Physical & Electromagnetic Optics: Diffraction Gratings

Optical Engineering

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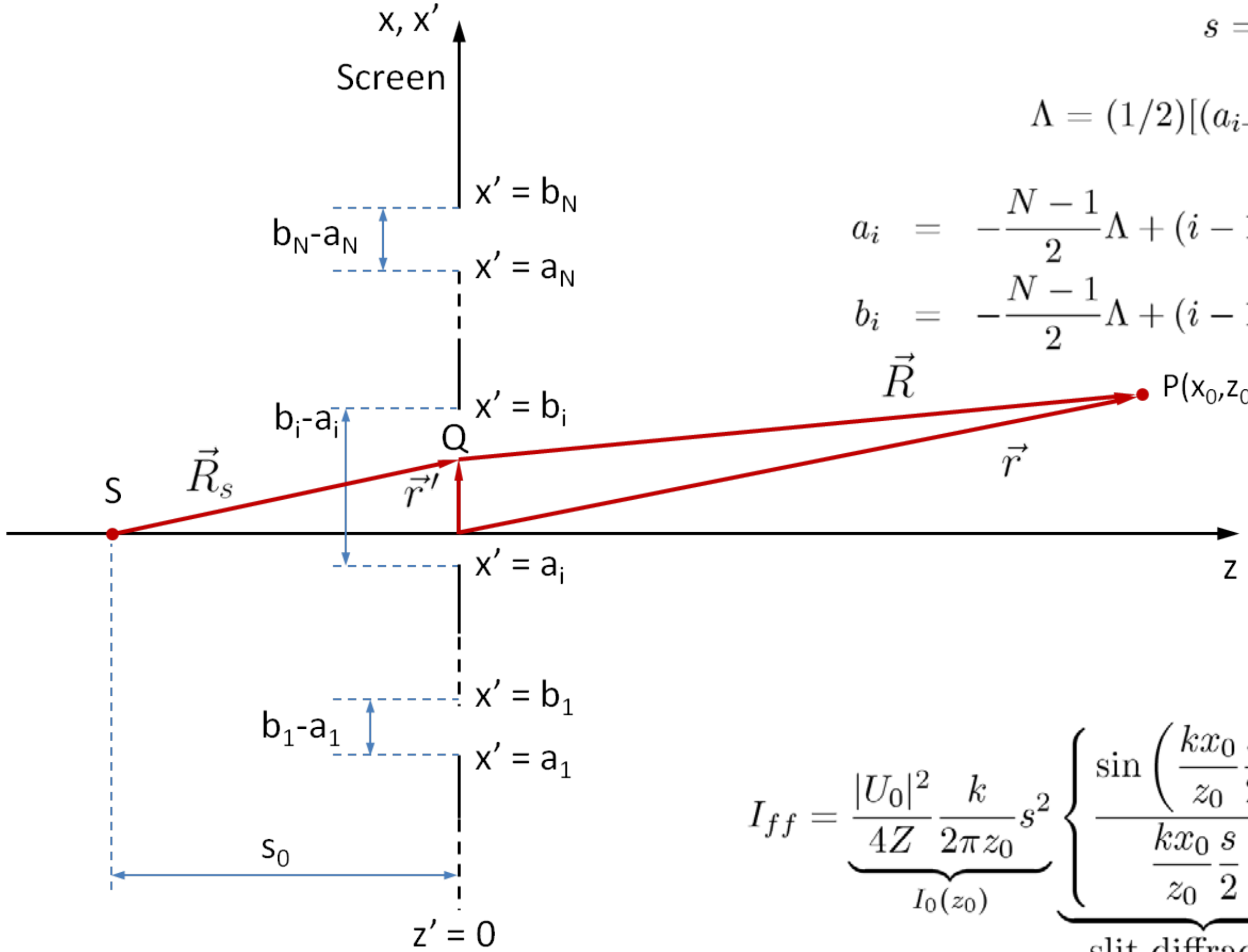
Multiple 1D-Slit Diffraction (General Case)

Periodic Slits



$$s = b_i - a_i$$

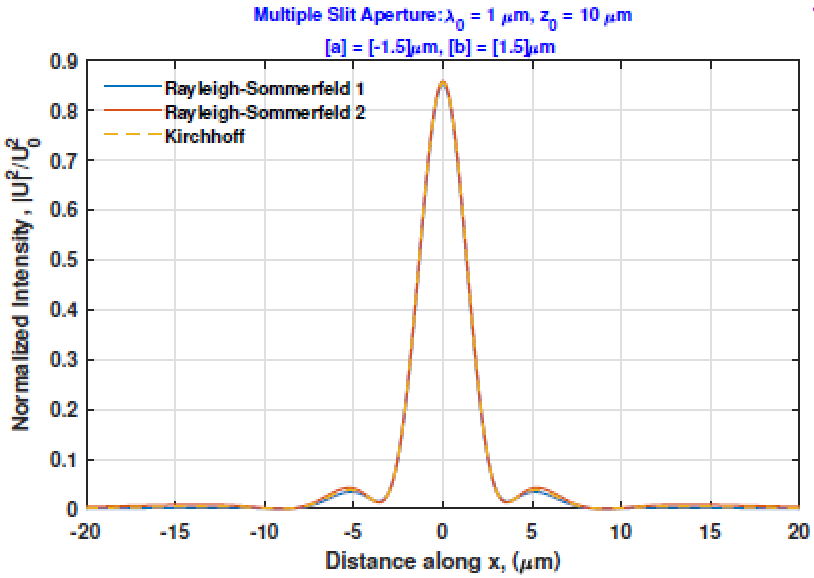
$$\Lambda = (1/2)[(a_{i+1} + b_{i+1}) - (a_i + b_i)]$$

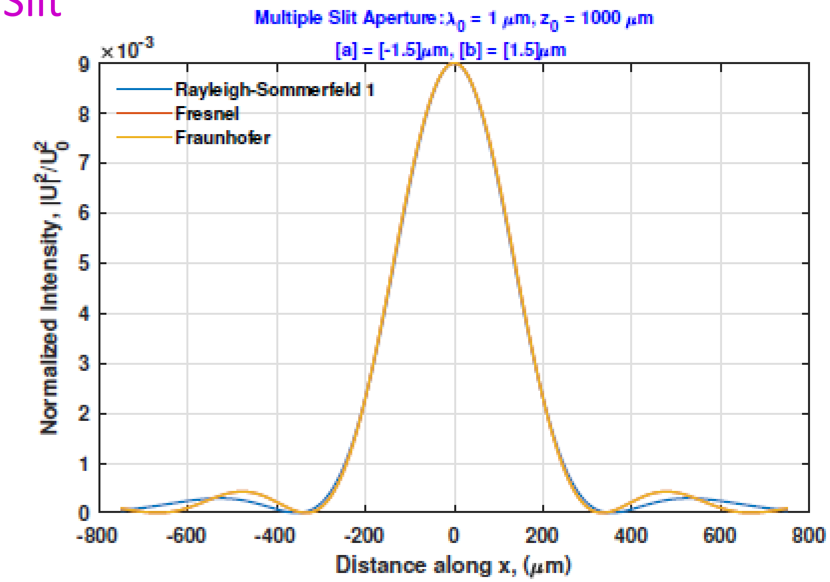
$$\begin{aligned} a_i &= -\frac{N-1}{2}\Lambda + (i-1)\Lambda - \frac{s}{2}, \\ b_i &= -\frac{N-1}{2}\Lambda + (i-1)\Lambda + \frac{s}{2}, \qquad \text{for} \quad i = 1, 2, \cdots, N \end{aligned}$$

$$I_{ff} = \underbrace{\frac{|U_0|^2}{4Z}\frac{k}{2\pi z_0}s^2}_{I_0(z_0)} \underbrace{\left\{ \frac{\sin\left(\frac{kx_0}{z_0}\frac{s}{2}\right)}{\frac{kx_0}{z_0}\frac{s}{2}} \right\}^2}_{\text{slit diffraction}} \underbrace{\left\{ \frac{\sin\left(N\frac{kx_0}{z_0}\frac{\Lambda}{2}\right)}{\sin\left(\frac{kx_0}{z_0}\frac{\Lambda}{2}\right)} \right\}^2}_{\text{array factor}}$$

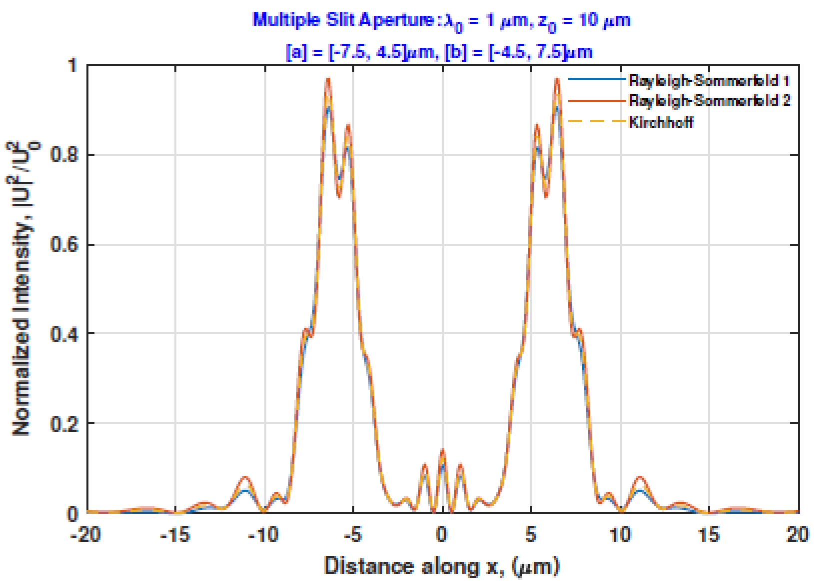
Multiple Slit Diffraction

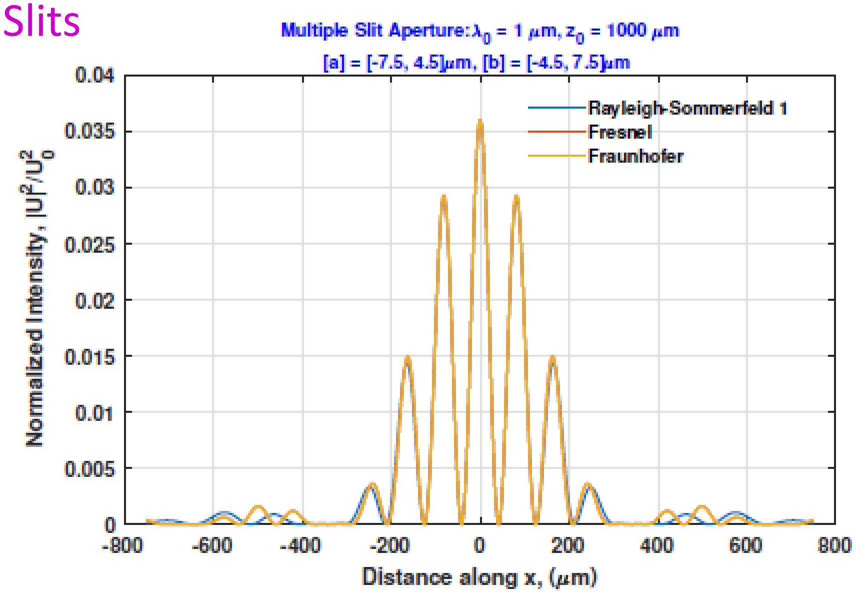
Single Slit





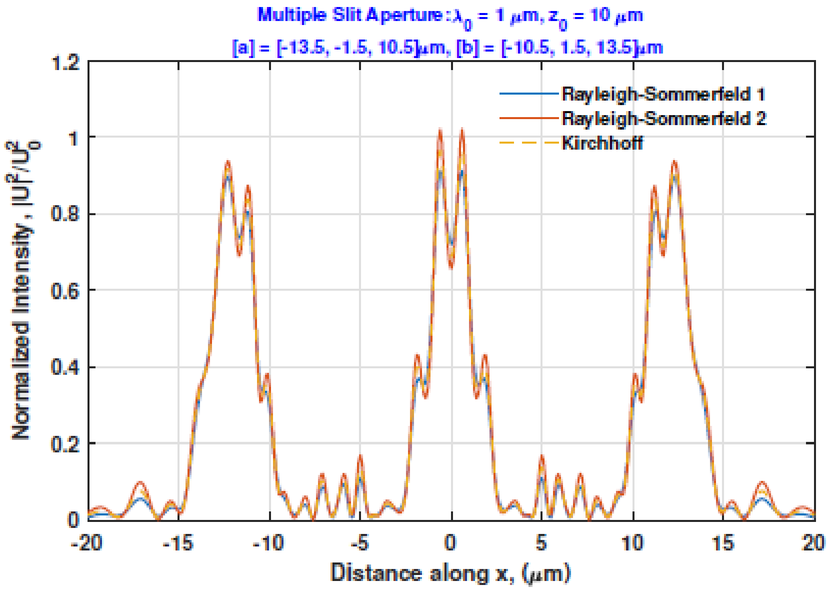
Two Slits

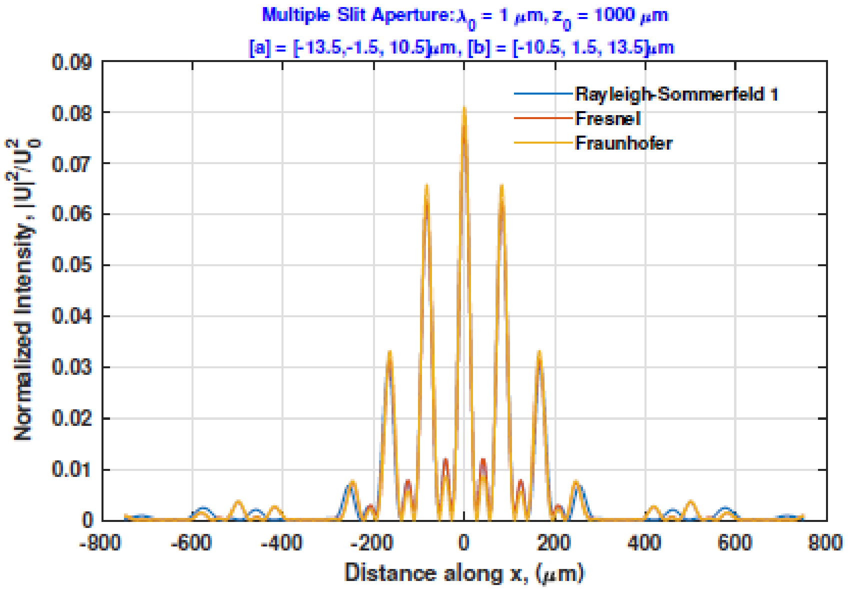




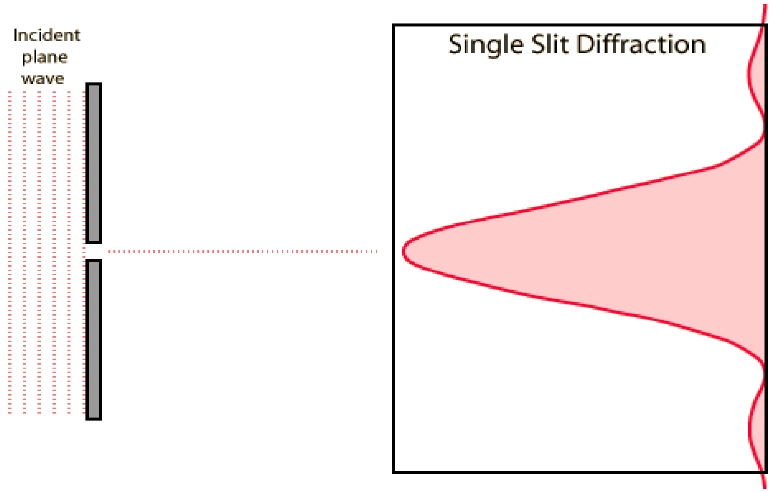
Multiple Slit Diffraction

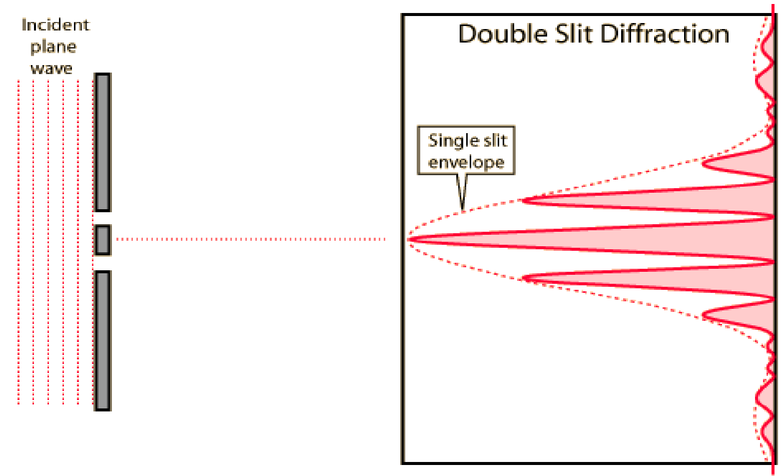
Three Slits

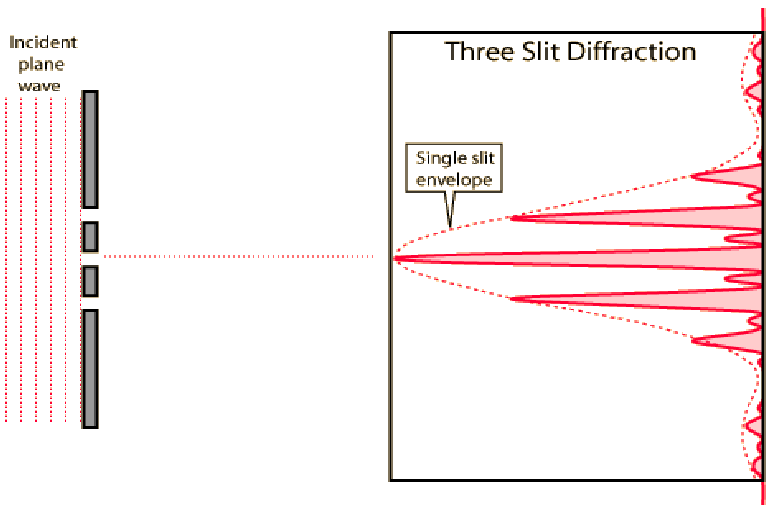


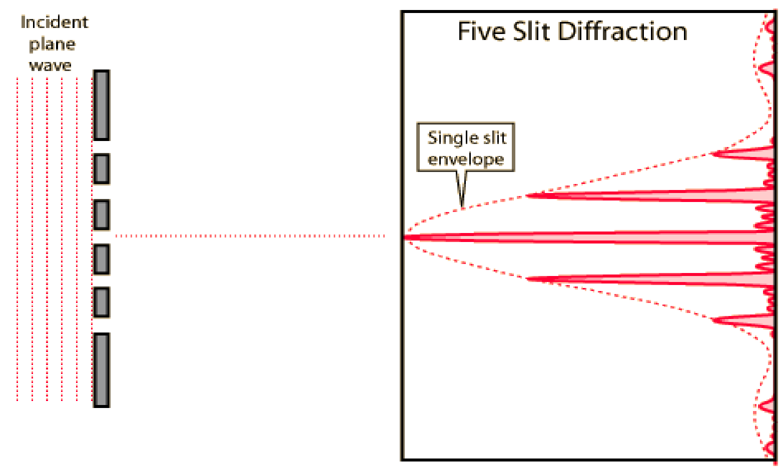


Multiple Slit Diffraction (Fraunhofer Approximation)



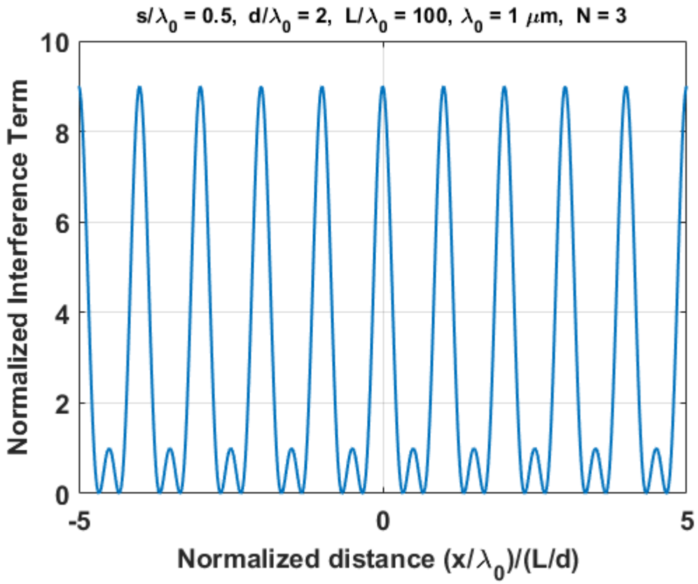


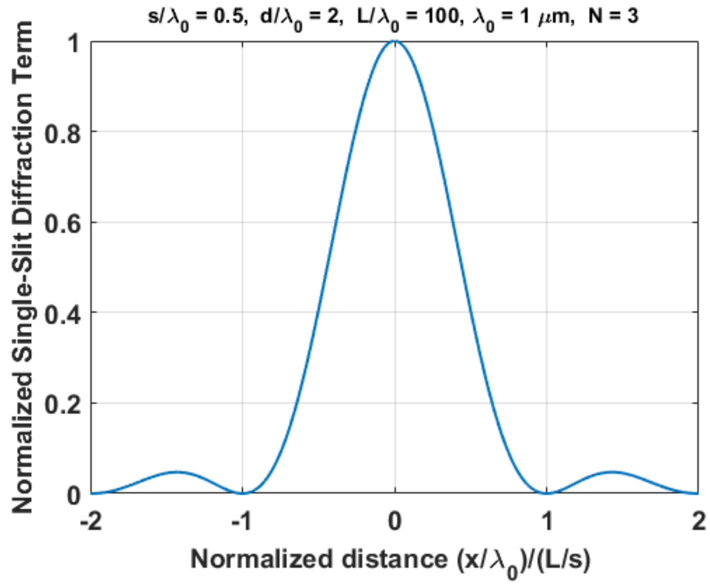


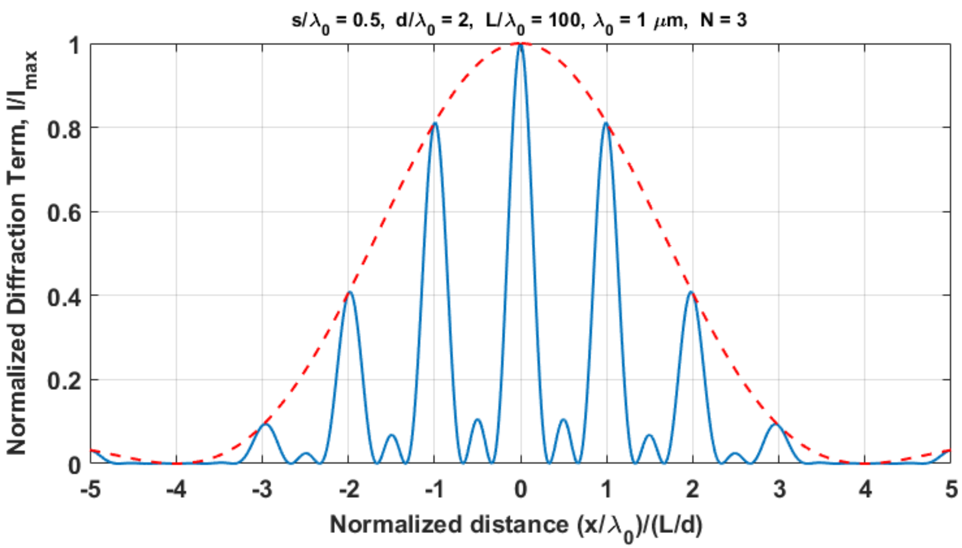


http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/mulslid.html#c2

Multiple Slit Diffraction (N=3) – Fraunhofer Approximation

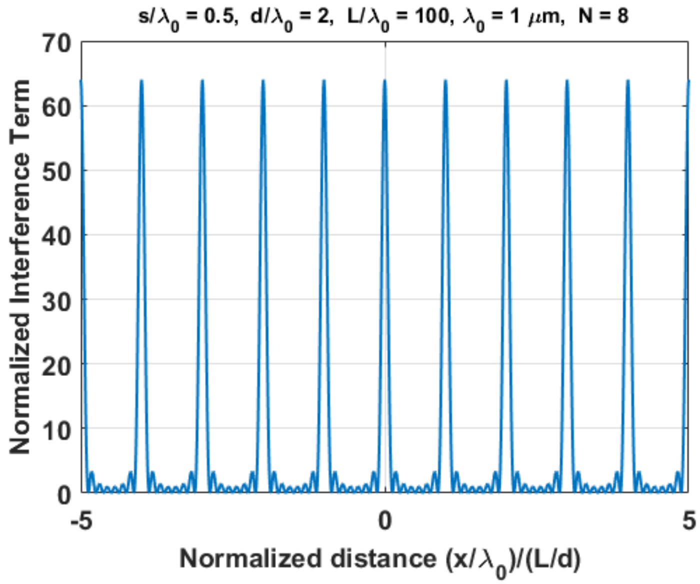


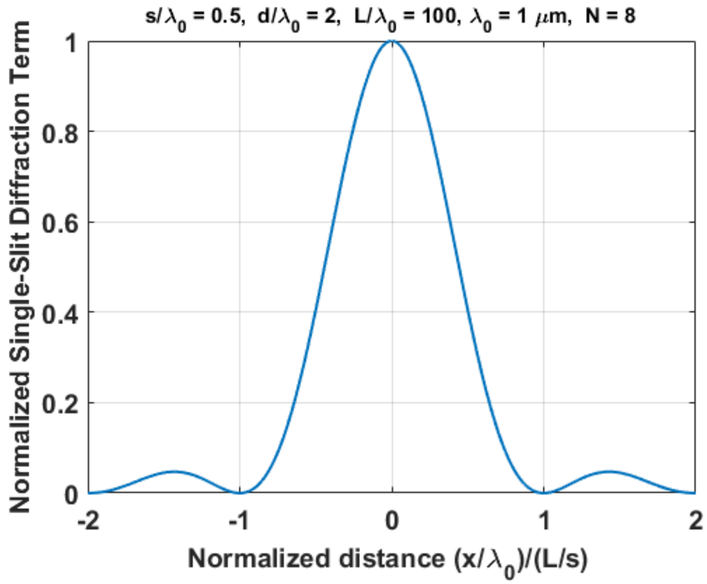


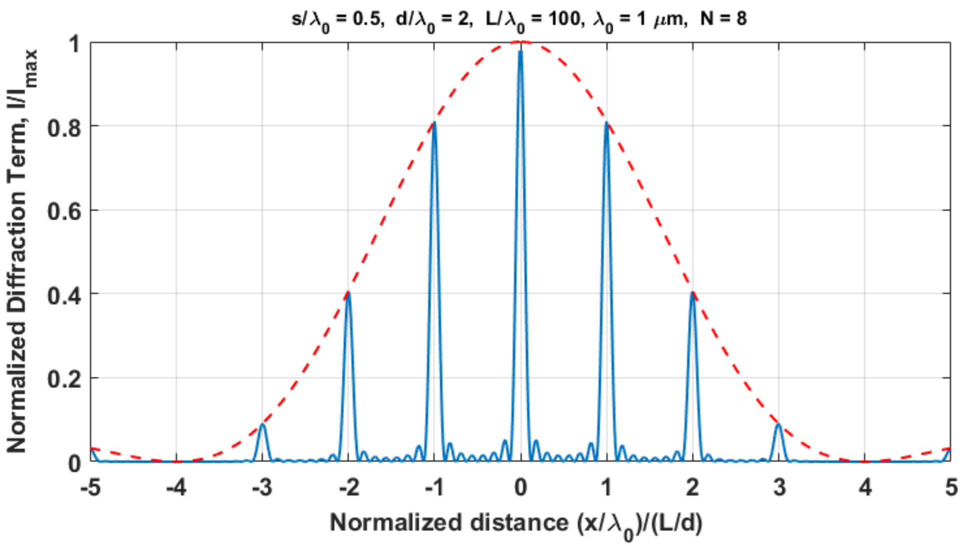


$$I_{ff} = \underbrace{\frac{|U_0|^2}{4Z}\frac{k}{2\pi z_0}s^2}_{I_0(z_0)} \underbrace{\left\{\frac{\sin\left(\frac{kx_0}{z_0}\frac{s}{2}\right)}{\frac{kx_0}{z_0}\frac{s}{2}}\right\}^2}_{\text{slit diffraction}} \underbrace{\left\{\frac{\sin\left(N\frac{kx_0}{z_0}\frac{\Lambda}{2}\right)}{\sin\left(\frac{kx_0}{z_0}\frac{\Lambda}{2}\right)}\right\}^2}_{\text{array factor}}$$

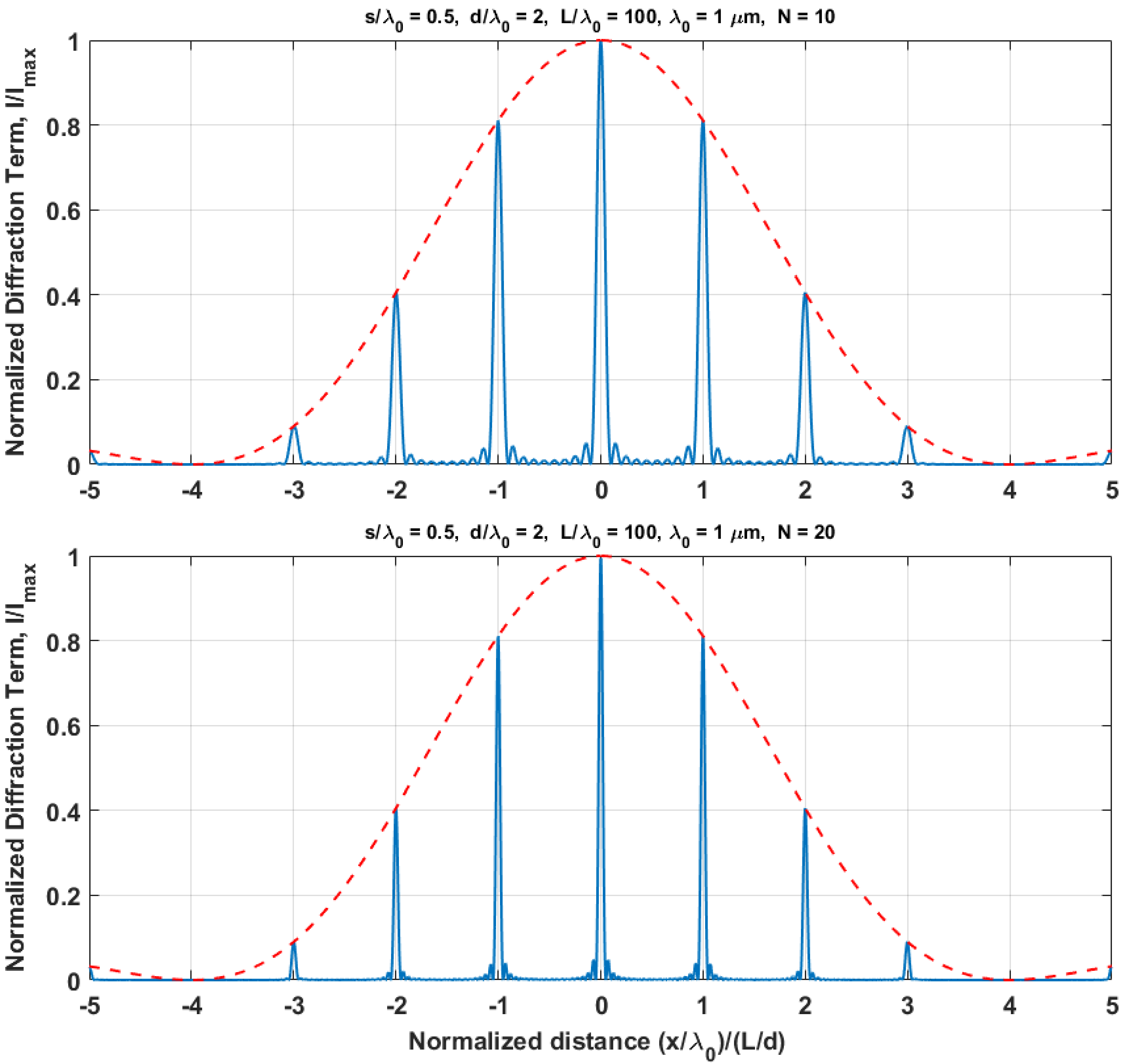
Multiple Slit Diffraction (N=8) – Fraunhofer Approximation



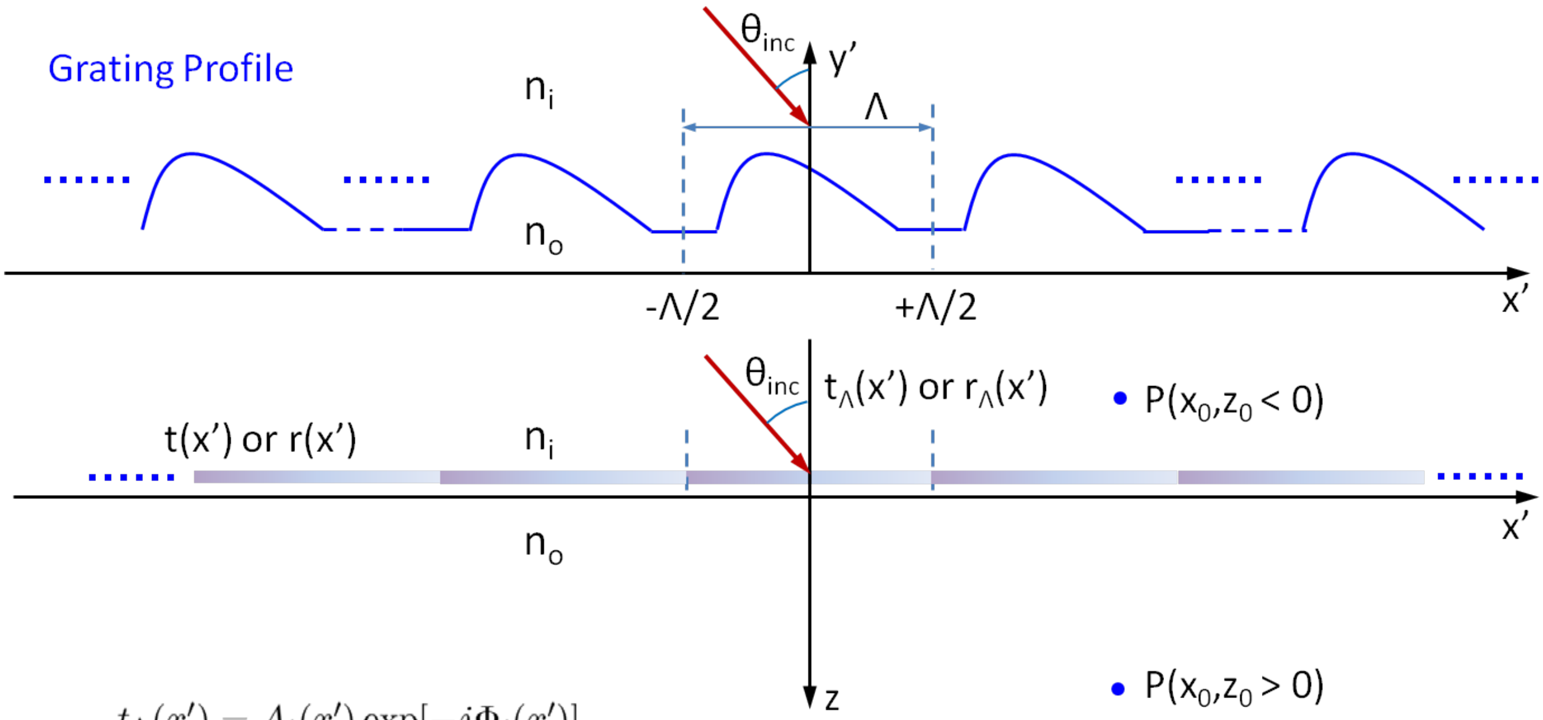




Multiple Slit Diffraction (N=10,20) – Fraunhofer Approximation



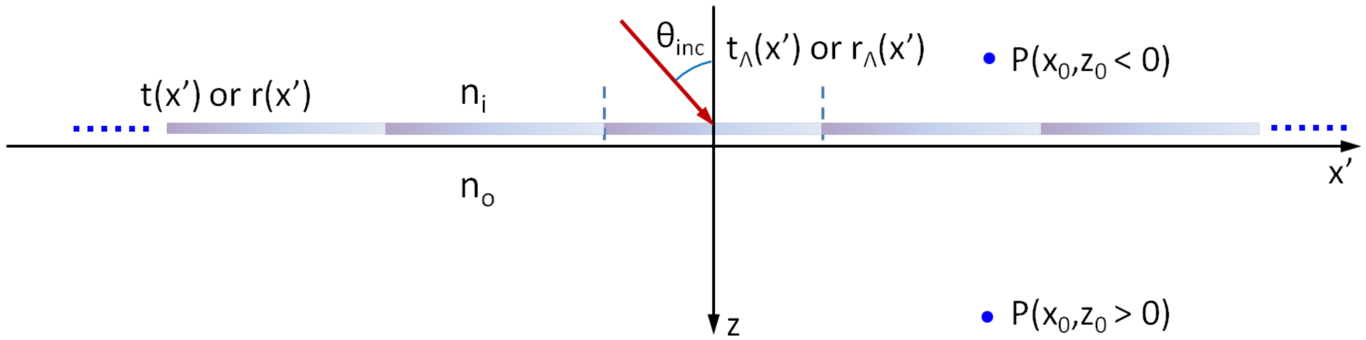
Scalar Theory of Grating Diffraction – Transmittance Approach



$$t_\Lambda(x') = A_t(x') \exp[-j\Phi_t(x')]$$

$$\begin{aligned} t(x') &= t(\vec{k}_{inc}) \sum_{m=-\infty}^{+\infty} t_m \exp[-jmKx'], \\ t_m &= \frac{1}{\Lambda}\mathcal{F}\{t_\Lambda(x')\}\Big|_{k_x} = \frac{1}{\Lambda} T_\Lambda(mK), \end{aligned}$$

$$U_{inc}(x', z) = U_0 \exp[-jk \sin\theta_{inc} x'] \exp[-jk\cos\theta_{inc} z].$$



Plane Wave Spectrum Approach

$$U(x_0, z_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{U}_t(k_x; z=0) e^{-j(k_x x_0 + k_z z_0)} dk_x, \qquad \text{where,}$$

$$\tilde{U}_t(k_x; z=0) = \mathcal{F}\{U_{inc}(x', z=0) t(x')\}\Big|_{k_x} = \mathcal{F}\{U_0 e^{-jk \sin\theta_{inc} x'} \sum_{m=-\infty}^{+\infty} t_m e^{-jmKx'}\}$$

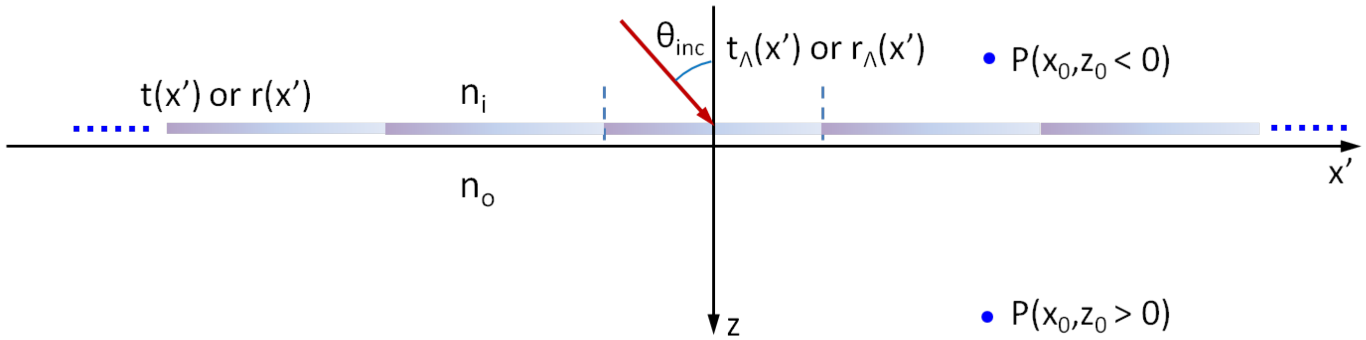
$$= U_0 \sum_{m=-\infty}^{+\infty} t_m 2\pi \delta\left(k_x - mK - k\sin\theta_{inc}\right) = 2\pi U_0 \sum_{m=-\infty}^{+\infty} t_m \delta\left(k_x - k_{xm}\right),$$

where $k_{xm} = mK + k\sin\theta_{inc}$.

$$U(x_0, z_0) = U_0 \sum_{m=-\infty}^{+\infty} t_m \exp[-jk_{xm}x_0] \exp[-jk_{zm}z_0], \qquad \text{where,}$$

$$k_{zm} = \begin{cases} \sqrt{k^2 - k_{xm}^2}, & \text{when} \quad k \geq k_{xm}, \\ -j\sqrt{k_{xm}^2 - k^2}, & \text{when} \quad k < k_{xm}. \end{cases}$$

Scalar Theory of Grating Diffraction – Transmittance Approach



$\text{comb}(x;\Lambda) = \sum_{m=-\infty}^{+\infty} \delta(x - m\Lambda)$

Finite-Number-of-Periods

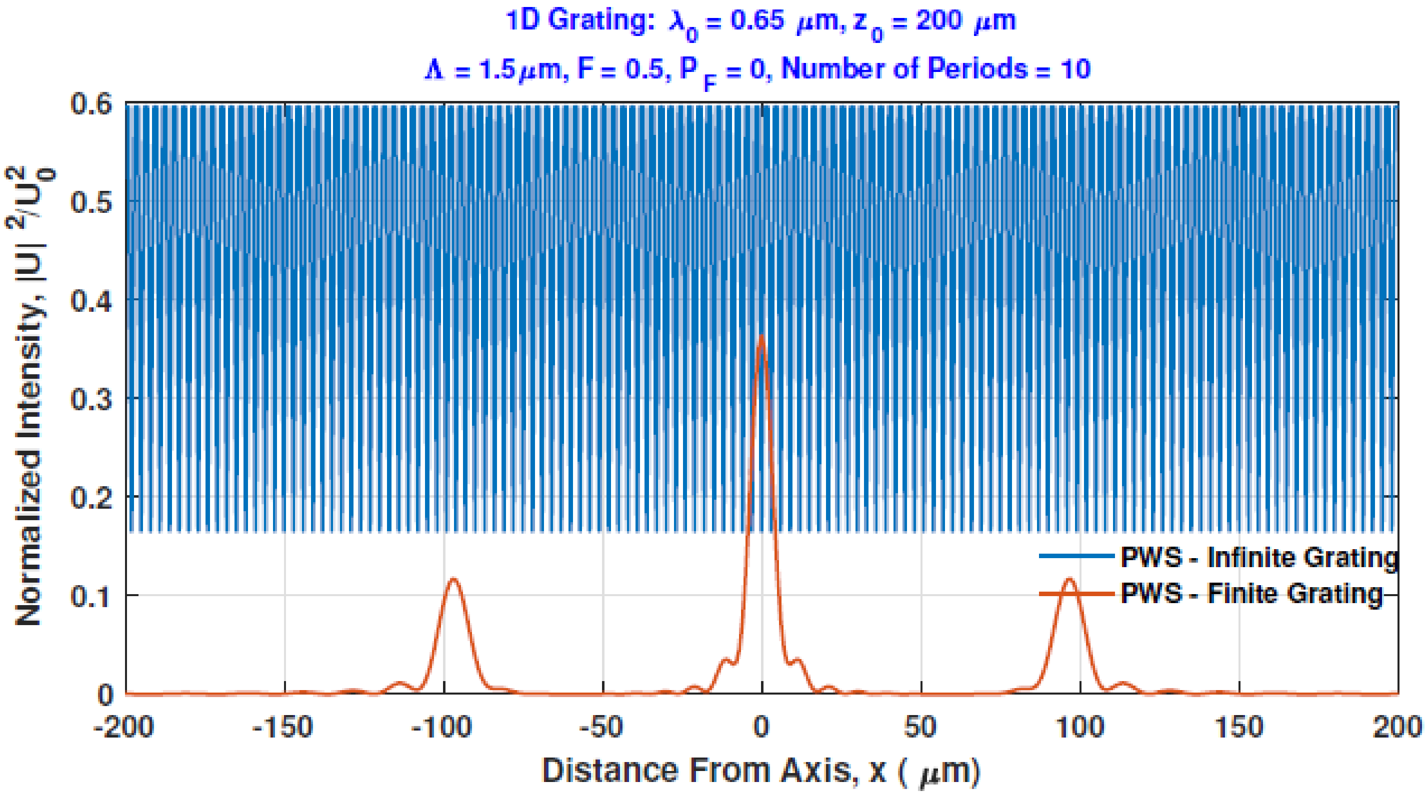
$$t(x') = [t_\Lambda(x') * \text{comb}(x';\Lambda)]\, \text{rect}\left(\frac{x'}{L}\right)$$

$$\begin{aligned} \mathcal{F}\{t(x')\}\Big|_{k_x} &= \frac{1}{2\pi}\left[T_\Lambda(k_x)\frac{2\pi}{\Lambda}\sum_{m=-\infty}^{+\infty}\delta(k_x - mK)\right] * \left[L\ \text{sinc}\left(\frac{k_x L}{2\pi}\right)\right] = \\ &= L\sum_{m=-\infty}^{+\infty}\frac{1}{\Lambda}T_\Lambda(mK)\ \text{sinc}\left(\frac{(k_x - mK)L}{2\pi}\right). \end{aligned}$$

$$\begin{aligned} U(x_0, z_0) &= \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{U}_t(k_x; z=0)e^{-j(k_x x_0 + k_z z_0)}dk_x, \qquad \text{where,} \\ \tilde{U}_t(k_x; z=0) &= \mathcal{F}\{U_{inc}(x', z=0)t(x')\}\Big|_{k_x} = U_0 L\sum_{m=-\infty}^{+\infty}\frac{1}{\Lambda}T_\Lambda(k_{xm})\ \text{sinc}\left(\frac{(k_x - k_{xm})L}{2\pi}\right) \\ &= U_0 L\sum_{m=-\infty}^{+\infty} t_m\ \text{sinc}\left(\frac{(k_x - k_{xm})L}{2\pi}\right), \qquad \text{where}\ \ k_{xm} = mK + k\sin\theta_{inc}. \end{aligned}$$

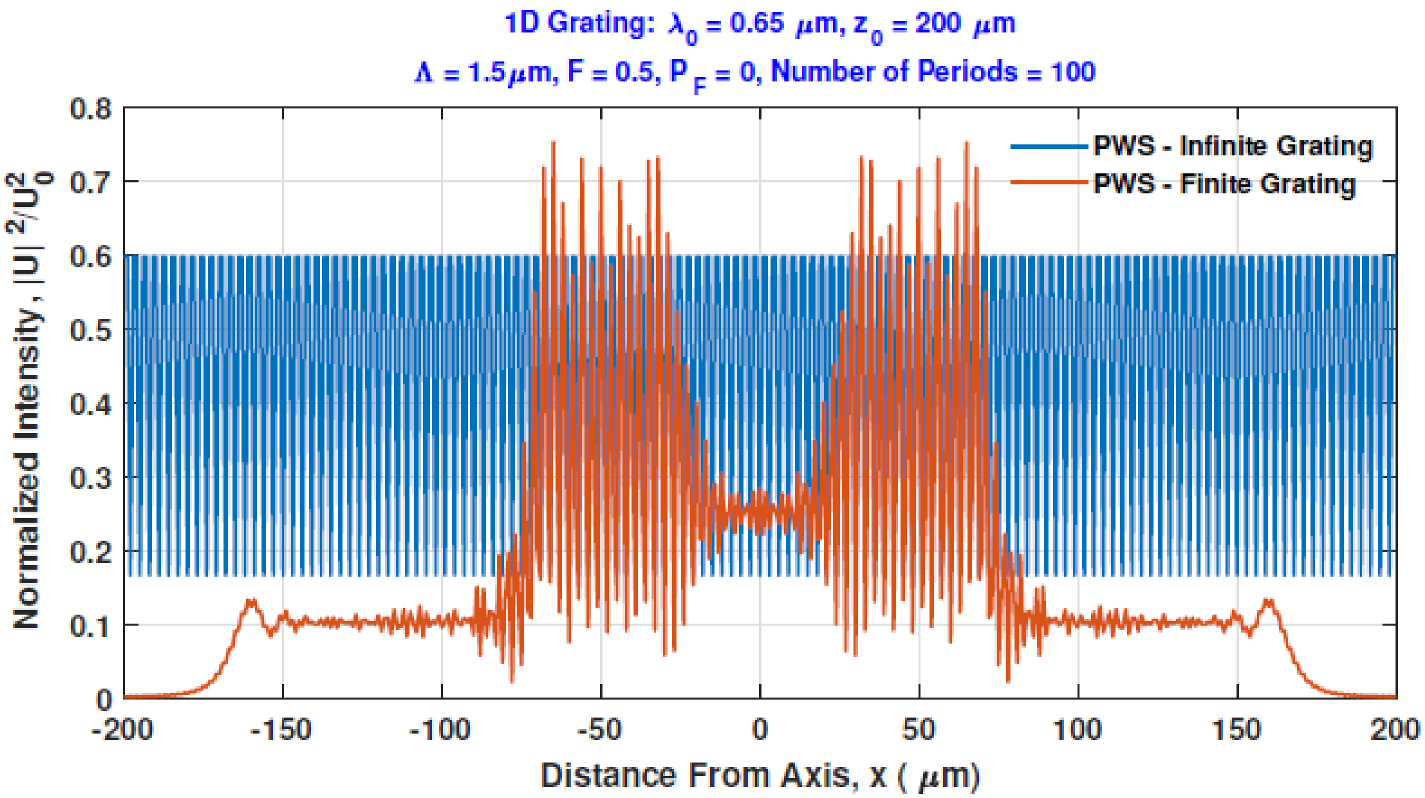
Scalar Theory of Grating Diffraction – Transmittance Approach

Multiple-Slit Grating



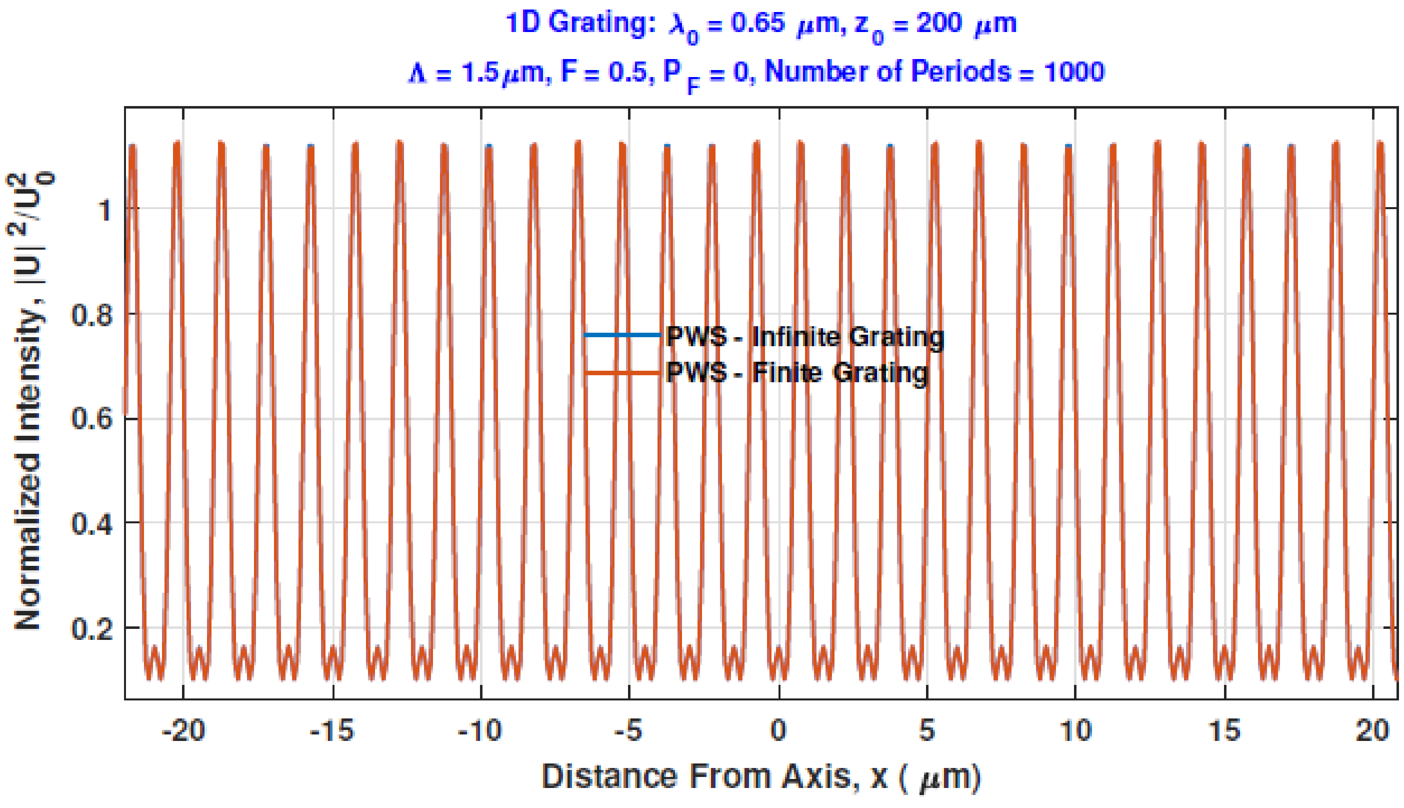
Scalar Theory of Grating Diffraction – Transmittance Approach

Multiple-Slit Grating



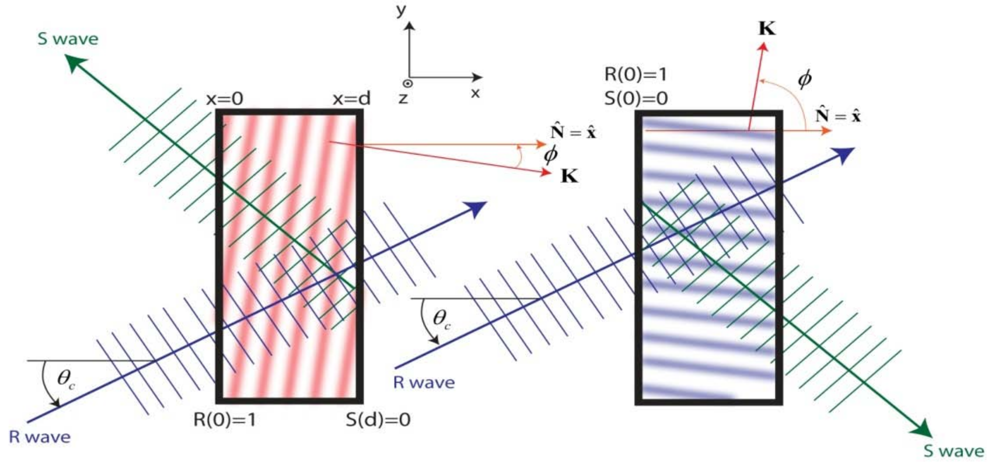
Scalar Theory of Grating Diffraction – Transmittance Approach

Multiple-Slit Grating



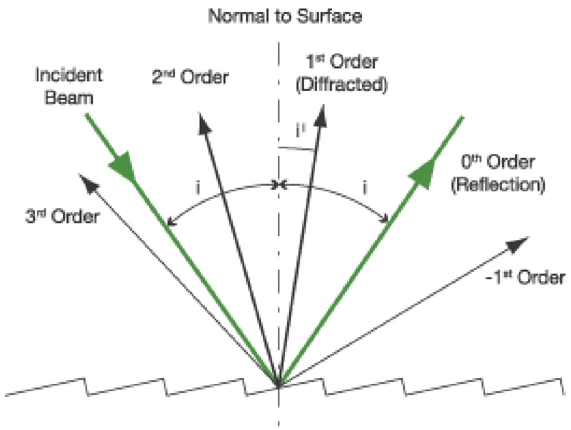
Diffraction Gratings

Example of a reflecting and transmitting holographic (volume) gratings



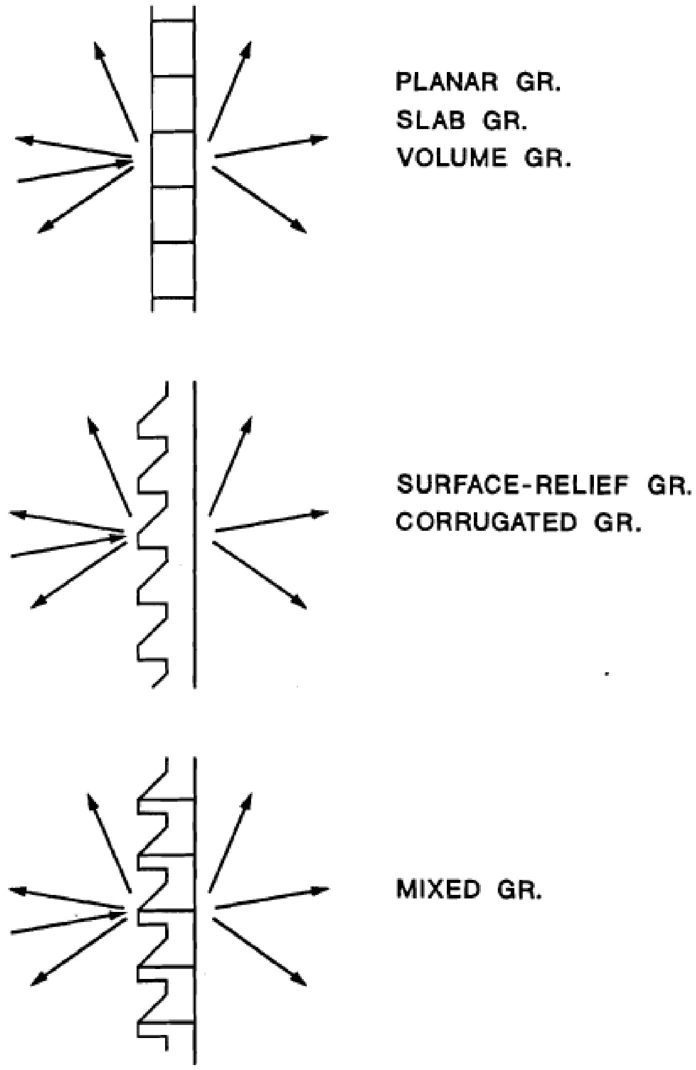
http://www.intechopen.com/books/holography-basic-principles-and-contemporary-applications/understanding-diffraction-in-volume-gratings-and-holograms

Example of a reflecting surface-relief grating



http://www.andor.com/learning-academy/diffraction-gratings-understanding-diffraction-gratings-and-the-grating-equation

Diffraction Grating Classification



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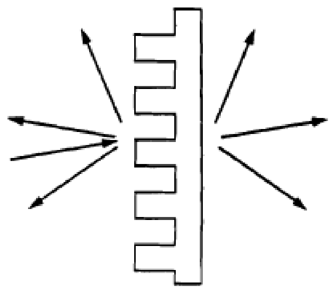
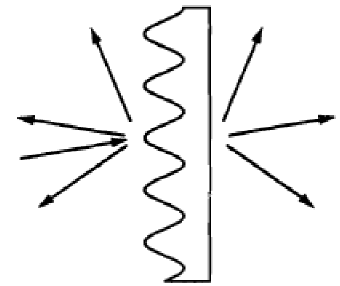
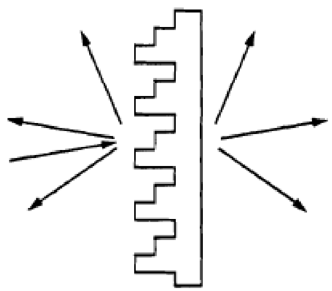
Surface-Relief Grating Types



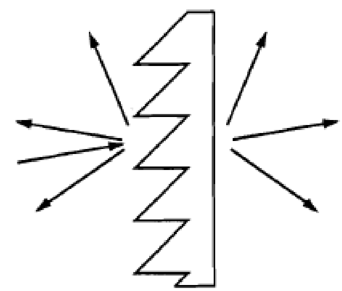
RECTANGULAR GR.
BINARY GR.
LAMELLAR GR.



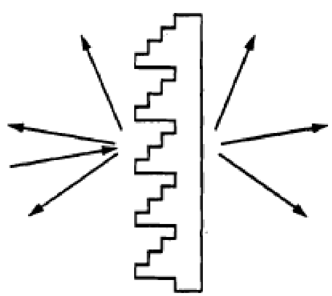
SINUSOIDAL GR.



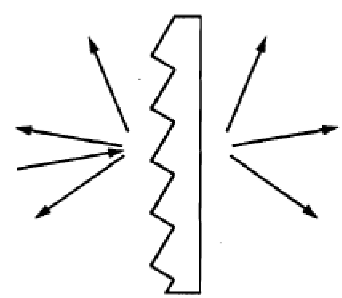
THREE-LEVEL GR.



SAWTOOTH GR.
BLAZED GR.



STAIRSTEP GR.
MULTILEVEL GR.
BINARY MASK GR.



ECHLETTE GR.

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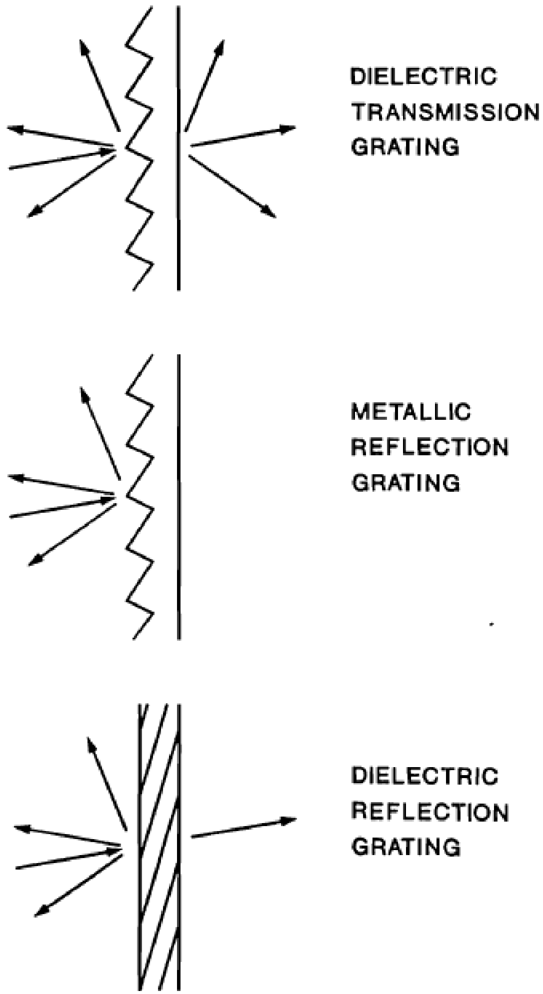
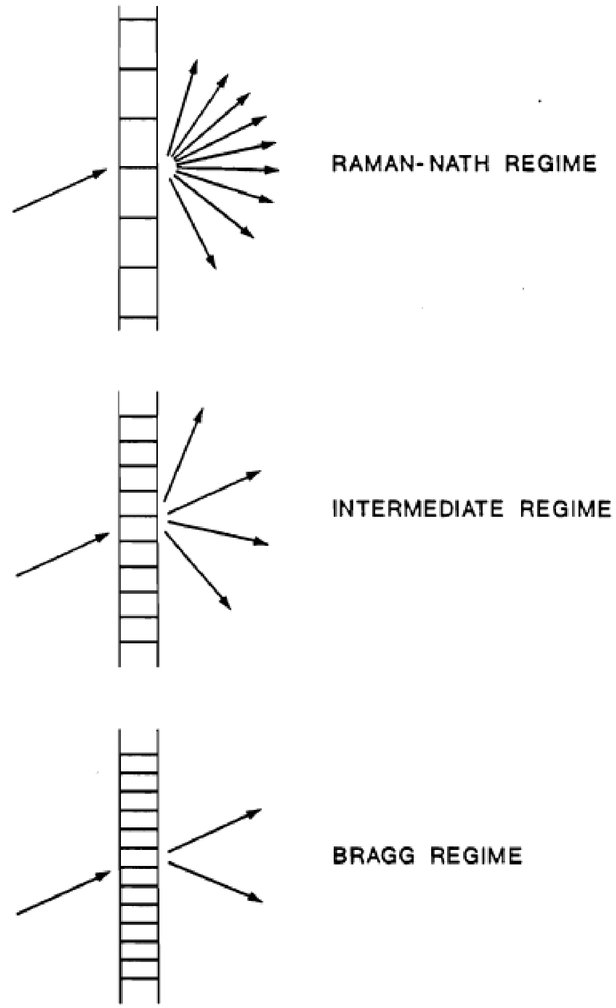
Diffraction Grating Classification

Transmission or Reflection



Classification based on Regime



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Diffraction by Gratings

- Acousto-Optics
- Diffractive Optics
- Integrated Optics
- Holography
- Optical Computing
- Optical Signal Processing
- Spectroscopy

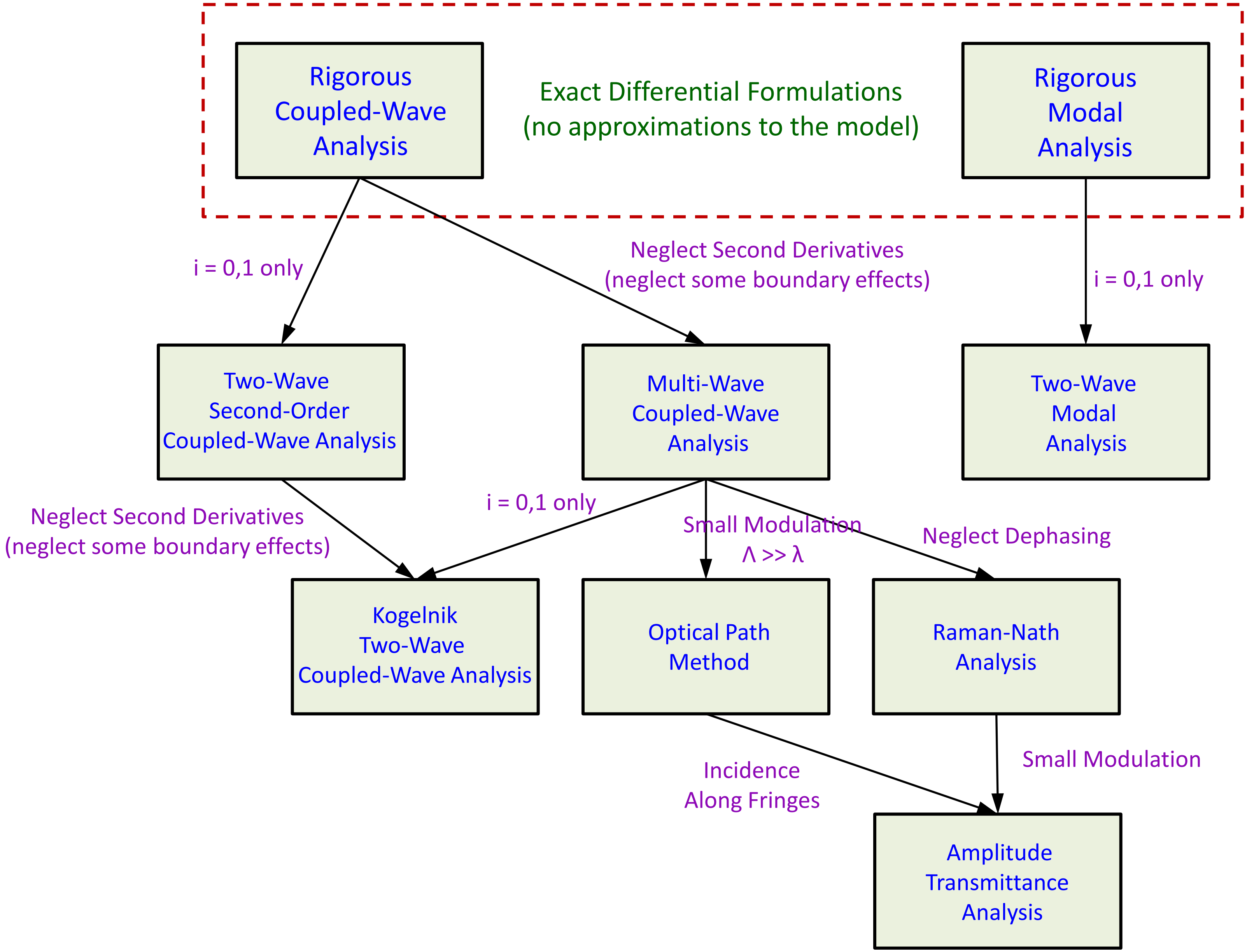
Grating Applications

- Acoustic-Wave Generation
- Antireflection Surfaces
- Beam Coding, Coupling, Detection, etc.
- Grating Lenses
- Grating Scanners
- Head-Up Displays
- Holographic Optical Elements
- Interferometry
- Instrumentation
- Mode Conversion
- Multiplexing / Demultiplexing
- Modulation / Switching
- Optical Interconnections
- Photonic Crystal Devices
- Spectral Analysis

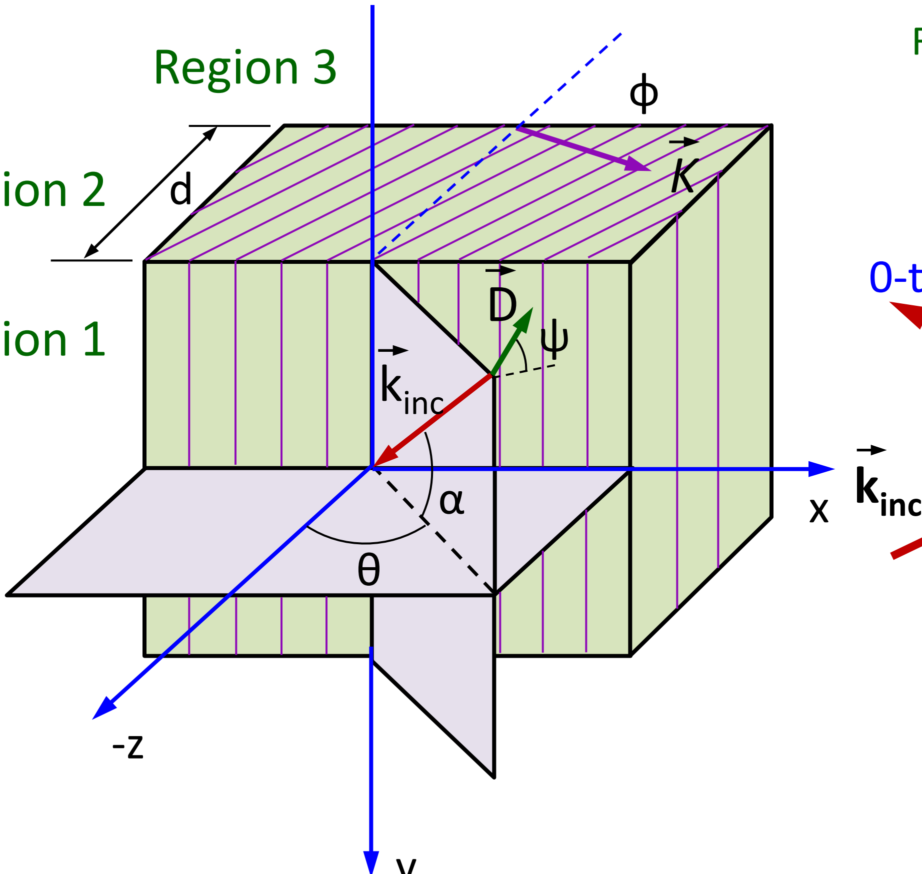
Methods Of Analysis Of Gratings

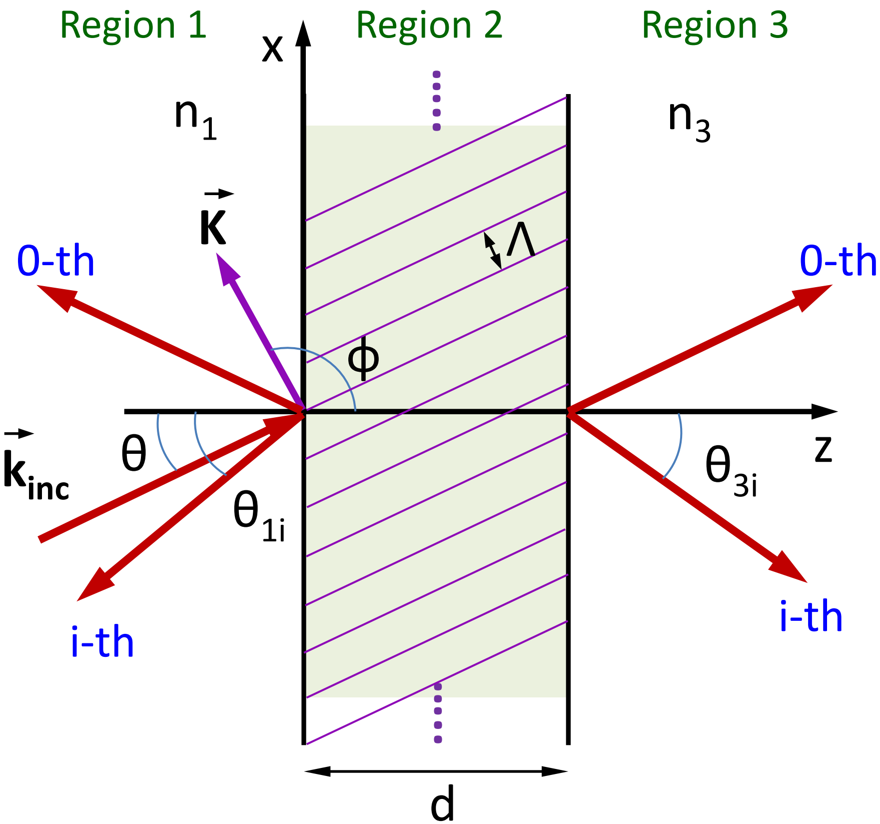
- Integral Methods
  - Finite Elements
  - Boundary Elements
- Differential Methods
  - Exact Methods
    - - Rigorous Coupled Wave Analysis (RCWA)
    - - Modal Analysis
  - Approximate Methods
    - - Two-Wave Coupled-Wave Analysis (Kogelnik's)
    - - Raman-Nath Analysis
    - - Others

Differential Grating Diffraction Analysis Hierarchy



Holographic Grating Diffraction Geometry





Electromagnetic Problem Formulation

Maxwell Equations

$$\vec{\nabla} \times \vec{E} = -j\omega\vec{B}$$

$$\vec{\nabla} \times \vec{H} = +j\omega\vec{D} + \vec{J}$$

Constitutive Relations

$$\vec{D} = \epsilon_0 \tilde{\varepsilon} \vec{E}$$

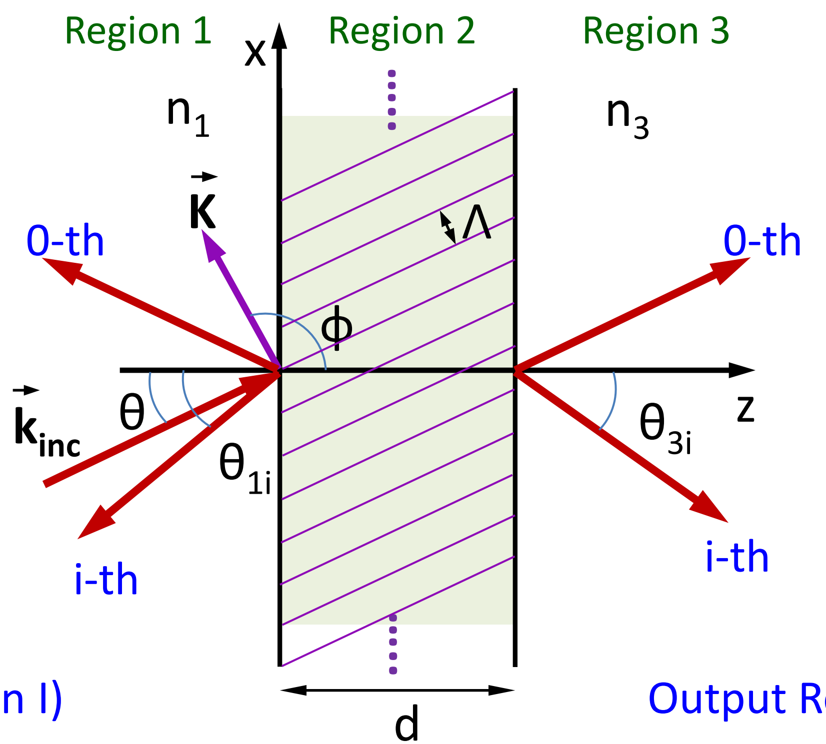
$$\vec{B} = \mu_0 \vec{H}$$

$$\vec{J} = \tilde{\sigma} \vec{E}$$

Medium Properties: Permittivity, Conductivity Tensors are Periodic

Electromagnetic Boundary Conditions: Continuity of Tangential
Electric and Magnetic Field Components

Electromagnetic Field Expansions
Rigorous Coupled Wave Analysis (RCWA)



Input Region (Region I)

$$\vec{E}_I = \vec{E}_{inc} + \sum_i \vec{R}_i \exp[-j\vec{k}_{1i} \cdot \vec{r}]$$

$$\vec{H}_I = -\frac{1}{j\omega\mu_0} \vec{\nabla} \times \vec{E}_I$$

Output Region (Region III)

$$\vec{E}_{III} = \sum_i \vec{T}_i \exp[-j\vec{k}_{3i} \cdot \vec{r}]$$

$$\vec{H}_{III} = -\frac{1}{j\omega\mu_0} \vec{\nabla} \times \vec{E}_{III}$$

Electromagnetic Field Expansions
Rigorous Coupled Wave Analysis (RCWA)

Grating Region (Region II)

$$\vec{E}_{II} = \sum_i \vec{S}_i(z) \exp[-j\vec{\sigma}_i \cdot \vec{r}] = \sum_i \vec{S}_i(z) \exp[-j(\vec{k}_{inc} - i\vec{K}) \cdot \vec{r}]$$

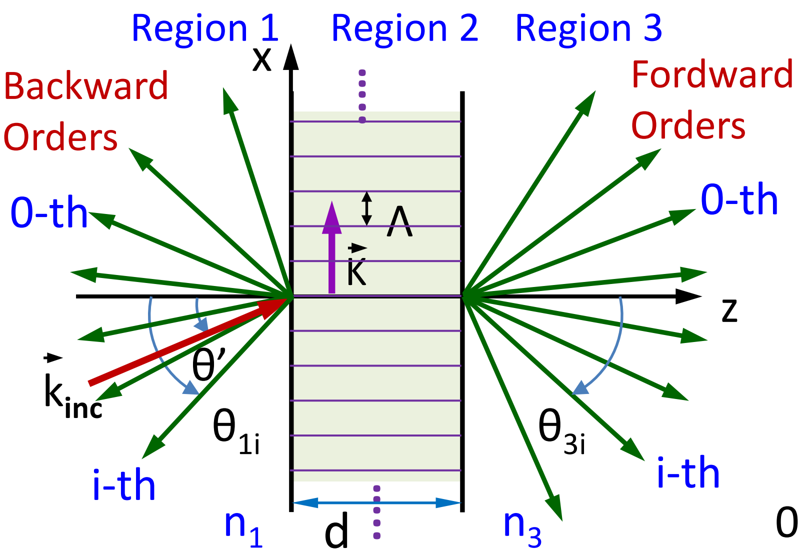
$$\vec{H}_{II} = \left(\frac{\epsilon_0}{\mu_0}\right)^{1/2} \sum_i \vec{U}_i(z) \exp[-j\vec{\sigma}_i \cdot \vec{r}] = \left(\frac{\epsilon_0}{\mu_0}\right)^{1/2} \sum_i \vec{U}_i(z) \exp[-j(\vec{k}_{inc} - i\vec{K}) \cdot \vec{r}]$$

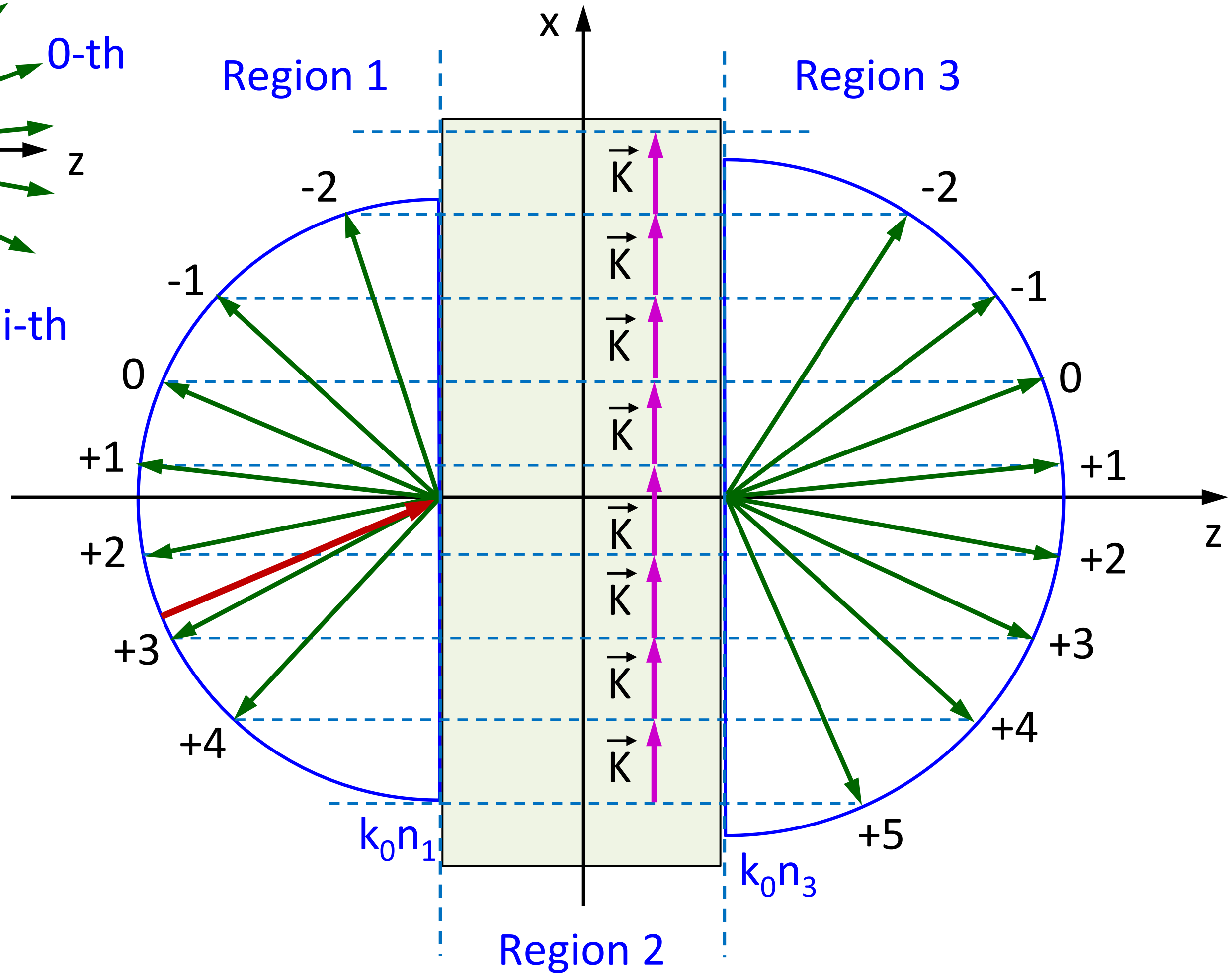
Complex Permittivity Tensor Expansions (Region II)

$$\tilde{\varepsilon} = \sum_h \tilde{\varepsilon}_h \exp[jh\vec{K} \cdot \vec{r}]$$

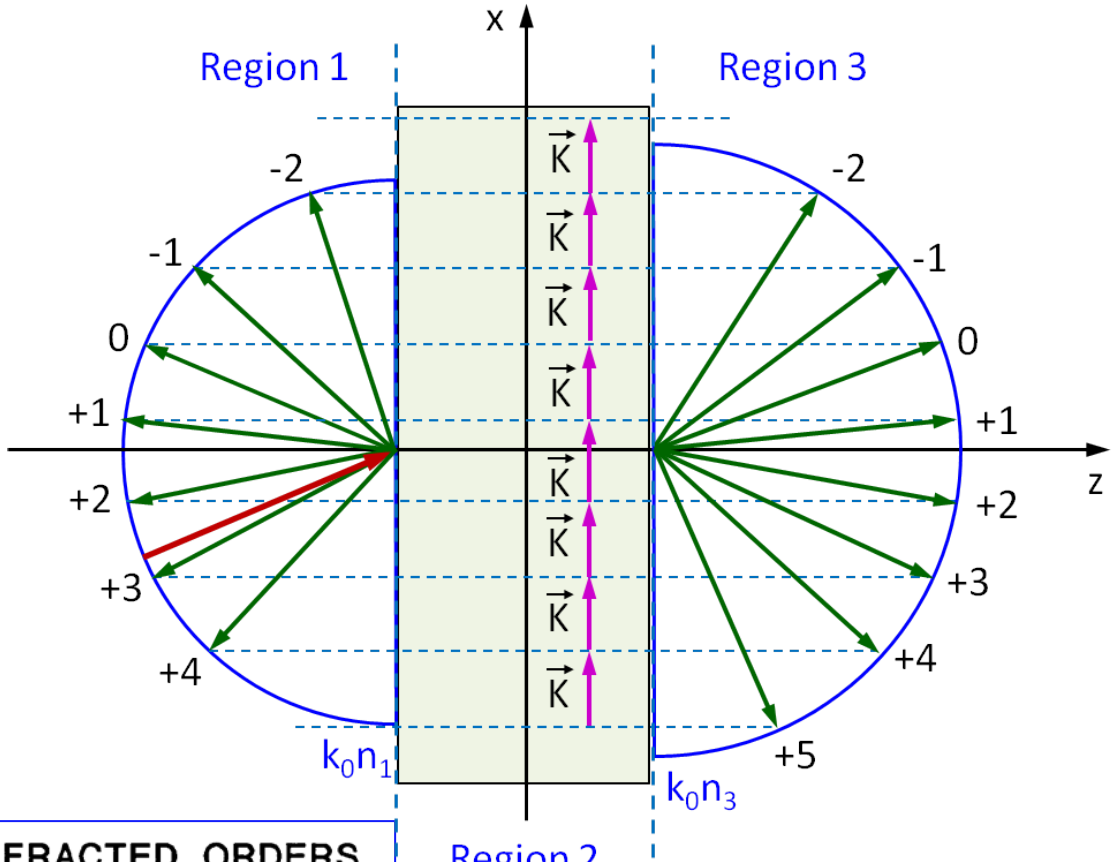
$$\tilde{\varepsilon}_h = [\tilde{\varepsilon} - j\tilde{\sigma}/\omega\epsilon_0]_h = \text{Fourier Tensor Component}$$

Floquet Condition





Grating Equation



FORWARD-DIFFRACTED ORDERS

$$n_1 \sin\theta' - n_3 \sin\theta_i'' = i\,\frac{\lambda}{\Lambda}$$

BACKWARD-DIFFRACTED ORDERS

$$\sin\theta' + \sin\theta_i' = i\,\frac{\lambda}{\Lambda n_1}$$

θ' = angle of incidence

θ_i'' = angle of forward diffraction

θ_i' = angle of backward diffraction

All are measured CCW from the normal in their respective regions.

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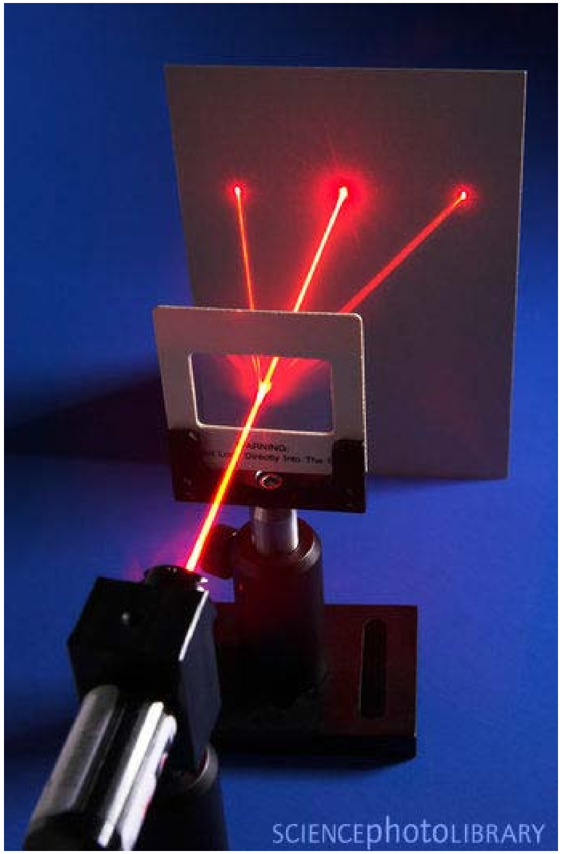
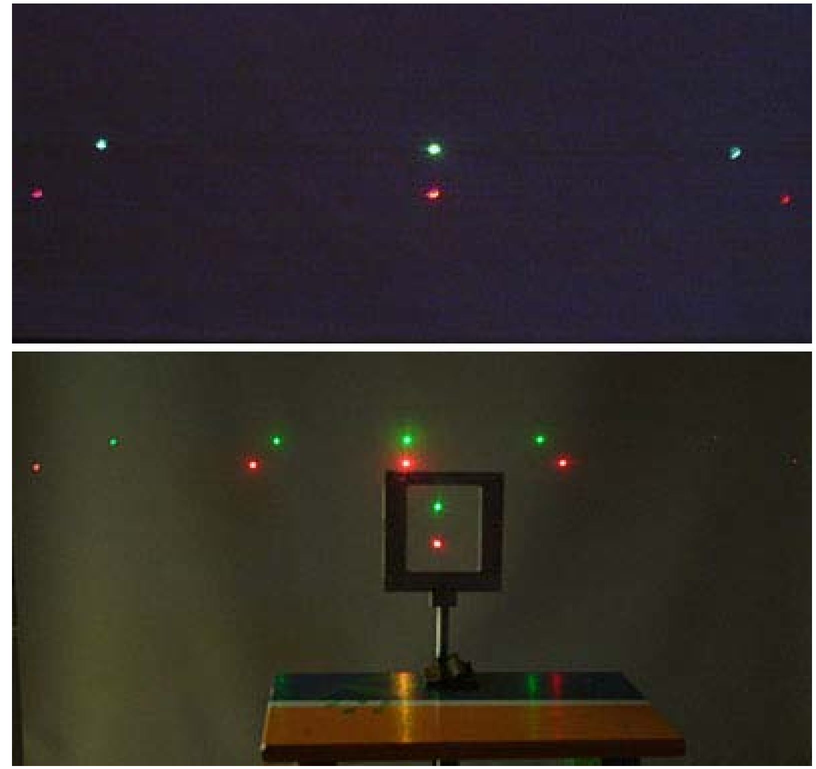
Grating Equation





Red laser beam split by a diffraction grating. Transmission diffraction gratings consist of many thin lines of either absorptive material or thin grooves on an otherwise transparent substrate. Light transmission through a diffraction grating occurs along discrete directions, called diffraction orders. Here a diode laser beam (635 nm) is split into three diffraction orders (+1, 0, -1). This grating's groove density is 500 lines/mm.

http://www.sciencephoto.com/media/92635/view

Rigorous Coupled Wave Analysis (RCWA) Numerical Implementation

Truncation to Arbitrary Number of Diffraction Orders: *M =2m+1*

Grating Region Equations

$$\frac{d\tilde{V}}{dz} = j\tilde{A}\tilde{V}$$

$$\tilde{V}^T = [\tilde{S}_x^T, \tilde{S}_y^T, \tilde{U}_x^T, \tilde{U}_y^T] \quad (4M \times 1)$$

$$\tilde{A} = \text{Coupling Matrix} \quad (4M \times 4M)$$

Standard Eigenvector/Eigenvalue Analysis

$$\tilde{V}(z) = \tilde{W} \exp[\tilde{\Lambda} z]\tilde{C}$$

$$\tilde{W} = \text{Matrix of Eigenvectors of } \tilde{A} \quad (4M \times 4M)$$

$$\tilde{\Lambda} = \text{Matrix of Eigenvalues (diagonal) of } \tilde{A} \quad (4M \times 4M)$$

$$\tilde{C} = \text{Vector of Unknown Coefficients} \quad (4M \times 1)$$

Boundary Conditions: Input and Output Regions Boundaries

Rigorous Coupled Wave Analysis (RCWA)
Numerical Implementation

System of Linear Equations (10M x 10M)

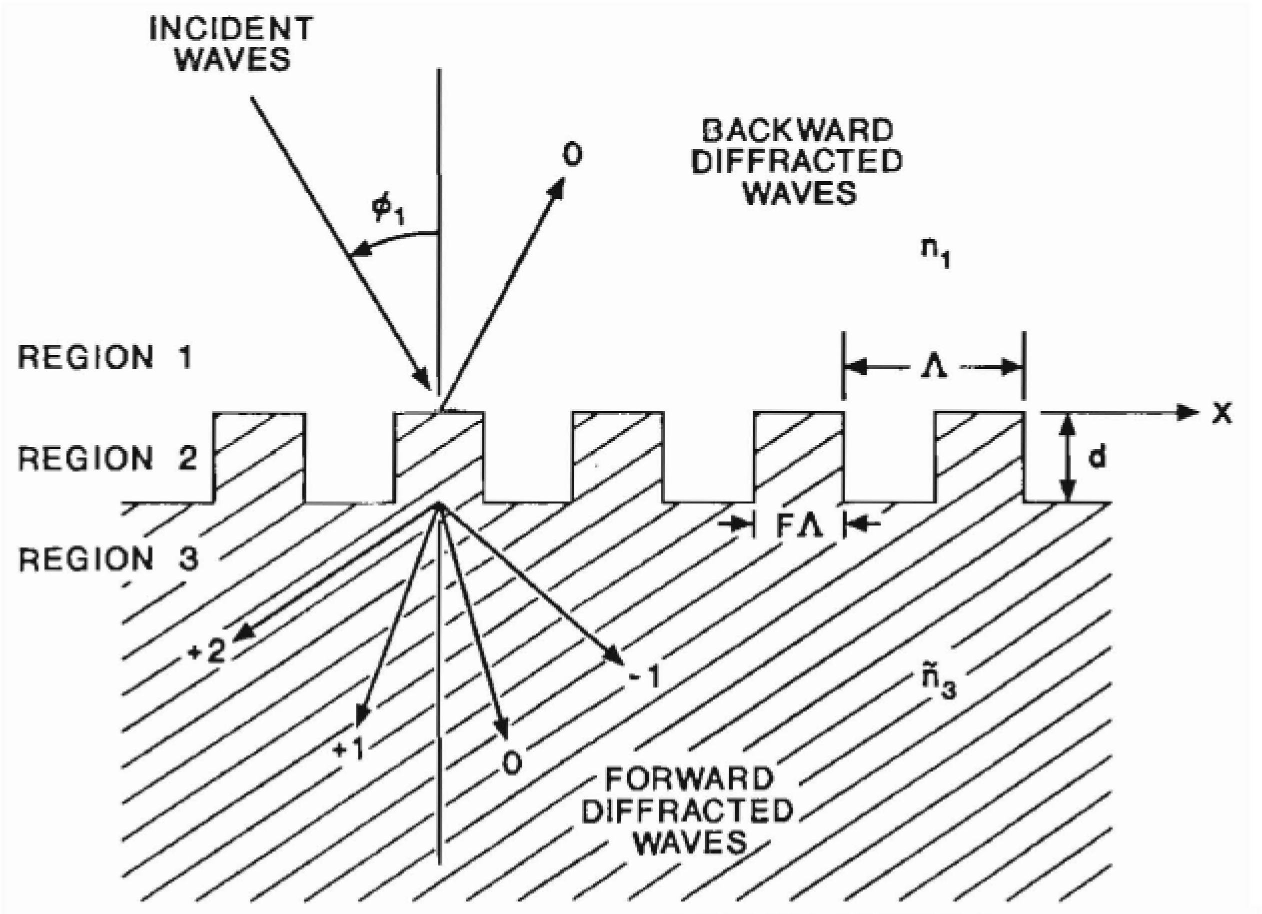
$$\tilde{L}\tilde{x} = \tilde{b}$$

$$\tilde{L} = \text{Matrix of Coefficients of Linear Equations} \quad (10M \times 10M)$$

$$\tilde{x}^T = [\vec{R}_i^T, \vec{T}_i^T, \tilde{C}^T] \quad (10M \times 1)$$

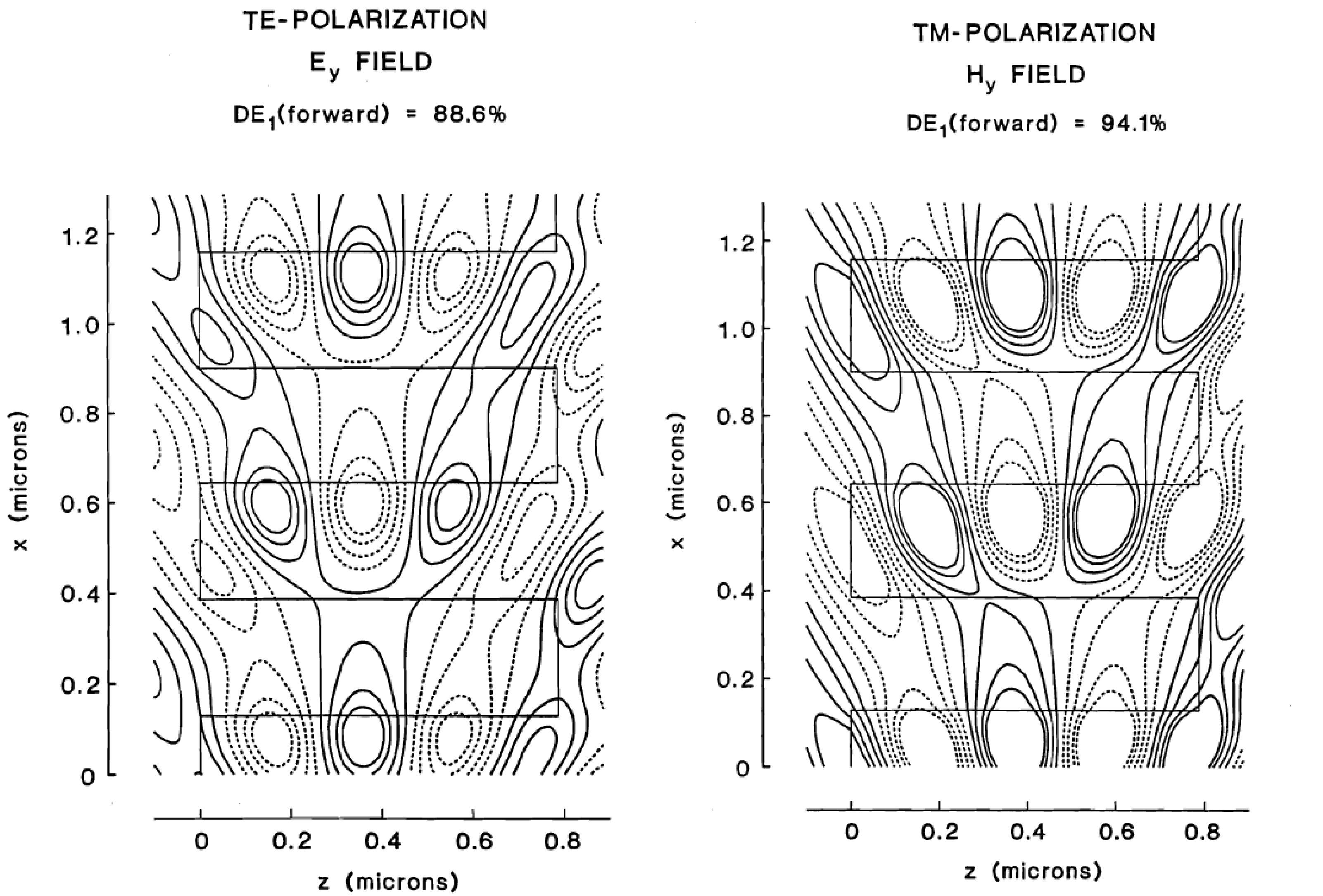
$$\tilde{b} = \text{Excitation Vector (depends on Incident Wave)} \quad (10M \times 1)$$

Rigorous Coupled Wave Analysis (RCWA)
Surface-Relief Grating



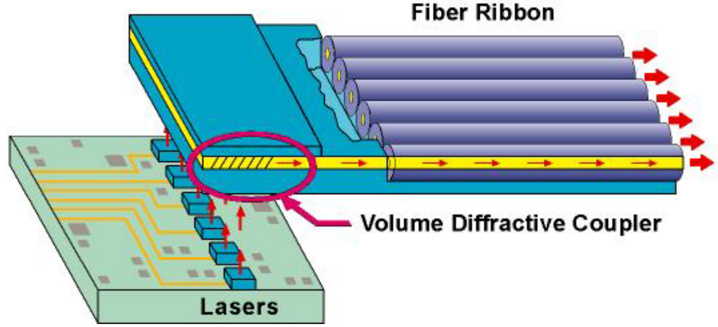
T. K. Gaylord, Notes on Gratings, Georgia Tech

Rigorous Coupled Wave Analysis (RCWA)
Surface-Relief Grating



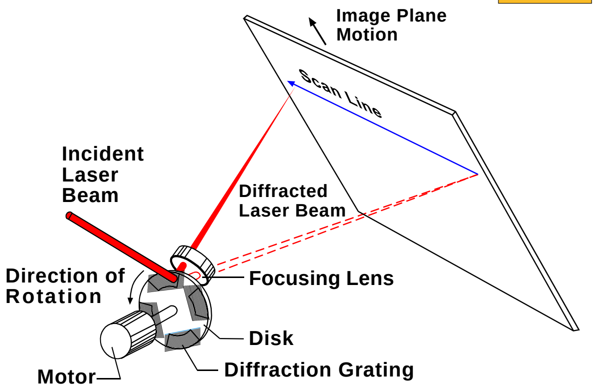
Gratings Applications Examples

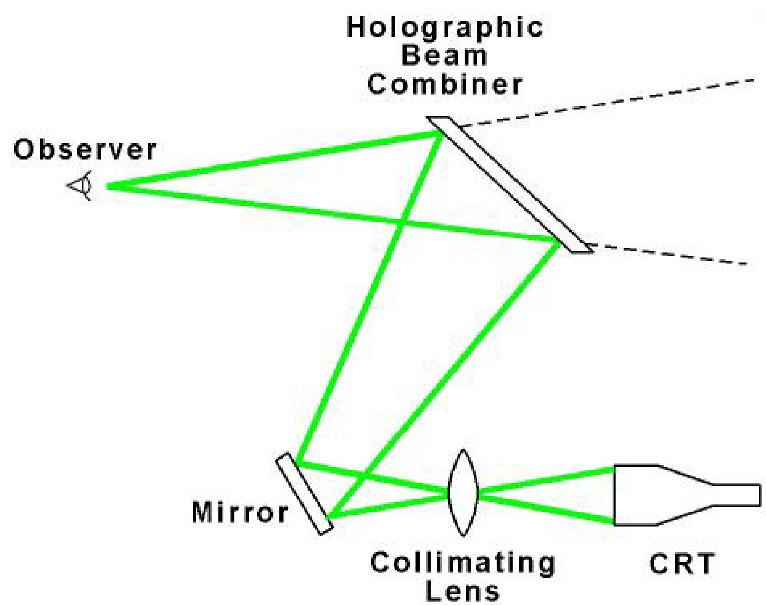
OPTICAL INTERCONNECTION



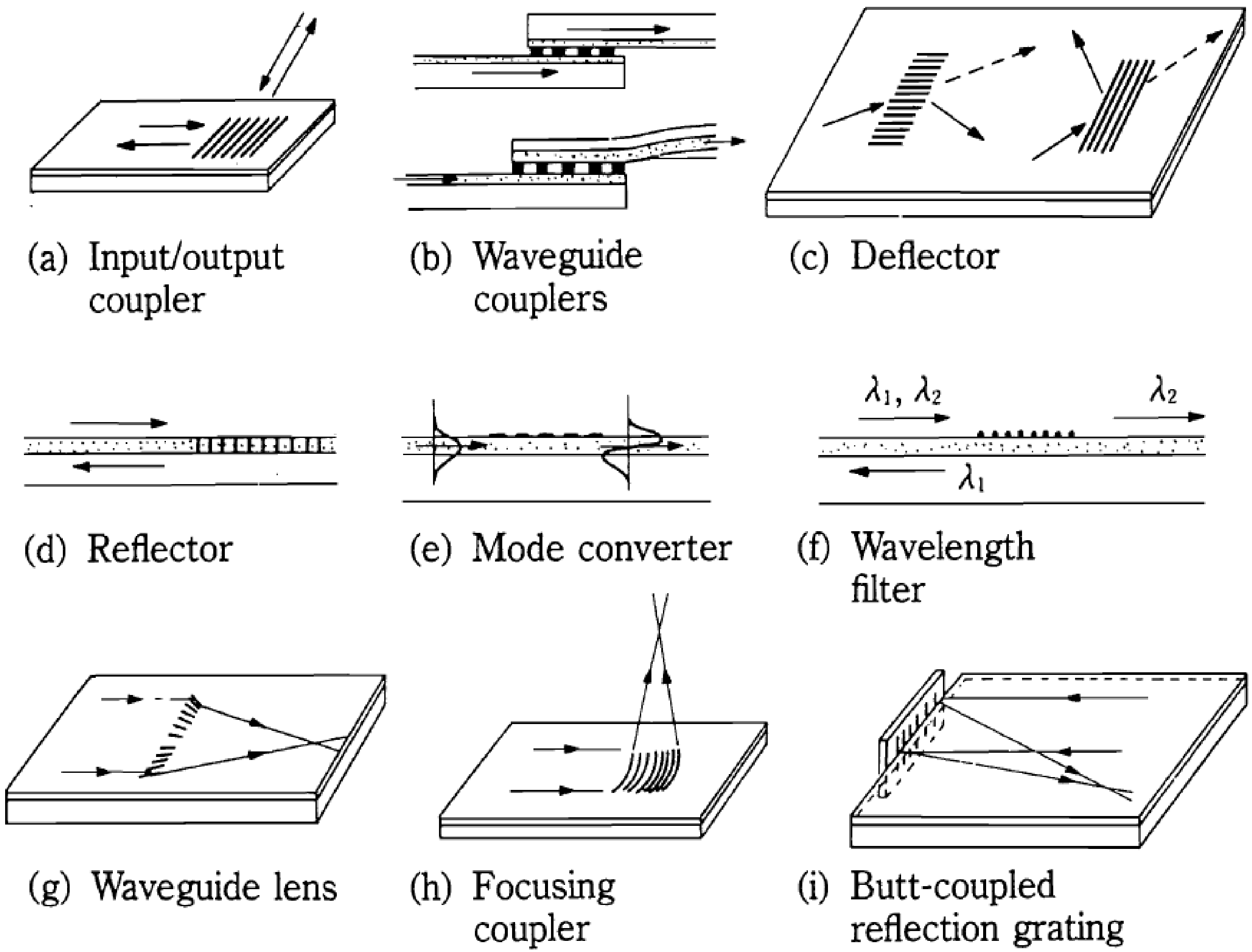
DIFFRACTIVE PRINTER SCANNER

- Linear Scan
- Uniform Intensity





Grating Applications in Integrated Optics



From “Optical Integrated Circuits”, Nishihara, Haruna, and Suhara, McGraw-Hill 1989