NATIONAL TECHNICAL UNIVERSITY OF ATHENS

SCHOOL OF ELECTRICAL & COMPUTER ENGINEERING

DIVISION OF ELECTROMAGNETIC APPLICATIONS, ELECTRO-OPTICS & ELECTRONIC MATERIALS

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OPTICAL ENGINEERING

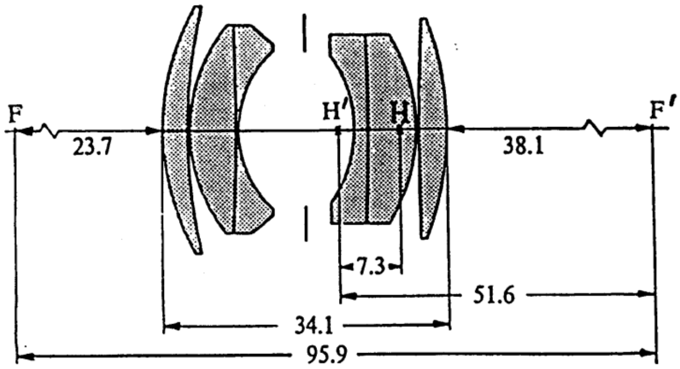
Problem Set No. 3

Due Date: April 23, 2024

(The problems that should be submitted have nonzero weighting factors)

Problem 1 (Nikkor-H Imaging): [0%]

A point object is located to the left of the Nikkor-H lens (its characteristics are shown in the figure below). The object is 400mm along the optical axis (as measured from the front surface of the first lens on the left) and 10mm transverse to the optical axis. (a) Calculate the position of the image along the axis (as measured from the back surface of the rear (rightmost) lens). (b) Calculate the transverse magnification. (c) Is the image real or virtual? (d) Draw a ray trace diagram showing the parallel and the focal rays.



Problem 2 (Imaging with a Practical Lens): [0%]

It is desired to inspect items on a production line conveyor belt. These items are to be imaged on the photocathode face of a vidicon tube at a reproduction ratio of 10 to 1. The Nikkor-H lens (shown in the previous figure) is to be used for the required imaging. The light travels through the lens in the direction for which the lens was designed (from left-to-right). (a) Calculate the distance from the items to the front glass surface of the lens. (b) Calculate the distance from the back glass surface of the lens to the photocathode (or CCD array) face.

Problem 3 (Spherical Aberration): [0%]

Assume a family of thin lenses of focal length f (for all lenses of the family). The shape factor [$\sigma = (R_2+R_1)/(R_2-R_1)$] of these lenses varies in the interval $[-2, +2]$. Calculate (with the help of a computer) the focal distance difference between the paraxial focal distance and the focal distance including the 3rd order spherical aberration, as a function of the shape factor (it should be a curve similar to the one shown below). The curve should be calculated using Equation A and suitably using Equation B (for both refractive surfaces of the lens). Assume that $n_1 = 1$, $n_2 = 1.517$, $f = 10$ cm. Assume that the object is a great ("infinite") distance and examine the cases that $h = 1$ cm, $h = 5$ cm, and $h = 10$ cm. Determine and plot the curves that correspond to Equation A (Pedrotti) and Equation B (Jenkins & White) for all three cases of h as a function of σ. Determine the optimum shape factor for the minimization of the spherical aberration and compare it with the curves that you have determined in the previous questions.

A. Pedrotti Equation for Spherical Aberration (3rd order correction) of a thins lens of refractive index n in air:

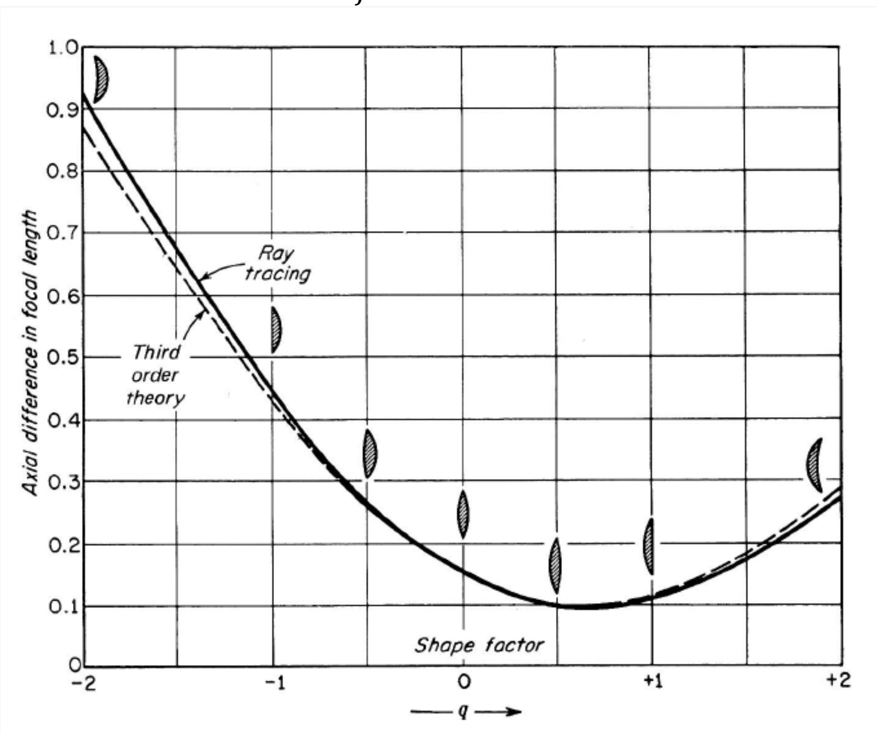
$$\frac{1}{s'(h)} - \frac{1}{s'(0)} = \frac{h^2}{8f^3}\frac{1}{n(n-1)}\left[\frac{n+2}{n-1}\sigma^2 + 4(n+1)p\sigma + (3n+2)(n-1)p^2 + \frac{n^3}{n-1}\right]$$

$$p = \frac{s'-s}{s'+s}\ , \quad \sigma = \frac{R_2+R_1}{R_2-R_1}$$

B. Jenkins and White Equation for Spherical Refractive Surface of Radius R (and refractive indices n_1 and n_2) (corrected):

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2-n_1}{R} + \frac{h^2}{2}\left[\frac{R}{f'}\frac{n_1^2}{n_2}\left(\frac{1}{s}+\frac{1}{R}\right)^2\left(\frac{1}{R}+\frac{n_2+n_1}{n_1 s}\right)\right]$$

$$\frac{n_2}{f'} = \frac{n_2-n_1}{R}$$



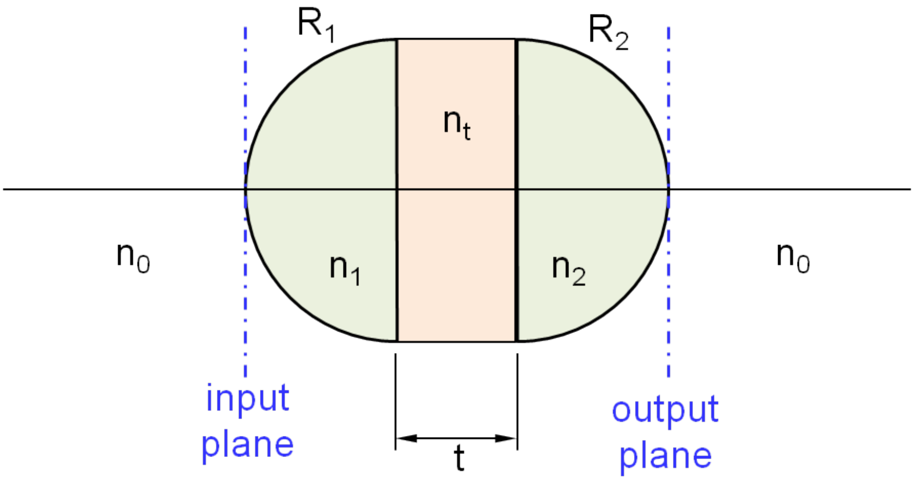
Problem 4 (Two Hemi-Spherical Lenses separated by a Distance): [0%]

Two hemispherical lenses of radii R_1 and R_2 have refractive indices n_1 and n_2 respectively. The two lenses are separated by a homogeneous region of thickness t and of refractive index n_t (as it can be seen in the follwoing figure). The material to the left and right of the hemisperical lenses is homogeneous with a refractive index of n_0. The input and output planes of this system are also shown in the figure.

(a) [4%] Find the matrix ABCD of this optical system (for this question the utilization of a symbolic computation program - as *MatLab* or *Mathematica* - could be useful in order to avoid manual matrix multiplications).

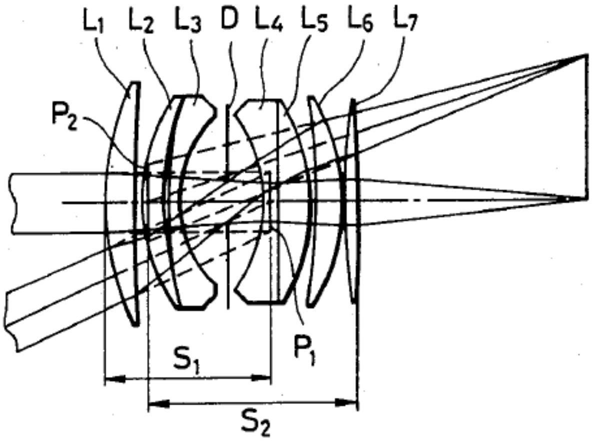
(b) [3%] If $|R_1| = 50$mm, $|R_2| = 50$mm, $n_1 = 1.50$, $n_2 = 1.50$, $n_t = 1.00$, $n_0 = 1.00$, and $0 \le t \le 100$mm, make a plot of the focal distance of the optical system, f_2, as a function of t. Furthermore, make a graphical representation of the distances of the principal points H_1 and H_2 as measured from the input plane as a function of t.

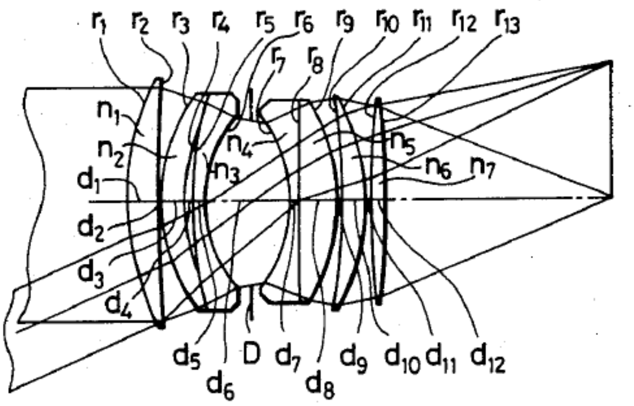
(c) [3%] Repeat (b) when $n_t = 1.50$ while all other parameters remain unchanged.



Problem 5 (Nikon Photographic Lens): [30%]

The Nikon photographic lens, <u>*Nikkor 100mm*</u>, is described in the US patent ***1984/4448497***. In this patent various implementations of the system (which is shown in the first two figures) are listed. The implementation to be used in this problem corresponds to the one described in the first embodiment. The system is comprised from 7 thick lenses. The data from the patent are shown in the following table:





First Embodiment

Focal length f = 100 F-number 1.4 Angle of view $2\omega = 46°$

		d-line	
$r_1 = +78.360$	$d_1 = 9.8837$	$n_1 = 1.79668$	$\nu_1 = 45.5$
$r_2 = +469.477$	$d_2 = 0.1938$		
$r_3 = +50.297$	$d_3 = 9.1085$	$n_2 = 1.77279$	$\nu_2 = 49.4$
$r_4 = +74.376$	$d_4 = 2.9457$		
$r_5 = +138.143$	$d_5 = 2.3256$	$n_3 = 1.67270$	$\nu_3 = 32.2$
$r_6 = +34.326$	$d_6 = 29.0698$		
$r_7 = -34.407$	$d_7 = 1.9380$	$n_4 = 1.7400$	$\nu_4 = 28.3$
$r_8 = -2906.977$	$d_8 = 12.4031$	$n_5 = 1.77279$	$\nu_5 = 49.4$
$r_9 = -59.047$	$d_9 = 0.3876$		
$r_{10} = -150.021$	$d_{10} = 8.3333$	$n_6 = 1.78797$	$\nu_6 = 47.5$
$r_{11} = -57.890$	$d_{11} = 0.1938$		
$r_{12} = +284.630$	$d_{12} = 5.0388$	$n_7 = 1.78797$	$\nu_7 = 47.5$
$r_{13} = -253.217$			

Back focal length 74.1
Full length of lens l = 81.8217
$f_1 = 116.8$
$f_2 = 172.6$
$f_6 = 115.0$

In the top figures the system of the 7 thick lenses is shown (**L1-L7** listed from left to right). The table contains the information of the lenses and their separations. Specifically, in the first column the spherical surface radius (in mm – positive or negative according to the convention for convex and concave surfaces) is listed (from left to right) for the left and right surface of each lens. The second column lists the separation in mm of the current spherical surface to the next. It is mentioned that lenses **L4** and **L5** are cemented and therefore the right surface of the **L4** coincides with the left surface of the **L5** lens. All the calculations of the problem are according to the paraxial optics approximation. The third column lists the refractive index of the corresponding lens for the wavelength of the *d-line* (λ_0=587.6nm). The last column includes the ***Abbe*** number of the lens material (however, material dispersion will no be used in this problem).

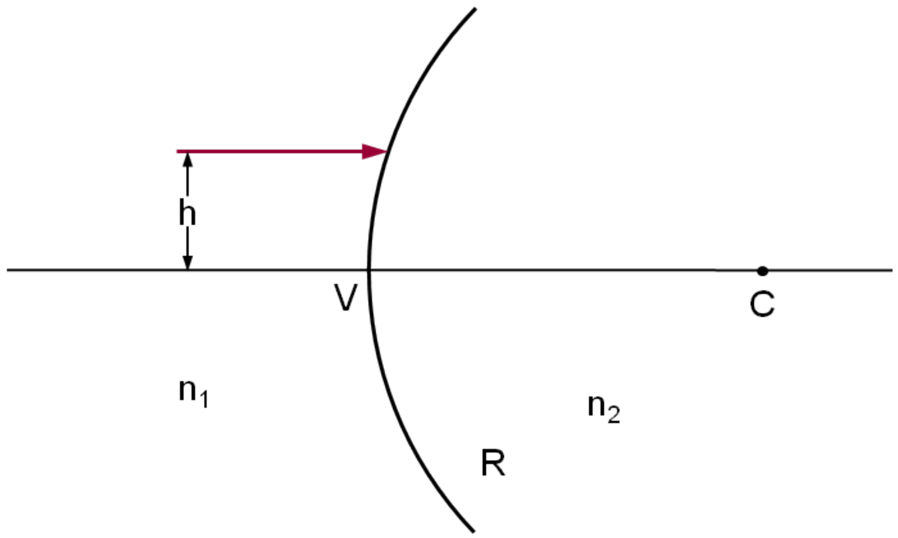
(a) [5%] At the bottom of the given table the focal lengths of lenses **L1**, **L2**, και **L6**, (**f_1**, **f_2**, και **f_6**), are listed in mm. Verify the results of the table.

(b) [18%] Find the *cardinal points* (H_1, F_1, N_1, H_2, N_2, F_2) of the lens system with input plane the left surface of the first lens (**L1**) and output plane the right surface of the last lens (**L7**). Find at what distance from the right surface of the **L7** lens the image of an object at infinity will be formed. Verify that the value that you find corresponds to the «back focal length» listed in the table.

(c) [7%] Repeat (b) for an object at a distance of 1m from the left surface of the **L1** lens. In this case determine how much the whole system of lenses needs to be shifted (to the left or to the right) such that the image will be formed correctly at the «back focal length» found (where either the film or the digital detector is positioned). Now assume that «full frame» format (of film or detector) is utilized and the **CoC** = 0.029mm. If an **f-number of 1.4** is used find the *hyperfocal distance* **H** and the *depth-of-field* (**DoF**) for the object distance of a meter.

Problem 6 (Spherical Aberration): [25%]

Assume a convex spherical refractive surface of radius *R* between two materials with refractive indices n_1 and n_2, respectively ($n_1 < n_2$). Consider a monochromatic ray impinging to the spherical interface from the left, parallel to the optical axis, and at a height of *h* as it is shown in the following figure.



(a) [5%] Find, without any approximation, at what distance the refracted ray will cross the optical axis and compare this result with the prediction of the paraxial approximation. I.e., if the refracted ray crosses the optical axis at point F find the VF distance without any approximations.

(b) [10%] If R = 100mm, n_1 = 1.0 (air) και n_2 = 1.5 (glass) find distance VF for values of h = 0.5mm, 1mm, 5mm, 10mm, 25mm, 40mm, 50mm, 75mm και 95mm. Furthermore, make a graphical representation of VF = VF(h) for the permitted range of h values. For the same interval show in the same graph the value of VF that is calculated based on the paraxial approximation. In addition, show in the same graph the third order approximation as it can be determined by the equation of Jenkins and White:

<u>Jenkins and White Equation for Spherical Refractive Surface of Radius R (and refractive indices n_1 and n_2) (corrected):</u>

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R}+\frac{h^2}{2}\left[\frac{R}{f'}\frac{n_1^2}{n_2}\left(\frac{1}{s}+\frac{1}{R}\right)^2\left(\frac{1}{R}+\frac{n_2+n_1}{n_1 s}\right)\right]$$

$$\frac{n_2}{f'}=\frac{n_2-n_1}{R}$$

(c) [10%] Now assume that the impinging ray is comprised of white light and therefore it contains all the wavelengths in the range from 380 to 750nm. The refractive index of the air can be considered constant as a function of the wavelength. However, the glass refractive index can be well approximated by the follwoing Sellmeier's formula (where the freespace wavelength is expressed in μm):

$$n^2 = A + \frac{G_1\lambda^2}{\lambda^2-\lambda_1^2} + \frac{G_2\lambda^2}{\lambda^2-\lambda_2^2} + \frac{G_3\lambda^2}{\lambda^2-\lambda_3^2},$$

where the various constants have the following values: A = 1, G_1 = 0.6961663, G_2 = 0.4079426, G_3 = 0.8974794, λ_1 = 0.0684043μm, λ_2 = 0.1162414μm, and λ_3 = 9.896161μm. If R = 100mm, n_1 = 1.0 (air) compute the distance VF = VF(λ) as a function of the freespace wavelength for h = 10 και 20 mm. Calculate also VF using the paraxial approximation. Make a graph of VF as a function of λ for three cases (h = 10mm, h = 20mm, and the paraxial approximation) in a common plot. Furthermore, show in a table the values of VF for h = 20mm along with its paraxial approximation for λ = 0.4, 0.5, 0.6, and 0.7μm.

Problem 7 (Spherical Aberration – Plano/Convex Lens): [15%]

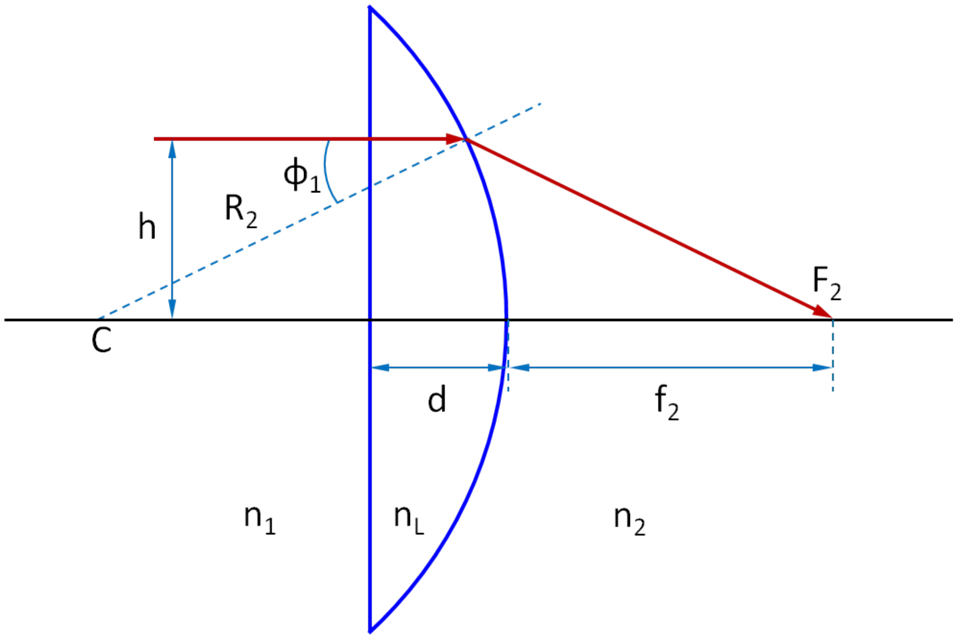
Assume a plano-convex lens of radius R_2 ($R_1 = \infty$) between two homogeneous materials of refractive indices n_1 and n_2 respectively. The refractive index of the lens is n_L ($n_L > n_1, n_2$). The lens thickness is d. A monochromatic ray is incident on the flat surface parallel to the optical axis at a height h as it is shown in the figure.

(a) [5%] Find, without any approximation, at what distance the refracted ray will cross the optical axis and compare this result with the prediction of the paraxial approximation. I.e., if the refracted ray crosses the optical axis at point F_2 find the f_2 distance without any approximations as a function of h.

(b) [3%] Find the range of lens thickness d for which total internal reflections from the spherical surface are avoided for all allowable values of h.

(c) [3%] If R_2 = 25mm, $n_1 = n_2 = 1.0$ (air), $n_L = 1.5$ (glass) and d = 5mm, find the distance f_2 for all allowable values of h. What is the maximum allowable value of h? Furthermore, make a graphical representation of $f_2 = f_2(h)$ for the permitted range of h values. Show the paraxial approximation also.

(d) [4%] Make a ray diagram for various allowable heights h making usage of the numerical data of part (b). Repeat the ray diagram for lens thickness d = 10mm.



Problem 8 (Keplerian and Galilean Telescope): [10%]

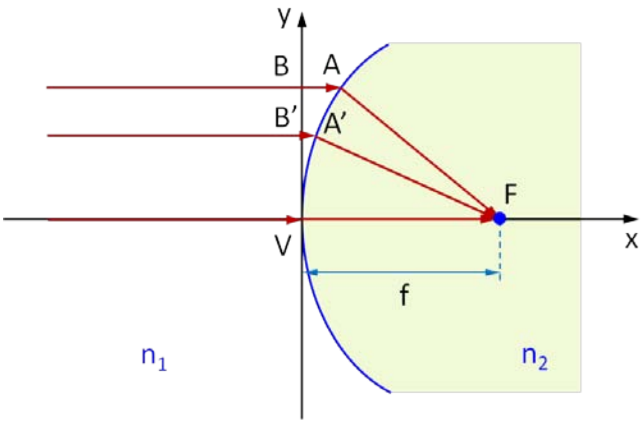
Assume two thin lenses that can be utilized to form a basic *Keplerian* or *Galilean* telescope.The focal distance of the objective lenses are f_o = 300mm (either for the *Keplerian* or the *Galilean*). The eyepiece lens has a focal length f_e = 25mm (for the *Keplerian* telescope) κand f_e = -25mm (for the *Galilean* telescope). If both telescopes are used for object at long distance (at infinity) then they have the same (in magnitude) angular magnification $-f_o/f_e$ with the *Keplerian* telescope forming an inverted virtual image and the *Galilean* telescope forming an upright virtual image. If both telescopes are used to image objects at finite images find for each telescope which is the minimum object distance in order to form a virtual image. If O_{min} is the larger of their minimum object distances (in order to form virtual images) then select an object distance of $2O_{min}$ for both telescopes and utilizing the ABCD matrix method draw a ray diagram showing the image formation for each telescope. Furthermore, in the latter case find the position and the transverse magnification of the final virtual image.

Problem 9 (Refractive Surface without Spherical Aberration): [10%]

The purpose of this problem is to design a refractive surface that does not suffer from spherical aberration. This means that parallel to the optical axis rays will focus to the same point (at distance f from the vertex of the surface) independently of the height that they incident on the refractive surface.

(a) [5%] Find the refractive surface $y=y(x)$ between two media with refractive indices n_1 and n_2 (where $n_2 > n_1$) that does not suffer from spherical aberration. Show that the surface is an ellipse and define its equation in the xy plane. In addition, plot the surface in the xy plane. Assume that $n_1 = 1.0$, $n_2 = 1.5$, and $f = 50$mm.

(b) [5%] Now in order to validate your design assume a ray incident on your design surface from the left at a height h. The ray is refracted at the refractive surface and intersects the optical axis at point F. Show ray diagrams of several rays at different heights h and verify that they all pass from the same point F which is located a distance f from the vertex of the surface. Use the previous numerical data for this question also.



Problem 10 (Human Eye and Accommodation): [10%]

The simplified diagram of the human eye is shown in the diagram below. The parameters given are:

Refractive Indices: $n_0 = 1$, $n_1 = 1.376$, $n_2 = 1.336$, $n_3 = 1.417$, and $n_4 = 1.336$.

Distances: d_0 = depends from the object distance from the, $d_1 = 0.55$mm, $d_2 = 3.26$mm, d_3 = depends on the accommodation level of the human lens (is the paraxial thickness of the human lens), $d_4 = 16.60$mm.

Radii of Curvature: $R_1 = 7.7$mm, $R_2 = 6.8$mm, R_3 and R_4 depend on the accommodation level of the human lens. From the book of Atchison & Smith (*Optics of the Human Eye*) the simplified approximate formula are given by:

$$x(A) = 1.052A - 0.00531A^2 + 0.000048564A^3$$

$$d_3 = 3.6 + 0.4\,\frac{x(A)}{10.87}\ \ (\text{in } mm)$$

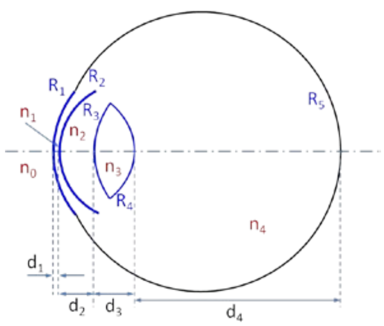
$$\frac{1}{R_3} = \frac{1}{10} - \left(\frac{1}{10} - \frac{1}{5.333}\right)\frac{x(A)}{10.87}\ \ (\sigma\varepsilon\ mm^{-1})$$

$$\frac{1}{R_4} = -\frac{1}{6} - \left(-\frac{1}{6} + \frac{1}{5.333}\right)\frac{x(A)}{10.87}\ \ (\sigma\varepsilon\ mm^{-1})$$

(a) [3%] Find the ABCD matrix with input plane the plane of the object (in distance d_0 from cornea) and output plane the plane of the image inside the vitreous humor (area of refractive index n_4). What is the image distance as measured from the right surface of the human lens? What is the refractive power n_4/f_2 of the human eye?

(b) [7%] Find the parameter A such that the image always forms on the retina surface, i.e. for $d_4 = 16.60$mm, for object distance (i) $d_0 = 25$m, (ii) $d_0 = 1.5$m, and (iii) $d_0 = 0.25$m. In these three cases find the radii of curvature R_3 and R_4 and the thickness d_3 as well as the refractive power n_4/f_2 of the human eye.

Human eye optical parameters



Note: **For all the problems** that you use some kind of software (such as *MatLab* or others) it is **mandatory** (in order to gain full credit for the corresponding problem) to include in your answers a printout of the code that you have written and used.