01/04/2024

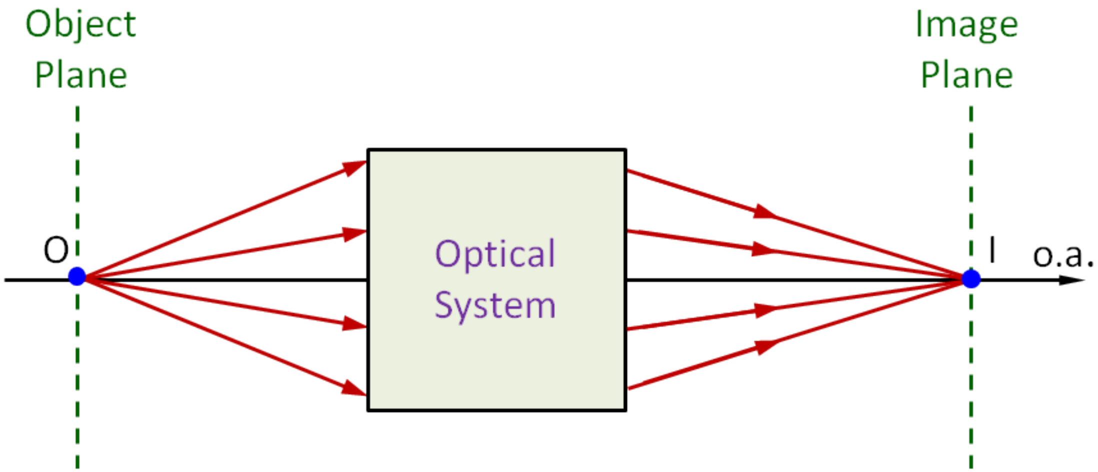
Image Formation Fundamentals

Optical Engineering

Prof. Elias N. Glytsis

School of Electrical & Computer Engineering
National Technical University of Athens

Imaging



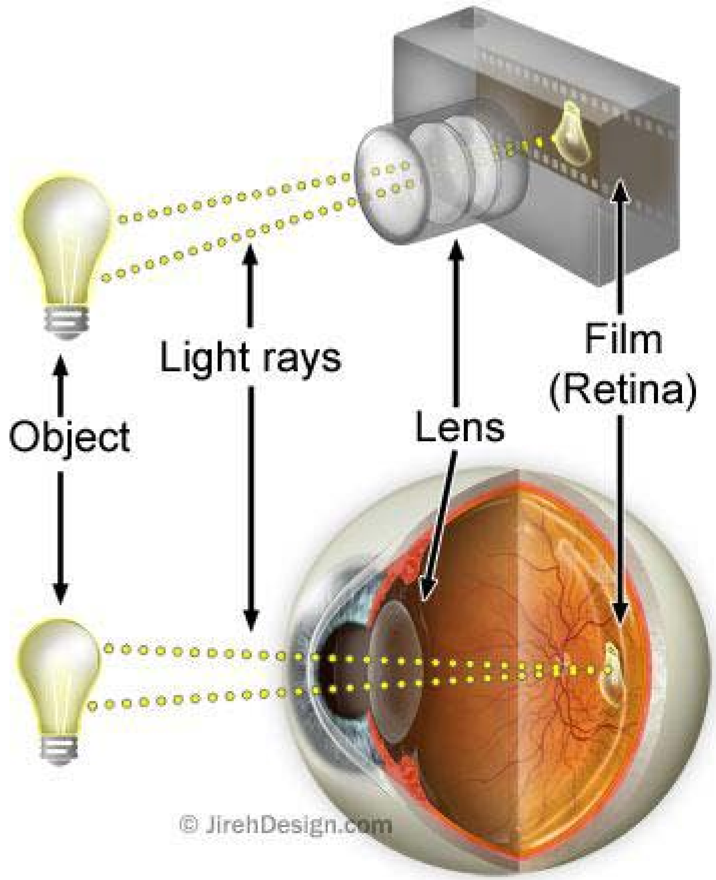
Conjugate Points

Imaging Limitations

- Scattering
- Aberrations
- Diffraction

F. L. Pedrotti and L. S. Pedrotti, Introduction to Optics, 2nd Ed., Prentice Hall, 1993.

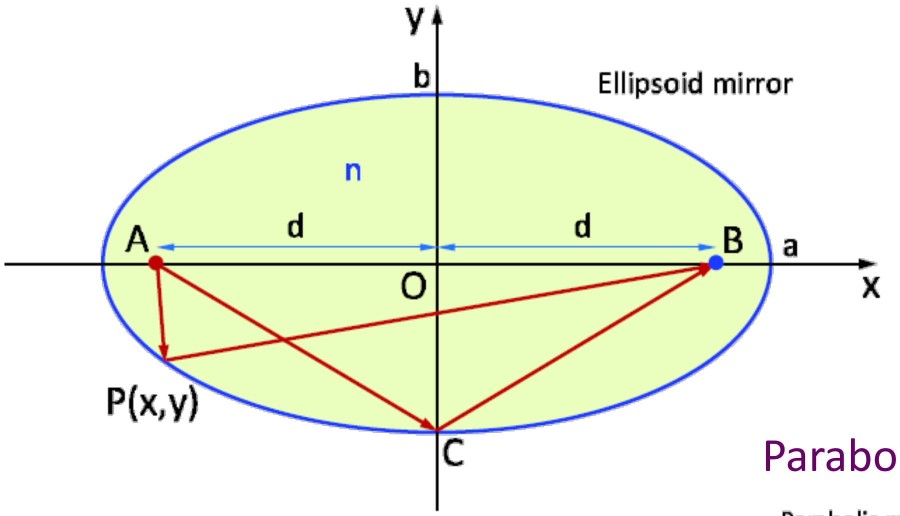
Example Optical Systems



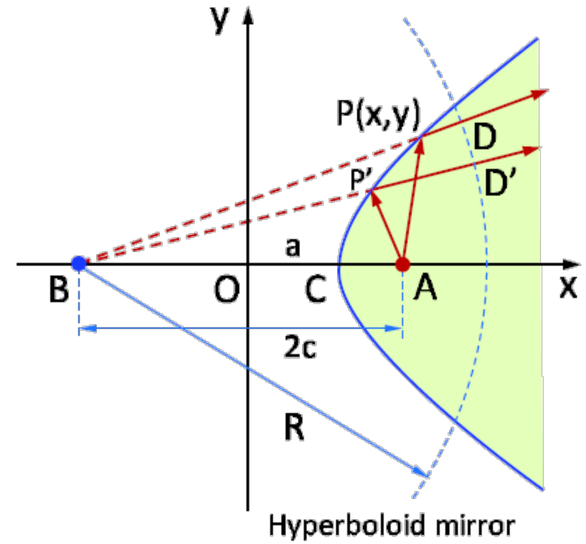
https://www.jirehdesign.com/images/tmc_eye_illustrations/eyeAnatomyCamera.jpg

Perfect Imaging Using Reflective Surfaces
(Cartesian Reflecting Surfaces)

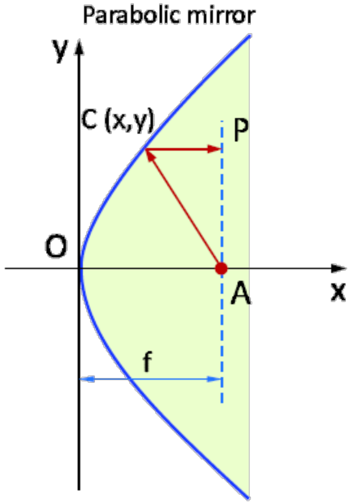
Ellipsoid



Hyperboloid

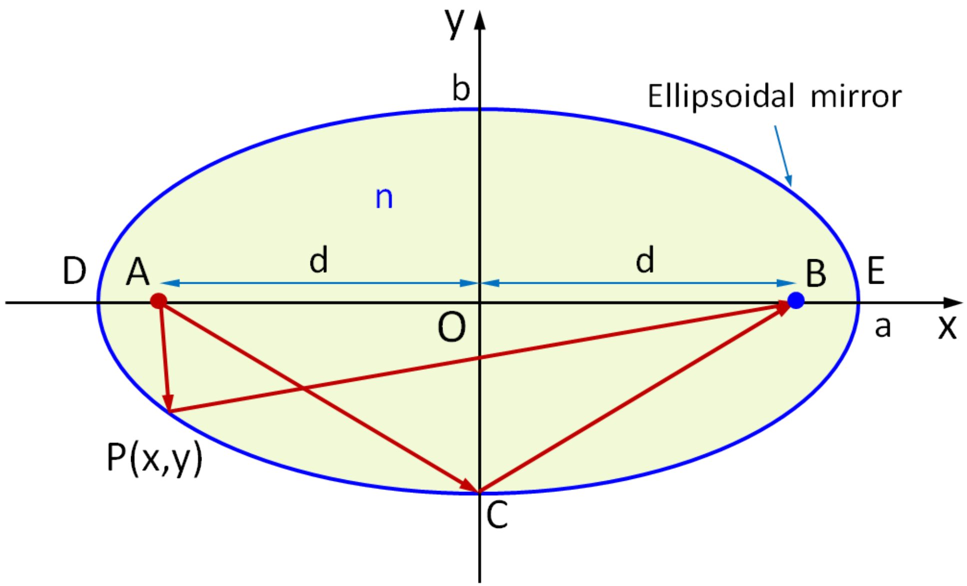


Paraboloid



Cartesian Reflecting Surfaces

Ellipsoidal Mirror



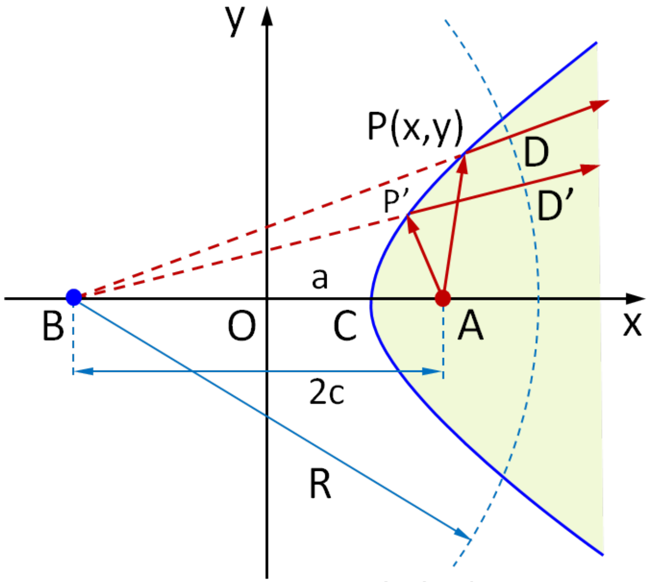
$$OPL = n(AP) + n(PB) = n\left[\sqrt{(x+d)^2+y^2} + \sqrt{(x-d)^2+y^2}\right] =$$

$$n(AC) + n(CB) = n2\left[\sqrt{b^2+d^2}\right] = n(AD) + n(DB) = n(AE) + n(EB) = n\,[2a] \Longrightarrow$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \qquad \text{with } \; b^2 + d^2 = a^2,$$

Cartesian Reflecting Surfaces

Hyperboloid Mirror

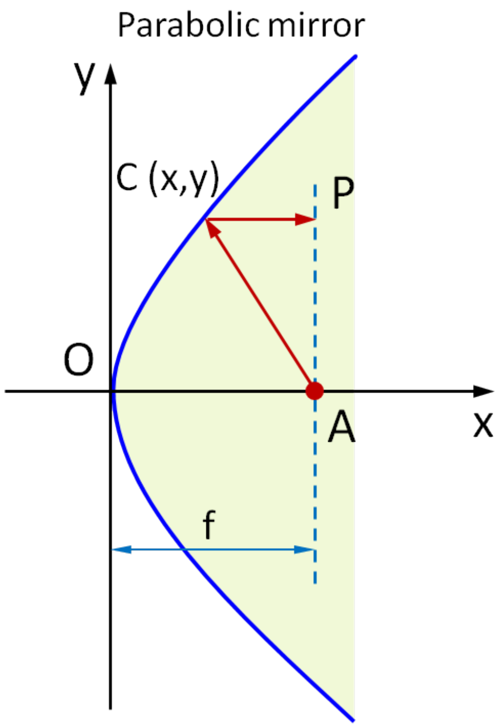


$$
\begin{aligned}
AP + PD = AP' + P'D' &= K \Longrightarrow \\
AP + (R - BP) = AP' + (R - BP') &= K \Longrightarrow \\
BP - AP = BP' - AP' &= R - K = K' \Longrightarrow \\
BP - AP &= K' \Longrightarrow \\
\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} &= K' = CB - CA = (c+a) - (c-a) = 2a \Longrightarrow
\end{aligned}
$$

$$
\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1, \qquad \text{with} \quad c^2 = a^2 + b^2,
$$

Cartesian Reflecting Surfaces

Parabolic Mirror



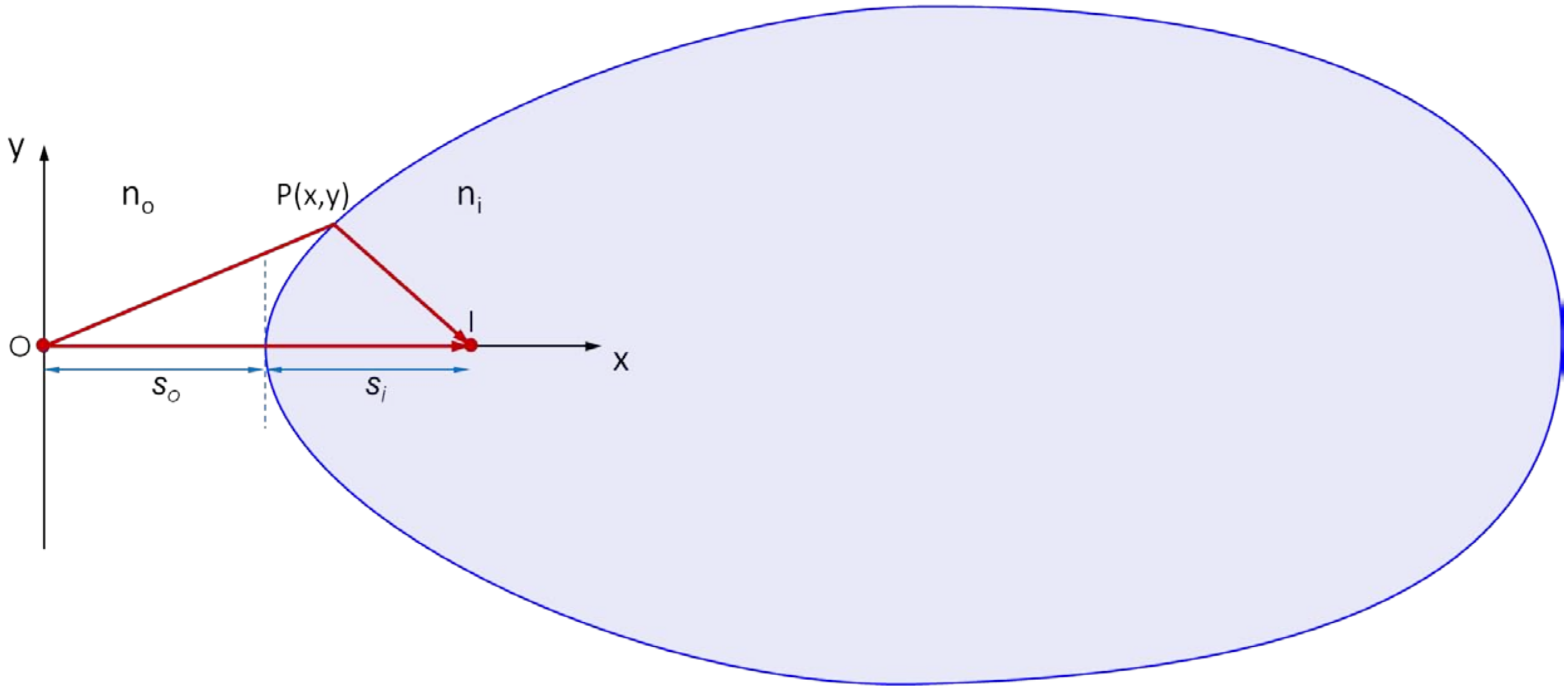
$$
\begin{aligned}
AC + CP &= 2OA = 2f \Longrightarrow \\
\sqrt{(x-f)^2 + y^2} + (f - x) &= 2f \Longrightarrow \\
y^2 - 4fx &= 0,
\end{aligned}
$$

Perfect Imaging Using Refractive Surfaces

(Cartesian Refracting Surfaces)

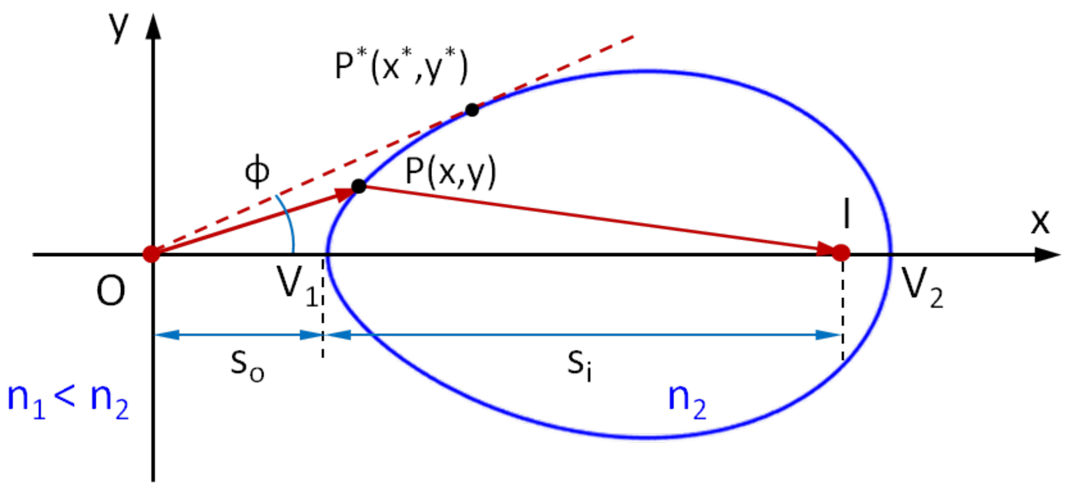
1. Each ray will travel in least time (Fermat).
2. All rays will take the same time (Isochronous).
3. Equal time implies equal *nd* (optical path length)

Cartesian Ovoid



Perfect Imaging Using Refractive Surfaces

Cartesian Ovoids – Real Image

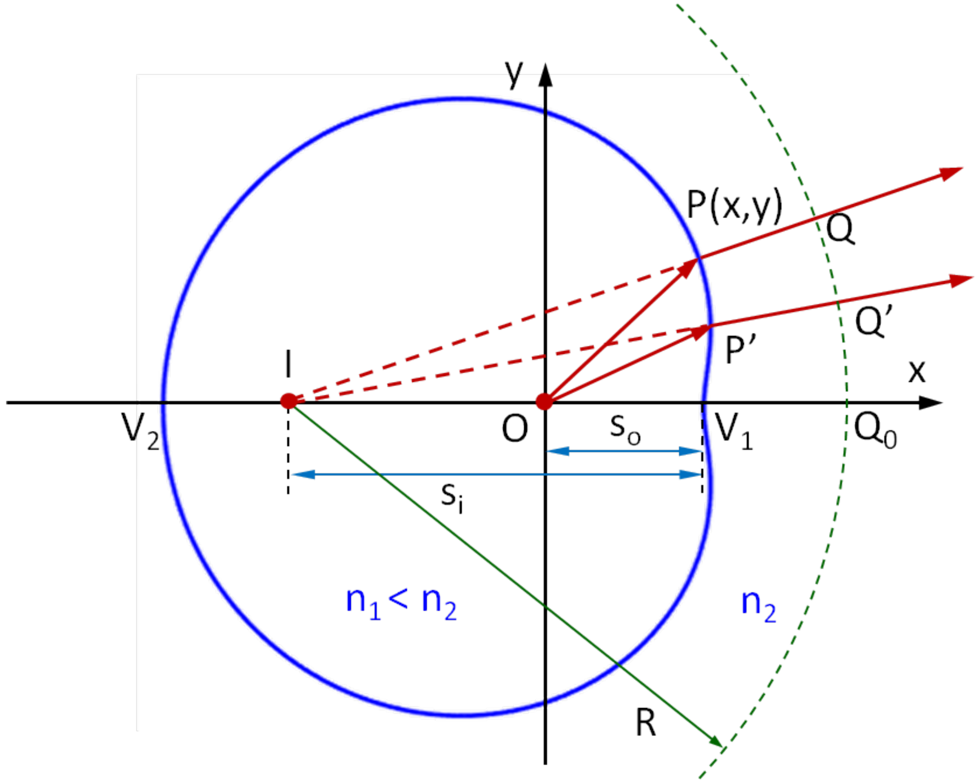


$$n_1(OP) + n_2(PI) \quad = \quad n_1 s_o + n_2 s_i \Longrightarrow$$

$$n_1\sqrt{x^2+y^2} + n_2\sqrt{(s_o+s_i-x)^2+y^2} \quad = \quad n_1 s_o + n_2 s_i,$$

Perfect Imaging Using Refractive Surfaces

Cartesian Ovoids – Virtual Image



$$n_1(OP) + n_2(PQ) \quad = \quad n_1(OP') + n_2(P'Q') = n_1 s_o + n_2(V_1 Q_0) \Longrightarrow$$

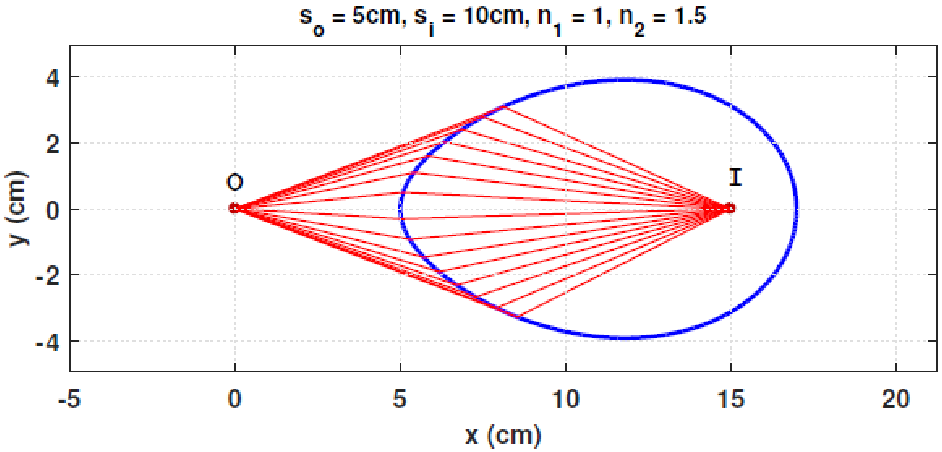
$$n_1(OP) + n_2[R - (IP)] \quad = \quad n_1(OP') + n_2[R - (IP')] = n_1 s_o + n_2(R - |s_i|) \Longrightarrow$$

$$n_1\sqrt{x^2 + y^2} - n_2\sqrt{(x + |s_i| - s_o)^2 + y^2} \quad = \quad n_1 s_o - n_2 |s_i|$$

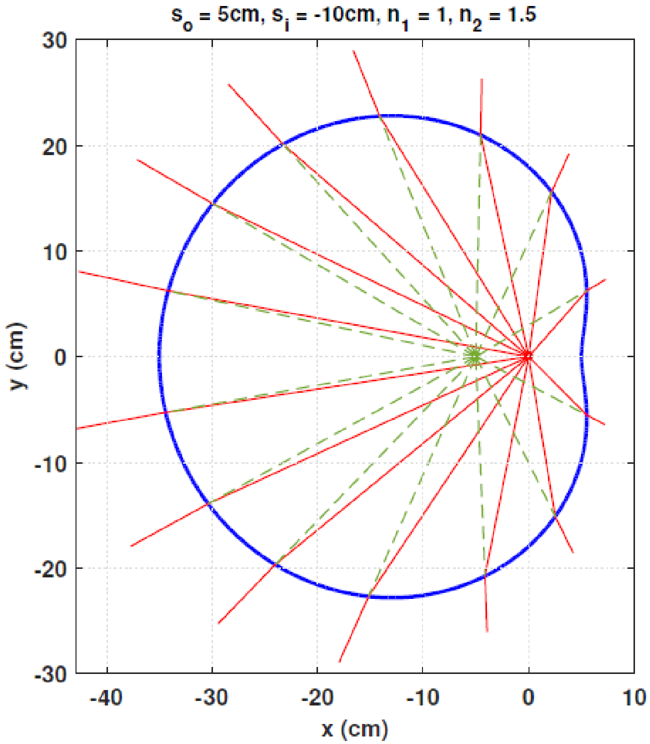
Perfect Imaging Using Refractive Surfaces

Cartesian Refracting Surfaces Examples

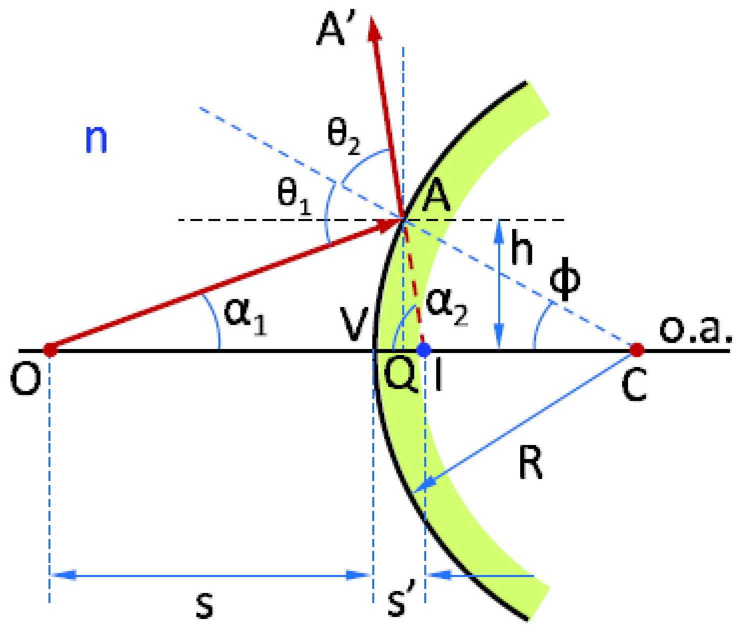
Real Image



Virtual Image



Reflection at a Spherical Mirror



Paraxial Approximation: $\sin\theta \simeq \theta$ $\tan\theta \simeq \theta$ $\cos\theta \simeq 1$

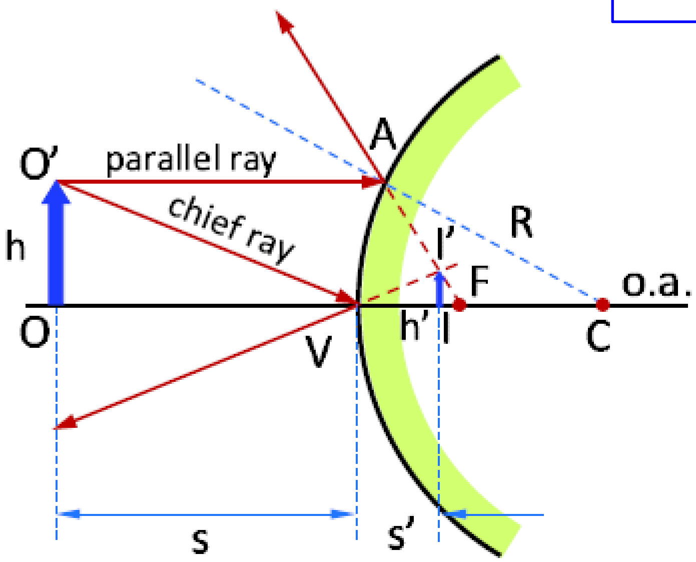
$$\theta_1 = \theta_2 \Longrightarrow (\alpha_1 + \phi) = (\alpha_2 - \phi) \Longrightarrow$$

$$\alpha_1 - \alpha_2 = -2\phi \Longrightarrow \frac{h}{s} - \frac{h}{s'} = -2\frac{h}{R} \Longrightarrow$$

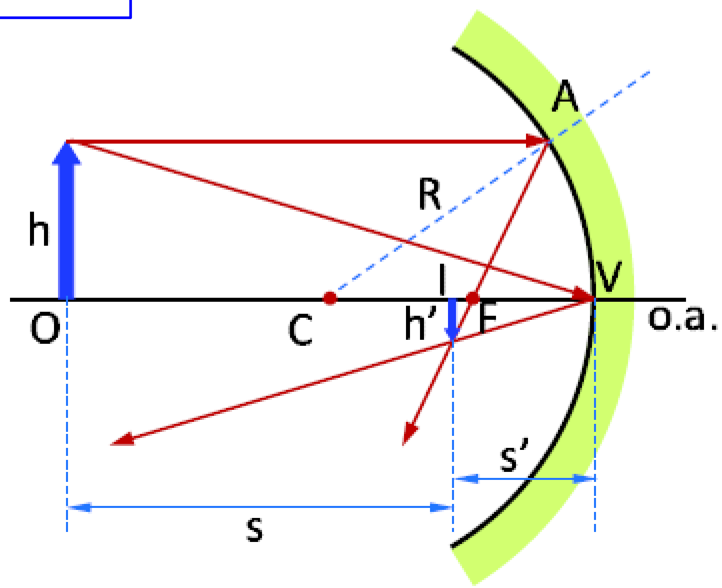
$$\boxed{\frac{1}{s} + \frac{1}{s'} = -\frac{2}{R},}$$

Reflection at a Spherical Mirror

$$\frac{1}{s} + \frac{1}{s'} = -\frac{2}{R},$$



Convex Spherical Mirror

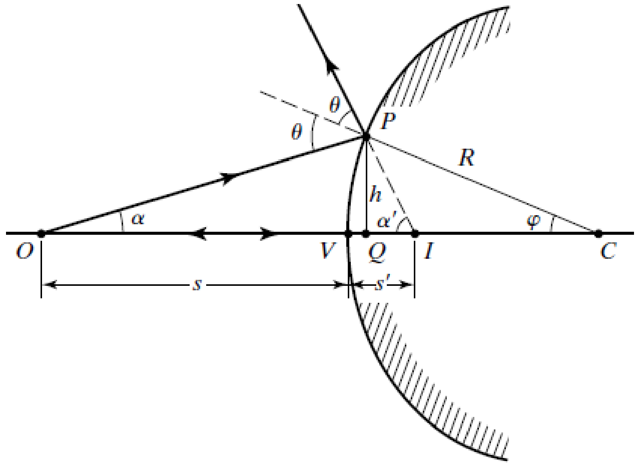


Concave Spherical Mirror

Magnification

$$m = \frac{h'}{h} = -\frac{s'}{s},$$

Reflection at a Spherical Mirror



$$\frac{1}{s} + \frac{1}{s'} = -\frac{2}{R}$$

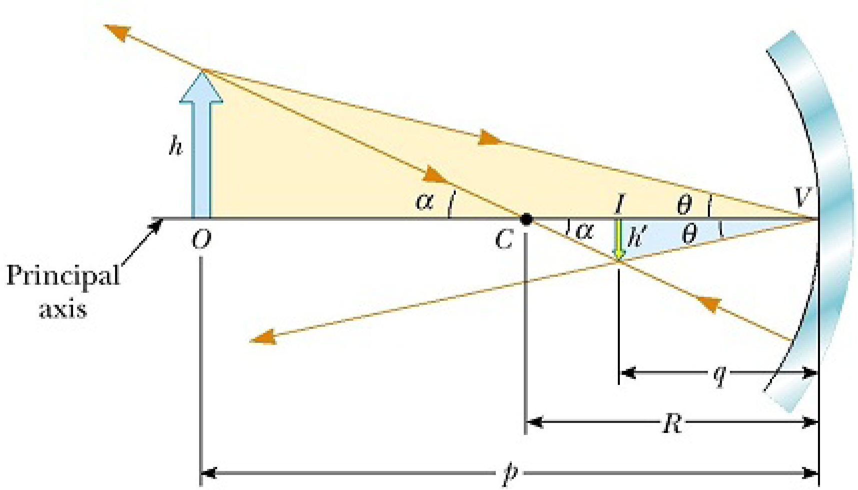
$$m = \frac{h'}{h} = -\frac{s'}{s},$$

Sign Conventions (light propagation from left to right)

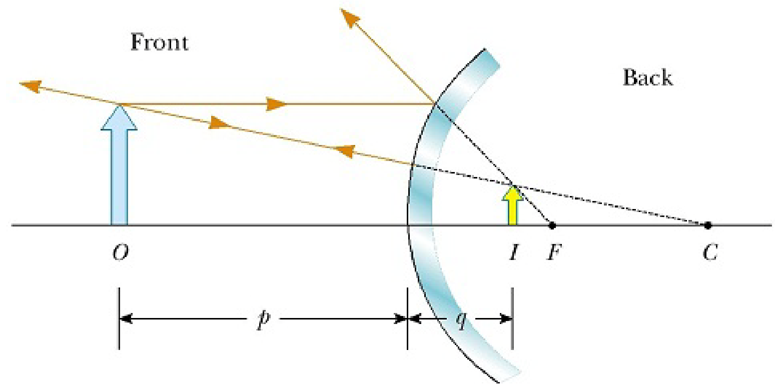
Spherical Mirrors (Light from Left-to-Right)		
R	s	s′
$R > 0$	$s > 0$	$s' > 0$
(convex)	real (left of V)	real (left of V)
$R < 0$	$s < 0$	$s' < 0$
(concave)	virtual (right of V)	virtual (right of V)
Magnification	$m > 0$ (upright)	$m < 0$ (inverted)
	$m = \frac{h'}{h} = -\frac{s'}{s}$	
Paraxial Equation	$\frac{1}{s} + \frac{1}{s'} = -\frac{2}{R} = \frac{1}{f}$	

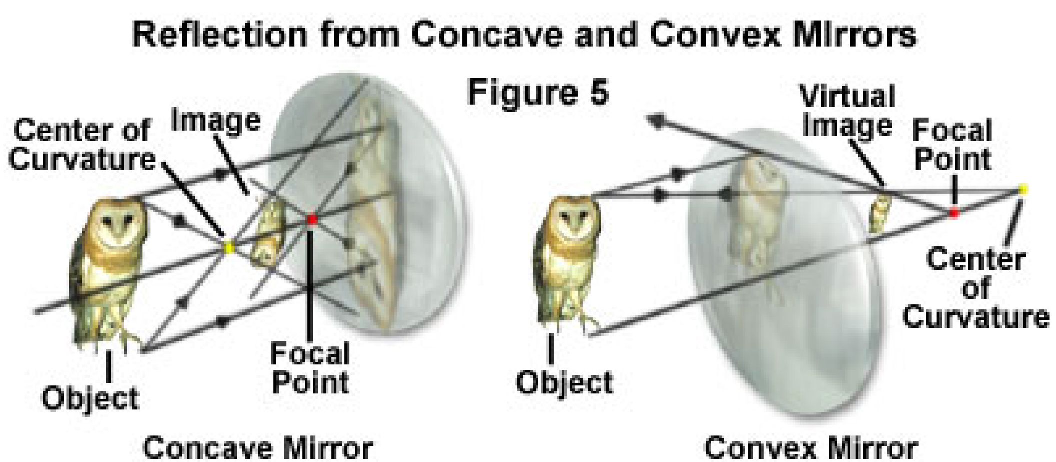
Image Formation by Spherical Mirrors

Concave



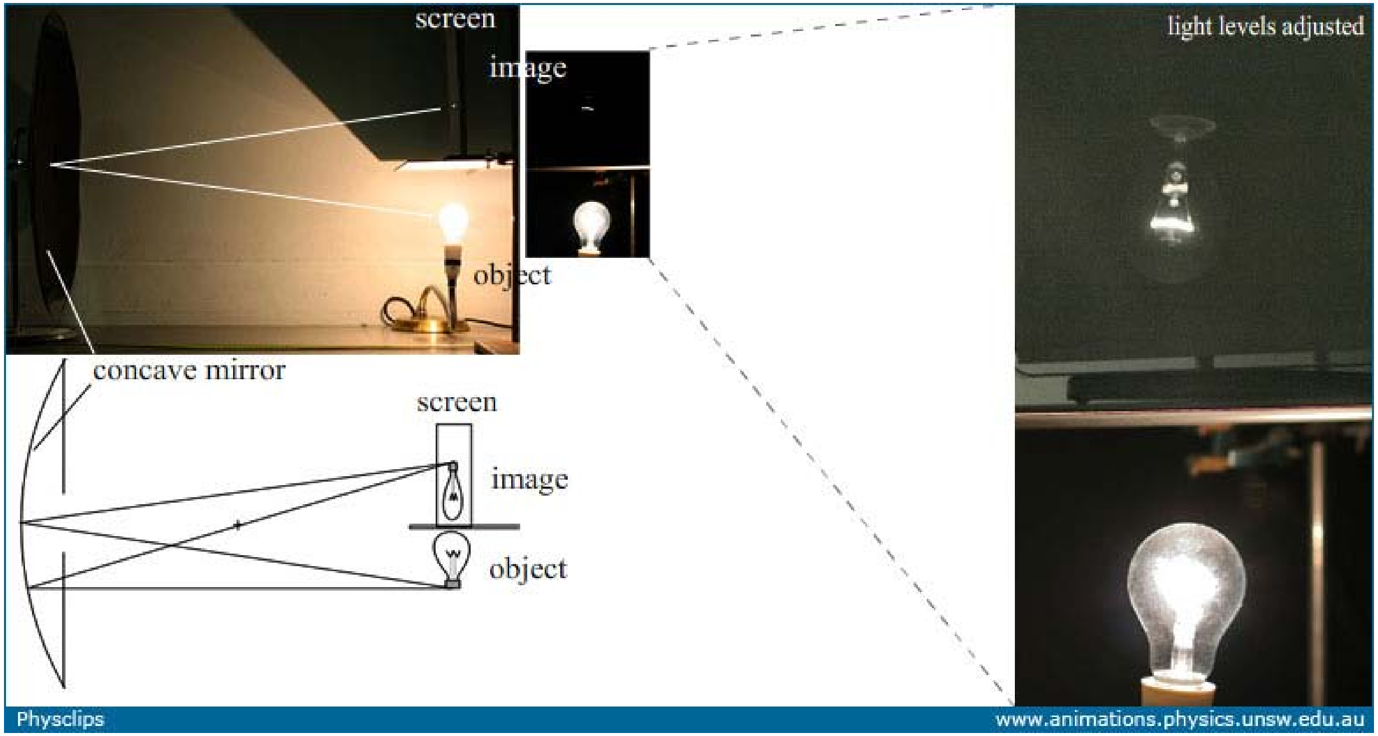
Convex





Real images in a concave mirror

A mirror can produce a real image, provided that it is a concave mirror. In this experiment, we use an incandescent lamp as the object, whose image we project onto a vertical white screen. There is a horizontal baffle between the lamp and the screen so that light from the lamp doesn't fall directly on the screen. Due to Aberrations, this cheap mirror is not a good approximation to a parabola, so using its whole area would produce a very distorted image. For that reason, we use a stop (a sheet of black paper) with a small hole to reduce the mirror area. The photo at top left shows a side view, and a schematic lies below. The middle photo was taken from above the mirror, looking towards the lamp and screen. A larger version of this photo is shown at right. In this version, the top half of the photo has been brightened, while the bottom half has been darkened, to show better the details of the lamp and to make it more obvious that the image is inverted. Note that rays of light really do meet at the position of this image, which is why we call it a **real image**.



An incandescent lamp is the object. Its (real) image is projected on a screen via a concave mirror.

http://www.animations.physics.unsw.edu.au/jw/light/mirrors-and-images.htm

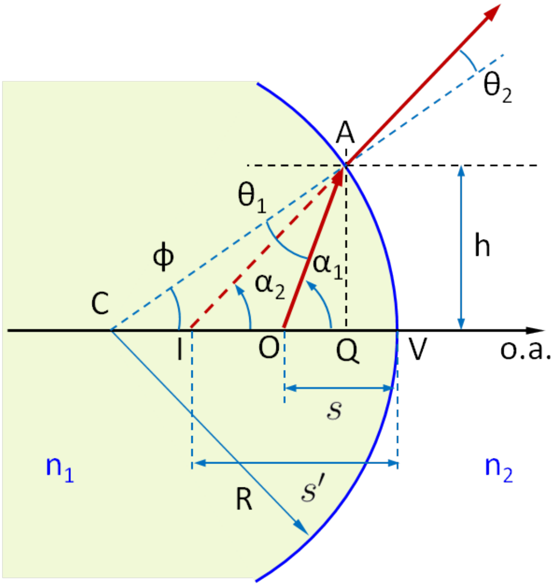
Spherical Mirrors

Convex | Concave

http://www.animations.physics.unsw.edu.au/jw/light/mirrors-and-images.htm

Convex Spherical Mirrors Applications

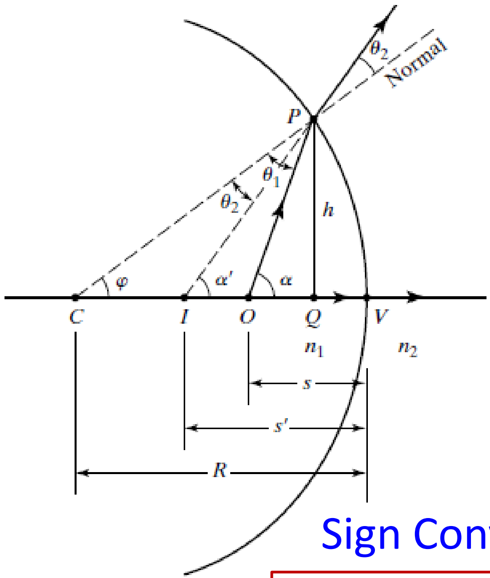
Refraction at a Spherical Interface



Paraxial Approximation: $\sin\theta \simeq \theta$ $\tan\theta \simeq \theta$ $\cos\theta \simeq 1$

$$
\begin{aligned}
n_1 \sin\theta_1 &= n_2 \sin\theta_2 \Longrightarrow n_1\theta_1 \simeq n_2\theta_2 \Longrightarrow \\
n_1(\alpha_1 - \phi) &= n_2(\alpha_2 - \phi) \Longrightarrow \\
n_1\left(\frac{h}{s} - \frac{h}{R}\right) &= n_2\left(\frac{h}{s'} - \frac{h}{R}\right) \Longrightarrow \\
\frac{n_1}{s} - \frac{n_2}{s'} &= \frac{n_1 - n_2}{R} \Longrightarrow \\
\frac{n_1}{s} + \frac{n_2}{s'} &= \frac{n_2 - n_1}{R}, \quad \text{(with sign convention)}
\end{aligned}
$$

Refraction at a Spherical Interface



$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

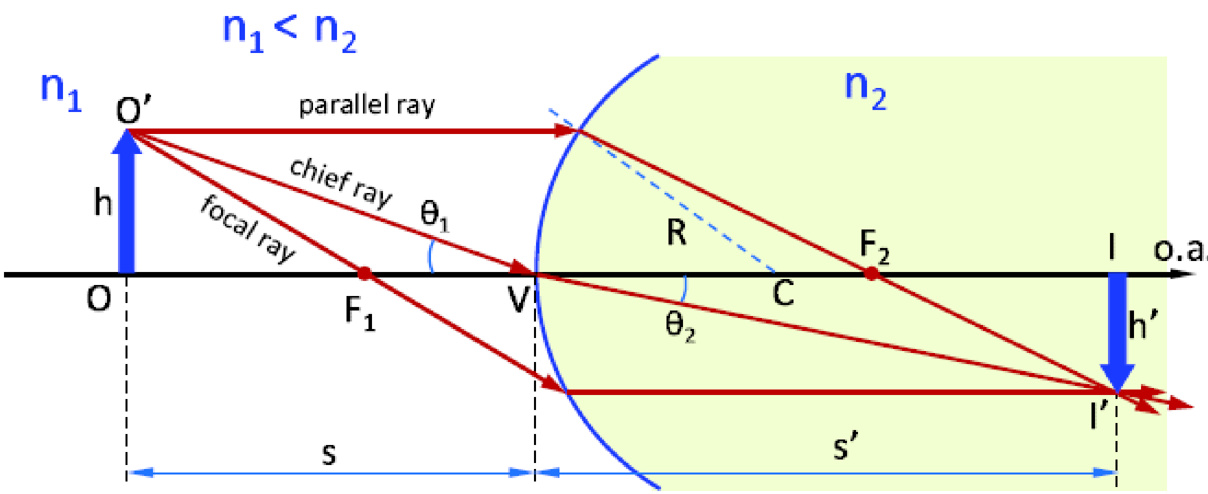
$$f_1 = \frac{n_1}{n_2 - n_1} R,$$

$$f_2 = \frac{n_2}{n_2 - n_1} R,$$

Sign Conventions (light propagation from left to right)

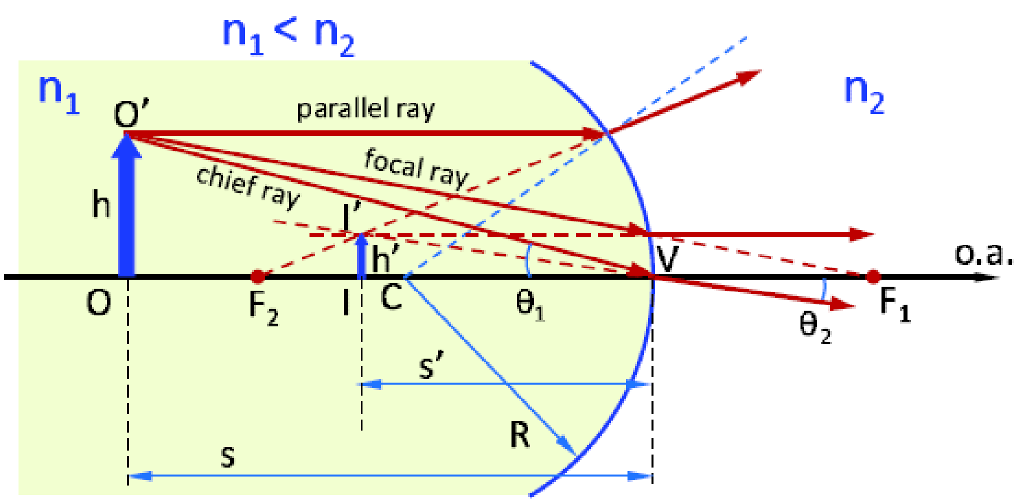
Spherical Refractive Surface (Light from Left-to-Right)		
R	s	s′
$R > 0$	$s > 0$	$s' > 0$
(convex)	real (left of V)	real (right of V)
$R < 0$	$s < 0$	$s' < 0$
(concave)	virtual (right of V)	virtual (left of V)
Magnification	$m > 0$ (upright)	$m < 0$ (inverted)
	$m = \frac{h'}{h} = -\frac{n_1}{n_2}\frac{s'}{s}$	
Paraxial Equation	$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$	

Refraction at a Spherical Interface



$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

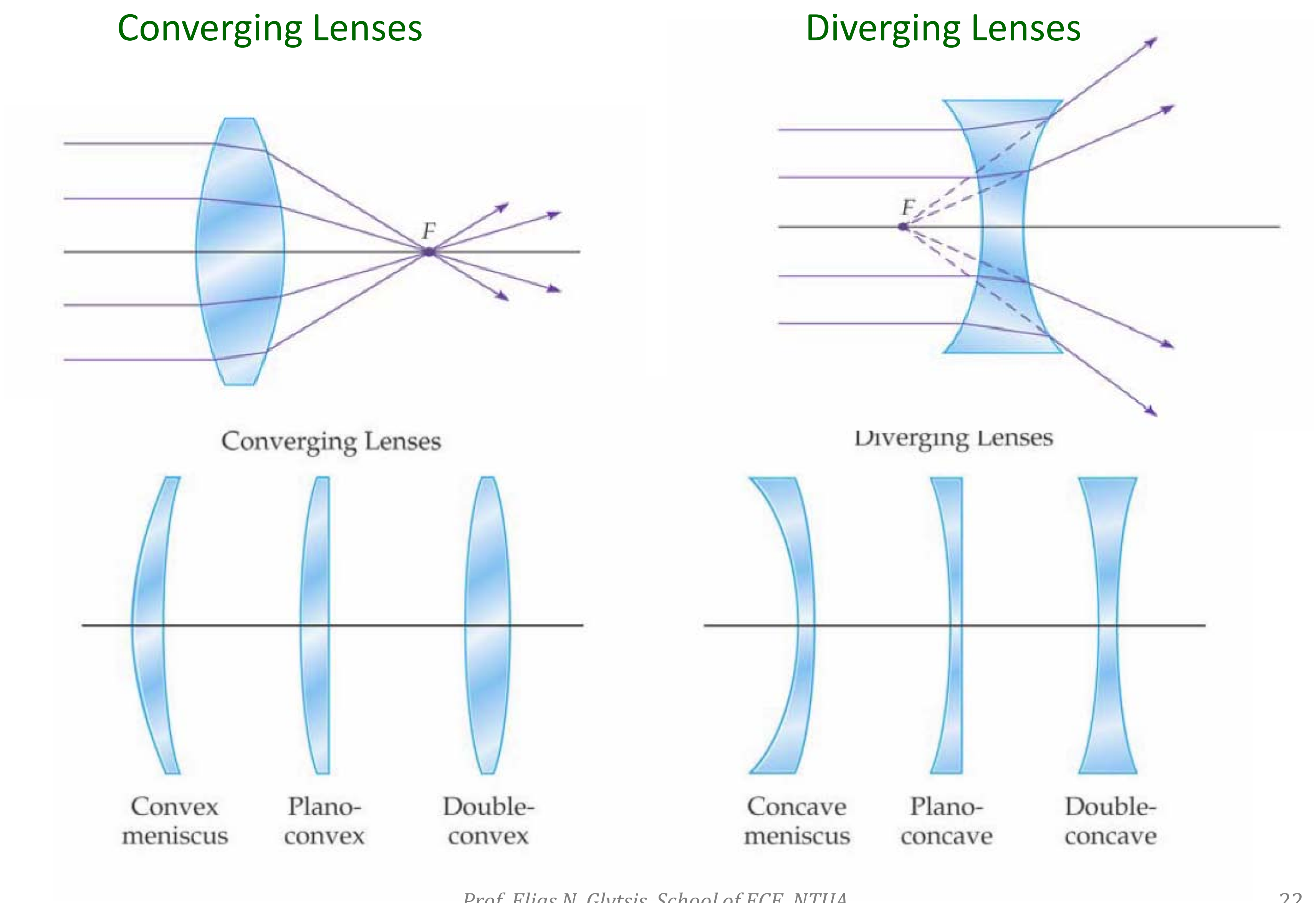
$$\begin{aligned} f_1 &= \frac{n_1}{n_2 - n_1} R, \\ f_2 &= \frac{n_2}{n_2 - n_1} R, \end{aligned}$$



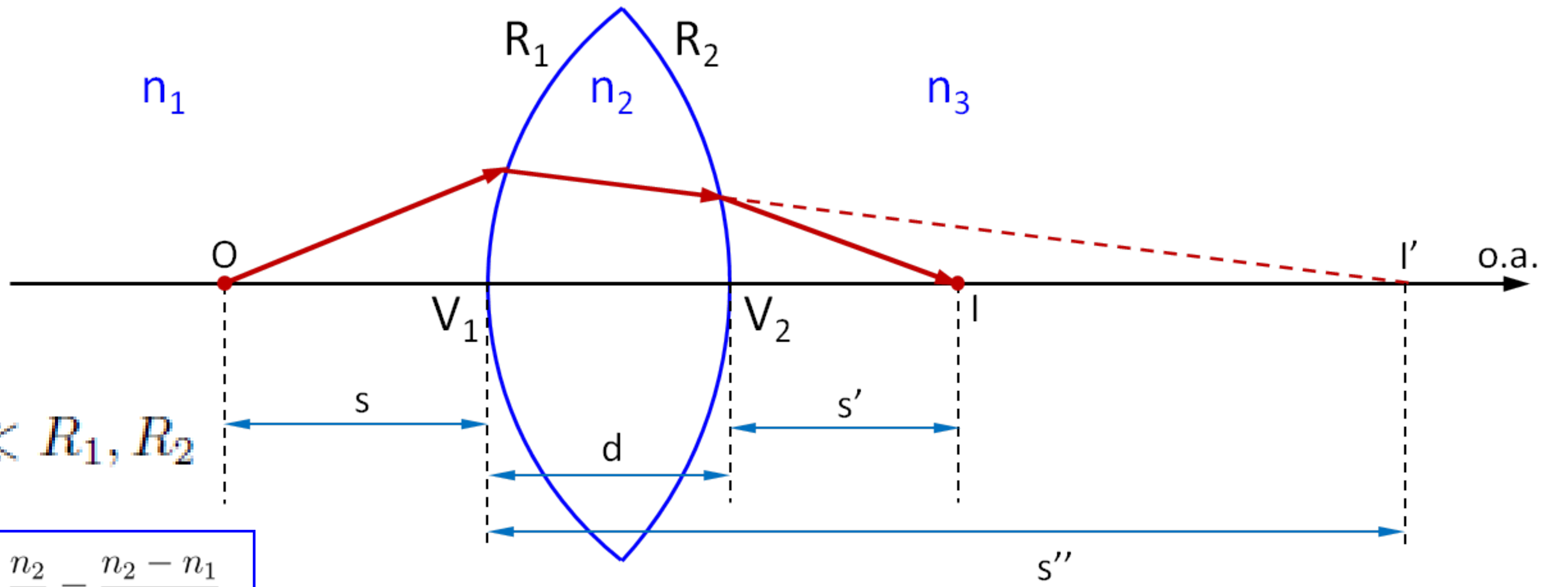
$$\begin{aligned} n_1 \sin\theta_1 &= n_2 \sin\theta_2 \Longrightarrow n_1\theta_1 \simeq n_2\theta_2 \Longrightarrow \\ n_1 \frac{h}{s} &= n_2 \frac{h'}{s'} \Longrightarrow \end{aligned}$$

$$m = \frac{h'}{h} = -\frac{n_1}{n_2}\frac{s'}{s},$$

Lenses Types



Thin Lens Equation (derivation)



$d \ll R_1, R_2$

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$$

$$\frac{n_1}{s} + \frac{n_2}{s''} = \frac{n_2 - n_1}{R_1},$$

$$\frac{n_2}{-(s'' - d)} + \frac{n_3}{s'} = \frac{n_3 - n_2}{R_2} \Longrightarrow$$

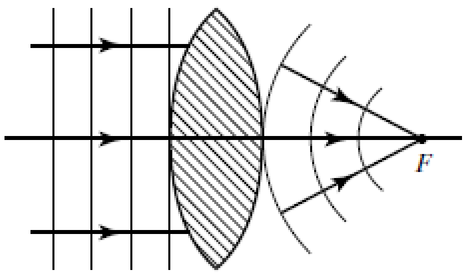
$$\frac{n_1}{s} + \frac{n_3}{s'} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2},$$

If $n_1 = n_3$

$$\frac{1}{s} + \frac{1}{s'} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f}$$

Thin Lens Equation

Conventional Converging Lens

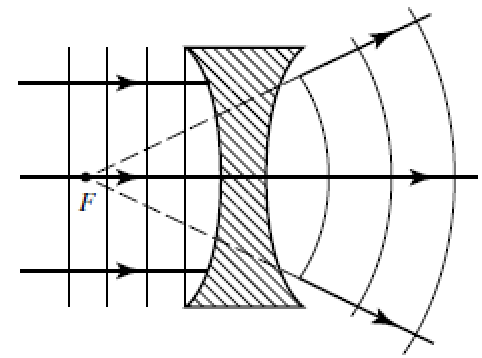


Surrounding medium
of the same index n_1

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Conventional Diverging Lens



Surrounding medium
of the different index n_1 (left) and n_3 (right)

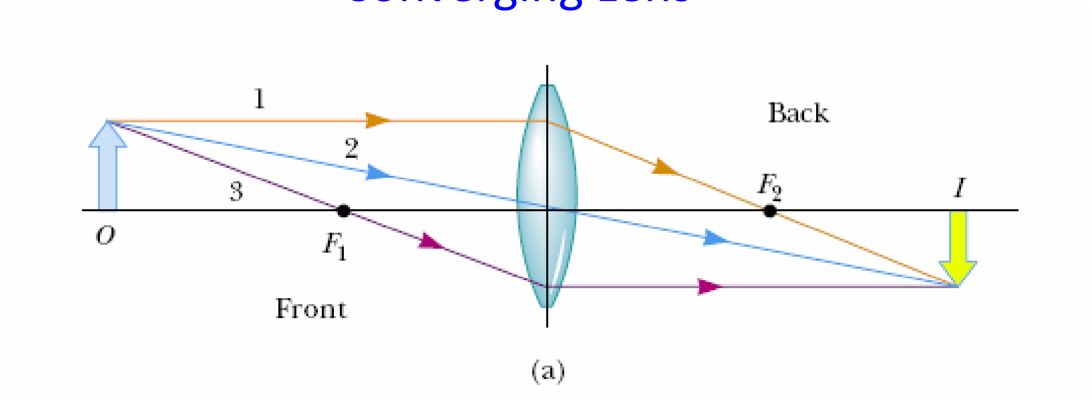
$$\frac{n_1}{s} + \frac{n_3}{s'} = \frac{n_2 - n_1}{R_1} - \frac{n_2 - n_3}{R_2}$$

$$\frac{1}{f_1} = \frac{1}{n_1}\left(\frac{n_2 - n_1}{R_1} - \frac{n_2 - n_3}{R_2}\right)$$

$$\frac{1}{f_2} = \frac{1}{n_3}\left(\frac{n_2 - n_1}{R_1} - \frac{n_2 - n_3}{R_2}\right)$$

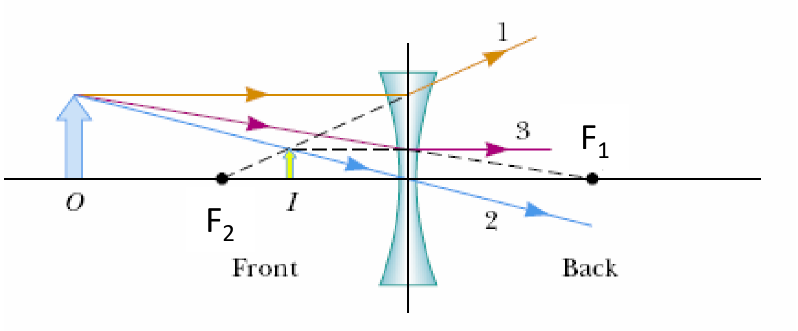
Thin Lenses

Converging Lens

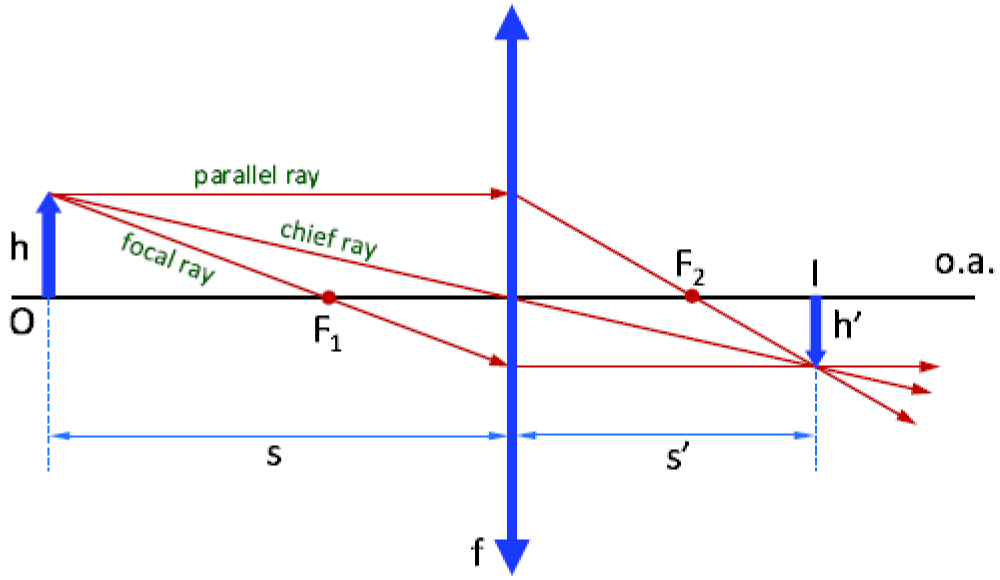


1: Parallel Ray
2: Chief Ray
3: Focal Ray

Diverging Lens



Thin Lens Imaging (positive lens)



s > 2f

Inverted

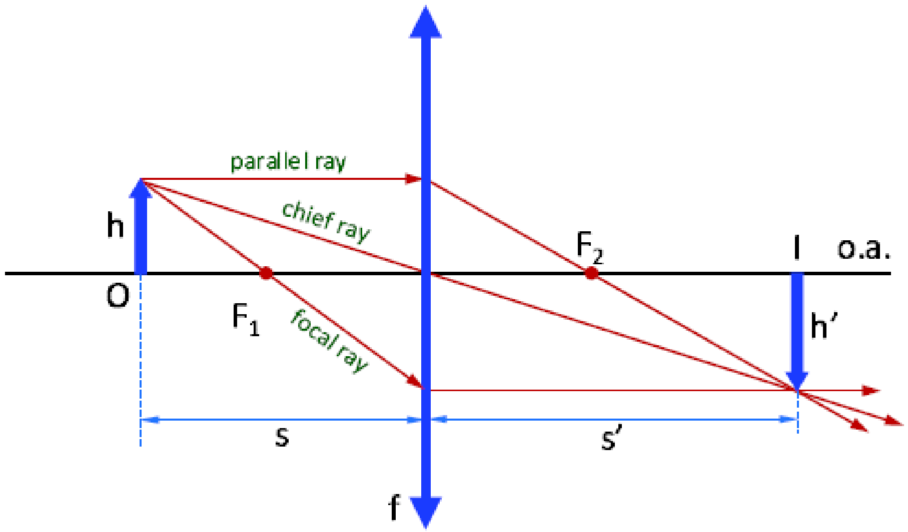
Reduced

Real

Magnification

$$m = \frac{h'}{h} = -\frac{n_1}{n_3}\frac{s'}{s}$$

Thin Lens Imaging (positive lens)

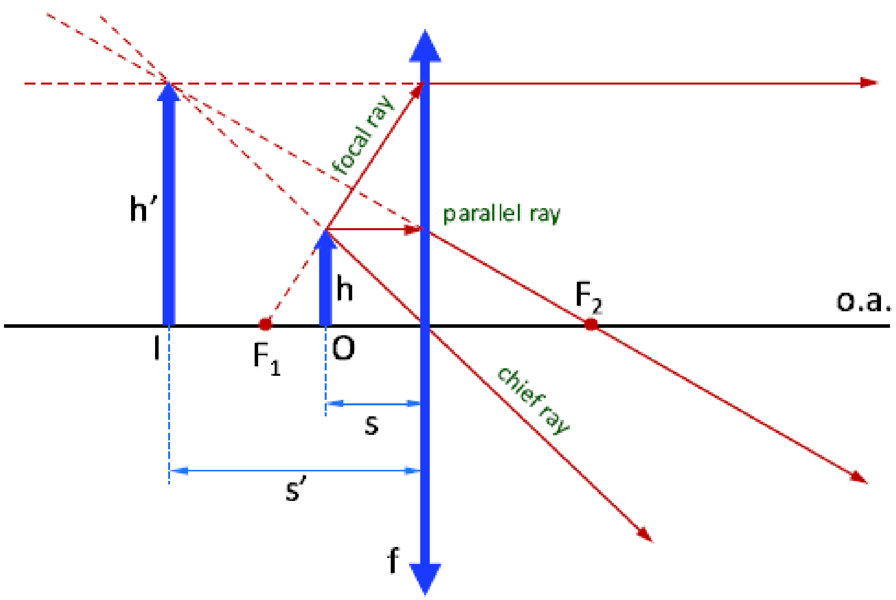


f < s < 2f

Inverted

Enlarged

Real



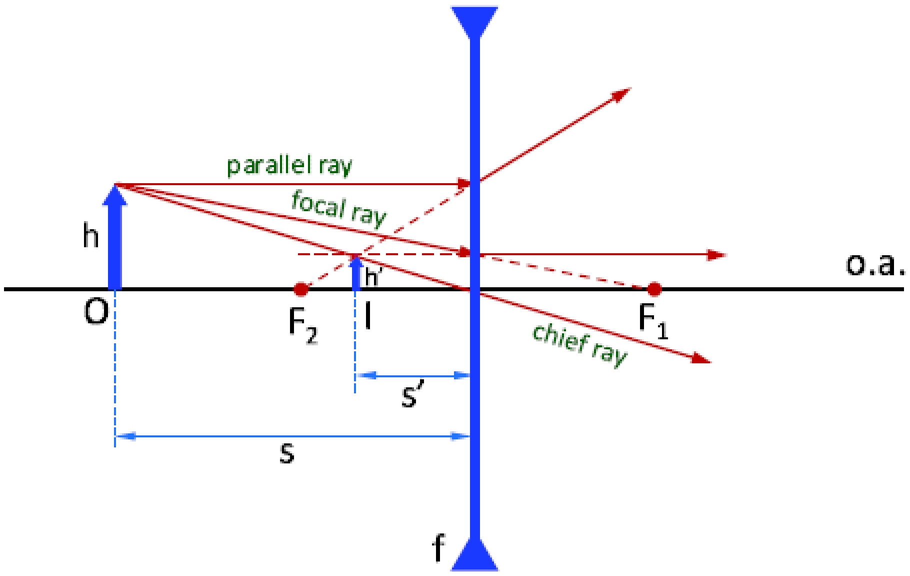
0 < s < f

Upright

Enlarged

Virtual

Thin Lens Imaging (negative lens)



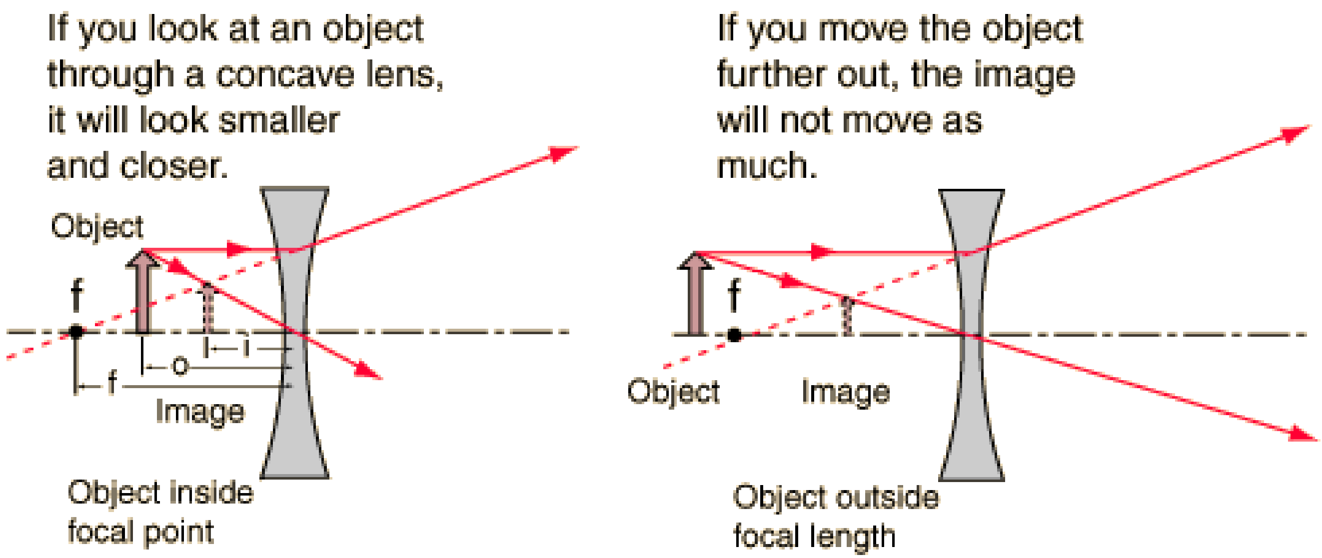
Upright

Reduced

Virtual

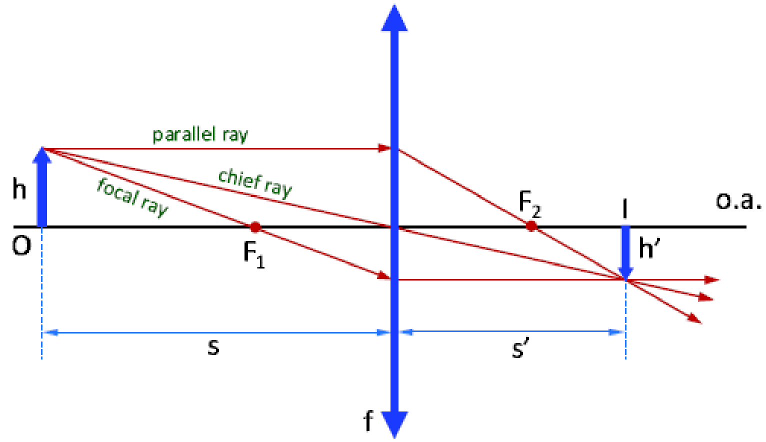
Image is always virtual, upright, and reduced

Negative Thin Lens



http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/raydiag.html

Thin Lens Imaging



Thin Lens Sign Conventions (Light from Left-to-Right)

$i = 1, 2$ $\quad R_i > 0$ (convex) $\quad R_i < 0$ (concave)

$s > 0$ (real object - left of V), $s < 0$ (virtual object - right of V)

$s' > 0$ (real image - right of V), $s' < 0$ (virtual image - left of V)

$f > 0$ (converging/positive lens), $f < 0$ (diverging/negative lens)

Thin-Lens Equation
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = \frac{n_2 - n_1}{n_1}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

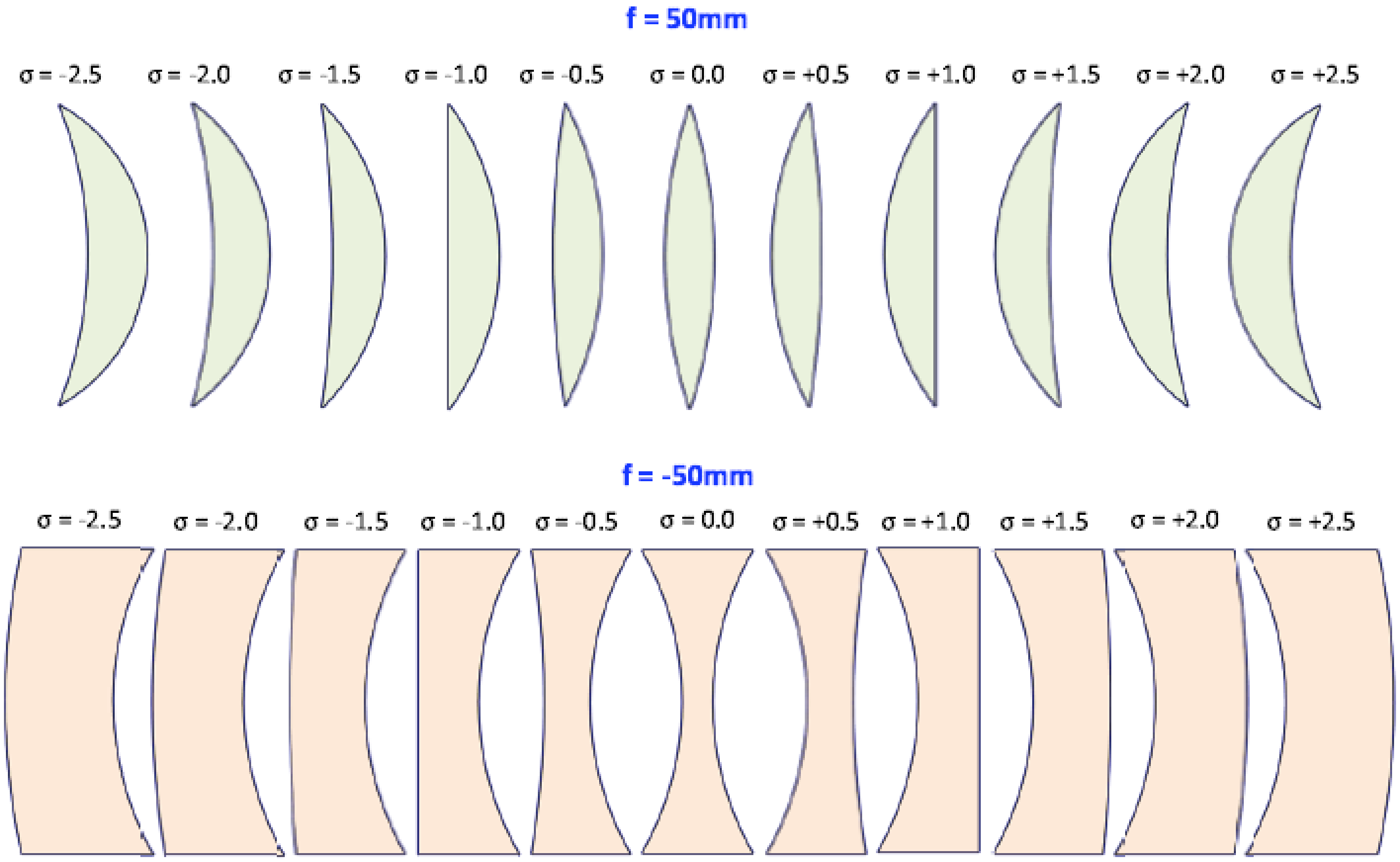
Magnification
$$m = \frac{h'}{h} = -\frac{s'}{s}$$

Longitudinal Magnification
$$m_L = \frac{\Delta s'}{\Delta s} = -\frac{s'^2}{s^2} = -m^2$$

Thin Lens Imaging

Positive Lenses						
Lens Type	**R_1**	**R_2**	**s**	**s′**	**m**	**f**
Biconvex	$+$	$-$	$+$	$\pm$	$\mp$	$+$
Plano-Convex	∞	$-$	$+$	$\pm$	$\mp$	$+$
Convex-Plano	$+$	∞	$+$	$\pm$	$\mp$	$+$
Positive Meniscus	$+$	$+$	$+$	$\pm$	$\mp$	$+$
$(R_1 < R_2)$						
Comment				s′	m	
if $s > 2f$				$+$	$-$	
if $f < s < 2f$				$+$	$-$	
if $0 < s < f$				$-$	$+$	
Negative Lenses						
Lens Type	**R_1**	**R_2**	**s**	**s′**	**m**	**f**
Biconcave	$-$	$+$	$+$	$-$	$+$	$-$
Plano-Concave	∞	$+$	$+$	$-$	$+$	$-$
Concave-Plano	$-$	∞	$+$	$-$	$+$	$-$
Negative Meniscus	$-$	$-$	$+$	$-$	$+$	$-$
$(\lvert R_1\rvert < \lvert R_2\rvert)$						

Example Lenses



Example Imaging with Convex Lenses

Image by a convex lens for object placed at different distance from it

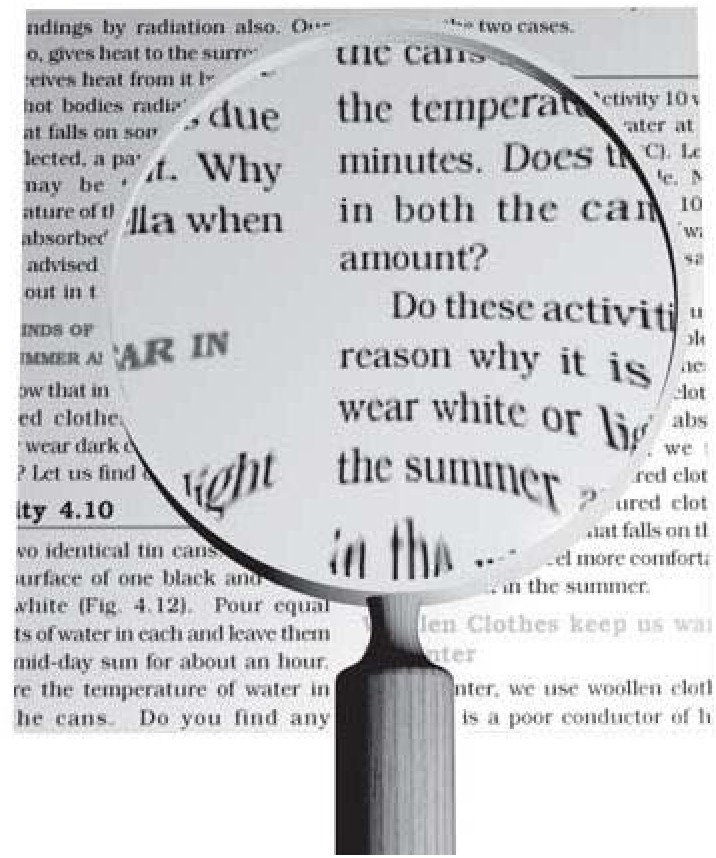
http://www.ekshiksha.org.in/eContent-Show.do?documentId=56

Example Imaging with Convex Lenses

Virtual image formed by the convex lens

http://www.ekshiksha.org.in/eContent-Show.do?documentId=56

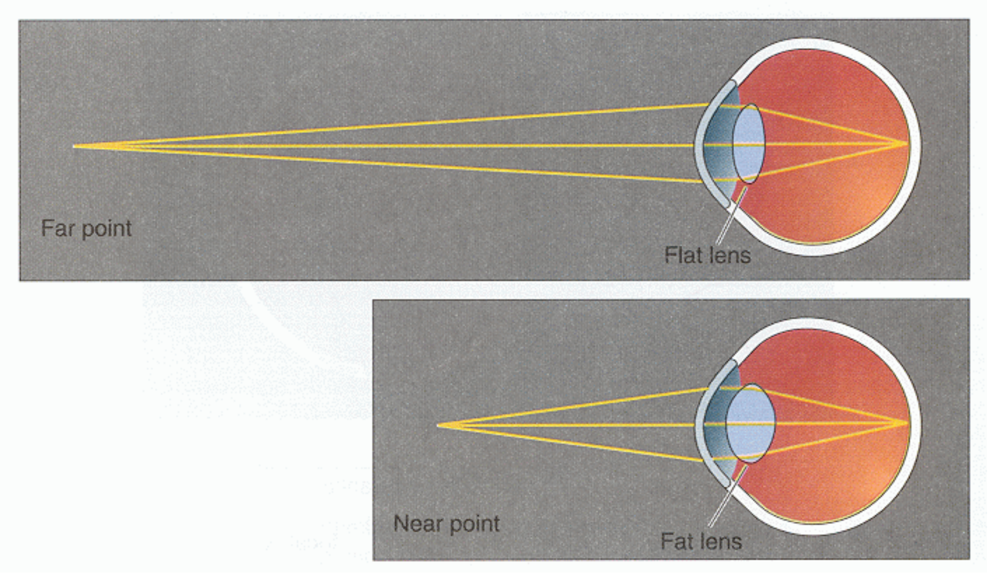
Magnifying Glass

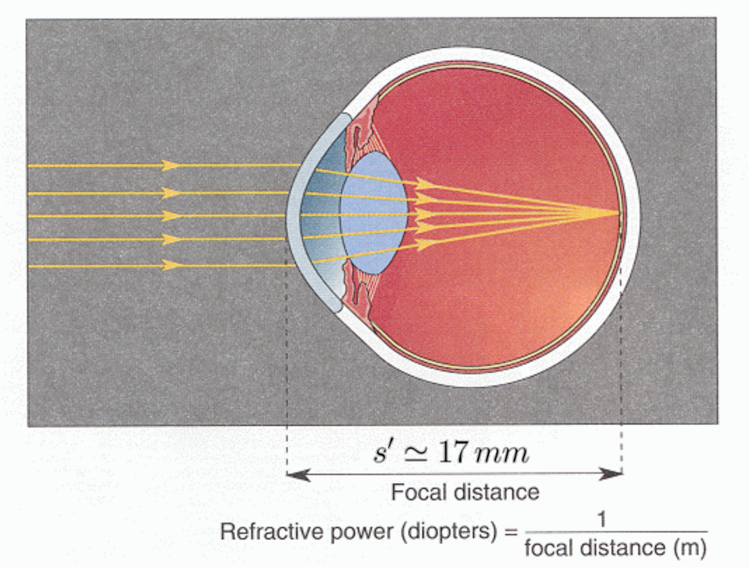


Example Imaging with Concave Lenses

http://philschatz.com/physics-book/contents/m42470.html

Optics of the Human Eye

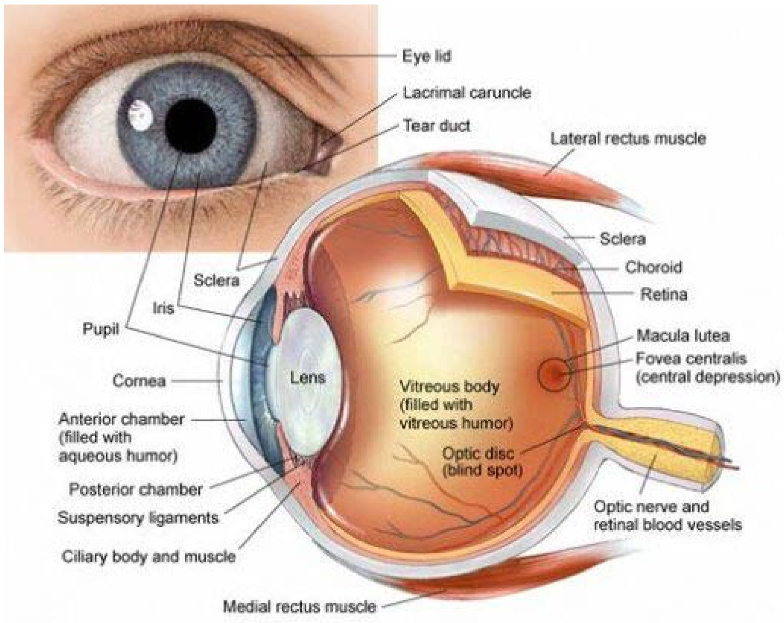




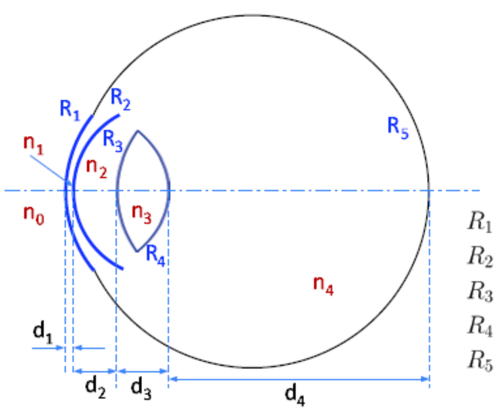
http://fourier.eng.hmc.edu/e180/lectures/eye/node5.html

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Longrightarrow f = \frac{s's}{s+s'} \qquad s' \simeq 17\,mm$$

Optics of the Human Eye



Human eye optical parameters



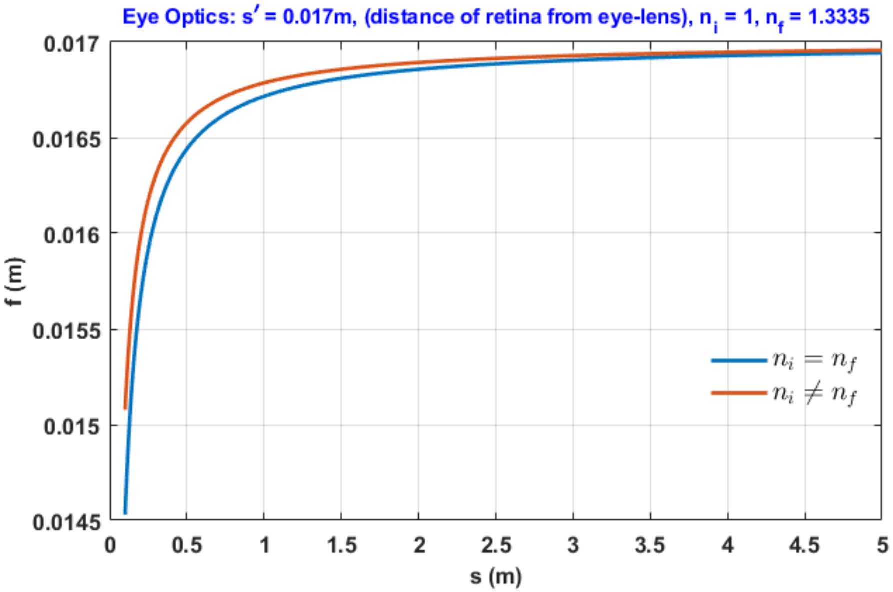
$$
\begin{aligned}
&R_1 = 7.8\,mm, && n_1 = 1.3771, && d_1 = 0.55\,mm,\\
&R_2 = 6.5\,mm, && n_2 = 1.3374, && d_2 = 3.05\,mm,\\
&R_3 = 10.2\,mm, && n_3 = 1.42, && d_3 = 4\,mm,\\
&R_4 = -6.0\,mm, && n_4 = 1.336, && d_4 = 16.6\,mm,\\
&R_5 = -13.4\,mm. && &&
\end{aligned}
$$

The relaxed eye has an approximate optical power of **60 diopters (D)** (ie, its focal length is 16.7 mm in air), with the corneal power being about 40 D, or two thirds of the total power. Due to orderly arrangement of collagen fibrils in the cornea, it is highly transparent with transmission above 95% in the spectral range of 400-900 nm. **The refractive index of the cornea is n≈1.3765±0.0005**. The amount of light reaching the retina is regulated by the pupil size, which can vary between 1.5 mm and 8 mm. The anterior chamber of the eye, which is located between the cornea and lens capsule, is filled with a clear liquid—**the aqueous humor having a refractive index n≈1.3335**. The crystalline lens of the eye, located behind the iris, is composed of specialized crystalline proteins with refractive index of **n=1.40-1.42**. The lens is about 4 mm in thickness and 10 mm in diameter and is enclosed in a tough, thin (5-15 mm), transparent collagenous capsule. In the relaxed eye, the lens has a power of about 20 D, while in the fully accommodated state, it can temporarily increase to 33 D. The **vitreous humor**—a transparent jelly-like substance filling the large cavity posterior to the lens and anterior to the retina—has a **refractive index n≈1.335.**

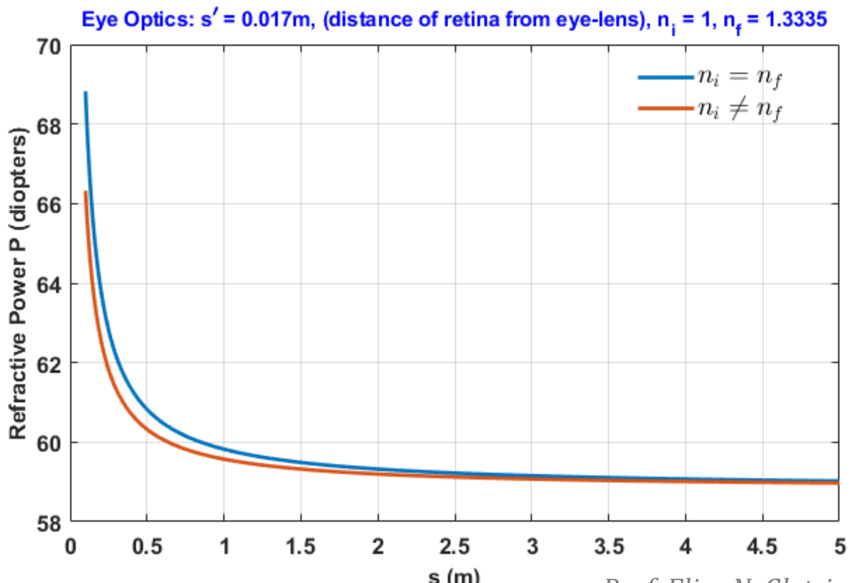
https://www.aao.org/munnerlyn-laser-surgery-center/optical-properties-of-eye

P. Artal, "Optics of the eye and its impact in vision: a tutorial," Adv. Opt. Photon., vol. 6, pp. 340–367, Sep 2014.

Optics of the Human Eye



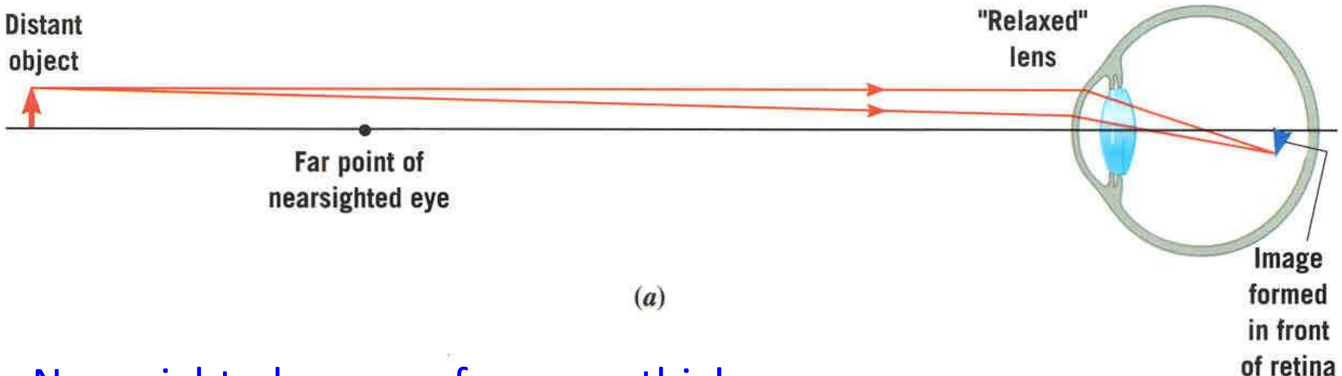
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Longrightarrow f = \frac{s's}{s+s'} \qquad s' \simeq 17\,mm$$



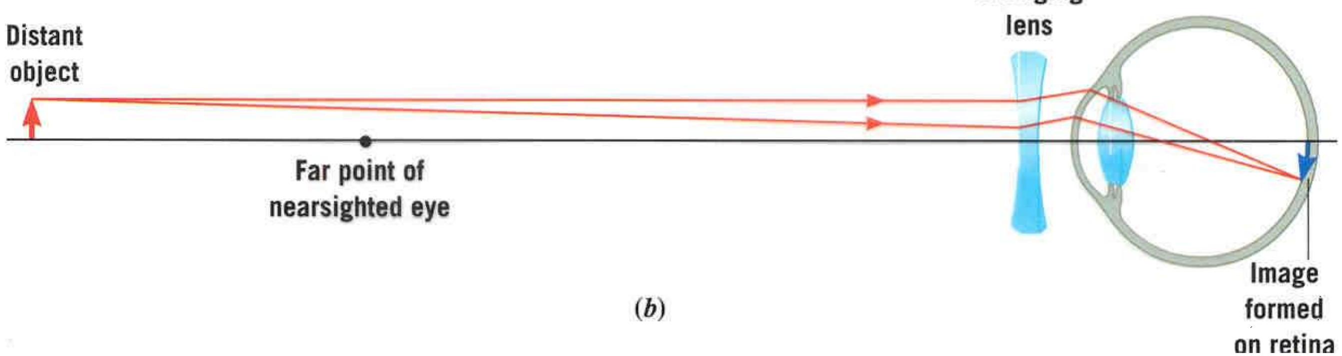
$$\frac{n_i}{s} + \frac{n_f}{s'} = \frac{n_f}{f_2} \Longrightarrow f_2 = \frac{s's}{(n_i/n_f)s+s'} \qquad s' \simeq 17\,mm$$

Near-Sighted Eye (Myopia) - Correction

Too far for near-sighted eye to focus

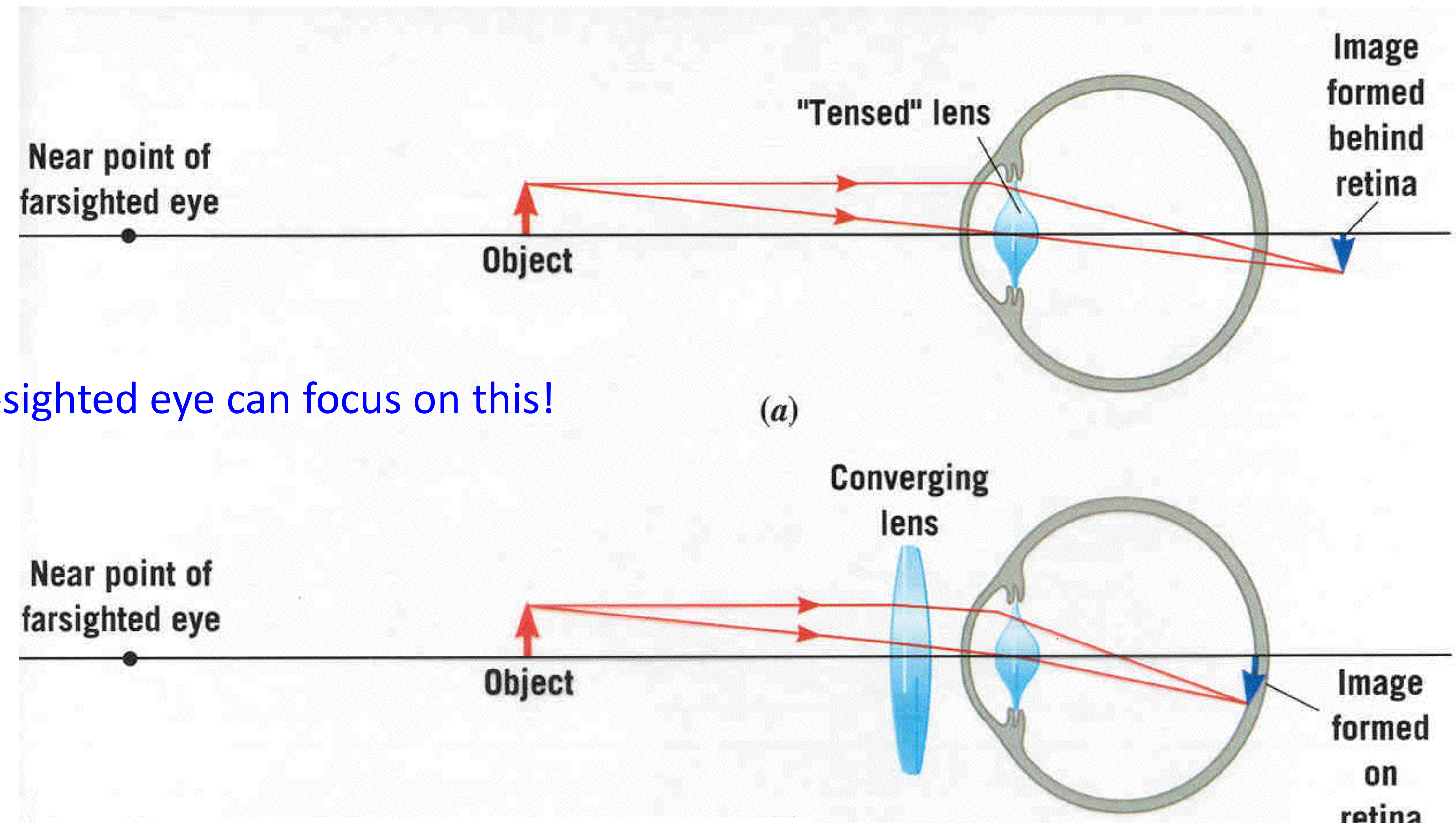


Near-sighted eye can focus on this!



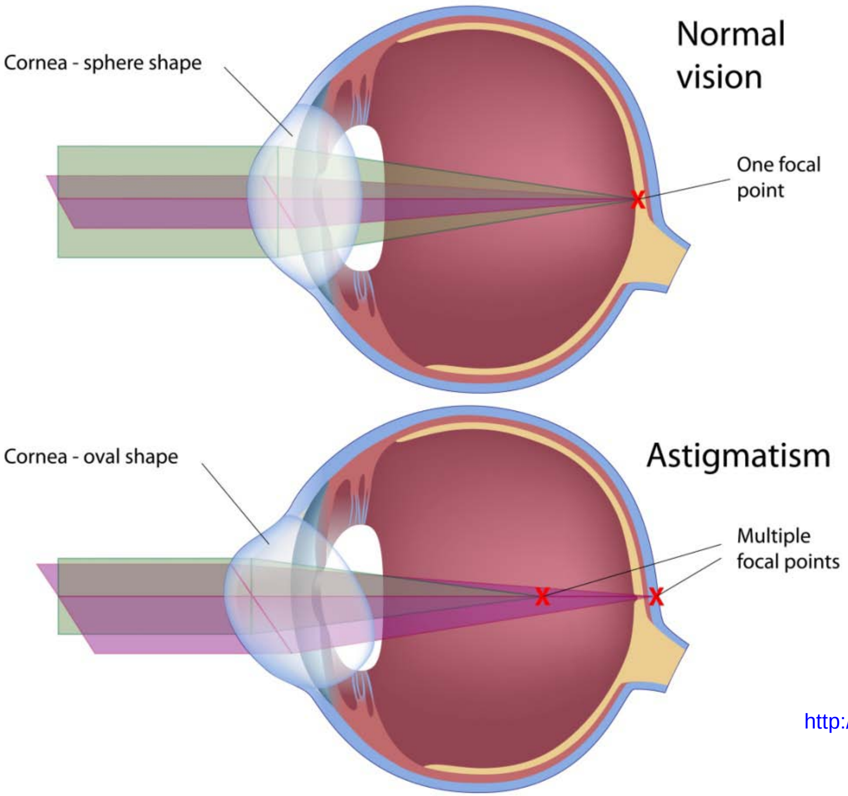
Far-Sighted Eye (Hyperopia/Presbyopia) - Correction

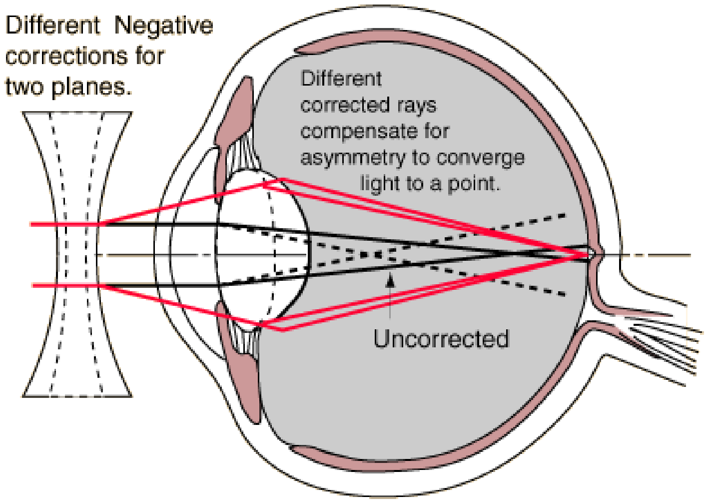
Too close for far-sighted eye to focus



Far-sighted eye can focus on this!

Astigmatic Eye - Correction

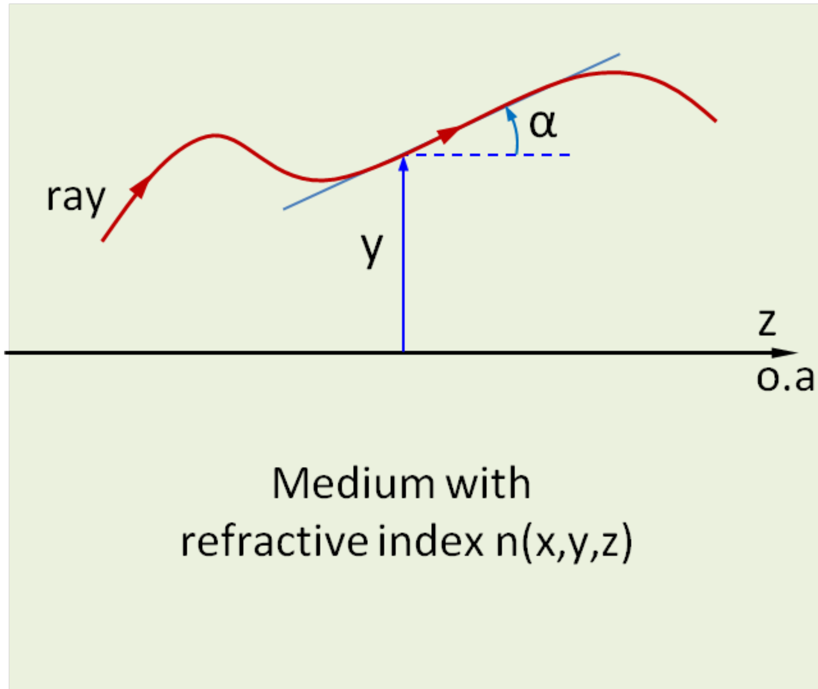




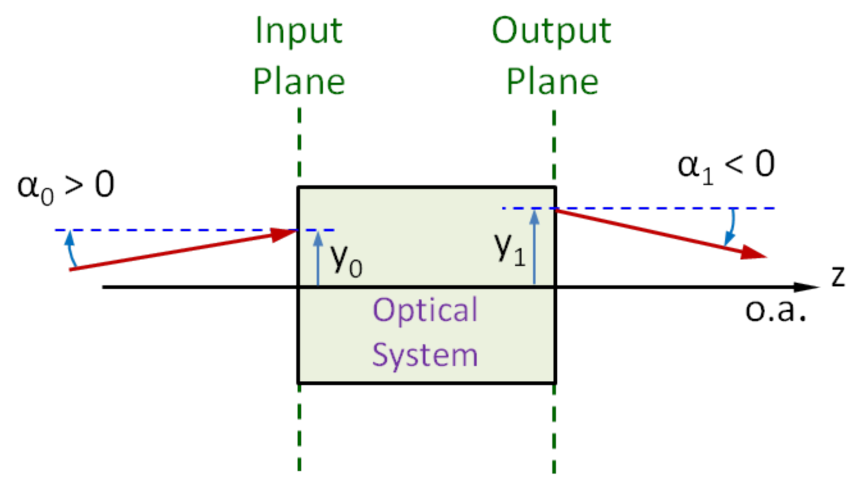
http://hydrogen.physik.uni-wuppertal.de/hyperphysics/hyperphysics/hbase/vision/imgvis/astig.gif

http://www.visionexcellence.com.au/WP1/wp-content/uploads/2013/05/astigmatism.jpg

Matrix Approach for Paraxial Rays



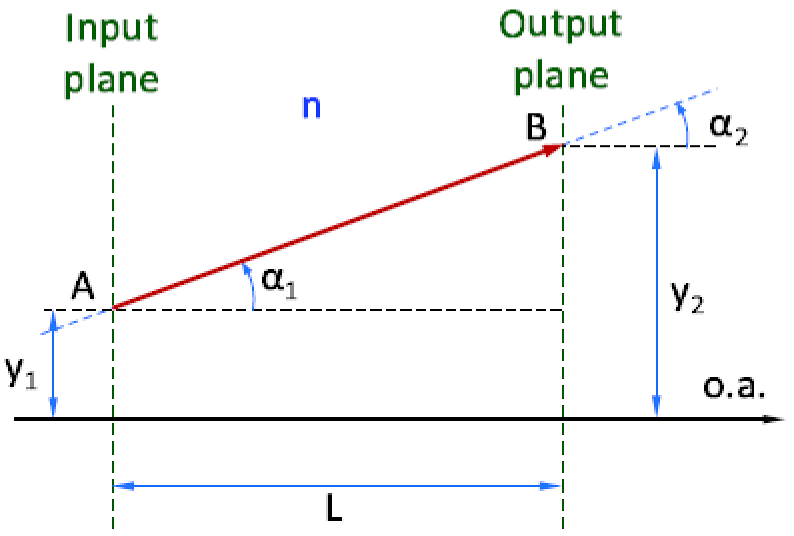
Ray description via y and α = dy/dz



$$\begin{bmatrix} y_1 \\ \alpha_1 = \dfrac{dy_1}{dz} = y_1' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_0 \\ \alpha_0 = \dfrac{dy_0}{dz} = y_0' \end{bmatrix}$$

Elementary ABCD Matrices

Translation Matrix



$$y_2 = y_1 + L\tan\alpha_1 \simeq y_1 + L\alpha_1,$$

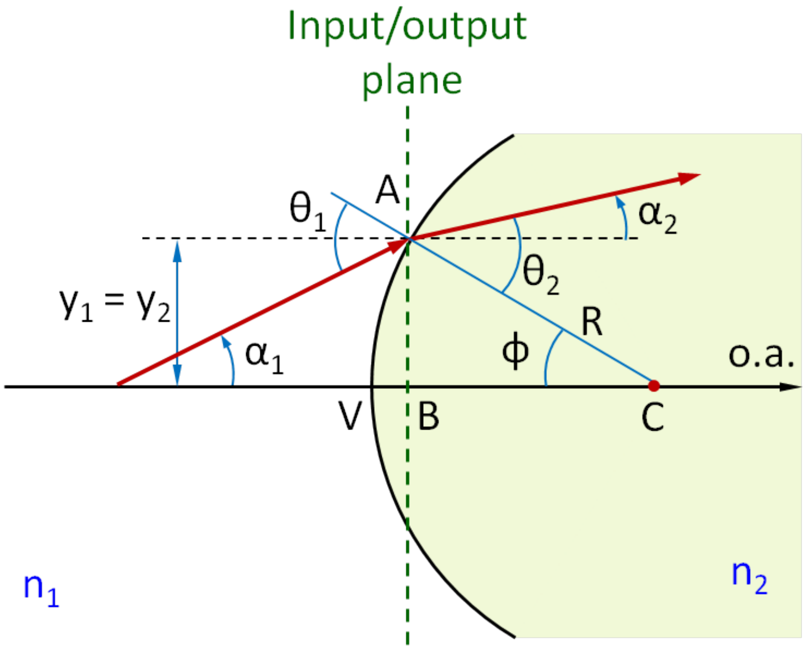
$$\alpha_2 = \alpha_1, \quad \text{and the resulting } ABCD \text{ matrix is}$$

$$\tilde{M} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}.$$

Paraxial Approximation: $\sin\theta \simeq \theta$ $\tan\theta \simeq \theta$ $\cos\theta \simeq 1$

Elementary ABCD Matrices

Spherical Refraction Matrix

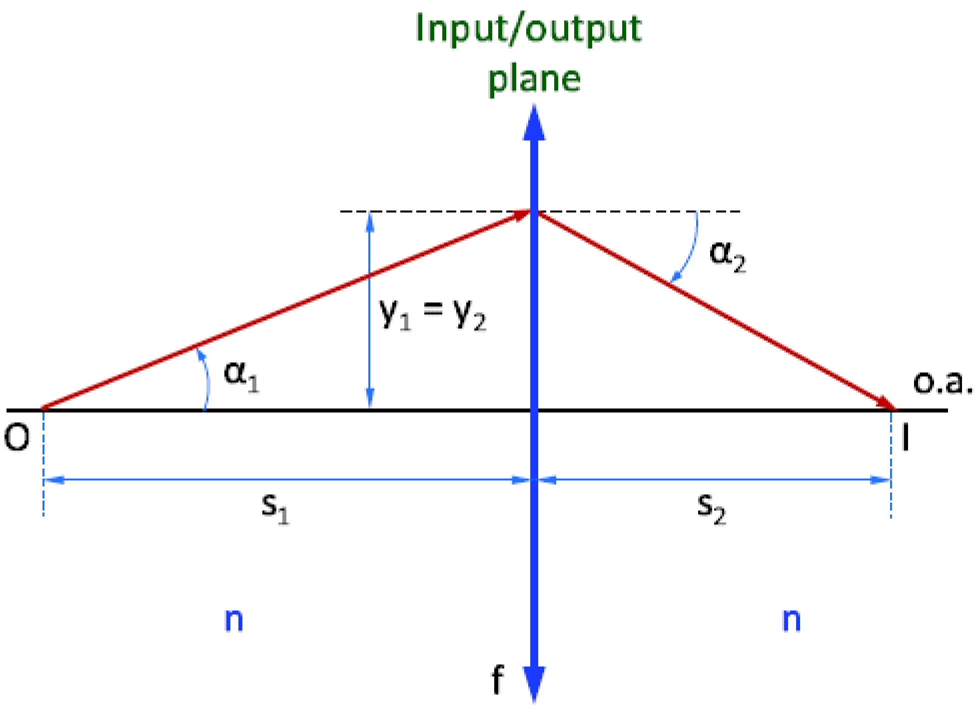


$$
\begin{aligned}
y_2 &= y_1 \\
n_1 \sin\theta_1 &= n_2 \sin\theta_2 \Longrightarrow n_1\theta_1 \simeq n_2\theta_2 \Longrightarrow \\
n_1(\alpha_1 + \phi) &= n_2(\alpha_2 + \phi) \Longrightarrow \\
\alpha_2 &= \left(\frac{n_1}{n_2} - 1\right)\phi + \frac{n_1}{n_2}\alpha_1 \Longrightarrow \\
\alpha_2 &= \left(\frac{n_1}{n_2} - 1\right)\frac{y_1}{R} + \frac{n_1}{n_2}\alpha_1 \qquad \text{resulting in} \\
\tilde{M} &= \begin{bmatrix} 1 & 0 \\ \left(\frac{n_1}{n_2} - 1\right)\frac{1}{R} & \frac{n_1}{n_2} \end{bmatrix}.
\end{aligned}
$$

Paraxial Approximation: $\sin\theta \simeq \theta$ $\quad \tan\theta \simeq \theta$ $\quad \cos\theta \simeq 1$

Elementary ABCD Matrices

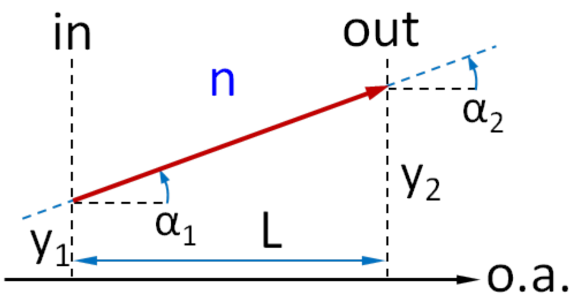
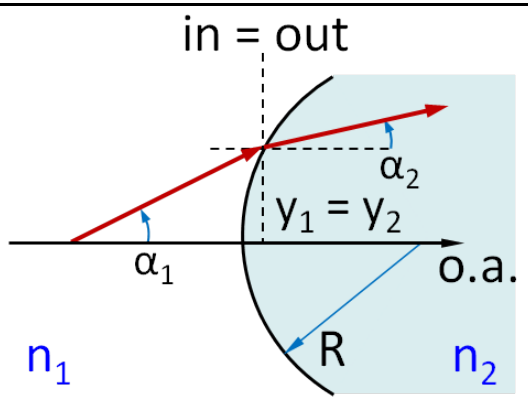
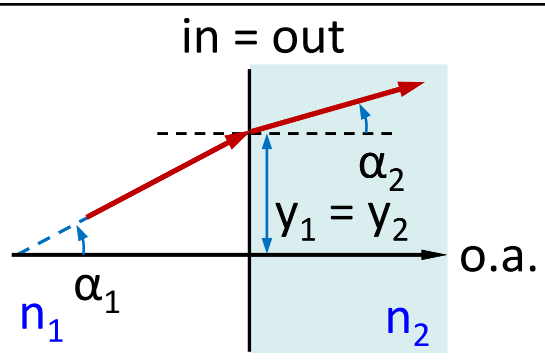
Thin Lens Matrix



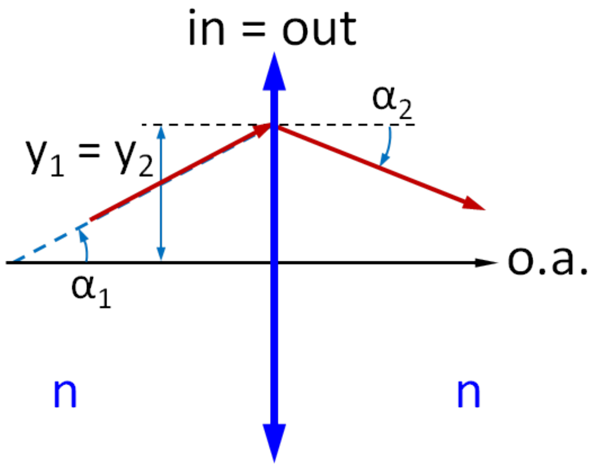
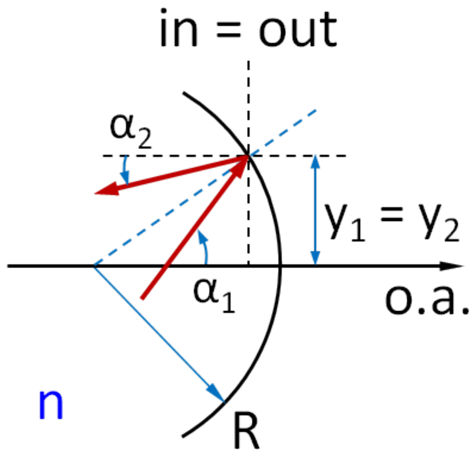
$$
\begin{aligned}
y_2 &= y_1 \\
|\alpha_2| &\simeq \frac{|y_1|}{s_2} = |y_1|\left(\frac{1}{f} - \frac{1}{s_1}\right) = \\
& \frac{|y_1|}{f} - \frac{|y_1|}{s_1} = \frac{|y_1|}{f} - \alpha_1 \Longrightarrow \\
\alpha_2 &= -\frac{y_1}{f} + \alpha_1, \qquad \text{resulting in} \\
\tilde{M} &= \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}.
\end{aligned}
$$

Paraxial Approximation:	$\sin\theta \simeq \theta$	$\tan\theta \simeq \theta$	$\cos\theta \simeq 1$

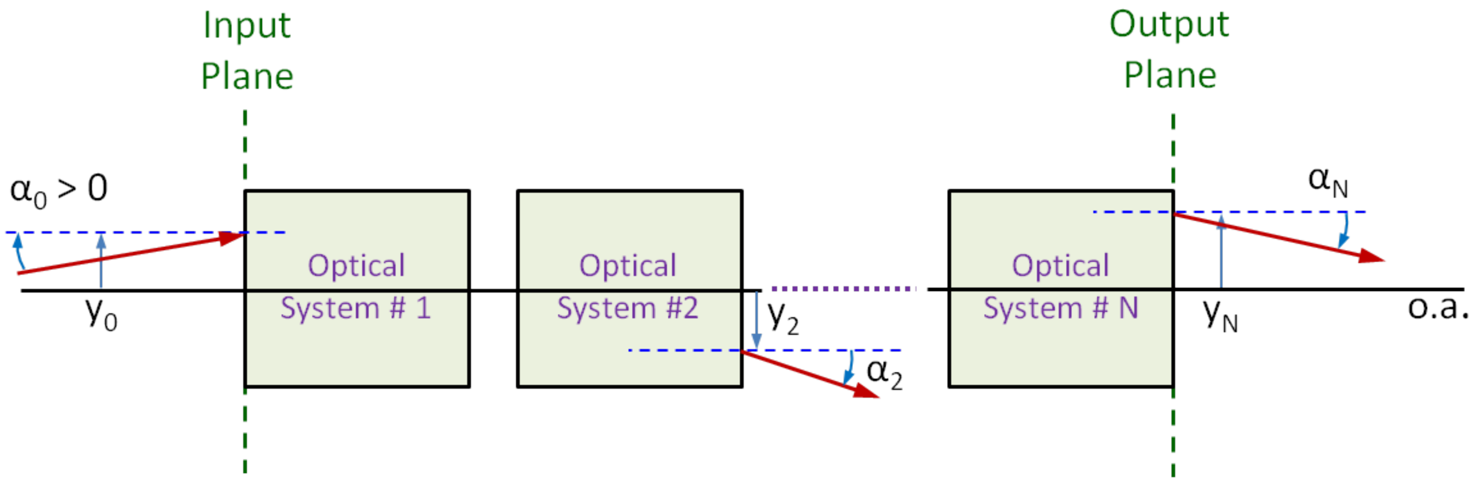
Elementary ABCD Matrices

Configuration	ABCD Matrix
	Translation Matrix $$M = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$$
	Spherical Refraction Matrix $$M = \begin{bmatrix} 1 & 0 \\ \left(\frac{n_1}{n_2} - 1\right)\frac{1}{R} & \frac{n_1}{n_2} \end{bmatrix}$$
	Planar Refraction Matrix $$M = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

Elementary ABCD Matrices

Configuration	ABCD Matrix
	Thin-Lens Matrix $M = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix}$
	Spherical Mirror Matrix $M = \begin{bmatrix} 1 & 0 \\ \dfrac{2}{R} & 1 \end{bmatrix}$

Combining ABCD Matrices

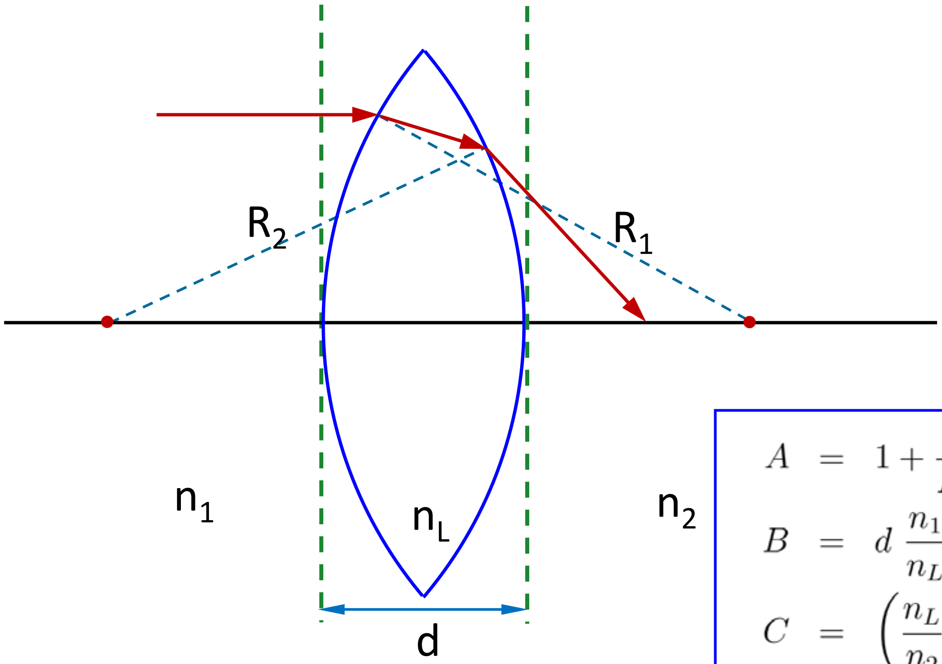


$$\begin{bmatrix} y_N \\ \alpha_N \end{bmatrix} = \begin{bmatrix} A_N & B_N \\ C_N & D_N \end{bmatrix} \begin{bmatrix} y_{N-1} \\ \alpha_{N-1} \end{bmatrix} = \begin{bmatrix} A_N & B_N \\ C_N & D_N \end{bmatrix} \begin{bmatrix} A_{N-1} & B_{N-1} \\ C_{N-1} & D_{N-1} \end{bmatrix} \begin{bmatrix} y_{N-2} \\ \alpha_{N-2} \end{bmatrix} =$$

$$= \begin{bmatrix} A_N & B_N \\ C_N & D_N \end{bmatrix} \begin{bmatrix} A_{N-1} & B_{N-1} \\ C_{N-1} & D_{N-1} \end{bmatrix} \cdots \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} y_0 \\ \alpha_0 \end{bmatrix} =$$

$$= \left[\prod_{i=1}^{N} \tilde{M}_{N+1-i} \right] \begin{bmatrix} y_0 \\ \alpha_0 \end{bmatrix} = \tilde{M}_{total} \begin{bmatrix} y_0 \\ \alpha_0 \end{bmatrix},$$

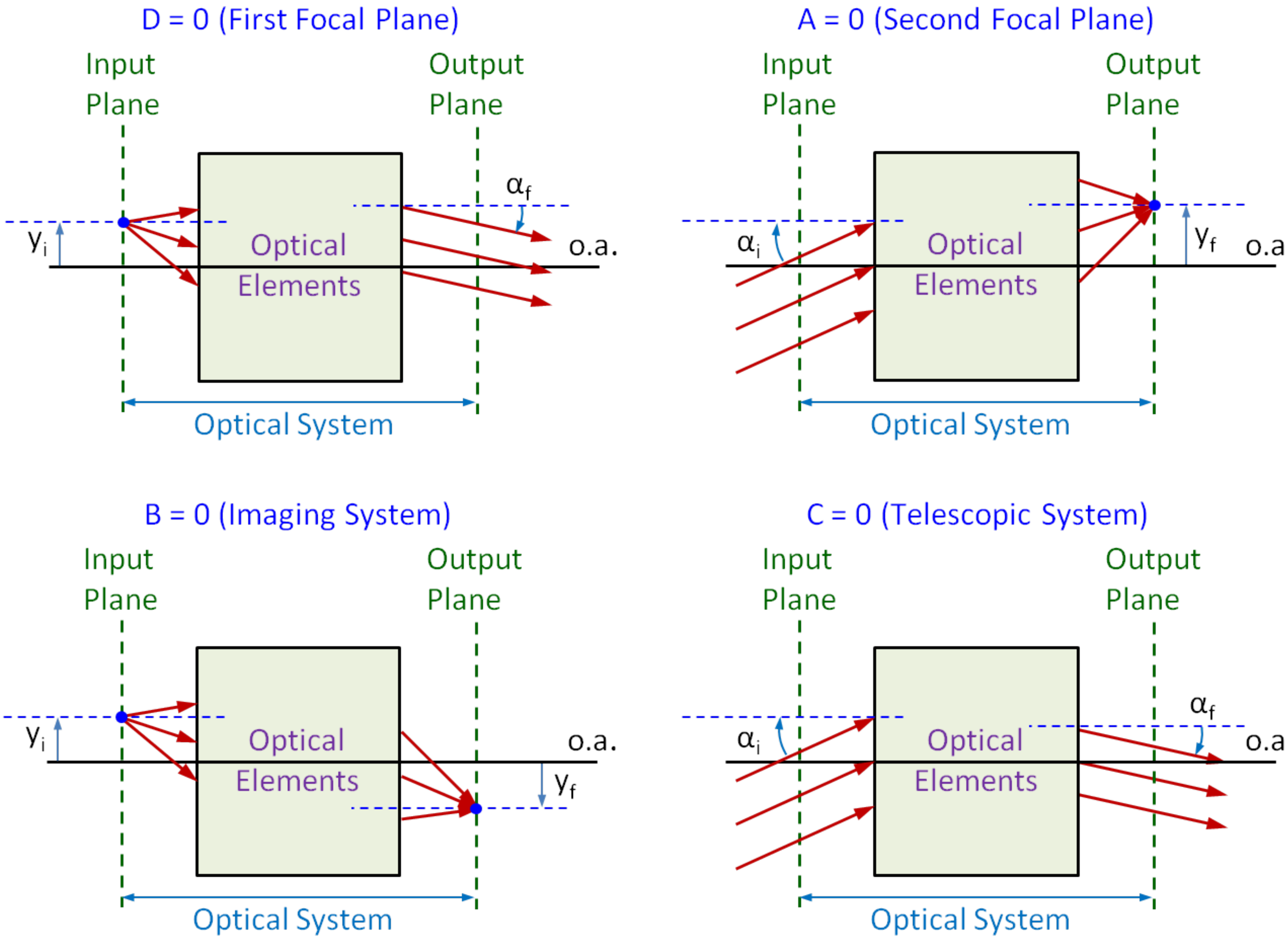
Example of Combining ABCD Matrices – Thick lens



$$
\begin{aligned}
A &= 1 + \frac{d}{R_1}\left(\frac{n_1}{n_L} - 1\right) \\
B &= d\,\frac{n_1}{n_L} \\
C &= \left(\frac{n_L}{n_2} - 1\right)\frac{1}{R_2} + \frac{n_L}{n_2}\left(\frac{n_1}{n_L} - 1\right)\frac{1}{R_1} + \frac{d}{R_1 R_2}\left(\frac{n_L}{n_2} - 1\right)\left(\frac{n_1}{n_L} - 1\right) \\
D &= d\,\frac{n_1}{n_L}\left(\frac{n_1}{n_L} - 1\right)\frac{1}{R_2} + \frac{n_1}{n_2}
\end{aligned}
$$

$$
M = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \left(\frac{n_L}{n_2} - 1\right)\frac{1}{R_2} & \frac{n_L}{n_2} \end{bmatrix} \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \left(\frac{n_1}{n_L} - 1\right)\frac{1}{R_1} & \frac{n_1}{n_L} \end{bmatrix}
$$

Significance of A, B, C, and D Elements

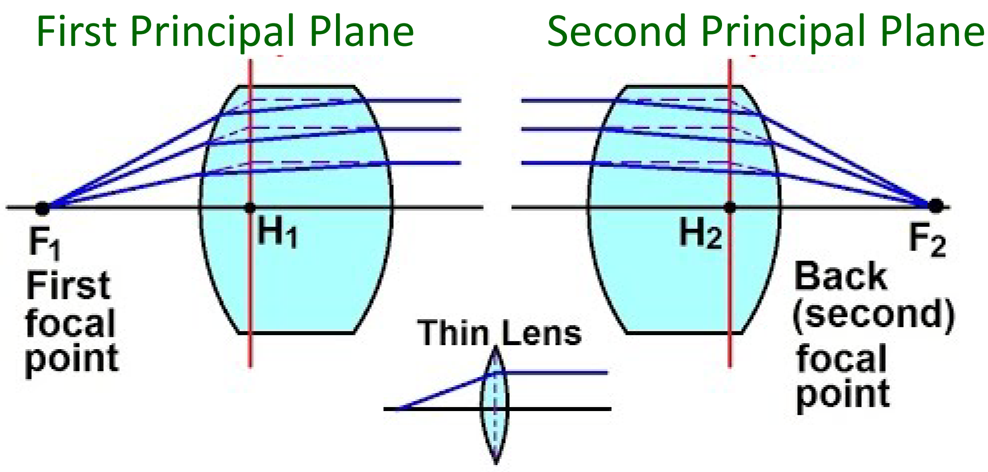


Significance of A, B, C, and D Elements

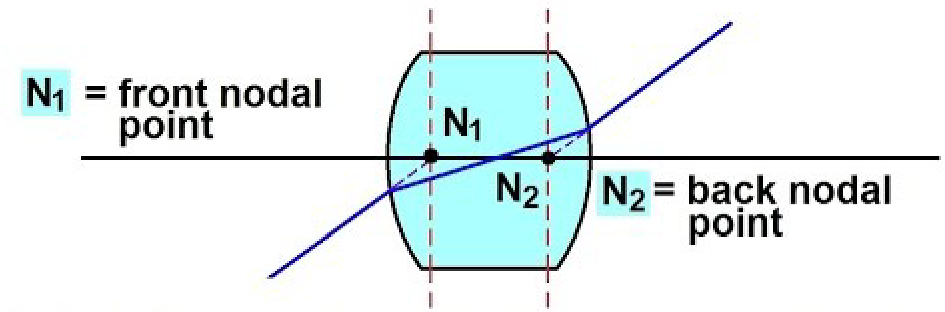
$$\begin{bmatrix} y_f \\ \alpha_f \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_i \\ \alpha_i \end{bmatrix}$$

- $\boldsymbol{D = 0}$: In this case $\alpha_f = Cy_i$, and if y_i is fixed, then all output rays have the same slope with respect to the optical axis. I.e. all rays leaving the object from y_i, at any slope, exit parallel (with slope α_f) from the output plane of the optical system. In this case the input plane coincides with the *First Focal Plane* of the optical system.
- $\boldsymbol{A = 0}$: In this case $y_f = B\alpha_i$, and if α_i is fixed, then all output rays pass from the same point y_f in the output plane independently of their height at the input plane. In this case the output plane coincides with the *Second Focal Plane* of the optical system.
- $\boldsymbol{B = 0}$: In this case $y_f = Ay_i$, and if y_i is fixed, then all output rays pass from the same point y_f in the output plane independently of their slope at the input plane. These points are called object and image points and the optical system is an *Imaging System*. In addition, $A = y_f/y_i$ is the transverse magnification of the optical system.
- $\boldsymbol{C = 0}$: In this case $\alpha_f = D\alpha_i$, and if α_i is fixed, then all output rays exit with the same slope α_f at the output plane independently of their heights at the input plane. The input and output planes form what is called as a *Telescopic System*. In addition, $D = \alpha_f/\alpha_i$ is the angular magnification of the optical system.

Cardinal Points and Planes



https://i.ytimg.com/vi/2EUzr8fP0TA/hqdefault.jpg



Note: if the reflective index in front and behind the lens is the same then the nodal points and the principal points are coincident.

https://www.youtube.com/watch?v=Wq0eMr_Lib0

Principal Planes and Cardinal Points of an Optical System

Cardinal Points Location

$$p = \frac{D}{C}$$

$$r = \frac{D - (n_i/n_f)}{C}$$

$$q = -\frac{A}{C}$$

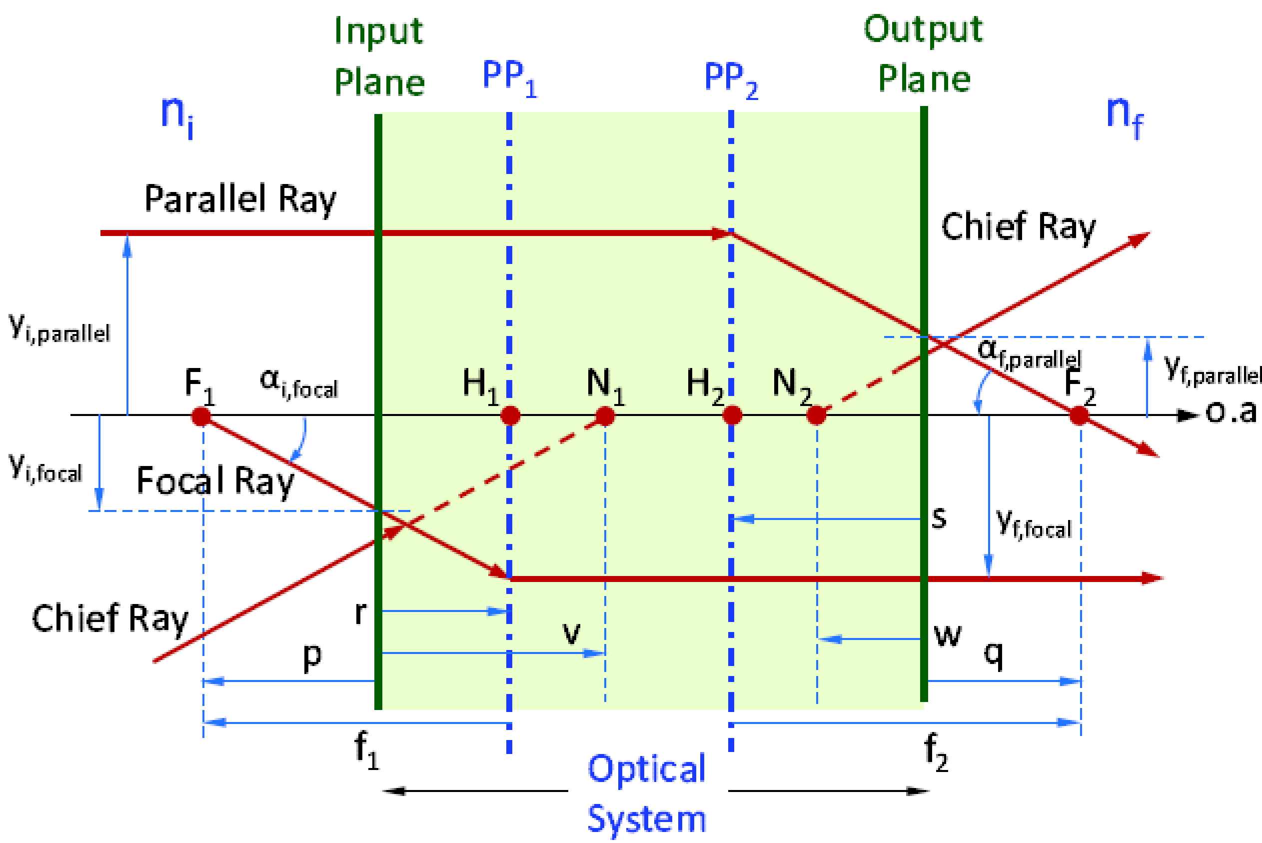
$$s = \frac{1 - A}{C}$$

$$v = \frac{D - 1}{C}$$

$$w = \frac{(n_i/n_f) - A}{C}$$

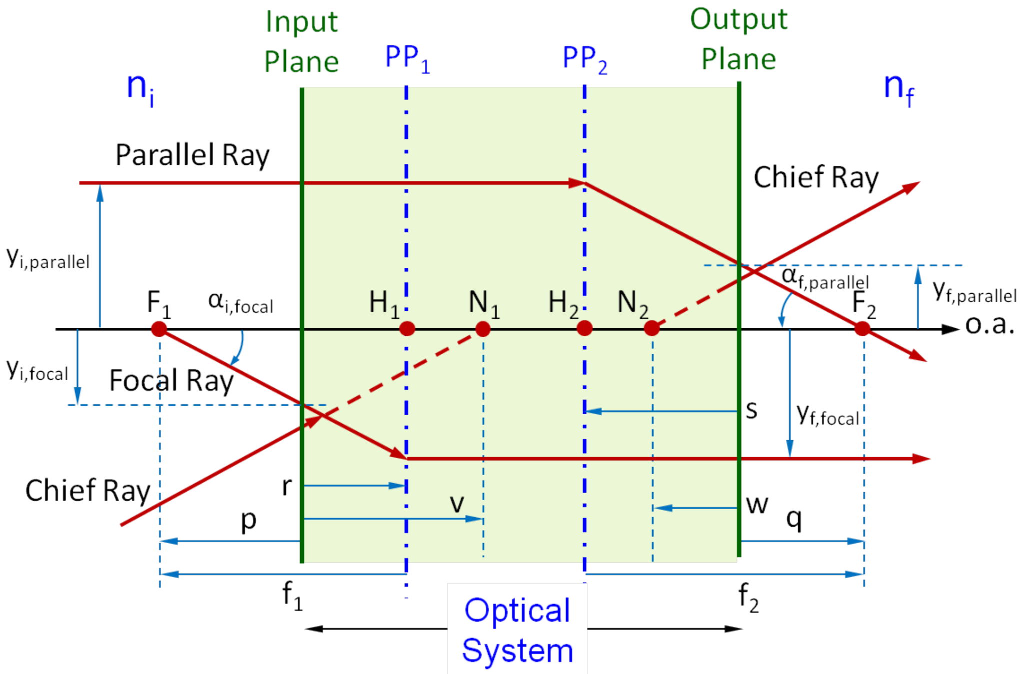
$$f_1 = \frac{n_i/n_f}{C}$$

$$f_2 = -\frac{1}{C}$$



F. L. Pedrotti, L. M. Pedrotti, and L. S. Pedrotti , Introduction to Optics, 3rd Ed., Pearson-Prentice Hall, 2007.

Principal Planes and Cardinal Points of an Optical System



$$\begin{bmatrix} y_{f,focal} \\ 0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_{i,focal} \\ \alpha_{i,focal} \end{bmatrix} \Longrightarrow$$

$$0 = Cy_{i,focal} + D\alpha_{i,focal} \Longrightarrow \frac{y_{i,focal}}{\alpha_{i,focal}} = -\frac{D}{C},$$

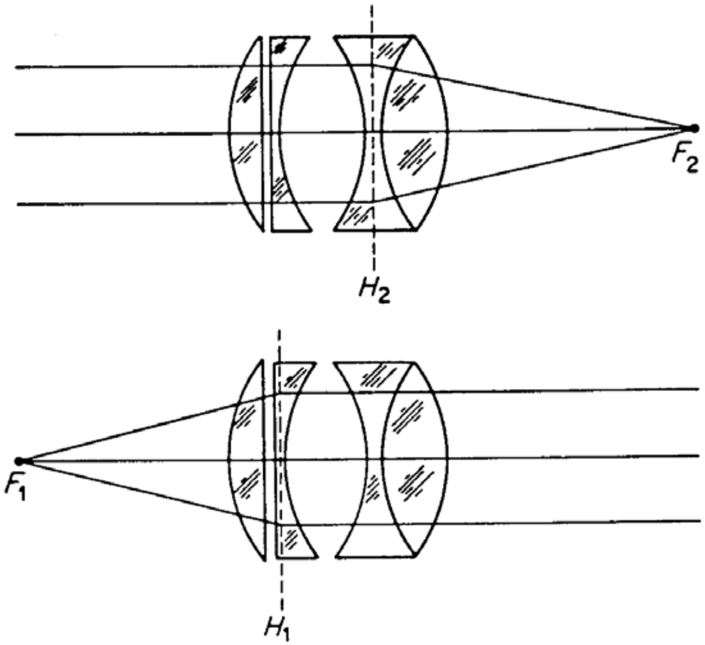
$$|p| = \frac{y_{i,focal}}{\alpha_{i,focal}} \Longrightarrow p = -\frac{y_{i,focal}}{\alpha_{i,focal}} = \frac{D}{C},$$

$$r = \frac{y_{f,focal} - y_{i,focal}}{\alpha_{i,focal}} = \frac{Ay_{i,focal} + B\alpha_{i,focal} - y_{i,focal}}{\alpha_{i,focal}} \Longrightarrow$$

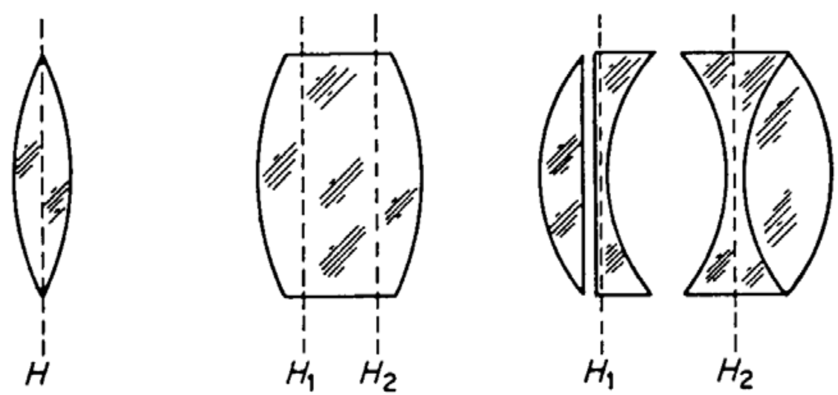
$$r = \frac{-AD + BC}{C} + \frac{D}{C} = \frac{D - (n_i/n_f)}{C},$$

$$f_1 = p - r = \frac{n_i/n_f}{C},$$

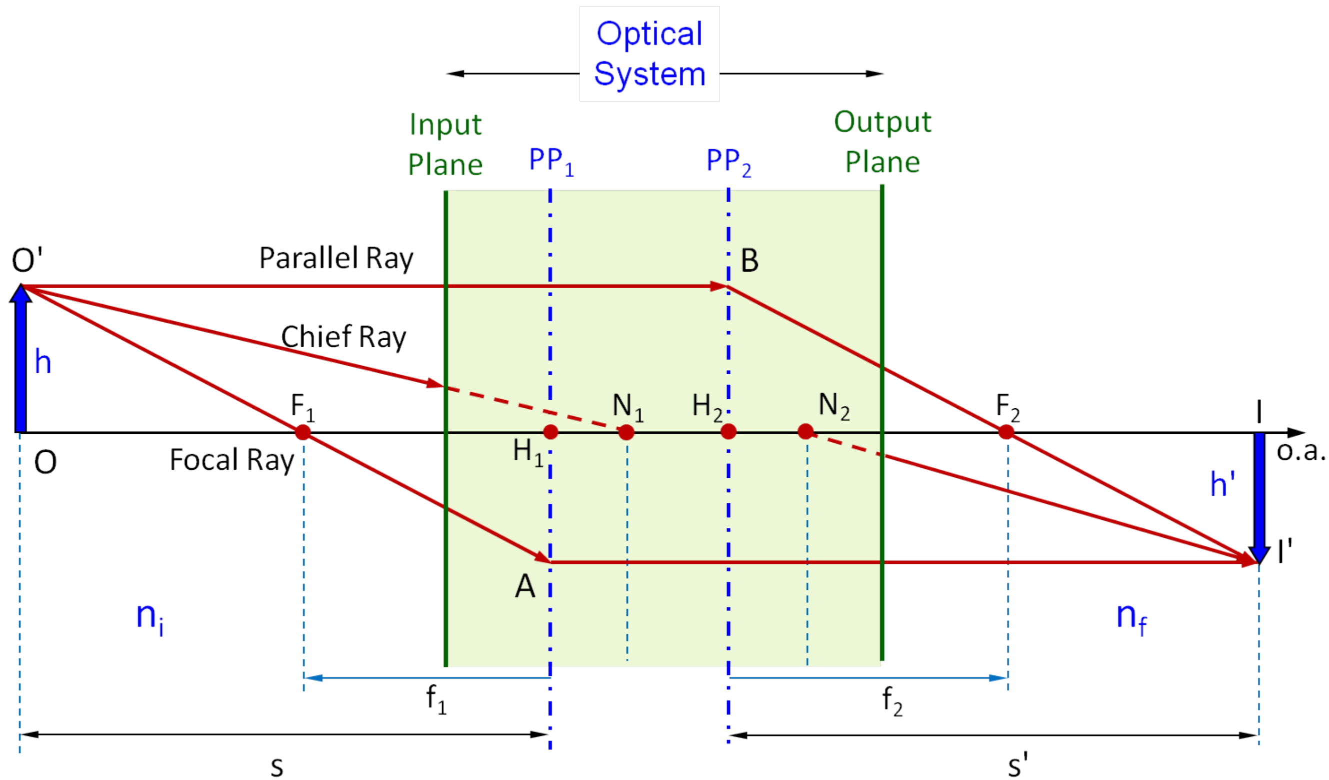
Principal Planes of a Converging Lens System



Principal Planes of a Thin Lens, a Thick Lens and a Complex Lens



Principal Planes and Cardinal Points



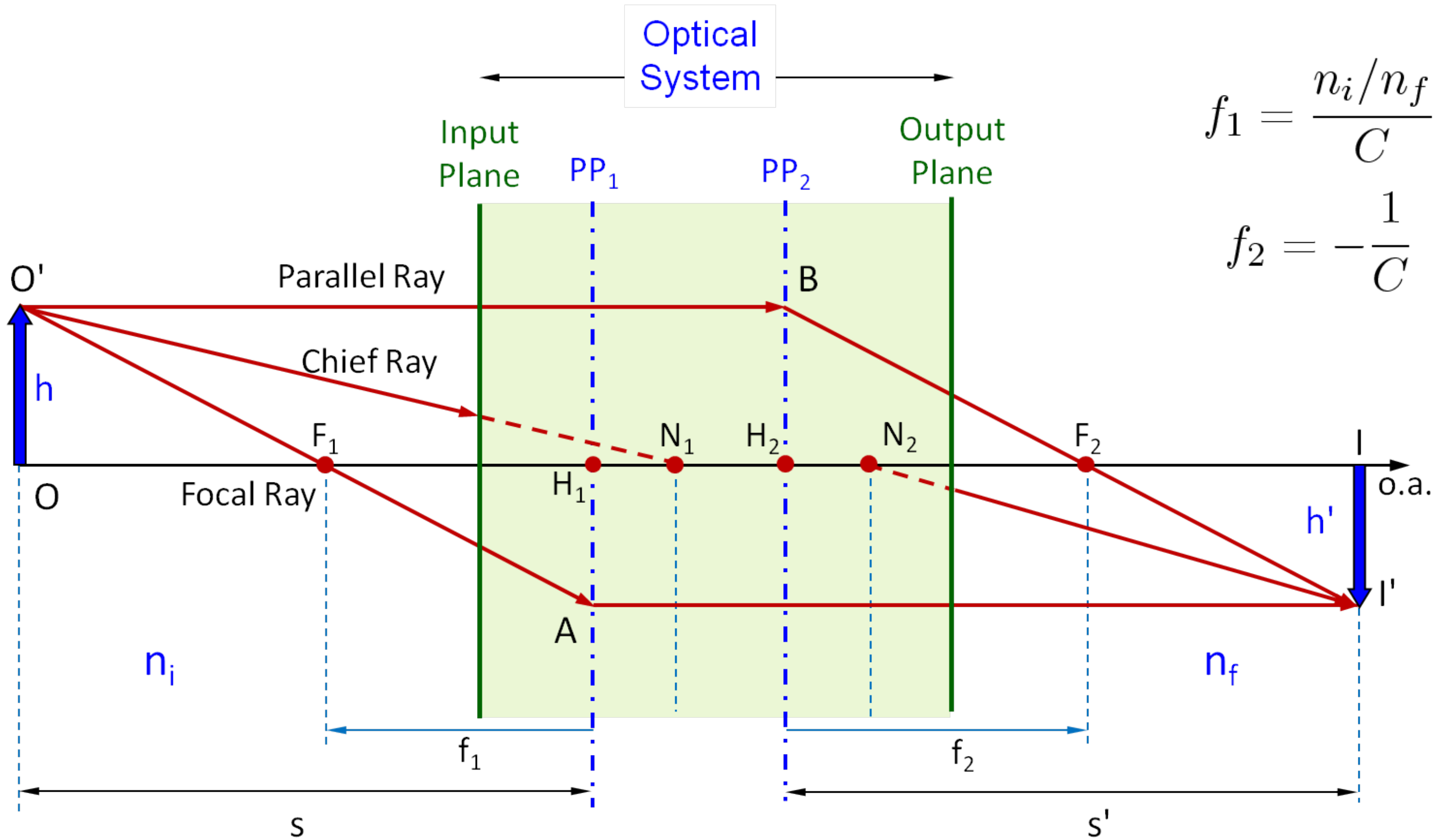
similar orthogonal triangles $O'OF_1$ and F_1H_1A and the H_2BF_2 and F_2II'

$$\frac{h}{s-f_1} = \frac{h'}{f_1},$$

$$\frac{h}{f_2} = \frac{h'}{s'-f_2}, \qquad \text{resulting in}$$

$$\frac{f_1}{s} + \frac{f_2}{s'} = 1.$$

Principal Planes and Cardinal Points



$$f_1 = \frac{n_i/n_f}{C}$$

$$f_2 = -\frac{1}{C}$$

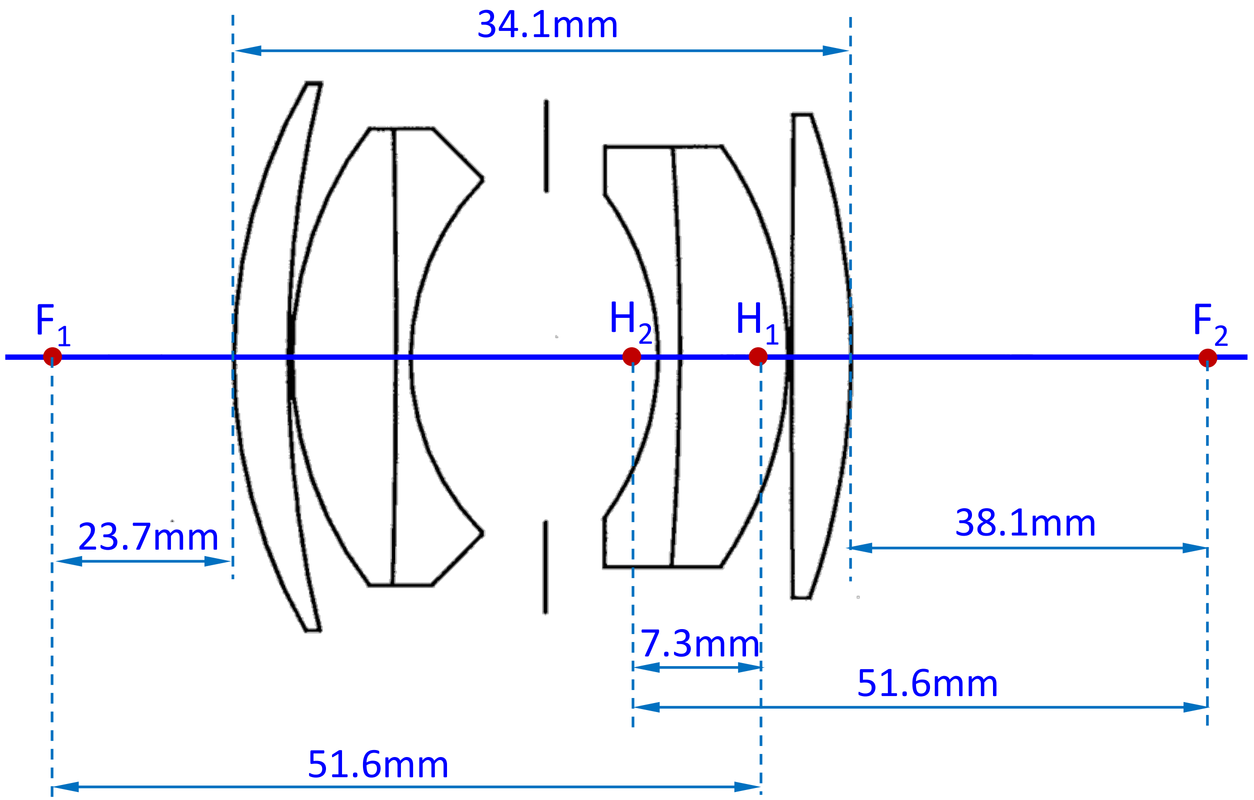
$$\frac{f_1}{s} + \frac{f_2}{s'} = 1$$

$$\text{If} \quad n_0 = n_f \Longrightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

General Purpose Imaging Lens System

Nikon 50mm (51.6mm) Nikkor-H f/2 Auto lens (on sale January 1964)

Distances are in millimeters



Nikon 50mm f/2
NIKKOR-H and AI (1964-1979)

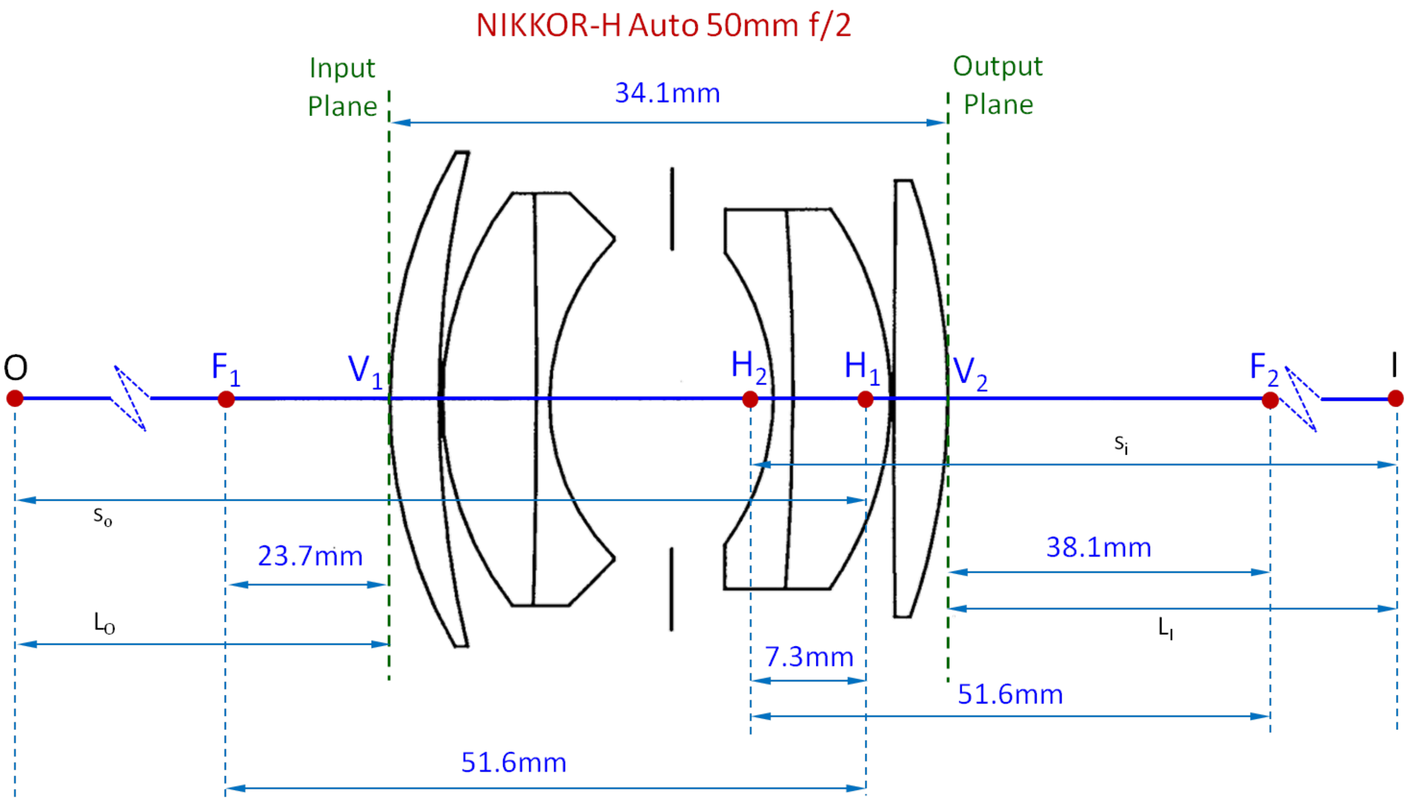
https://imaging.nikon.com/history/story/0002/index.htm

https://www.kenrockwell.com/nikon/images1/50mm-f2/50mm-f2-KEN_4011.jpg

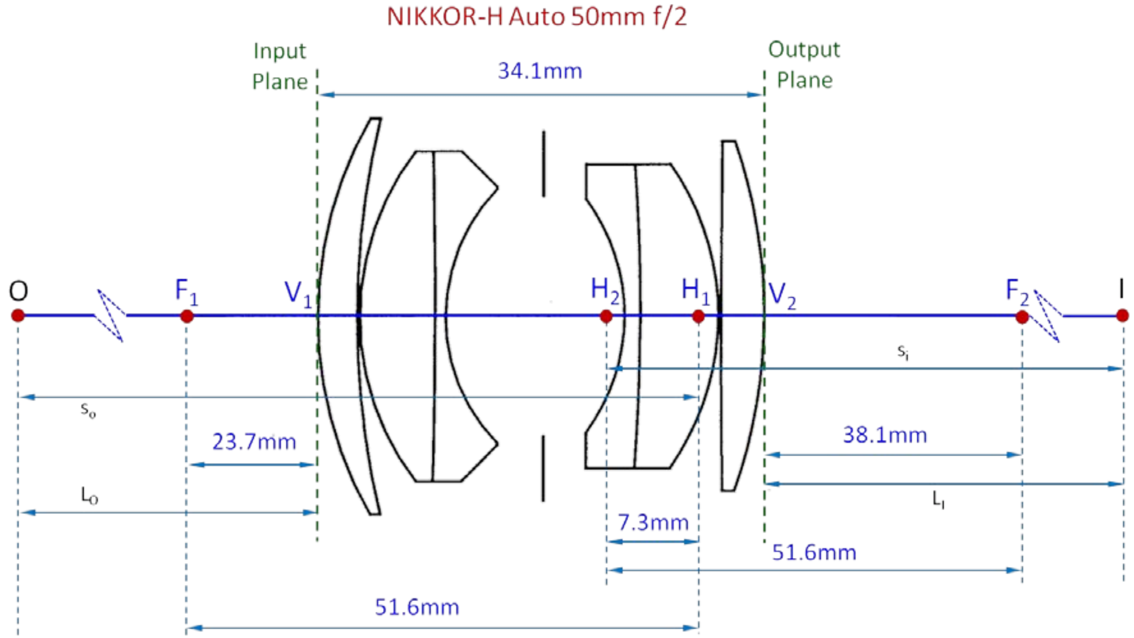
Example Imaging with

Nikon 50mm (51.6mm) Nikkor-H f/2 Auto lens (on sale January 1964)

In this example a real optical lens is used for imaging. The lens is shown in the figure where the cardinal points are also denoted. An object, O, is placed at a distance $L_O = 100$mm in front of the front lens surface (from vertex V1) of the Nikkor-H auto 50mm f/2 photographic lens. It is sought to calculate the location of the image, I, with respect to the rear surface of the rear lens (distance L_I), as well as its magnification.



Example Imaging with
Nikon 50mm (51.6mm) Nikkor-H f/2 Auto lens (on sale January 1964)



$$s_o = L_O + [34.1 - (51.6 - 7.3 - 38.1)] = L_O + 27.9 = 127.9\,mm \Longrightarrow$$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \Longrightarrow \frac{1}{127.9} + \frac{1}{s_i} = \frac{1}{51.6} \Longrightarrow s_i = 86.5\,mm.$$

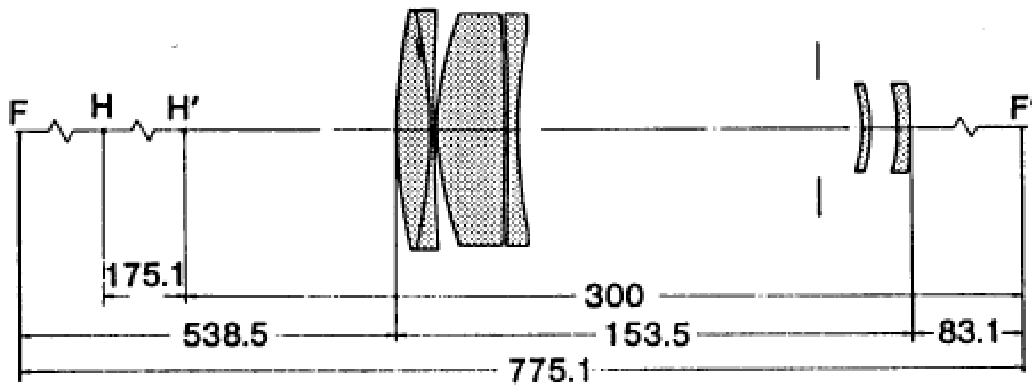
The distance of the image from the right vertex V_2, L_I is easily determined from s_i as follows: $L_I = s_i - (51.6 - 38.1) = 86.5 - 13.5 = 73\,\text{mm}$. Since $s_i > 0$ the image is real and inverted. The transverse magnification $m = -s_i/s_o = -86.5/127.9 = -0.676$. This example shows how with the knowledge of the cardinal points paraxial imaging can be performed very easily using appropriately the thin lens equation.

Principal Planes of a Telephoto Lens

Nikon – 300mm f/4 ED-IF AF Nikkor (1987-2000)

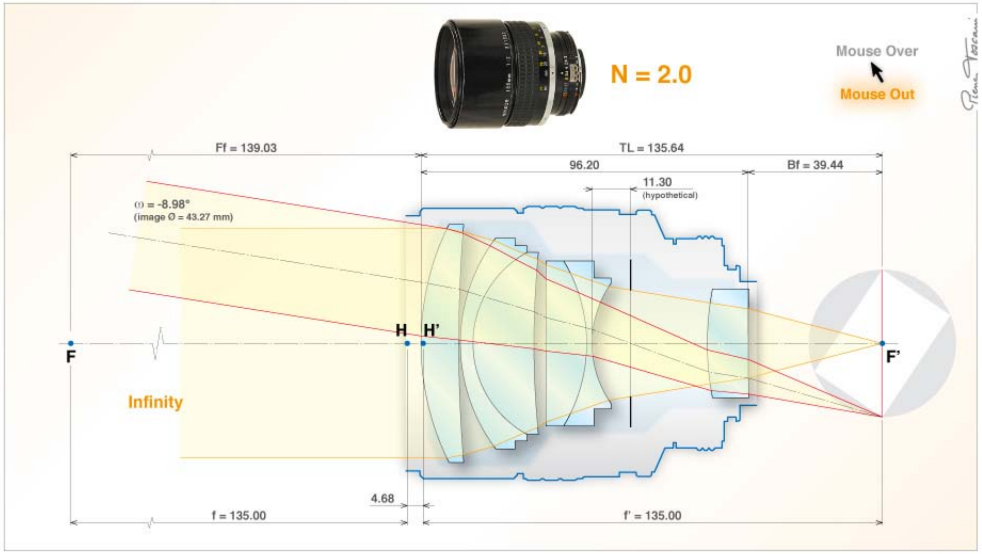
Distances are in millimeters

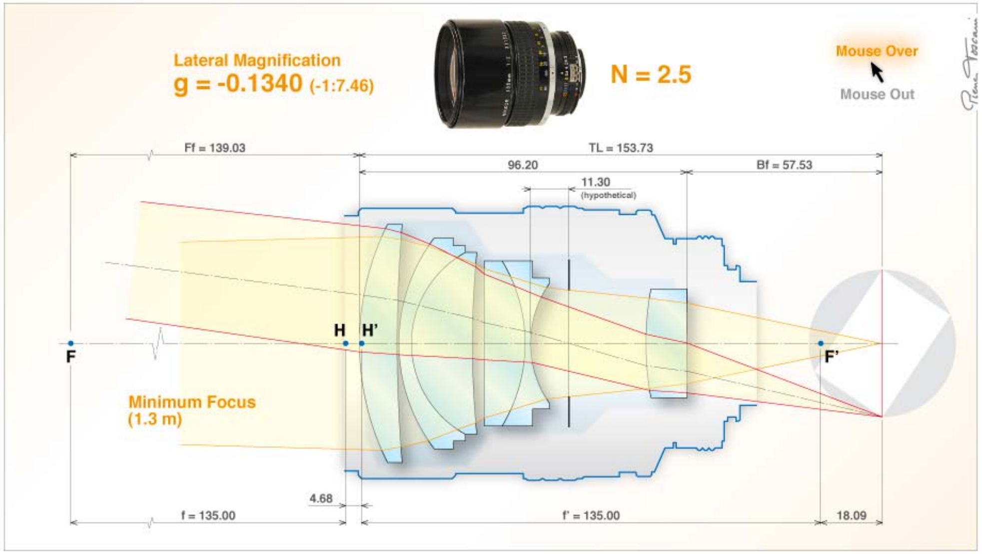
https://www.kenrockwell.com/nikon/3004af.htm



ED: Extra Low Dispersion Glass (reduce chromatic aberration)
IF: Internal Focusing (movement of group of elements with respect to other groups, allows focusing on closer objects
AF: Automatic Focusing (Rotating drive shaft through lens mounts moves lens with respect to camera)

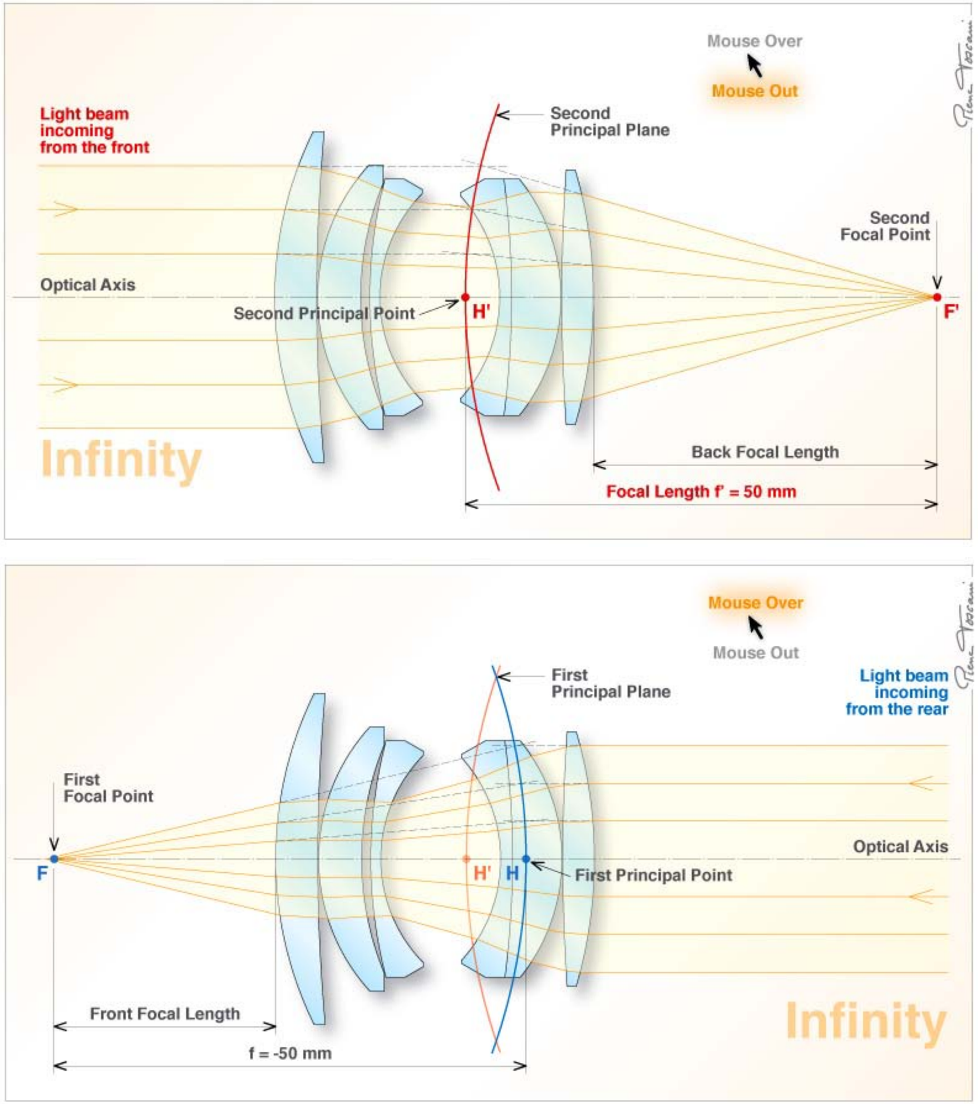
Nikkor 135mm f/2.0 Ais.





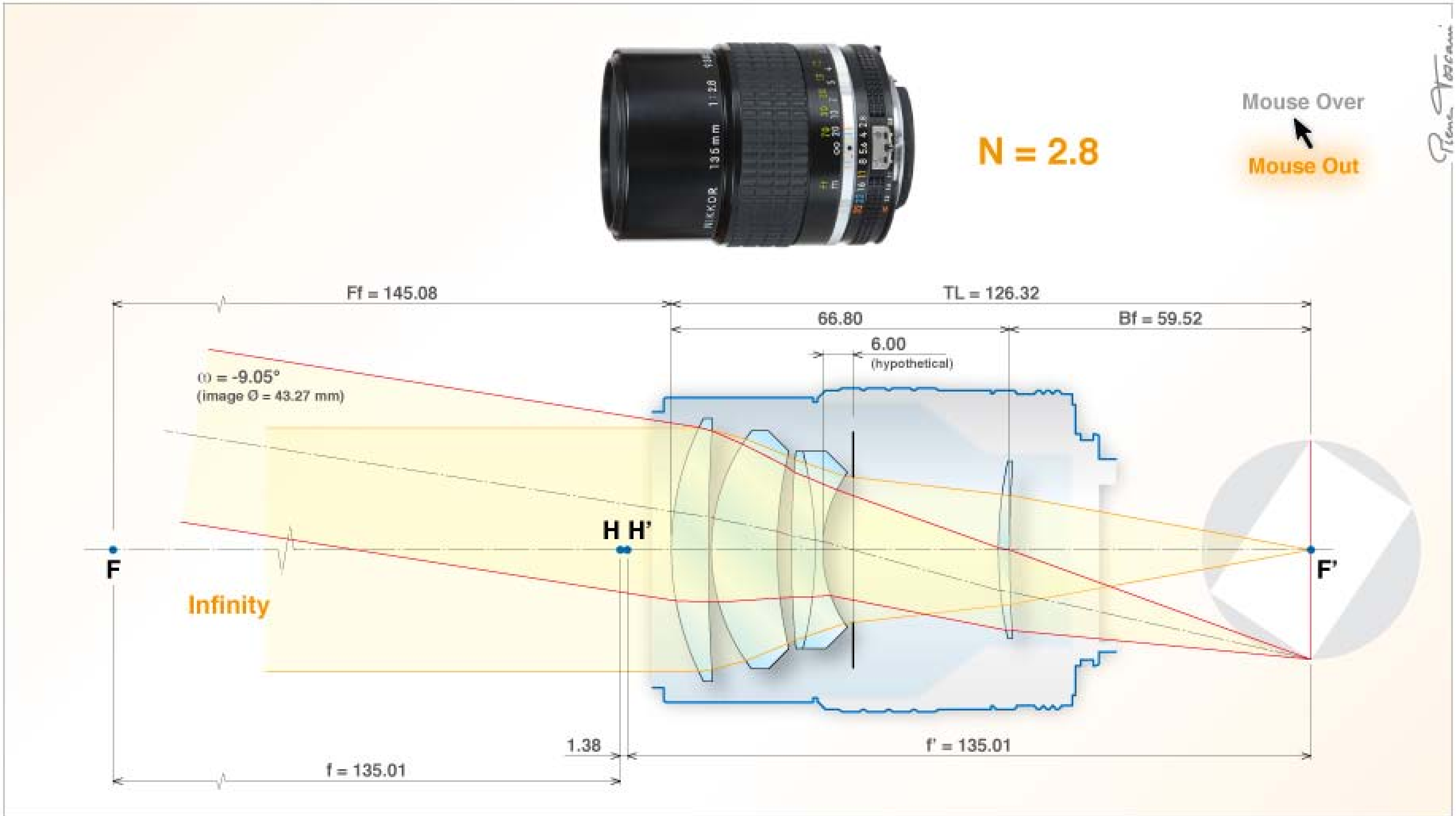
http://www.pierretoscani.com/

Optical system of a real photographic lens (50mm f/1.8)



http://www.pierretoscani.com/

Nikkor 135mm f/2.8 Ais.

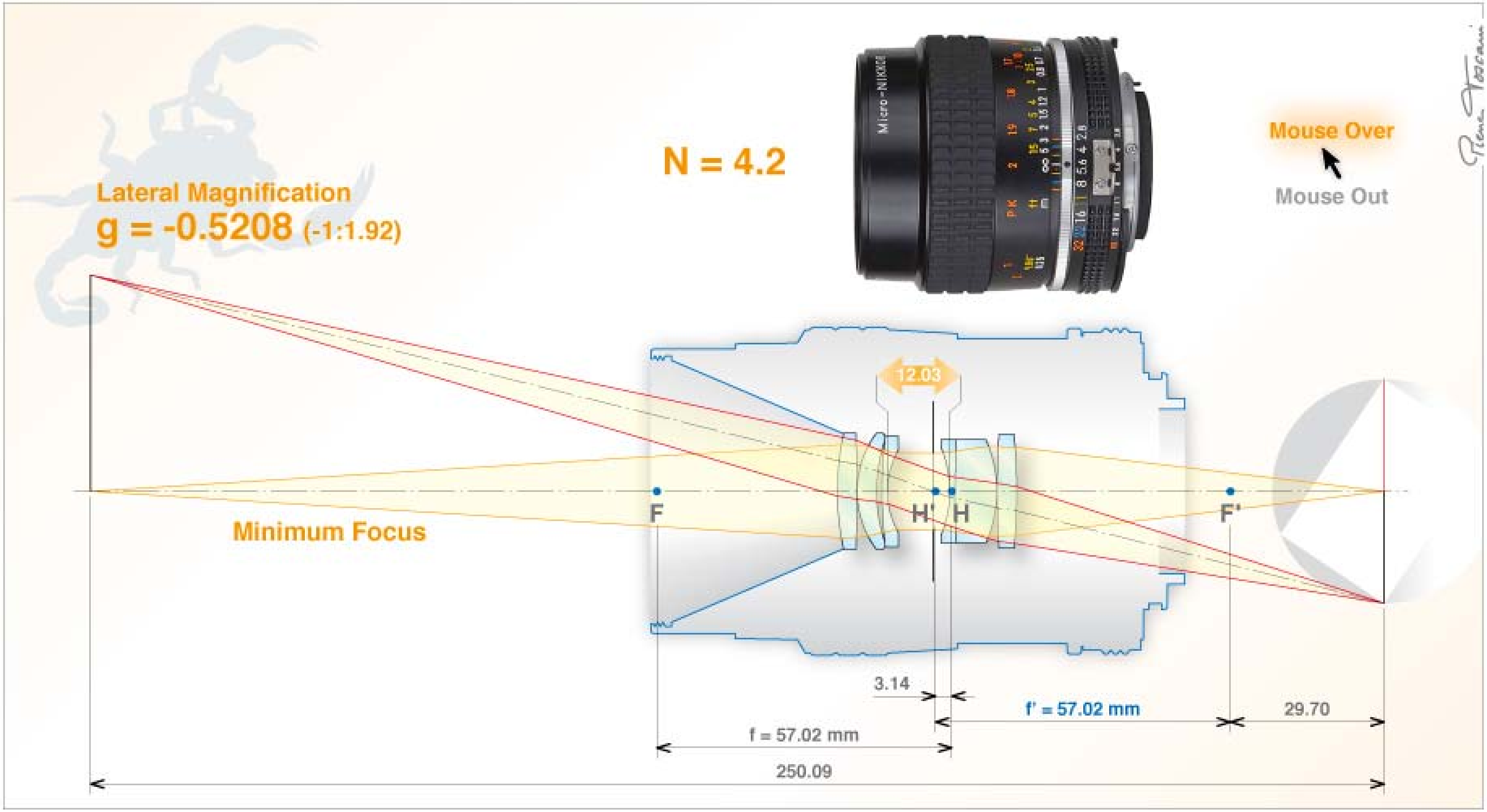


http://www.pierretoscani.com/images/echo_shortpres/

Nikkor 135mm f/2.8 Ais.

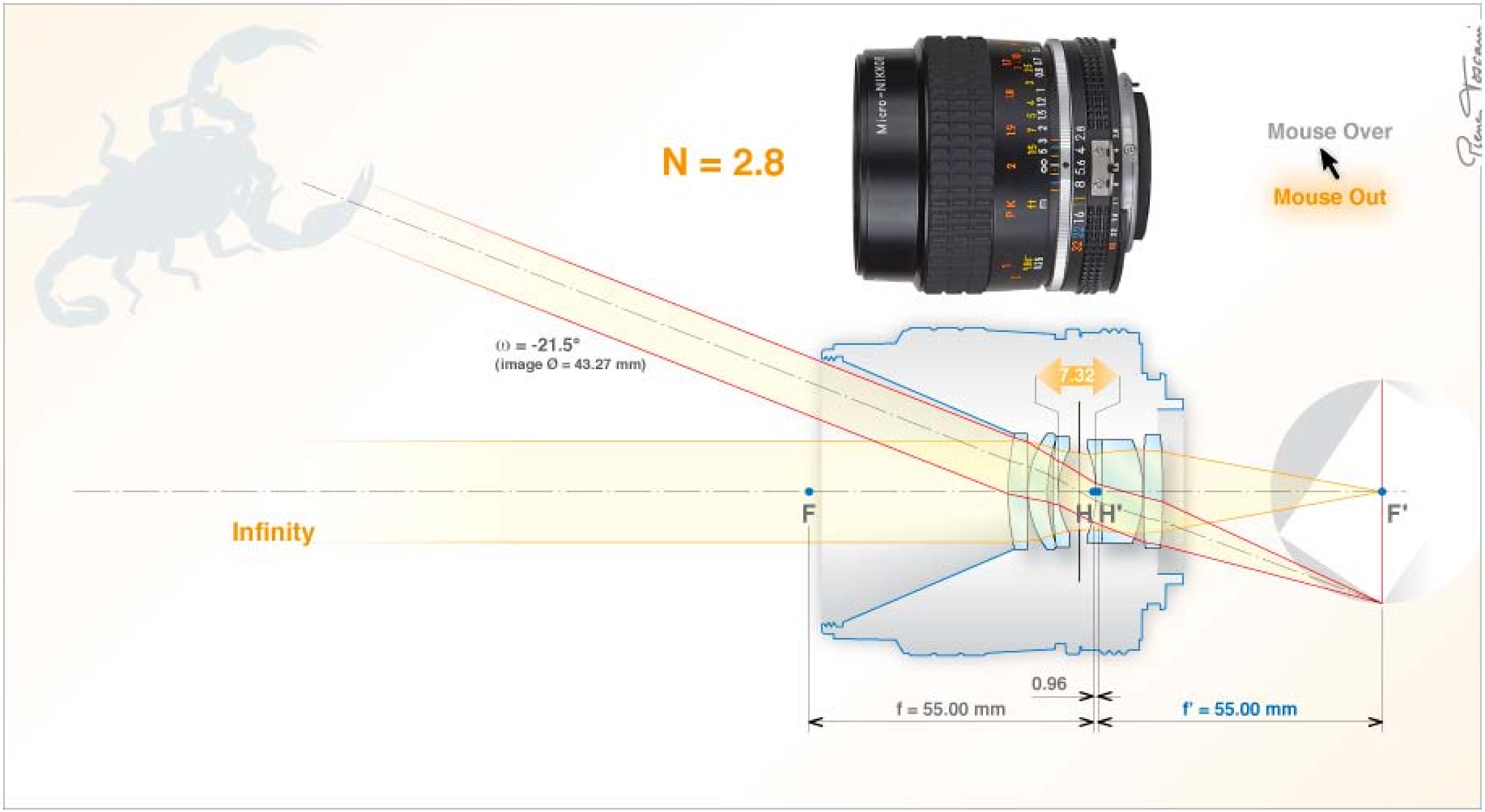
http://www.pierretoscani.com/images/echo_shortpres/

Micro-Nikkor 55mm f/2.8 Ais and AF Micro-Nikkor 55mm f/2.8



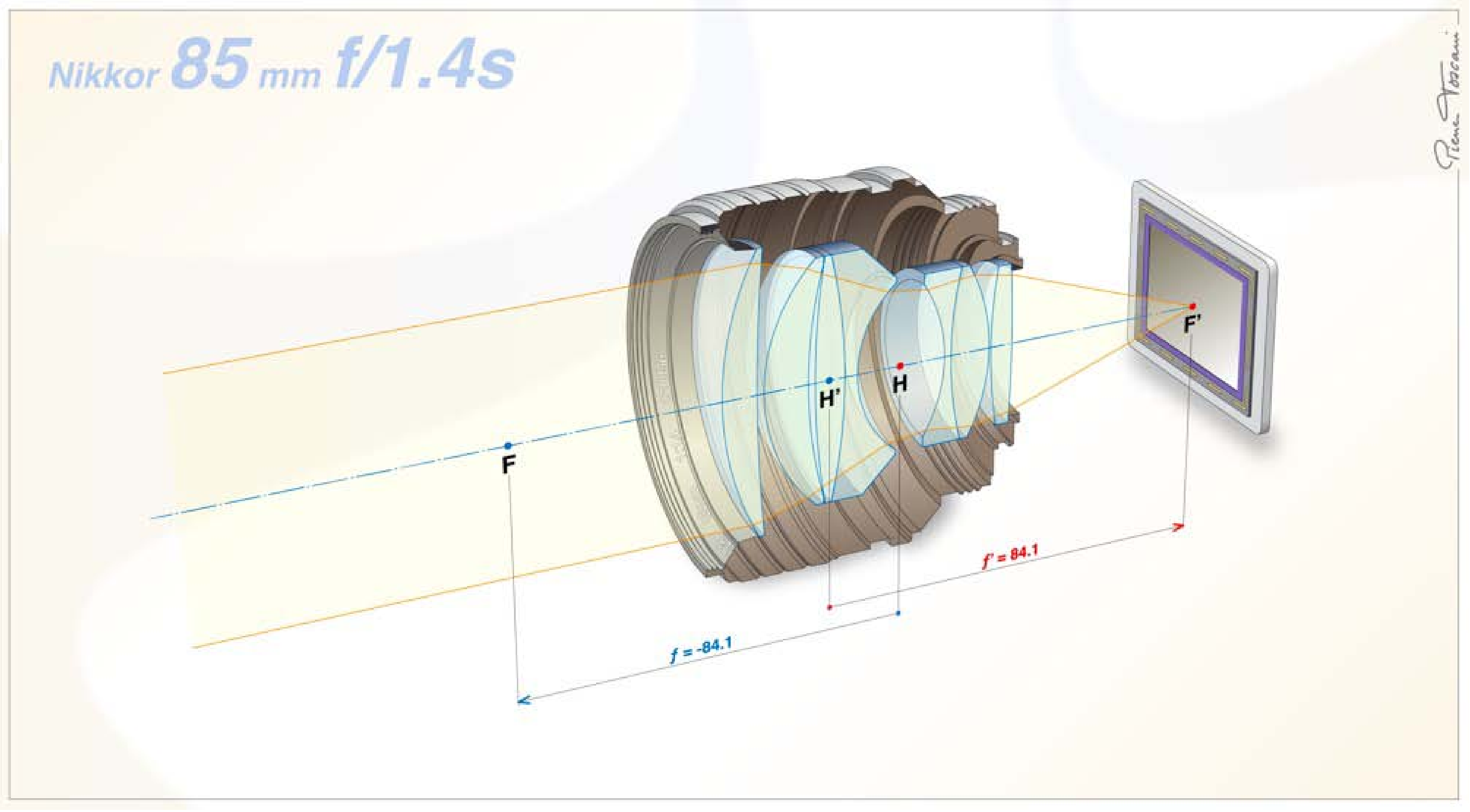
http://www.pierretoscani.com/images/echo_shortpres/

Micro-Nikkor 55mm f/2.8 Ais and AF Micro-Nikkor 55mm f/2.8



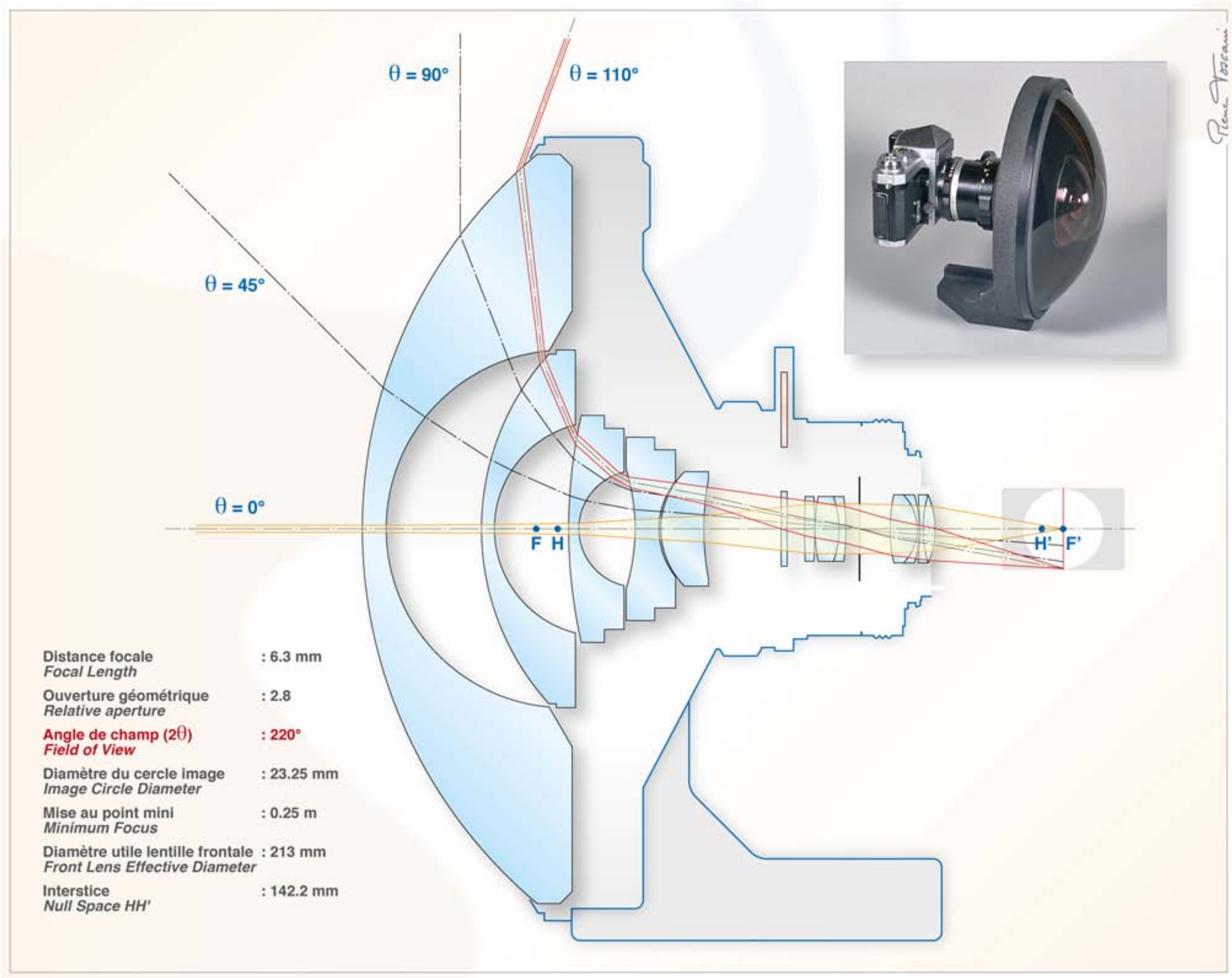
http://www.pierretoscani.com/images/echo_shortpres/

Nikkor 85mm f/2.4 s



http://www.pierretoscani.com/images/fig-focale-02a.jpg?crc=4231534395

The Fisheye-Nikkor 6 mm f/2.8



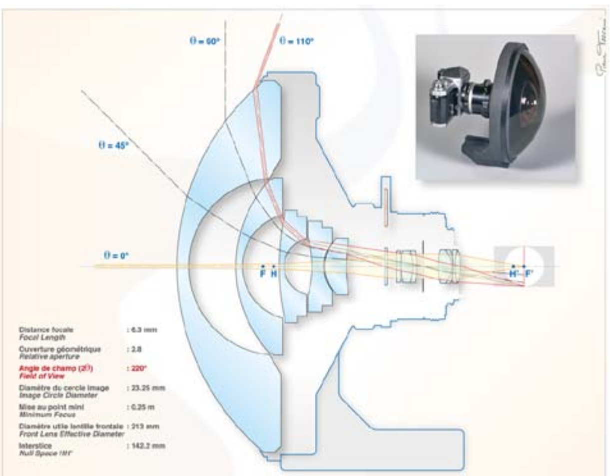
http://www.pierretoscani.com/fisheyes-(in-english).html

The Fisheye-Nikkor 6 mm f/2.8

The Fisheye-Nikkor 6 mm f/2.8.

Just out of curiosity...

The optical system of this lens is not only impressive because of its size —the effective diameter of the first two menisci are respectively 213 mm (8.4 inches) and 100 mm (4 inches), and the distance between the vertex of the front lens and the image plane is 208 mm (8.2 inches)—, but also because of the fact that the first three menisci are made of BK7 glass.

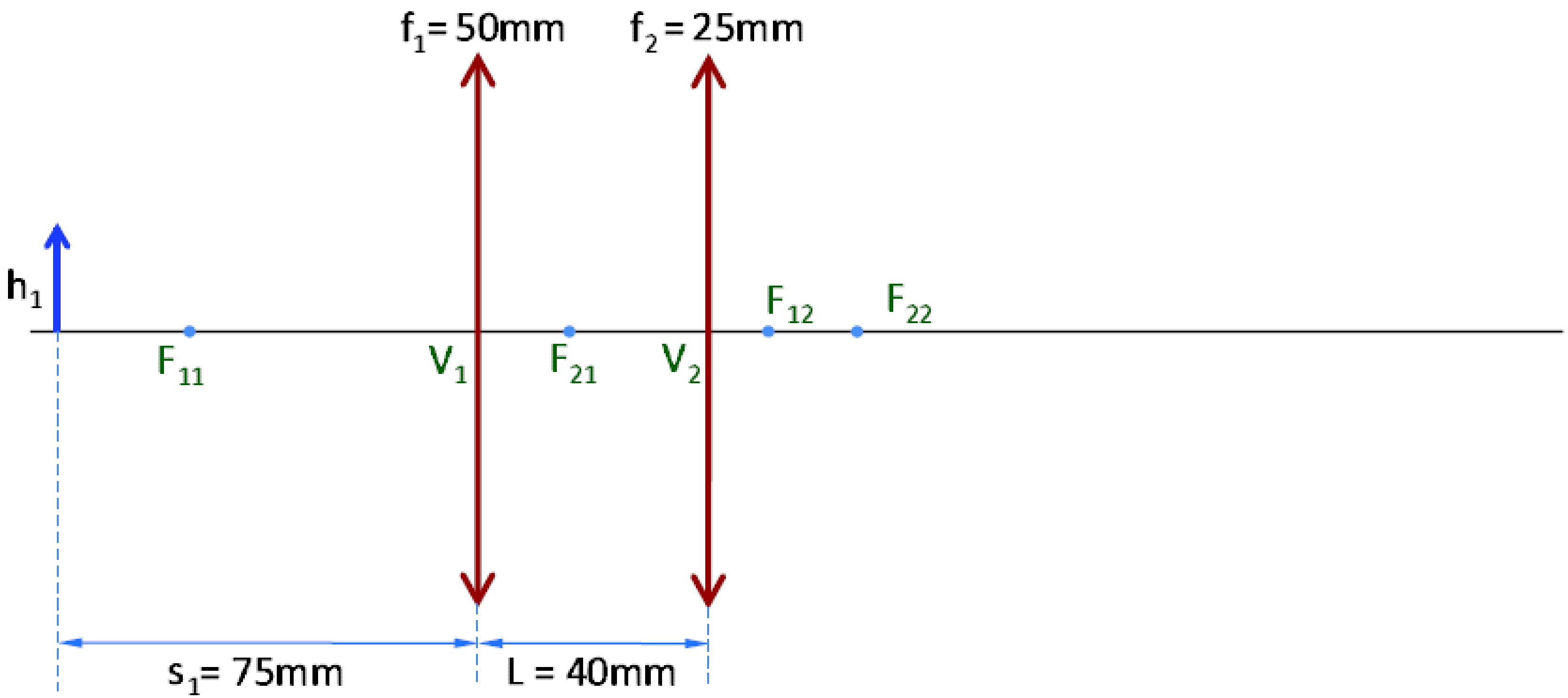


http://www.pierretoscani.com/fisheyes-(in-english).html

EXAMPLE I

Surface Number	r	d	νd	nd	
1	143.470	7.0	64.2	1.51680	# 01 Hikari E-BK7
2	52.500	28.0		1.00000	
3	76.400	3.8	64.2	1.51680	# 02 Hikari E-BK7
4	31.521	21.8		1.00000	
5	150.000	3.0	64.2	1.51680	# 03 Hikari E-BK7
6	17.100	16.5		1.00000	
7	-60.000	7.0	60.3	1.62041	# 04 Hikari E-SK16
8	22.625	0.6		1.00000	
9	23.900	12.6	28.3	1.72825	# 05 Hikari E-SF10
10	78.988	22.7		1.00000	
11	∞	1.8	59.0	1.51823	# 06 Hikari E-K3 (filter)
12	∞	5.1		1.00000	
13	278.333	3.0	33.8	1.64831	# 07 Hikari E-SF2
14	-185.420	0.1		1.00000	
15	52.030	7.0	55.4	1.53375	# 08 ?
16	-28.500	2.0	49,5	1.77250	# 09 Hikari E-LASF016
17	-77.000	13.0		1.00000	
18	45.000	8.0	59.0	1.51823	# 10 Hikari E-K3
19	-14.400	0.6	40.8	1.79631	# 11 ?
20	-34.500	0.1		1.00000	
21	-110.000	1.0	29.5	1.71736	# 12 Hikari E-SF1
22	35.000	3.5	64.2	1.51680	# 13 Hikari E-BK7
23	-25.763	Bf		1.00000	

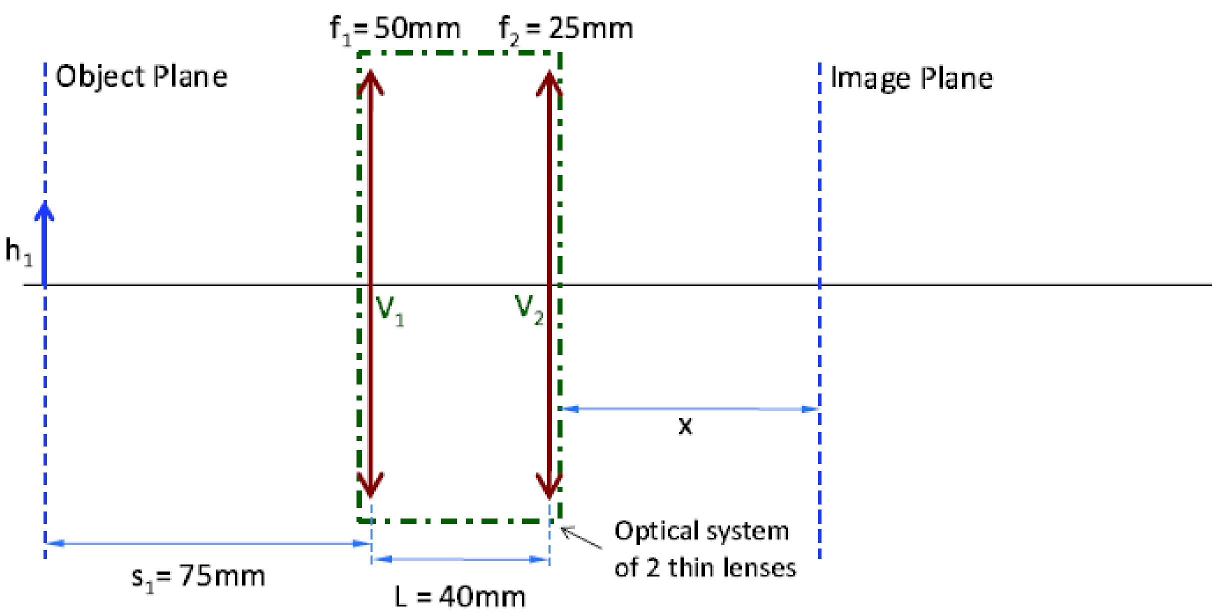
Two Thin-Lenses Example



A system of two thin lenses is given as shown in Fig. 1. The left thin lens has a focal distance of $f_1 = 50\,\mathrm{mm}$ (converging) and the right thin lens has a focal distance of $f_2 = 25\,\mathrm{mm}$ (converging also). The two thin lenses are separated by 40 mm. An object is placed at a distance of 75 mm to the left of the left thin lens. Find the position and magnification of the final image using (a) the method of matrices, (b) the thin lens equation, and (c) the method of the cardinal points.

Two Thin-Lenses Example

Method of ABCD Matrices



$$\begin{bmatrix} A_{TL} & B_{TL} \\ C_{TL} & D_{TL} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{25} & 1 \end{bmatrix} \begin{bmatrix} 1 & 40 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{50} & 1 \end{bmatrix} = \begin{bmatrix} 0.20 & 40\,mm \\ -0.028\,mm^{-1} & -0.6 \end{bmatrix}$$

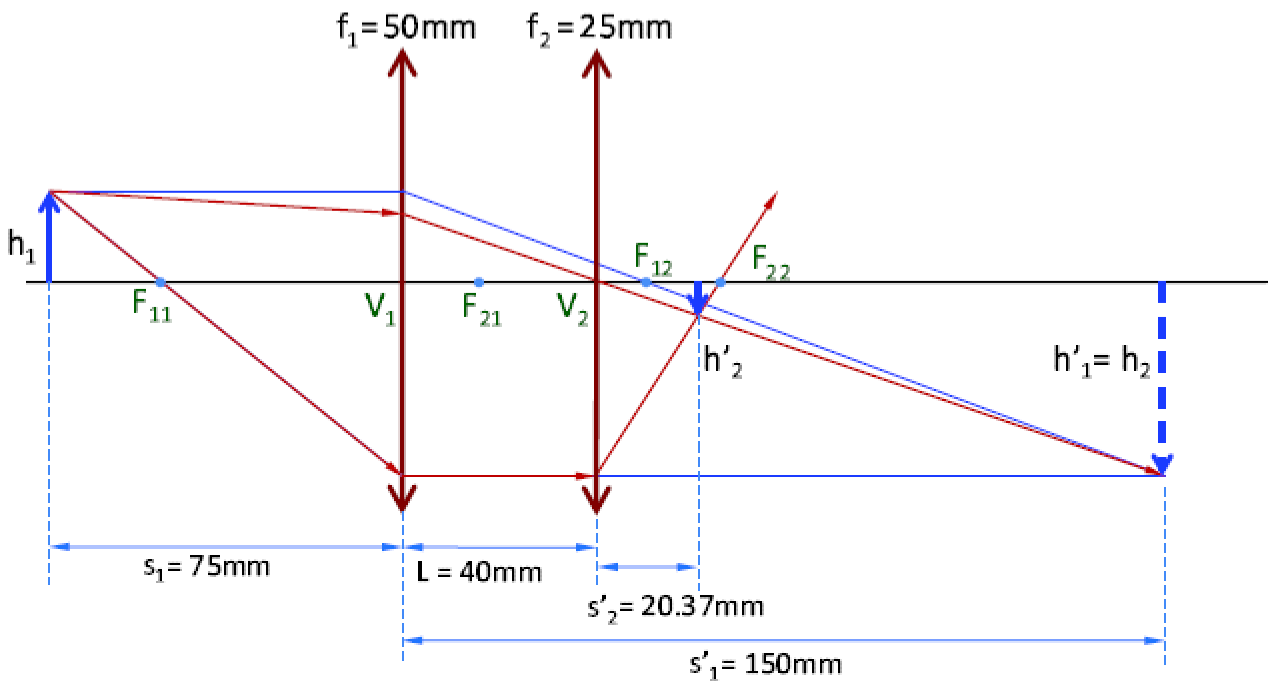
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} A_{TL} & B_{TL} \\ C_{TL} & D_{TL} \end{bmatrix} \begin{bmatrix} 1 & 75 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0.20 - 0.028x & 55 - 2.7x(mm) \\ -0.028\,mm^{-1} & 2.7 \end{bmatrix}.$$

$$B = 0 \Longrightarrow 55 - 2.7x = 0 \Longrightarrow x = 20.37\,mm$$

$$m = A(x = 20.37) = 0.2 - 0.028x = -0.37$$

Two Thin-Lenses Example

Method of Cascaded Thin Lenses



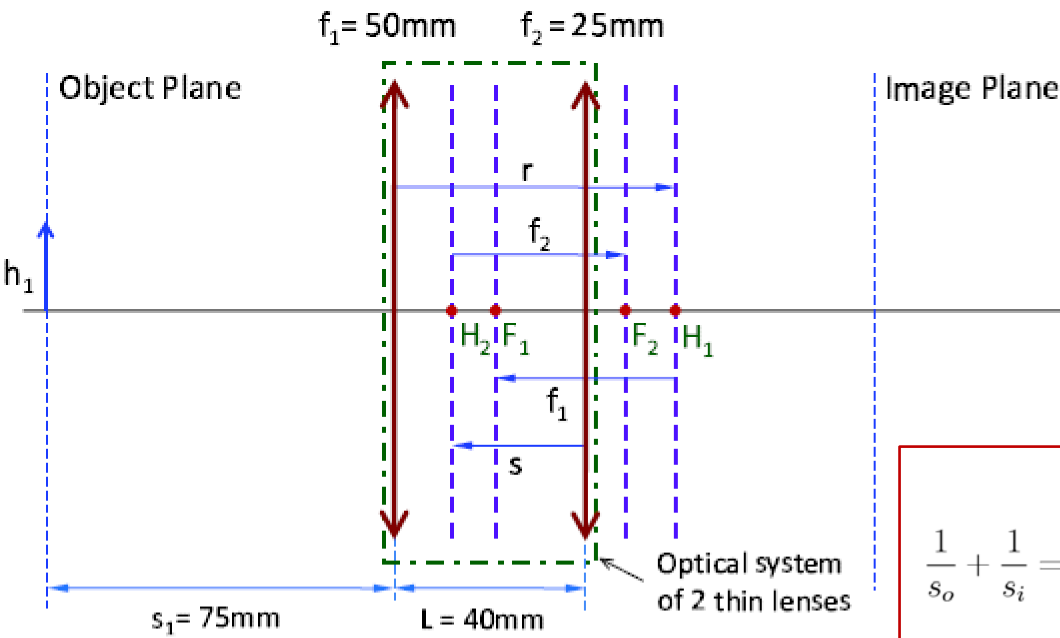
$$\frac{1}{s_1} + \frac{1}{s_1'} = \frac{1}{f_1} \Longrightarrow \frac{1}{75} + \frac{1}{s_1'} = \frac{1}{50} \Longrightarrow s_1' = 150\,mm.$$

$$\frac{1}{s_2} + \frac{1}{s_2'} = \frac{1}{f_2} \Longrightarrow \frac{1}{-110} + \frac{1}{s_2'} = \frac{1}{25} \Longrightarrow s_2' = 20.37\,mm.$$

$$m = \frac{h_2'}{h_1} = \frac{h_1'}{h_1}\frac{h_2'}{h_2 = h_1'} = \left(-\frac{s_1'}{s_1}\right)\left(-\frac{s_2'}{s_2}\right) = -\frac{150}{75}\left(-\frac{20.37}{-110}\right) = -0.37$$

Two Thin-Lenses Example

Method of Cardinal Points



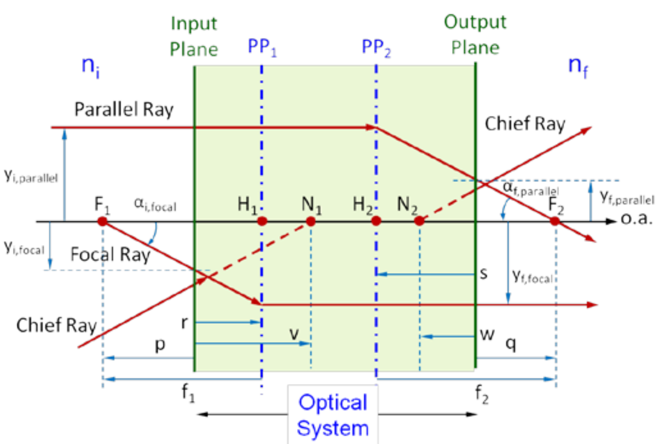
$$\begin{bmatrix} A_{TL} & B_{TL} \\ C_{TL} & D_{TL} \end{bmatrix} = \begin{bmatrix} 0.20 & 40\,mm \\ -0.028\,mm^{-1} & -0.6 \end{bmatrix}$$

$$s_o = 75 + r = 132.1428\,\text{mm}.$$

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \Longrightarrow \frac{1}{132.1428} + \frac{1}{s_i} = \frac{1}{35.714} \Longrightarrow s_i = 48.9412\,mm.$$

$$s_i - |s| = 48.9412 - 28.5714 = 20.37\,\text{mm}.$$

$$m = -s_i/s_o = -48.9412/132.1428 = -0.37$$



$$f_1 = \frac{n_0/n_f}{C_{TL}} = \frac{1}{-0.028} = -35.714\,mm,$$

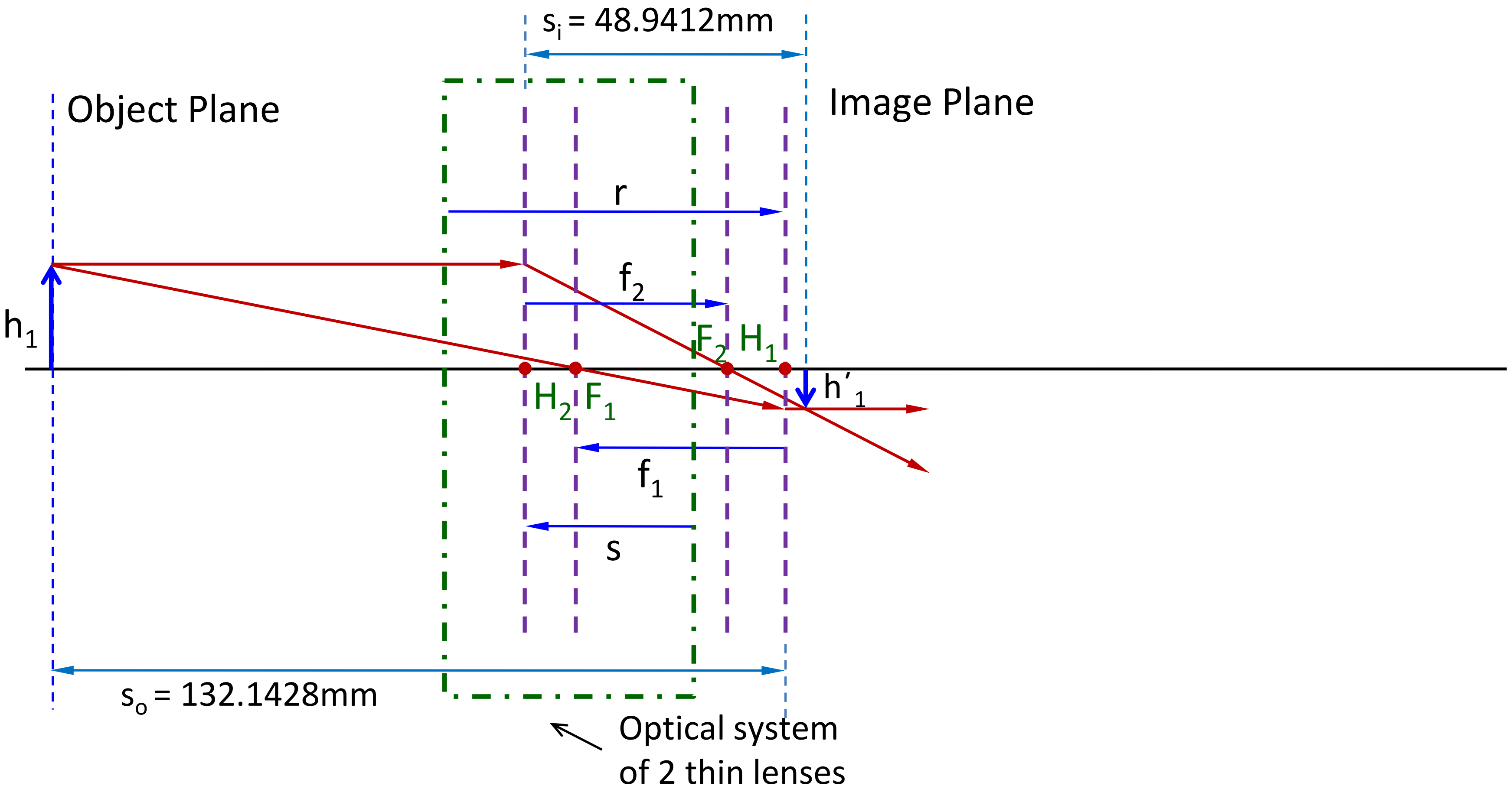
$$f_2 = -\frac{1}{C_{TL}} = -\frac{1}{-0.028} = +35.714\,mm,$$

$$r = \frac{D_{TL} - (n_0/n_f)}{C_{TL}} = \frac{-0.6 - 1}{-0.028} = +57.1428\,mm,$$

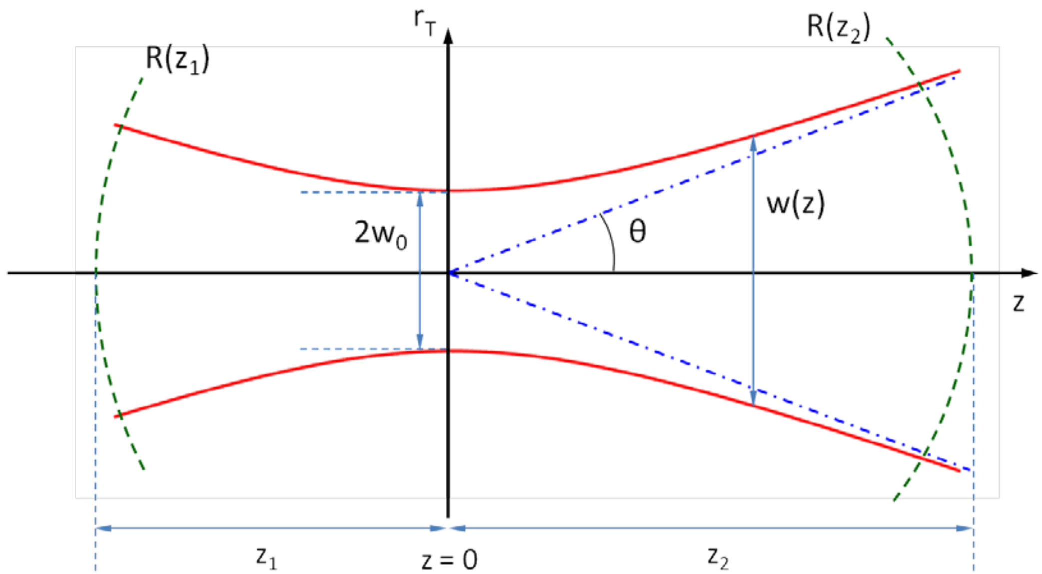
$$s = \frac{1 - A_{TL}}{C_{TL}} = \frac{1 - 0.2}{-0.028} = -28.5714\,mm,$$

Two Thin-Lenses Example

Method of Cardinal Points



Gaussian Beams



$$\frac{E(x,y,z)}{E_0} = \left\{ \frac{w_0}{w(z)} \exp\left[-\frac{r^2}{w^2(z)} \right] \right\} \quad \text{amplitude factor}$$

$$\times \exp\left\{ -j\left[kz - \tan^{-1}\left(\frac{z}{z_0} \right) \right] \right\} \quad \text{longitudinal phase}$$

$$\times \exp\left[-j\frac{kr^2}{2R(z)} \right] \quad \text{radial phase}$$

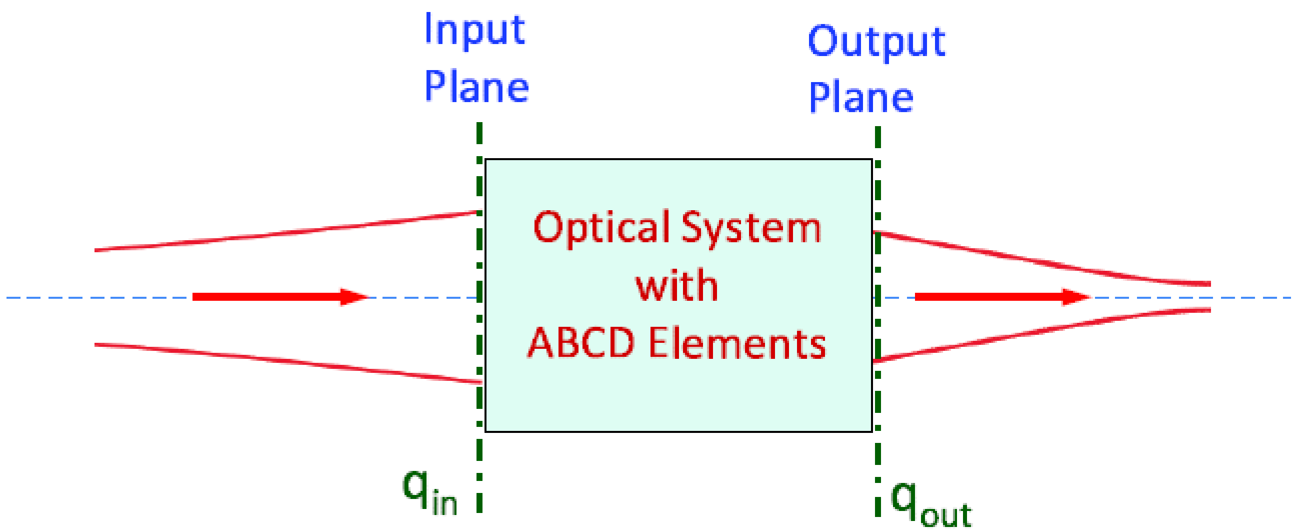
where

$$w^2(z) = w_0^2\left[1 + \left(\frac{\lambda_0 z}{\pi n w_0^2} \right)^2 \right] = w_0^2\left[1 + \left(\frac{z}{z_0} \right)^2 \right]$$

$$R(z) = z\left[1 + \left(\frac{\pi n w_0^2}{\lambda_0 z} \right)^2 \right] = z\left[1 + \left(\frac{z_0}{z} \right)^2 \right]$$

$$z_0 = \frac{\pi n w_0^2}{\lambda_0}$$

Gaussian Beams and ABCD Law

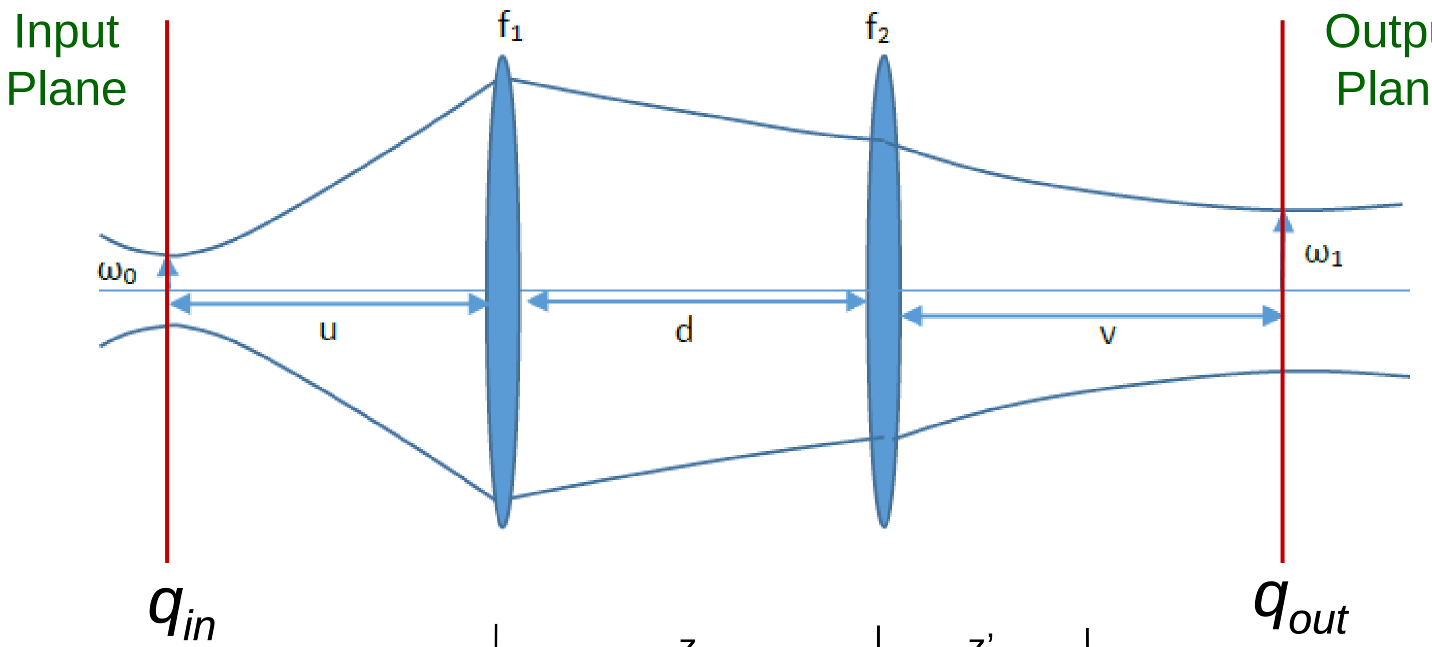


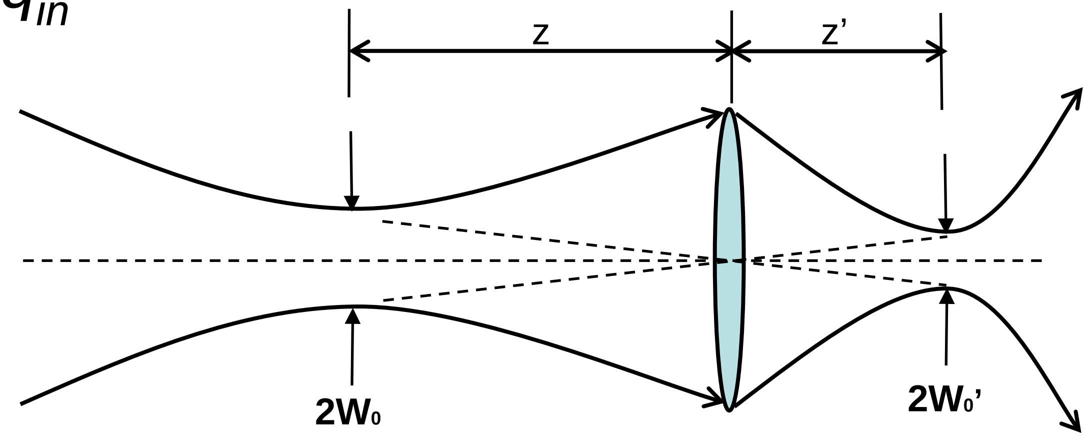
$$q_{out} = \frac{Aq_{in} + B}{Cq_{in} + D}$$

$$\frac{1}{q_{out}} = \frac{C + D(1/q_{in})}{A + B(1/q_{in})}$$

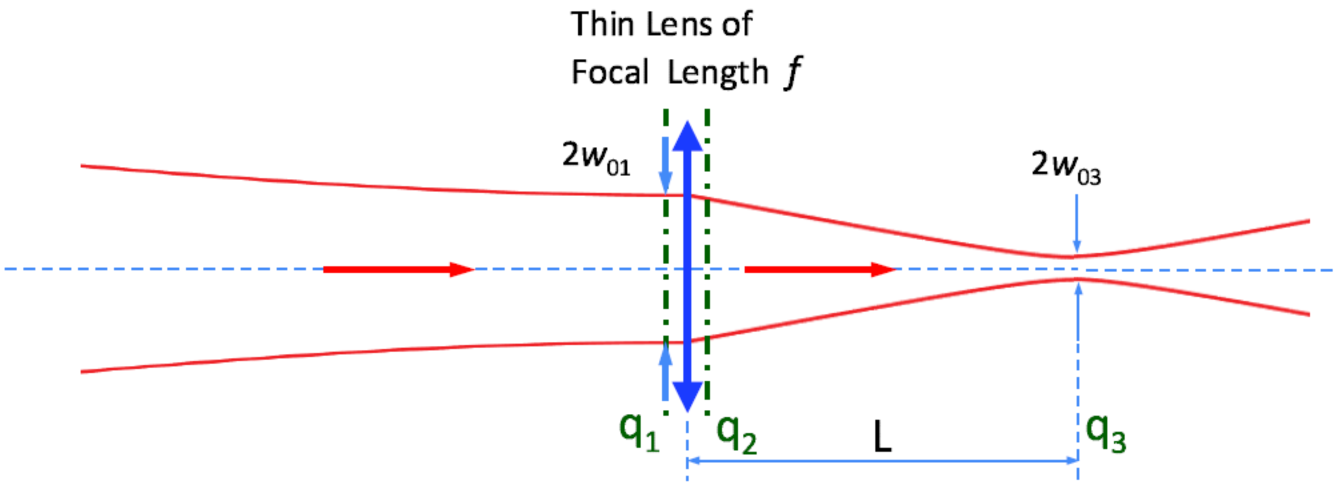
$$\frac{1}{q(z)} = \frac{z}{z^2 + z_0^2} - j\frac{z_0}{z^2 + z_0^2} = \frac{1}{R(z)} - j\frac{\lambda_0}{\pi n w^2(z)}$$

Gaussian Beams and ABCD Law Examples





Example: Gaussian Beams and ABCD Law Examples



$A = 1,\ B = 0,\ C = -1/f,$ and $D = 1.$

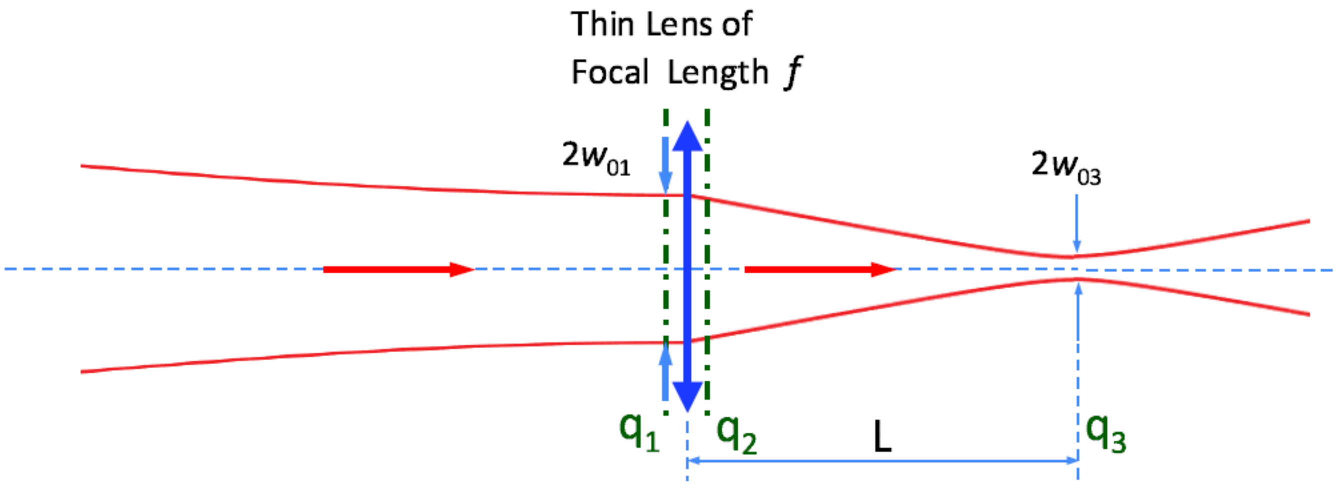
$$\frac{1}{q_2} = \frac{C + D(1/q_1)}{A + B(1/q_1)} = -\frac{1}{f} + \frac{1}{q_1} = -\frac{1}{f} - j\frac{\lambda_0}{\pi n w_{01}^2} = a + jb,$$

$$a = -1/f \text{ and } b = -\lambda_0/\pi n w_{01}^2 = -1/z_{01},$$

$A = 1,\ B = L,\ C = 0,$ and $D = 1.$

$$\begin{aligned}\frac{1}{q_3} &= \frac{C + D(1/q_2)}{A + B(1/q_2)} = \frac{1/q_2}{1 + L/q_2} = \frac{a + jb}{(1 + aL) + jbL} = \\ &= \frac{a(1 + aL) + Lb^2}{(1 + aL)^2 + L^2b^2} - j\frac{-b}{(1 + aL)^2 + L^2b^2} = \\ &= \frac{1}{R_3} - j\frac{\lambda_0}{\pi n w_{03}^2}.\end{aligned}$$

Example: Gaussian Beams and ABCD Law Examples



$$a(1 + aL) + Lb^2 = 0 \implies L = \frac{-a}{a^2 + b^2} \implies$$

$$L = \frac{f}{1 + \left(\frac{f}{z_{01}}\right)^2}.$$

$$w_{03} = w_{01} \frac{\frac{f}{z_{01}}}{\left[1 + \left(\frac{f}{z_{01}}\right)^2\right]^{1/2}}.$$

<u>Numerical Example</u>

f = 10cm

10mW He-Ne, $\lambda = 0.6328\mu m$

w_{01} = 1cm

z_{01} = 496.459m ≈ 0.5km

L ≈ 10cm

w_{03} = 2.0143μm

P_3 ≈ 78.45 kW/cm^2