $\sigma = 5.67 \cdot 10^{-8}\ \frac{W}{m^2}$

$$M_s^e = 5.67 \cdot 10^{-8} (5779.8)^4 \left(\frac{696000}{149600000}\right)^2 =$$

$$= 1369.7\ W/m^2$$

(β)



For thermodynamic equilibrium $\rightarrow$

Power absorbed from the sun (incident power) =

Power emitted by earth

$M_e = \sigma T_e^4$ (W/m^2) ~ The power emitted by earth is $M_e (4\pi R_e^2)$

Power incident from Sun $= (\pi R_e^2) M_s^e$, and consequently

$$\pi \cancel{R_e^2} M_s^e = (\sigma T_e^4) 4\cancel{\pi} \cancel{R_e^2} \Rightarrow$$

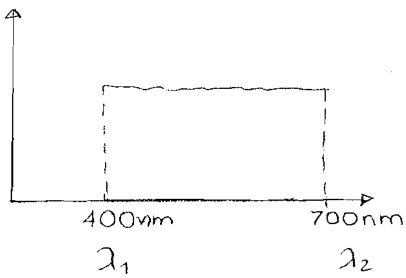
$$T_e = \left(\frac{M_s^e}{4} \frac{1}{\sigma}\right)^{1/4} = 278.8° K$$

(γ) If we take into account earth's albedo then the power absorbed is

$$(1-\alpha)\pi R_e^2 M_s^e \quad \text{where } \alpha = 30\%.$$

Then $$T_e = \left((1-\alpha)\frac{M_s^e}{4}\frac{1}{\sigma}\right)^{1/4} = 254.98° K$$

Problem No. 2: (Coherence)



$$t_c = \frac{1}{\Delta\nu} \qquad \ell_c = t_c \cdot c$$

$$\nu = \frac{c}{\lambda} \sim \Delta\nu = \nu_1 - \nu_2 = c\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

Thus $t_c = \dfrac{1}{c} \dfrac{1}{\frac{1}{\lambda_1} - \frac{1}{\lambda_2}} \Rightarrow$

$$\boxed{t_c = \frac{1}{c} \cdot \left(\frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}\right) \qquad \ell_c = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1}}$$

(a) $\lambda_1 = 400$ nm $\quad \lambda_2 = 700$ nm

$$\ell_c = \frac{400 \cdot 700}{700 - 400} \text{ nm} = 933{,}33 \text{ nm} = \underline{0{,}9333\,\mu m.}$$

$$t_c = \frac{1}{c}\ell_c = \frac{0.9333 \cdot 10^{-6} \text{ m}}{3 \cdot 10^{8} \text{ m/sec}} = 0.3111 \cdot 10^{-14} \text{sec} = \underline{0.003111 \text{ psec}}$$

(b) $\lambda_1 = 693.4 - 0.00001 = 693.39999$ nm

$\lambda_2 = 693.4 + 0{,}00001 = 693.40001$ nm

$$\ell_c = \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} = \frac{693.39999 \cdot 693.400001}{0.00002} \text{ nm} = 2.404 \cdot 10^{10} \text{ nm} \Rightarrow$$

$$\underline{\ell_c = 24.04 \text{ m}}$$

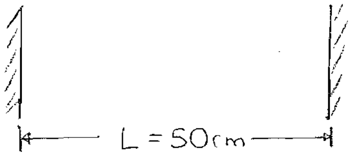
$$t_c = \frac{\ell_c}{c} = \frac{24.04 \text{ m}}{3 \cdot 10^{8} \text{ m/sec}} = 8.013 \cdot 10^{-8} \text{sec} = \underline{0.08013\,\mu\text{sec}}$$

Problem No.3: (Laser Modes)

$\lambda_0 = 632.8\ nm$

$$c = \lambda_0 \nu_0 \Rightarrow \quad \nu_0 = \frac{c}{\lambda_0} = \frac{3 \cdot 10^8\ \cancel{m}/sec}{632.8 \cdot 10^{-9}\ \cancel{m}} = \frac{3}{632.8}\, 10^{17}\ Hz \Rightarrow$$

$$\underline{\nu_0 = 4.74 \cdot 10^{14}\ Hz}$$



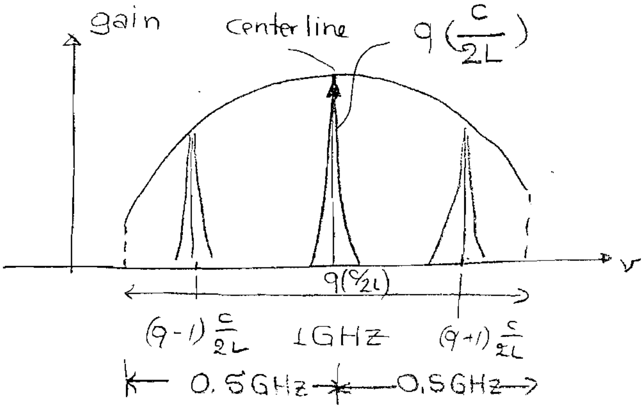
$$\nu = q\left(\frac{c}{2L}\right) \Rightarrow$$

$$q = \frac{\nu_0}{(c/2L)}$$

$$\Delta\nu = \frac{c}{2L} = \frac{3 \cdot 10^8\ \cancel{m}/sec}{2 \cdot 50 \cdot 10^{-2}\ \cancel{m}} = 3 \cdot 10^8\ Hz = 300\ MHz = 0{,}3\ GHz$$

$$q = \frac{4.74 \cdot 10^{14}\ Hz}{3 \cdot 10^8\ Hz} = 1{,}580278129 \cdot 10^6 = 1580278{,}129$$

Since q must be integer the nearest to ν_0 frequency corresponds to $\underline{q = 1580278}$



since the separation between longitudinal modes is $\frac{c}{2L} = 0.3\ GHz$ there are 2 additional longitudinal modes that can fit in the gain curve. These two frequencies correspond to $(q-1)\frac{c}{2L}$ and $(q+1)\frac{c}{2L}$ where q as above.

Thus the <u>number of longitudinal modes = 3,</u> for that cavity.

Problem No. 4: (Photometry-Brightness Comparison)

Laser # 1 : He-Cd : $P_1 = 10 mW$, $\lambda_1 = 441.6$ nm

Laser # 2 : He-Ne : $P_2 = 1.5 mW$, $\lambda_2 = 632.8$ nm

We have to compare the luminous powers of the two lasers.

#1)
$$\frac{V(441.6) - V(440)}{V(450) - V(440)} = \frac{441.6 - 440}{450 - 440} \Rightarrow$$ using Table values

$$V(441.6) = 0.023 + (0.038 - 0.023)\frac{1.6}{10} = 0.0254$$

Luminous power of Laser #1 $= 10 \cdot 10^{-3} W \cdot 683 \frac{lm}{W} \, 0.0254 =$

$$= \underline{0.17348 \text{ lm}}$$

#2) Again using interpolation from the Table values we get:

$$V(632.8) = 0.265 + (0.175 - 0.265) \cdot \frac{632.8 - 630}{640 - 630} = 0.2398$$

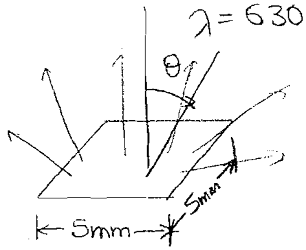
Luminous power of Laser #2 $= 1.5 \cdot 10^{-3} W \; 683 \frac{lm}{W} \cdot 0.2398 =$

$$= \underline{0.24568 \text{ lm}}$$

Since the luminous power of the He-Ne laser is higher the <u>He-Ne laser appears brighter</u>.

Problem No. 5: (Lighted Panel Switch)

(a)



$A = 5 \cdot 5\, mm^2 = 25\, mm^2$

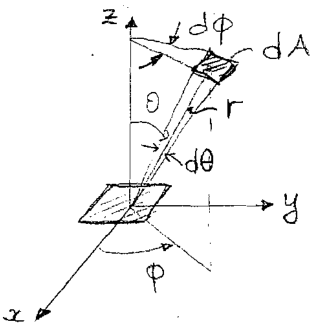
$\Phi_e = 1.10^{-6}\, W$

$\Phi_v = \phi_e \dfrac{683\, \ell m}{1W} V(630nm) =$

$= 10^{-6} W \dfrac{683\, \ell m}{W} 0.265 = \underline{1.801 \cdot 10^{-4} \ell m}$

(b) The luminous intensity satisfies the Lambert's Law

$I_v(\theta) = I_o \cos\theta$. The luminous intensity normal to the surface of the switch is I_o.



$$I_v(\theta) = \frac{d\phi_v}{d\Omega_s} \Longrightarrow$$

$$\phi_v = \int I_v(\theta)\, d\Omega_s = \int I_v(\theta) \frac{dA}{r^2} =$$

But dA in spherical coordinates is

$$dA = (r\sin\theta\, d\phi)(r\, d\theta) = r^2 \sin\theta\, d\theta\, d\phi$$

$$\phi_v = \int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} I_o \cos\theta \frac{\cancel{r^2}\sin\theta\, d\theta\, d\phi}{\cancel{r^2}} = 2\pi I_o \int_0^{\pi/2} \frac{\sin 2\theta}{2} d\theta =$$

$$= \frac{2\pi}{4} I_o \left[-\cos 2\theta \right]_0^{\pi/2} = \pi I_o \rightarrow I_o = \frac{\Phi_v}{\pi} = \frac{1.801 \cdot 10^{-4} \ell m}{\pi \quad (sr)}$$

$\underline{I_o = 5.736 \cdot 10^{-5}\ \ell m / sr = 5.736 \cdot 10^{-5}\ \text{candles}}$

(c)

$$L_v = \frac{I_v}{A_{s\perp}} = \frac{I_o \cancel{\cos\theta}}{A_s \cancel{\cos\theta}} = \frac{I_o}{A_s} = \frac{5.736 \cdot 10^{-5} \ell m/sr}{25 \cdot 10^{-6}\ m^2} \Rightarrow$$

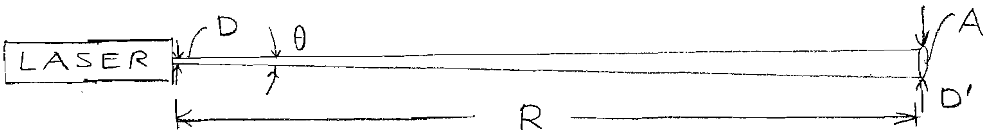
$$\underline{L_v = 2.294 \frac{\ell m}{m^2 sr}}$$

Problem 6 (Photometric Quantities): (10%)

An argon ion laser emits 2.8 watts of power at a wavelength of λ_0 =514.5 nm. The beam diameter at the output of the laser is 1.4 mm. The beam diverges and is incident upon a screen that is 10 m away. The axis of the beam is normal to the screen. (a) Calculate the luminous power at the screen. (b) Calculate the illuminance on the screen in footcandles within the illuminated region. Assume a uniform intensity within the cross section of the beam (instead of a Gaussian profile) for simplifying your calculations.

Problem No. 6: (Photometric Quantities)

$\phi_e = 2.8\,W \qquad \lambda = 514.5\,nm \qquad D = 1.4\,mm \qquad R = 10\,m$



(a) $\phi_v = \phi_e \dfrac{683\,lm}{1W} \cdot V(514.5)$

From the table: $\dfrac{V(514.5) - V(510)}{V(520) - V(510)} = \dfrac{514.5 - 510}{520 - 510} \Rightarrow$

$$V(514.5) = 0,503 + \frac{514.5 - 510}{520 - 510}(0,710 - 0,503) = 0,59615$$

$$\phi_v = 2,8\,W \frac{683\,lm}{1\,W}\,0,59615 = \underline{1140.07\,lm}$$

(b)

$$E_V = \frac{\phi_v}{A}$$

$$\theta = \frac{4}{\pi}\frac{\lambda}{D} \Rightarrow \theta = \frac{4}{\pi} \cdot \frac{514.5 \cdot 10^{-9}\,m}{1.4 \cdot 10^{-3}\,m} = 0.468\,mrad$$

$$D' = R \cdot \theta = 10\,m \cdot 0,468 \cdot 10^{-3}\,rad = 4.68\,mm.$$

$$A = \frac{\pi D'^2}{4} = 1.7195 \cdot 10^{-5}\,m^2$$

Thus $E_V = \dfrac{1140.07\,lm}{1.7195 \cdot 10^{-5}\,m^2} = 663.02 \cdot 10^5 \dfrac{lm}{m^2} =$

$$= 663.02\ 10^5 \frac{lm}{10.764\,ft^2} = 6.1596\ 10^6\ \text{footcandles}$$

Problem 7 (Blackbody Radiation): (0%)

The radiation that our Sun emits can be resembled to the radiation emitted by a blackbody with average surface temperature of 5505° C. If it is assumed that each planet of our solar system is at constant temperature (in thermodynamic equilibrium), and emits radiation as a blackbody what each planet mean temperature in degrees Kelvin? Take into account the average power reflection coefficient of the Sun radiation by each planet (its ***albedo***). Neglect the greenhouse effect, the planetary atmospheric pressure, and any other phenomenon that could contribute to the average planet temperature. All the data that are needed are shown in the following table:

	Average Radius (km)	**Average Distance from the Sun ($\times 10^6$ km)**	**Average *albedo***
Sun	695700	-	-
Mercury	2439.5	57.9	0.12
Venus	6052	108.2	0.75
Earth	6378	149.6	0.30
Mars	3396	227.9	0.16
Jupiter	71492	778.6	0.34
Saturn	60268	1433.5	0.34
Uranus	25559	2872.5	0.30
Neptune	24764	4495.1	0.29
Pluto	1185	5906.4	0.40

Problem No. 7:

$T_s = 5505° + 273° \text{ K} = 5778° \text{ K}$

Radiation emitted by the sun: $M_s = (\sigma T_s^4)$ (W/m²)

Radiation received by a planet: $M_{sp}(\pi R_p^2)$

$$M_{sp}\, 4\pi D_p^2 = M_s\, 4\pi R_s^2 \Rightarrow M_{sp} = M_s \left(\frac{R_s}{D_p}\right)^2$$

M_s = radiation emitted by the sun per unit area (on its surface)

M_{sp} = radiation " " " " " " " (at a distance of a planet)

D_p = planet distance from the sun

R_p = radius of a planet

A_p = planet Albedo

$$M'_{sp} = (1 - A_p) M_s \left(R_s/D_p\right)^2$$ (exclude the power reflected due to planet Albedo). (W/m²)

Planet emits all radiation that receives to remain in a thermodynamic equilibrium:

Total Power received by planet: $M'_{sp}\, \pi R_p^2 \Rightarrow$

$$(4\cancel{\pi R_p^2})\, \sigma T_p^4 = M'_{sp}\, \cancel{\pi R_p^2} \Rightarrow$$

$$4\cancel{\sigma} T_p^4 = (1 - A_p)\, \cancel{\sigma} T_s^4 \left(\frac{R_s}{D_p}\right)^2 \Rightarrow T_p = \left\{ \frac{1-A_p}{4}\, T_s^4 \left(\frac{R_s}{D_p}\right)^2 \right\}^{1/4}$$

$$T_p = \left(\frac{1-A_p}{4}\right)^{1/4} \sqrt{\frac{R_s}{D_p}}\; T_s$$

Applying the last equation for all planets of the solar system (Pluto is not officially a planet anymore) the following temperatures are obtained:

Planet:	Temperature (°K)	Temperature (°C)
Mercury	433.77	160.77
Venus	231.66	-41.34
Earth	254.85	-18.15
Mars	216.11	-56.89
Jupiter	110.08	-162.92
Saturn	81.13	-191.87
Uranus	58.16	-214.84
Neptune	46.66	-226.34
Pluto	39.03	-233.97

Of course the average temperature of earth is 14.6°C due to greenhouse effects.

Measured temperatures: (http://www.universetoday.com/35664/temperature-of-the-planets/

Mercury: +465° C (sun side) -184°C (dark side)

Venus : +460° C (severe greenhouse effect)

Earth : 14.6°C

Mars : -55° C

Jupiter : -145° C

Saturn : -178° C

Uranus : -224° C

Neptune : -218°C

Pluto : -229°C