NATIONAL TECHNICAL UNIVERSITY OF ATHENS

SCHOOL OF ELECTRICAL & COMPUTER ENGINEERING

DIVISION OF ELECTROMAGNETIC APPLICATIONS, ELECTRO-OPTICS & ELECTRONIC MATERIALS

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OPTICAL ENGINEERING

Problem Set No. 4

Due Date: May 22, 2024

(The problems that should be submitted have nonzero weighting factors)

Problem 1 (Reflected and Transmitted Powers): [0%]

Sellmeier's formula expresses the dependence of the refractive index of a material $n(\lambda)$ as a function of the freespace wavelength. For your convenience Sellmeier's formula is given below along with a table of the related constants B and C, for 3 separate materials (sapphire crystal is anisotropic and both the ordinary and the extraordinary indices are included).

$$n^2(\lambda) = 1 + \sum_i \frac{B_i \lambda^2}{\lambda^2 - C_i},$$

Table of coefficients of Sellmeier equation

Material	B_1	B_2	B_3	C_1	C_2	C_3
borosilicate crown glass (known as *BK7*)	**1.03961212**	**0.231792344**	**1.01046945**	**$6.00069867\times10^{-3}\mu m^2$**	**$2.00179144\times10^{-2}\mu m^2$**	**$1.03560653\times10^{2}\mu m^2$**
sapphire (for ordinary wave)	**1.43134930**	**0.65054713**	**5.3414021**	**$5.2799261\times10^{-3}\mu m^2$**	**$1.42382647\times10^{-2}\mu m^2$**	**$3.25017834\times10^{2}\mu m^2$**
sapphire (for extraordinary wave)	**1.5039759**	**0.55069141**	**6.5927379**	**$5.48041129\times10^{-3}\mu m^2$**	**$1.47994281\times10^{-2}\mu m^2$**	**$4.0289514\times10^{2}\mu m^2$**
fused silica(quartz)	**0.696166300**	**0.407942600**	**0.897479400**	**$4.67914826\times10^{-3}\mu m^2$**	**$1.35120631\times10^{-2}\mu m^2$**	**$97.9340025\ \mu m^2$**

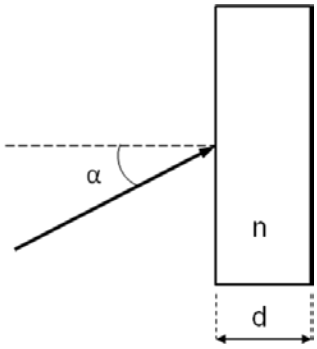
For this problem assume that light from a monochromator is incident at an angle θ (from air) on a planar and smooth surface of **quartz (fused silica)**. The polarization of the incident wave could be either TE or TM.

(a) The freespace wavelength of the incident wave is 633nm. Give the graphical representation of the reflection and transmission coefficients, as well as the percentages of the reflected and transmitted powers as functions of the angle of incidence ($0 \le \theta \le 90$deg) for both polarizations. (You could use the program *fresnel_equations.m* – matlab – that is provided in the course webpage).

(b) In this case the angle of incidence is constant and equal to $\theta = 30$deg. Give the graphical representation of the reflection and transmission coefficients, as well as the percentages of the reflected and transmitted powers as functions of the freespace wavelength ($0.1\mu m \le \lambda \le 2.0\mu m$) for both polarizations.

Problem 2 (Interference Filter): [0%]

In some application it is desired to use a one-half wavelength thick interference filter ($d = \lambda/2$ ή $d = (2m+1)\lambda/2$ where m is an integer) to scan the wavelength range of wavelengths around λ_0. The interference filter is specified to operate at λ_s where $\lambda_s > \lambda_0$. The half wavelength layer in the interference filter has an index of refraction of n. Calculate the angular tuning rate of the filter (the change in resonant wavelength for a change in angle) at λ_0. If the interference filter is designed for λ_s = 520nm and has n=1.4, calculate these quantities for the argon laser wavelength 514.5nm.



Problem 3 (Fabry-Perot Interference Filter): [0%]

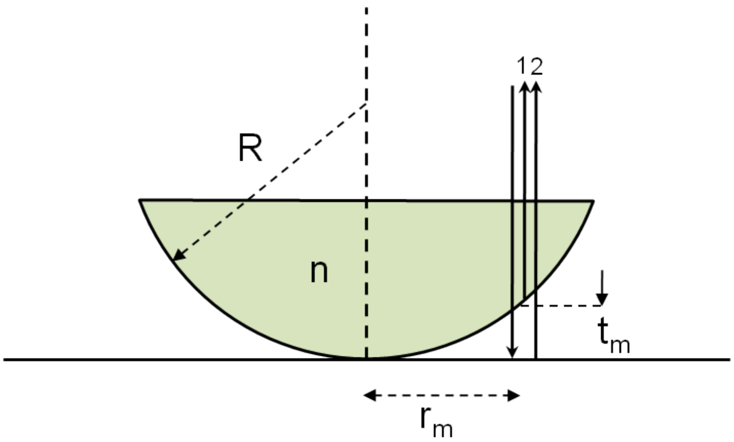
A beam of white light (which has a continuous spectrum between 400*nm* to 700*nm*) is incident at an angle of 45 *deg* on to a set of parallel glass plates between a thin air layer of thickness of 0.001*cm*. The reflected light is passing through a prism spectrometer. How many and which dark lines are seen in the spectrum?

Problem 4 (Fabry-Perot Interference Filter): [0%]

A Fabry-Perot interferometer is comprised of a transparent plate of refractive index n = 4.50 and thickness 2*cm*. The parallel surfaces of the plate have a power reflectivity of (r^2) 90%. If this interferometer is used in the range of freespace wavelength of 546*nm* determine (a) the highest order bright fringe (largest value of integer m), (b) the ration T_{max}/T_{min}, and (c) the resolving power of the interferometer.

Problem 5 (Newton's Rings): [0%]

Newton rings are being formed between a plano-convex lens and a flat planar optical surface as it is shown in the following figure. The fringes are formed due to interference between the partially reflected beams 1 and 2 as it is indicated in the figure. If the diameter of the 10^{th} bright ring is 7.89*mm* when the freespace wavelength of the incident light is 546.1*nm* determine the radius of curvature R of the spherical surface of the lens. What is its focal distance if the refractive index of the lens is n = 1.50?



Problem 6 (Antireflection Layer): [25%]

In this problem the scope is to design an anti-reflective layer between two materials with refractive indices n_1 and n_2 respectively. The incident plane wave has a freespace wavelength of λ_d and is incident at an angle θ_1 as it is evident from the figure below. From the electromagnetic analysis of this problem (using the transmission line theory), it is straightforward to determine the reflection coefficient Γ (of the wave that is reflected by the combination of the anti-reflective layer and the region of refractive index n_2) which is given by the following equations (for TE and TM polarization and angle of incidence θ_1):

$$\Gamma = \frac{Z_{in} - \tilde{Z}_1}{Z_{in} + \tilde{Z}_1}$$

$$Z_{in} = \tilde{Z}_c \frac{\tilde{Z}_2 \cos(k_o n_c d_c \cos\theta_c) + j\tilde{Z}_c \sin(k_o n_c d_c \cos\theta_c)}{\tilde{Z}_c \cos(k_o n_c d_c \cos\theta_c) + j\tilde{Z}_2 \sin(k_o n_c d_c \cos\theta_c)}$$

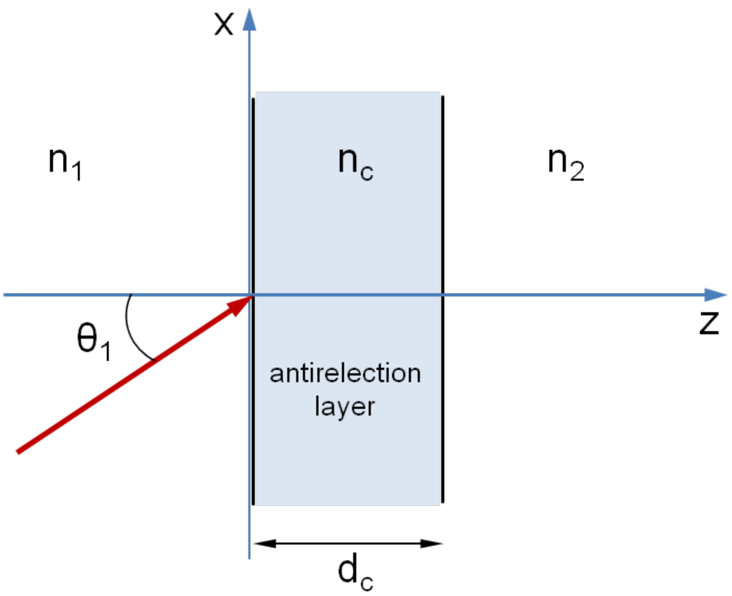
$$\tilde{Z}_i = Z_i / \cos\theta_i \quad (i = 1, 2, c) \qquad \text{TE Polarization}$$

$$\tilde{Z}_i = Z_i \cos\theta_i \quad (i = 1, 2, c) \qquad \text{TM Polarization}$$

In the above equations Z_i correspond to the wave impedances of the media involved (i = 1, 2, c, where c corresponds to the anti-reflective layer of thickness d_c). The angles θ_i correspond to the propagation angles inside each material. For the numerical implementation assume that λ_d = 500*nm* (design wavelength), n_1 = 1.0, and n_2 = 2.0.

(a) Initially tackle the design of the anti-reflective layer for normal incidence, i.e. for θ_1 = 0 *deg*. Determine the minimum thickness d_c as well as the refractive index n_c of the anti-reflective layer. Make a graphical representation of the reflected power as a function of the freespace wavelength in the range 300*nm* $\leq \lambda_0 \leq$ 700*nm* when the anti-reflective layer was designed for λ_d. Furthermore, make a graphical representation of the reflected power as a function of the angle of incidence θ_1 (-90*deg* $\leq \theta_1 \leq$ +90*deg*) when the designed wavelength is used.

(b) Now tackle the design of the anti-reflective for an angle of incidence θ_1 = 45 *deg*. Determine the minimum thickness d_c as well as the refractive index n_c of the anti-reflective layer for either TE or TM polarization. Make a graphical representation of the reflected power as a function of the freespace wavelength in the range 300*nm* $\leq \lambda_0 \leq$ 700*nm* when the anti-reflective layer was designed for λ_d (separate for each polarization). Furthermore, make a graphical representation of the reflected power as a function of the angle of incidence θ_1 (-90*deg* $\leq \theta_1 \leq$ +90*deg*) for both polarizations when the designed was done for TE polarization and correspondingly when the designed was done for TM polarization.



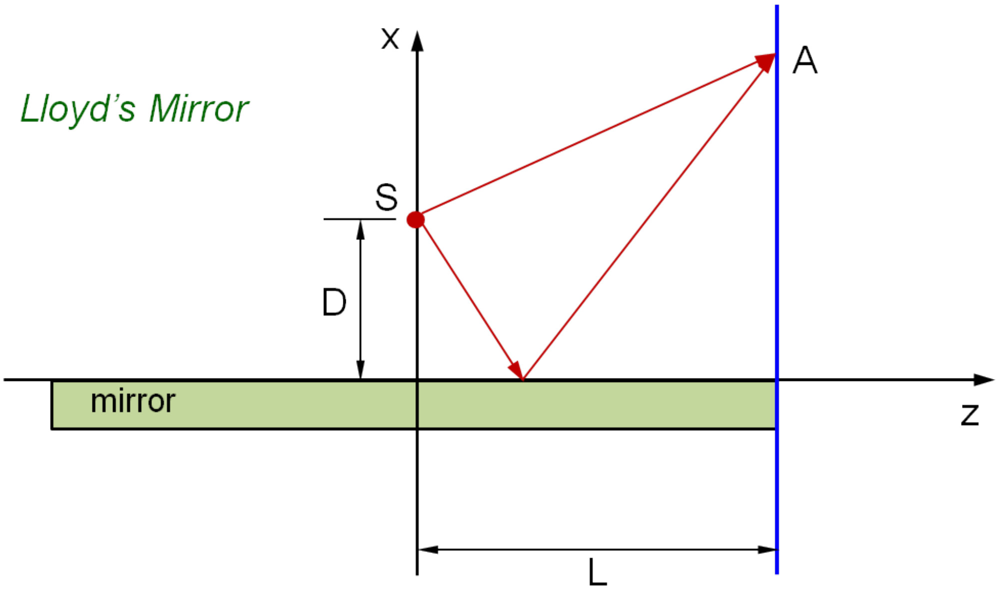
Problem 7 (Lloyd's Mirror): [20%]

Lloyd's mirror is a setup that interference can be observed similarly to the Young's experiment. In the setup shown in the figure below rays are emanating from the monochromatic point source S and interfere at point A either coming propagating directly to A from the source or via reflection from the mirror. Assume that the surrounding medium is air.

(a) Determine an equation in the *xyz* coordinate system that defines the positions of the interference (fringe) maxima and an equation for the corresponding minima in the space of wave interference of the source S. The equations should have the form $f(x,y,z) = 0$.

(b) Determine in the *xz* plane (y=0) the loci of points where the maxima and the minima of the interference are observed. For the numerical implementation assume that D = 5μm and λ=1μm. Make a graphical representation of the loci for -20μm < z < 20μm and 0μm < x < 15μm.

(c) If z = L = 20μm determine (numerically according to the date of (b)) the first 5 maxima (constructive interference) and the first 5 minima (destructive interference) at the plane $z=L$ (with y=0). Compare the exact numerical values with their approximate ones.



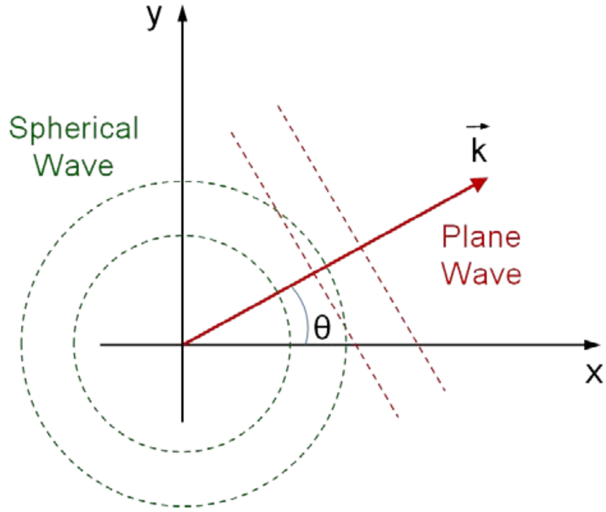
Problem 8 (Spherical and Plane Wave Interference): [20%]

A spherical wave and a plane wave of the same freespace wavelength interfere in freespace. The spherical wave emanates from the origin (0,0,0) of the coordinate system and has the form $E_{sw} = (E_0/r)\exp(-jk_0 r)$ where k_0 is the wavenumber of freespace and $r = (x^2+y^2+z^2)^{1/2}$. The plane wave has a wavevector of $\vec{k} = k_0(\cos\theta\,\hat{\imath}_x + \sin\theta\,\hat{\imath}_y)$ and has the form $E_{pw} = E_0 \exp[-jk_0(x\cos\theta + y\sin\theta)]$. It is assumed that the polarization of both waves are collinear in order to avoid vectorial considerations.

(a) [10%] Determine an equation, in the *xyz* coordinate system, that defines the positions of the interference (fringe) maxima and an equation for the corresponding minima. The equations should have the form *f(x,y,z)* = 0.

(b) [20%] Determine in the *xy* plane (z = 0) the loci of the points where the maxima and the minima of the interference are observed. For the numerical implementation assume that the freespace wavelength is λ_0=1μm. Make a graphical representation of the loci for -20μm < x < 20μm and -20μm < y < 20μm. Determine the loci for several values of the optical path difference.

(c) [10%] Determine (numerically, using the data of (b)) along the line x = 20μm (z=0) the points where maxima occur (constructive interference) and where minima occur (destructive interference) along the y direction (use the parameters you have selected in (b)).



Problem 9 (Young's Double Slit Experiment): [10%]

White spatially coherent light (380nm-780nm) is sent through two slits in a Young's Double Slit Experiment setup. The separation between the slits is 0.5mm and the observation screen is 50cm away from the slits. There is a hole in the screen at a point 1mm away from the central line.

(a) [5%] Which wavelengths will be absent in the light coming from the hole?

(b) [5%] Which wavelengths will have strong intensity in the light coming from the hole?

Problem 10 (Three-Plane-Wave Interference): [25%]

Three coherent plane waves of the same frequency interfere inside a homogeneous, isotropic, and linear medium. This problem is based on information included in journal paper: ***J. L. Stay and T. K. Gaylord,``Three-beam-interference lithography: contrast and crystallography,'' Appl. Opt., vol. 47, no. 18, pp. 3221-3230, Jun. 20, 2008.***

The wavevectors of the three plane waves are $\vec{k}_i$ (where i = 1,2,3). For example, one plane wave is shown in the figure below. Every plane wave can be written in the following phasor form:

$$\vec{E}_i = E_i \hat{e}_i \exp[-j(\vec{k}_i \cdot \vec{r} + \varphi_{0i})],$$

where $\vec{r}$ is the position vector, E_i is the amplitude of plane wave i (assume that are real numbers), φ_{0i} is a phase, and $\hat{e}_i$ is the polarization vector of plane wave i (assume that these vectors are real so all plane waves are linearly polarized).

(a) [8%] Find the intensity, $I(x,y,z) = |\vec{E}|^2$, which results from the interference of the three plane waves in space (*x*,*y*,*z*).

(b) [8%] From question (a) is obvious that the interference is characterized by three terms that correspond to the interference of the plane waves every two. Specifically, there is one term that corresponds to the interference of $\vec{k}_1$ with $\vec{k}_2$, a second terms which corresponds to the interference of $\vec{k}_1$ with $\vec{k}_3$, and a third term that corresponds to the interference of $\vec{k}_2$ with $\vec{k}_3$. Find the equations for the maximum interference of each of these three terms. Show that the equation of the third term could result from the equations of the first two. Therefore, the maximum total interference of the three plane waves is comprised from lines that are the intersections of the equations (which are planes) of the maximum interference of the two terms discussed previously. If it is desired these lines to be parallel to the z axis what would be the condition that the three wavevectors should satisfy?

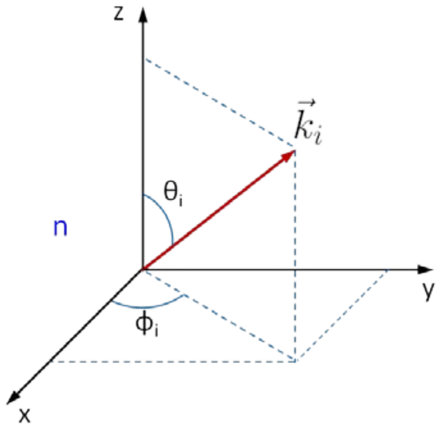
(c) [9%] Now assume that the three plane waves are interfering in air (n = 1) and they have the following parameters ($k_0 = 2\pi/\lambda_0$, and $\lambda_0 =$ 1μm):

$$\vec{k}_1 = k_0\left[-\frac{1}{3},-\frac{\sqrt{3}}{3},-\frac{\sqrt{5}}{3}\right],\ \vec{k}_2 = k_0\left[+\frac{2}{3},0,-\frac{\sqrt{5}}{3}\right],\ \vec{k}_3 = k_0\left[-\frac{1}{3},+\frac{\sqrt{3}}{3},-\frac{\sqrt{5}}{3}\right],$$

$$\hat{e}_1 = \left[+\frac{\sqrt{3}}{2},-\frac{1}{2},0\right],\ \hat{e}_2 = [0.2821,-0.9256,0.2523],\ \hat{e}_3 = [0.9426,\ 0.2185,-0.2523],$$

$E_1 = 1$, $E_2 = \frac{\sqrt{2}}{2}$, $E_3 = \frac{\sqrt{2}}{2}$, $\varphi_{01} = \varphi_{02} = \varphi_{03} = 0$.

Make a graphical representation (with the help of *MatLab* or equivalent software) of the intensity I in the xy plane. You could utilize the Matlab function **surface(x,y,I), shading interp** for the colored representation of the intensity in the xy plane. Calculate numerically the visibility of the fringes produced, $V = (I_{max}-I_{min})/(I_{max}+I_{min})$ as well as the angles θ_i and φ_i of the three plane waves and verify that their polarizations are compatible with plane waves.



Note: **For all the problems** that you use some kind of software (such as *MatLab* or others) it is **mandatory** (in order to gain full credit for the corresponding problem) to include in your answers a printout of the code that you have written and used.