NATIONAL TECHNICAL UNIVERSITY OF ATHENS

SCHOOL OF ELECTRICAL & COMPUTER ENGINEERING

DIVISION OF ELECTROMAGNETIC APPLICATIONS, ELECTRO-OPTICS & ELECTRONIC MATERIALS

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OPTICAL ENGINEERING

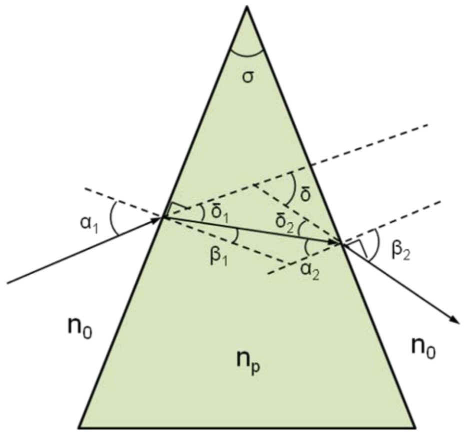
Problem Set No. 2

Due Date: April 9, 2024

(The problems that should be submitted have nonzero weighting factors)

Problem 1 (Deviation by a Prism): [0%]

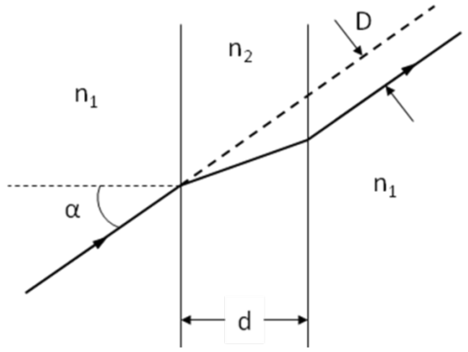
A ray of light is incident at an angle α_1 upon a prism of index of refraction n_p. The apex angle of the prism (defined by the two refractive surfaces) is σ. The medium surrounding the prism has a refractive index n_0. Compute the angular deviation δ as it has been defined in class (and shown in the figure). Find what is the minimum value of δ_{min} and for what value of α_1 the value of δ_{min} is obtained? If $\sigma = 20$ deg and $n_0 = 1$, $n_p = 2$, make a graphical representation of angular deviation as a function of angle α_1 (between 0 και 89.9 deg) and find δ_{min} as well as for which value of α_1 occurs. Repeat the last graphical representation when $\sigma = 25$ deg και $n_0 = 1$, $n_p = 2.50$. What do you observe?



Problem 2 (Plane Parallel Plate): [0%]

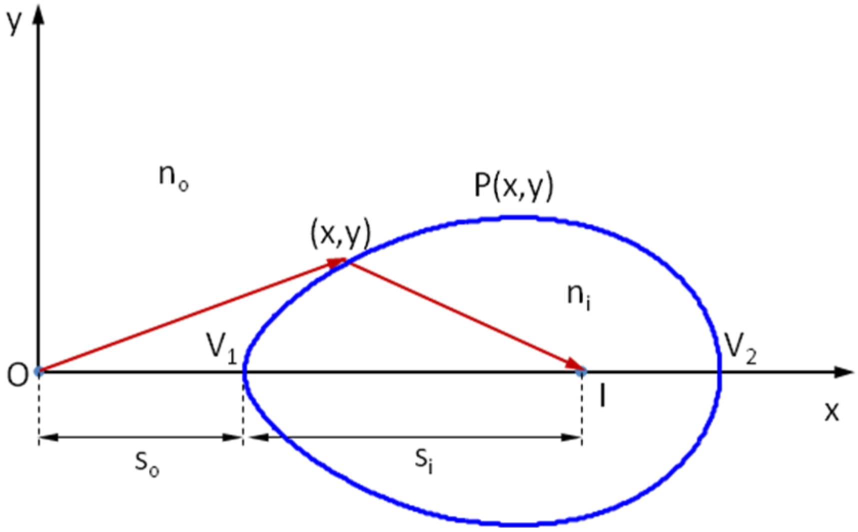
A ray of light is incident upon a plane parallel plate of index of refraction n_2 with an angle of incidence α. The thickness of the plate is d. The medium surrounding the plate has a refractive index of n_1. Find the displacement D of the light ray upon passing through the plate as a function of d, n_1, n_2, and α.

For fused quartz (n_2=1.46) plane parallel plate in air (n_1=1.0), calculate the displacement for an angle of incidence of 67.5deg and a thickness of 0.525 inches. (1 inch = 2.54cm).



Problem 3 (Cartesian Ovoid): [0%]

As it was discussed in class, to achieve perfect imaging between an object point O (in material of refractive index n_o) and its image point I (in material of refractive index n_i), a refractive boundary surface (between two materials with refractive indices n_o and n_i) is needed which is called "Cartesian Ovoid." In the following figure such a complete (closed) Cartesian ovoid is shown that images point O in image I. The distance of O from V_1 is s_o while the distance of I from V_1 is s_i. In this problem the s_o, s_i, n_o and n_i are considered known. (a) Determine the position of point V_2. (b) Determine numerically the refractive boundary surface *P(x,y)* for $n_o = 1.0$, $n_i = 1.5$, s_o = 5cm, s_i,= 10cm, 15cm, and 20cm. Make a graphical representation of the resulting Cartesian ovoids in a common coordinate axis system (usage of some software tool such as MatLab would be necessary). (c) Assume, as it is reasonable, that the ray with the maximum slope that emanates from O and ends in I is tangential to the ovoid. Determine the maximum slopes for the cases of the distances s_i of question (b). For this case a numerical approach will be again necessary.



Problem 4 (Thin Lenses System): [0%]

An object is located 12 cm to the left of a negative thin lens of -20cm focal length. Another positive thin lens of focal length of +4cm is placed 2cm to the right of the first lens. Find the image distance, as measured from the second lens. Find the transverse magnification and determine if the image is real or virtual. Make a ray diagram of the system.

Problem 5 (Thin Lenses System): [0%]

A small object is placed 20cm from the first of a series of three thin lenses with focal lengths -10, +15, and -20cm respectively. The first two lenses are separated by 30 cm and the last two are separated by 20 cm. Calculate the final image position relative to the last lens and its transverse magnification relative to the original object. Is the image real or virtual? In addition, provide a ray diagram.

Problem 6 (Cartesian Ovoid): [15%]

(a) [5%] Determine the Cartesian ovoid for the perfect imaging with a refractive surface when the object point O is located at a distance (in air with $n_o = 1.0$) s_o = 5cm to the left of vertex V_1, and the real image point I is at a distance s_i = 7.5cm to the right of V_1 (inside the refractive material with n_i = 1.35). Both the object and the image are located along the x-axis. Determine the equation describing the intersection of the ovoid with the xy-plane. Assume that the point (x=0,y=0) corresponds to the position of the object. Determine the position of V_2 and make a graphical representation of the ovoid in the xy-plane. (b) [10%] Repeat (a) in the case of virtual image I, i.e., when distance s_i = -7.5cm (to the left of V_1).

Problem 7 (Dispersing Prism): [10%]

Sellmeier's formula gives a good approximation of the dependence of the refractive index of a material, n(λ), on the freespace wavelength. Sellmeier's formula is give below for your convenience along with the necessary constants B_i and C_i for two different materials:

$$n^2(\lambda) = 1 + \sum_i \frac{B_i \lambda^2}{\lambda^2 - C_i},$$

Table of coefficients of *Sellmeier* equation						
Material	**B_1**	**B_2**	**B_3**	**C_1**	**C_2**	**C_3**
Schott (multiple purpose glass)	1.3182408	0.0244	1.08915181	$8.79\times10^{-3}\mu m^2$	$6.09\times10^{-2}\mu m^2$	$110\ \mu m^2$

An equilateral prism from **Schott multiple purpose glass** is used in a spectrometer.
(a) [3%] Determine the minimum angular deviation of the prism as a function of freespace wavelength in the visible spectrum (i.e., between 400nm and 700nm). Make a graphical representation of your findings.
(b) [3%] Determine the dispersive power of the prism (approximate and exact).
(c) [4%] Determine the minimum angular dispersion $d\delta_{min}/d\lambda$ (in deg/μm) as a function of freespace wavelength and make its graphical representation.

Problem 8 (Thin Lenses System): [15%]

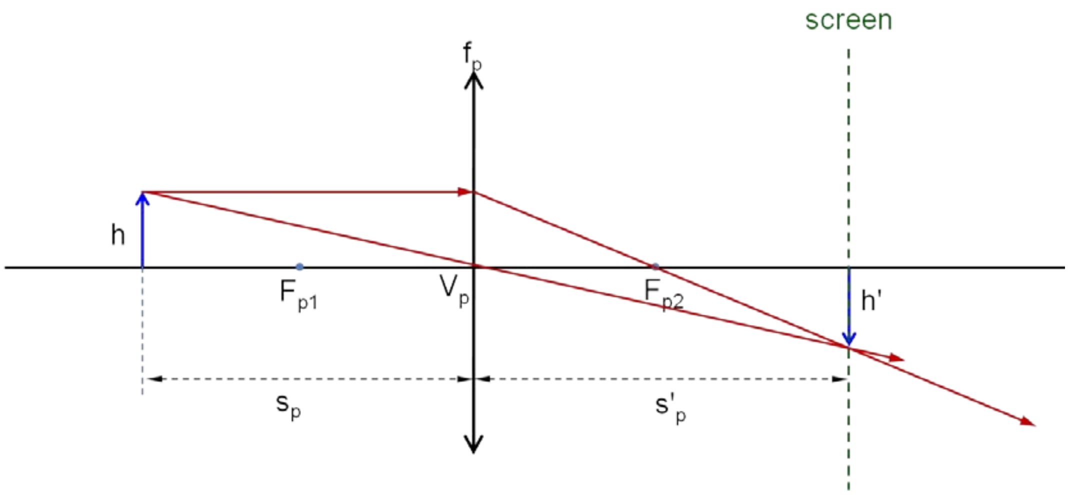
(a) [5%] An thin object 10mm tall is placed at a distance of 150mm to the left of a positive thin lens of focal length of 100mm. Another negative thin lens, of focal length of -75mm, is located 250mm to the left of the positive lens. Determine where the image is located (measuring its distance from the negative lens). What is the transverse magnification? Is the image real or virtual? Make a ray diagram to show the imaging by the two lenses system. (β) [10%] Now assume the the distance L between the two thin lenses is varied with movement only of the right lens and constant the position of the left thin lens. Let $0 < L < 400$mm. Make a graph of the image position as it is measured from the center of the right moving lens. Find the ranges of L that the final image is real or virtual.

Problem 9 (Focal Length Measurement for a Positive Thin Lens): [10%]

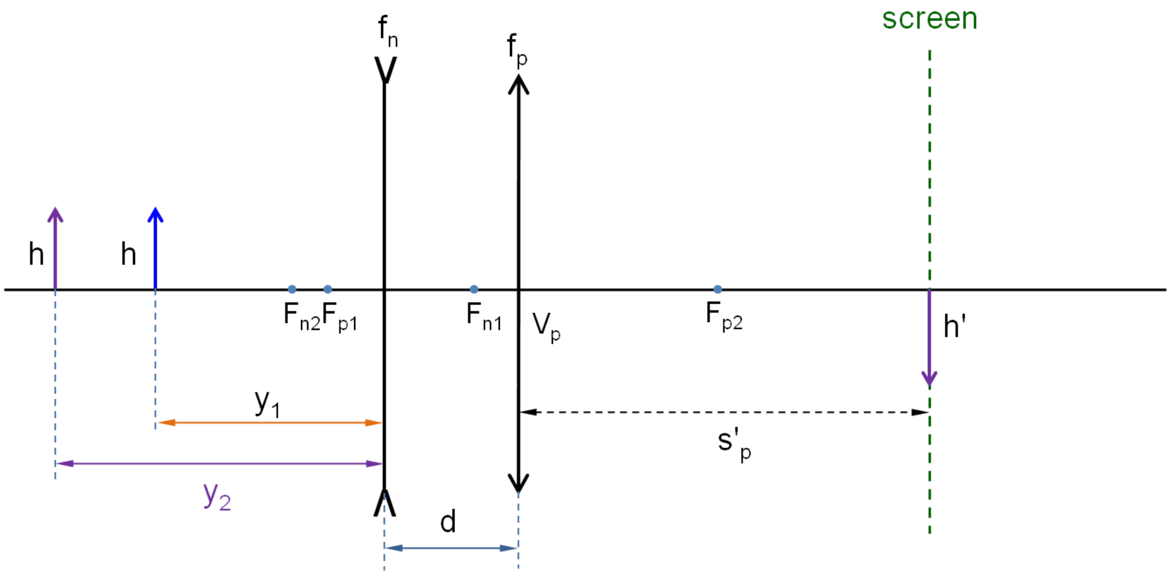
A convenient way to measure the focal length of a positive thin lens maks use of the following fact. If a pair of conjugate object and real image points (O and I) are separated by a distance $L > 4f$, there will be two locations of the lens, a distance d apart, for which the same pair of conjugate points can be obtained. Determine the focal distance of the lens as a function of L and d.

Problem 10 (Focal Length Measurement for a Negative Thin Lens): [10%]

The following procedure comprised a manner that can be used to measure the focal distance of a negative lens. In this process a positive lens (of focal distance f_p) is used at a distance s_p from an object of height h. The initial real image of height h' is formed at a distance s'_p on a screen (as it is shown in the figure below).



Retaining the positions of the positive lens and of the screen constant the thin negative lens (of the unknown focal length f_n) is inserted into the system between the positive lens and the object. The negative lens is positioned to the left of the positive lens and at a distance *d* and is closer to the positive lens than to the object. This is shown in the figure below.

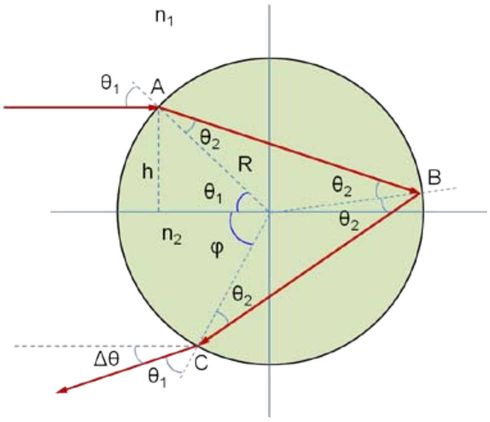


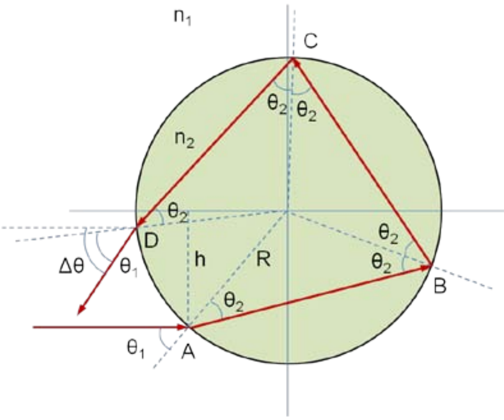
The object is initially at a distance y_1 from the negative lens. However, with the insertion of the negative lens into the system, the image is not formed anymore on the screen at distance s'_p from the positive lens. In order to force the image to form again on the screen the object is moved at a distance y_2 from the negative lens. Find the focal distance f_n of the negative lens using the distances y_1 and y_2 (which can be measured). Furthermore, make a typical ray diagram to show the formation of the image by the system of the two lenses.

Problem 11 (Basics of Rainbow Formation): [20%]

This problem is an effort to understand the basics of the formation of rainbows in the air. A sample rainbow is shown in the figure below where both the primary (lower and with brighter colors) and the secondary (higher and in faint colors) rainbows are shown. Observe that the separation of colors is opposite in the two rainbows. The primary rainbow is formed by incident light rays that undergo two refractions and one reflection inside each water droplet. This is also schematically shown in the left figure of a water droplet. The secondary rainbow is formed by incident light rays that undergo two refractions and two reflections inside a water droplet as it is shown schematically in the right figure.

Photograph of a rainbow (http://atgbcentral.com/data/out/98/4799387-rainbow-pics.jpg)

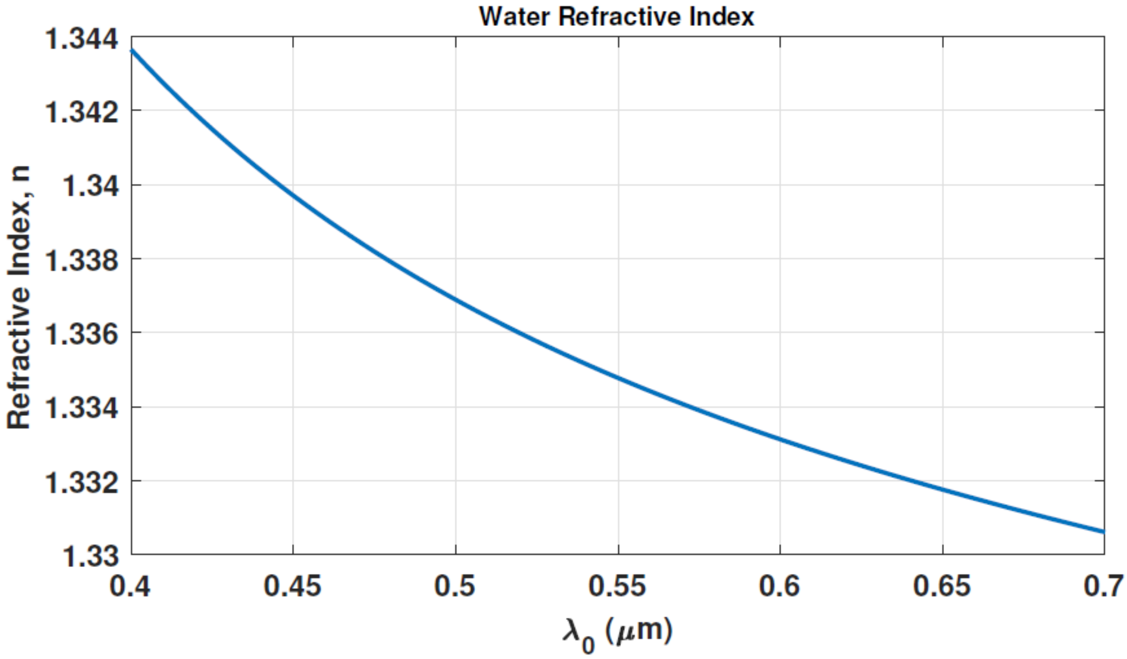




In order to justify these effects it is necessary to have the frequency (wavelength) dependence of the refractive index of water. The refractive index of the water as a function of the freespace wavelength λ_0 (expressed in microns) is given by the following Sellmeier's equation while a plot of the refractive index is also shown in the figure below:

$$n^2(\lambda_0) = 1 + \frac{5.672526103 \times 10^{-1}\lambda_0^2}{\lambda_0^2 - 5.085550461 \times 10^{-3}} + \frac{1.736581125 \times 10^{-1}\lambda_0^2}{\lambda_0^2 - 1.814938654 \times 10^{-2}} +$$

$$\frac{2.121531502 \times 10^{-2}\lambda_0^2}{\lambda_0^2 - 2.617260739 \times 10^{-2}} + \frac{1.138493213 \times 10^{-1}\lambda_0^2}{\lambda_0^2 - 1.073888649 \times 10^{+1}}$$

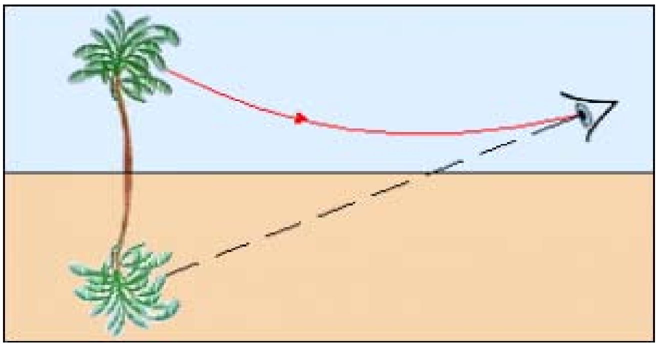


(a) Evaluate the angular deviation $\Delta\theta$ shown in the water droplet refraction figures above as a function of the ratio h/R (which is related to the incident angle θ_1) for a water droplet of radius R. For simplicity consider that all light rays belong to the same meridian of the spherical droplet so 3D incident can be avoided. Vary h/R in the range between 0 and 1 for both cases. Plot $\Delta\theta$ as a function of the ratio h/R for various wavelengths in the visible range (0.380μm to 0.720μm). What do you observe? Consider that the refractive index of the air n_1=1.

(b) Find the equation that defines the extremum $\Delta\theta$ and the corresponding angle of incidence θ_1 for both the primary and the secondary rainbow. Make a plot of both $\Delta\theta_{ext}$ and $\theta_{1,ext}$ as a function of the wavelength for the above given range and with the given Sellmeier equation of water.

Problem 12 (Mirage Simulation): [20%]

Assume that in very hot days the air near ground is thinner due to higher temperature and it gets thicker with the distance from the ground since the temperature reduces. The refractive index of the air (neglecting dispersion) under these conditions can be approximated by the equation $n(r) = n_0[1 + kr]$ (where $k > 0$). Of course the distance $r = \sqrt{x^2 + y^2}$ is the radial distance from the ground. The *z*-axis is on the ground as it is shown in the last diagram.



http://images.math.cnrs.fr/Le-chant-de-la-Terre.html?lang=es

For the numerical implementation assume that $n_0 = 1.00022$ and $k = 1.3333 \times 10^{-5} m^{-1}$.
(a) Assume that a ray emanates at height *h* with an angle θ as shown in the figure. The angle θ is positive when the ray points to negative *x* (as in the figure) and is negative when it points to the positive *x*. Assume that θ varies between -3/60deg – 30/60deg with a step of 3/60deg. Furthermore, assume that h = 2m. Find all the routes that each ray follows due to the varying refractive index, for a maximum distance of 1 km along the z direction.
(b) Repeat (a) when h = 3m.
Hints: If a ray route hits the ground it is interrupted i.e. the ray is absorbed by the ground. For the solution of the eikonal equation the boundary conditions $(\frac{dr}{dt})(t = 0) = n(r)\left(\frac{dr}{ds}\right)(t = 0)$ are needed. The eikonal equation, in the general case, can be solved only numerically using Runge-Kutta techniques. For example, Matlab's ``ode45'' function can easily solve nonlinear systems of differential equations (other similar routines in various computational packages can be used). Check also references:
A. Sharam et al., “Tracing rays through graded-index media: a new method,” Appl. Opt., vol. 21, No. 6, pp. 984-987, Mar. 15, 1982.
A. Sharam, “Computing optical path length in gradient-index media: a fast and accurate method,” Appl. Opt., vol. 24, No. 24, pp. 4367-4370, Dec. 15, 1985.
In the case that the refractive index can be described by the relationship $n(r) = n_0[1 + kx]$ (which is more reasonable for the given problem) then there exist an analytical solution of the eikonal equation. In this case specify (using a graphical represeantation) the difference between the analytical and the numerical (via Runge-Kutta) solutions as function of the distance z for various values of *h* and θ.