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Fourier Optics

Optical Engineering

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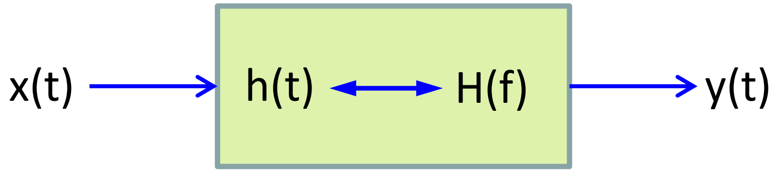
Linear Systems



$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$
$$Y(f) = H(f)X(f)$$

Fourier Transforms

$$f(x) = \int_{-\infty}^{+\infty} F(u) e^{-j2\pi ux} du$$

$$F(u) = \int_{-\infty}^{+\infty} f(x) e^{+j2\pi ux} dx$$

Convolution

$$f(x) * g(x) = \int_{-\infty}^{+\infty} f(x')g(x-x')dx'$$

1-D Fourier Transform Properties

Function $f(x)$	Fourier Transform $F(u)$
$f(x) + g(x)$	$F(u) + G(u)$
$f(ax)$	$(1/\|a\|)F(u/a)$
$(1/\|a\|)f(x/a)$	$F(au)$
$f(x-a)$	$e^{+j2\pi au}F(u)$
$e^{-j2\pi ax}f(x)$	$F(u-a)$
$f((x-a)/b)$	$\|b\|F(bu)e^{+j2\pi au}$
df/dx	$-j2\pi uF(u)$
$j2\pi xf(x)$	dF/du
$f(x) * g(x)$	$F(u)G(u)$
$f(x)g(x)$	$F(u) * G(u)$
$f^*(x)$	$F^*(-u)$
$f^*(-x)$	$F^*(u)$
$f(-x)$	$F(-u)$
$F(x)$	$f(-u)$
$F(-x)$	$f(u)$

Fourier Transforms

Basic 1-D Fourier Transform Pairs

Function $f(x)$	Fourier Transform $F(u)$
$\mathrm{rect}(x) = \begin{cases} 1 & \text{for } \lvert x\rvert < 1/2 \\ 0 & \text{for } \lvert x\rvert > 1/2 \end{cases}$	$\mathrm{sinc}(u) = \sin(\pi u)/(\pi u)$
$\mathrm{tri}(x) = \begin{cases} 1 - \lvert x\rvert & \text{for } \lvert x\rvert < 1 \\ 0 & \text{for } \lvert x\rvert > 1 \end{cases}$	$\mathrm{sinc}^2(u) = \sin^2(\pi u)/(\pi u)^2$
$e^{-\pi x^2}$	$e^{-\pi u^2}$
$\delta(x)$	1
$\delta(x-a)$	$e^{+j2\pi a u}$
$e^{-j2\pi a x}$	$\delta(u-a)$
$\cos(2\pi a x)$	$(1/2)[\delta(u+a)+\delta(u-a)]$
$\sin(2\pi a x)$	$(1/2j)[\delta(u+a)-\delta(u-a)]$
$\mathrm{comb}(x) = \sum_{n=-\infty}^{\infty} \delta(x-n)$	$\mathrm{comb}(u) = \sum_{n=-\infty}^{\infty} \delta(u-n)$

2-D Fourier Transforms

$$
\begin{aligned}
f(x,y) &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(u,v)e^{-j2\pi(ux+vy)}dudv \\
F(u,v) &= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)e^{+j2\pi(ux+vy)}dxdy
\end{aligned}
$$

2-D Convolution

$$
f(x,y) ** g(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x',y')g(x-x',y-y')dx'dy'
$$

2-D Fourier Transforms

2-D Fourier Transform Properties

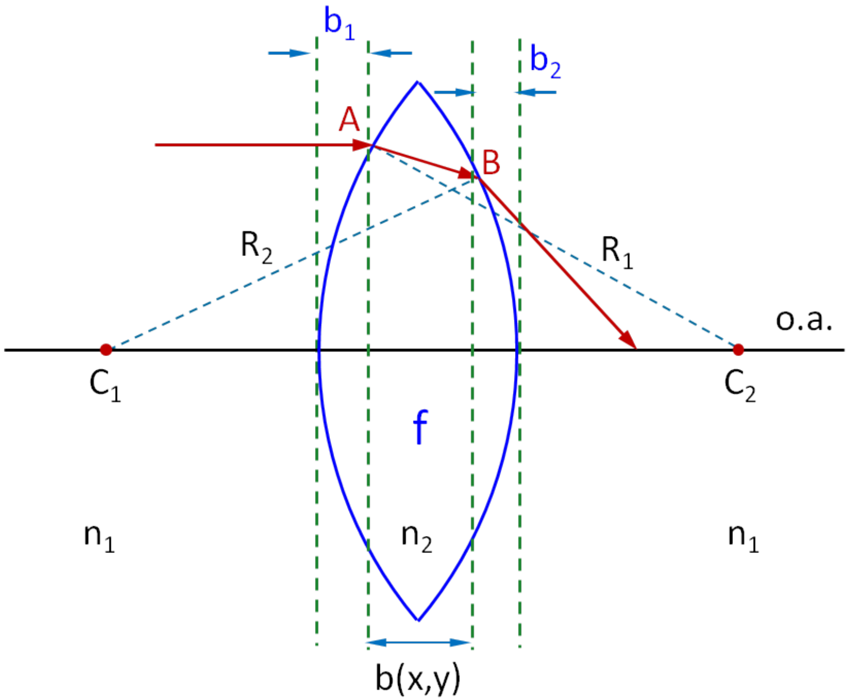
Function $f(x,y)$	Fourier Transform $F(u,v)$
$f(x,y)+g(x,y)$	$F(u,v)+G(u,v)$
$f(ax,by)$	$(1/\lvert a\rvert\lvert b\rvert)F(u/a,v/b)$
$(1/\lvert a\rvert)f(x/a)$	$F(au)$
$f(x-a,y-b)$	$e^{+j2\pi(au+bv)}F(u)$
$e^{-j2\pi(ax+by)}f(x,y)$	$F(u-a,v-b)$
$f(x,y)g(x,y)$	$F(u,v)**G(u,v)$
$f(x,y)**g(x,y)$	$F(u,v)G(u,v)$
$f^*(x,y)$	$F^*(-u,-v)$
$f^*(-x,-y)$	$F^*(u,v)$
$f(-x,-y)$	$F(-u,-v)$

2-D Fourier Transforms

Basic 2-D Fourier Transform Pairs

Function $f(x)$	Fourier Transform $F(u)$
$\mathrm{rect}(x,y) = \mathrm{rect}(x)\mathrm{rect}(y)$	$\mathrm{sinc}(u,v) = \mathrm{sinc}(u)\mathrm{sinc}(v) = \dfrac{\sin(\pi u)}{\pi u}\dfrac{\sin(\pi v)}{\pi v}$
$\mathrm{tri}(x,y) = \mathrm{tri}(x)\mathrm{tri}(y)$	$\mathrm{sinc}^2(u,v) = \mathrm{sinc}^2(u)\mathrm{sinc}^2(v)$
$e^{-\pi(x^2+y^2)}$	$e^{-\pi(u^2+v^2)}$
$\delta(x,y)$	1
$\delta(x-a, y-b)$	$e^{+j2\pi au}e^{+j2\pi bv}$
$e^{-j2\pi(ax+by)}$	$\delta(u-a, v-b)$
$\cos(2\pi(ax+by))$	$(1/2)[\delta(u+a,v+b)+\delta(u-a,v-b)]$
$\sin(2\pi(ax+by))$	$(1/2j)[\delta(u+a,v+b)-\delta(u-a,v-b)]$
$\mathrm{comb}(x,y) = \mathrm{comb}(x)\mathrm{comb}(y)$	$\mathrm{comb}(u,v) = \mathrm{comb}(u)\mathrm{comb}(v)$

Thin Lens Transmittance Function



$$b_0 = b_1 + b_2 + b(x,y)$$

$$\phi = k_0 n_2 b(x,y) + k_0 n_1 \left[b_0 - b(x,y) \right]$$

$$b_1 = |R_1| - \sqrt{R_1^2 - (x^2+y^2)} \simeq \frac{x^2+y^2}{2|R_1|}$$

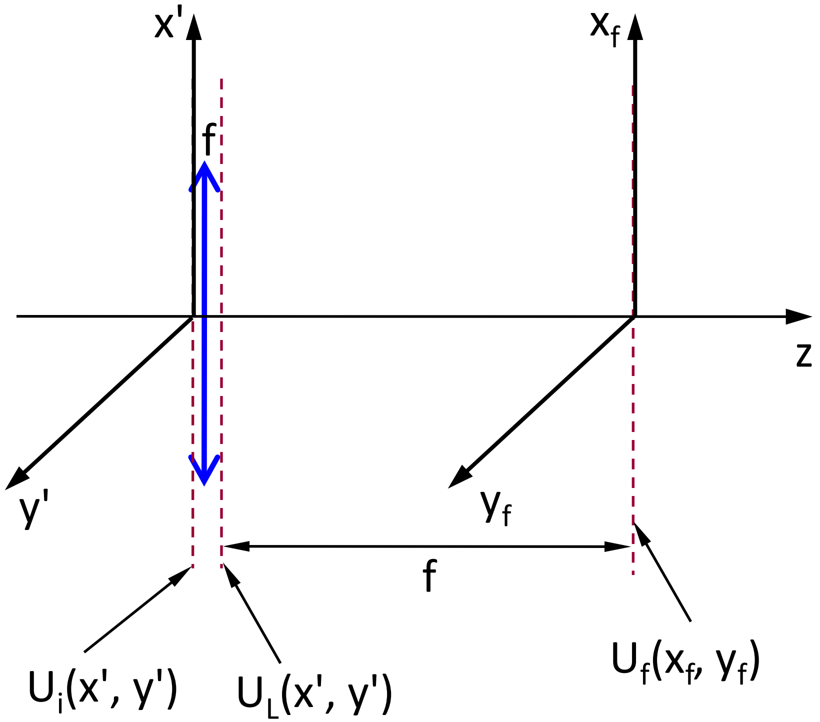
$$b_2 = |R_2| - \sqrt{R_2^2 - (x'^2+y'^2)} \simeq \frac{x'^2+y'^2}{2|R_2|} \simeq \frac{x^2+y^2}{2|R_2|}$$

$$b(x,y) = b_0 - b_1 - b_2 = b_0 - \frac{x^2+y^2}{2}\left(\frac{1}{|R_1|} + \frac{1}{|R_2|}\right) = b_0 - \frac{x^2+y^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\phi = k_0 n_2 b_0 - k_0 \frac{x^2+y^2}{2}(n_2 - n_1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = k_0 n_2 b_0 - k_0 \frac{x^2+y^2}{2f}$$

$$\boxed{t(x,y) = e^{-jk_0 n_2 b_0} e^{+jk_0 \frac{x^2+y^2}{2f}}}$$

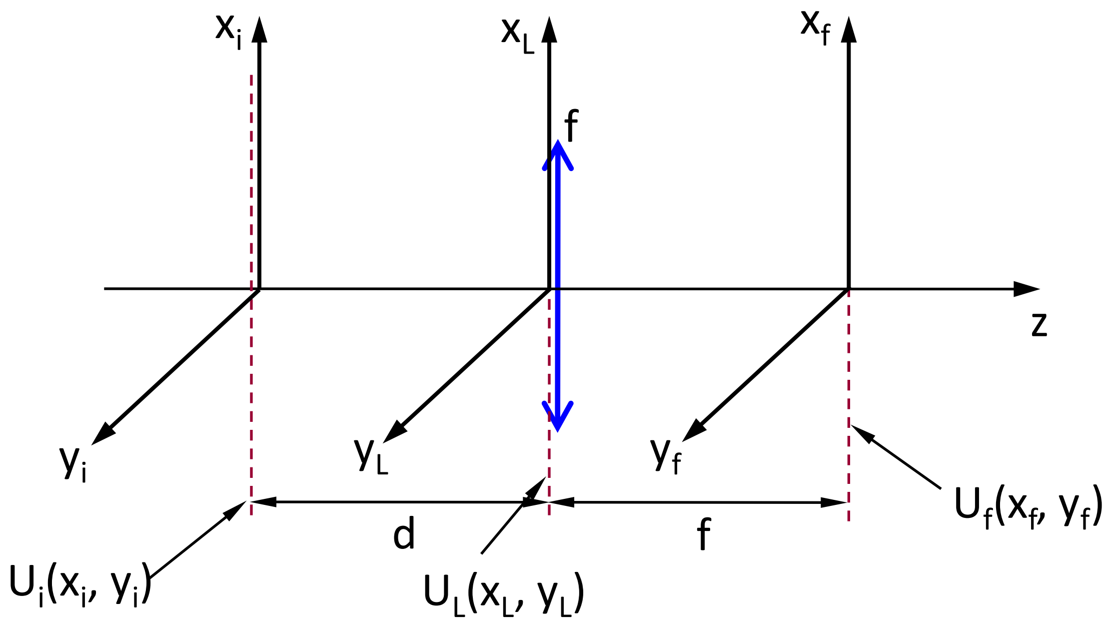
Thin Lens Fourier Transforming Property



$$U_L(x', y') = t(x', y')U_i(x', y') = e^{-jk_0 n_2 b_0} e^{+jk_0 \frac{x^2+y^2}{2f}} U_i(x', y')$$

$$U_f(x_f, y_f) = jk_0 \frac{e^{-jk_0 f}}{2\pi f} e^{-jk_0 n_2 b_0} e^{-jk_0 \frac{x_f^2+y_f^2}{2f}} \mathcal{F}\left\{U_i(x', y')\right\}\Bigg|_{u=x_f/\lambda f, v=y_f/\lambda f}$$

Thin Lens Fourier Transforming Property

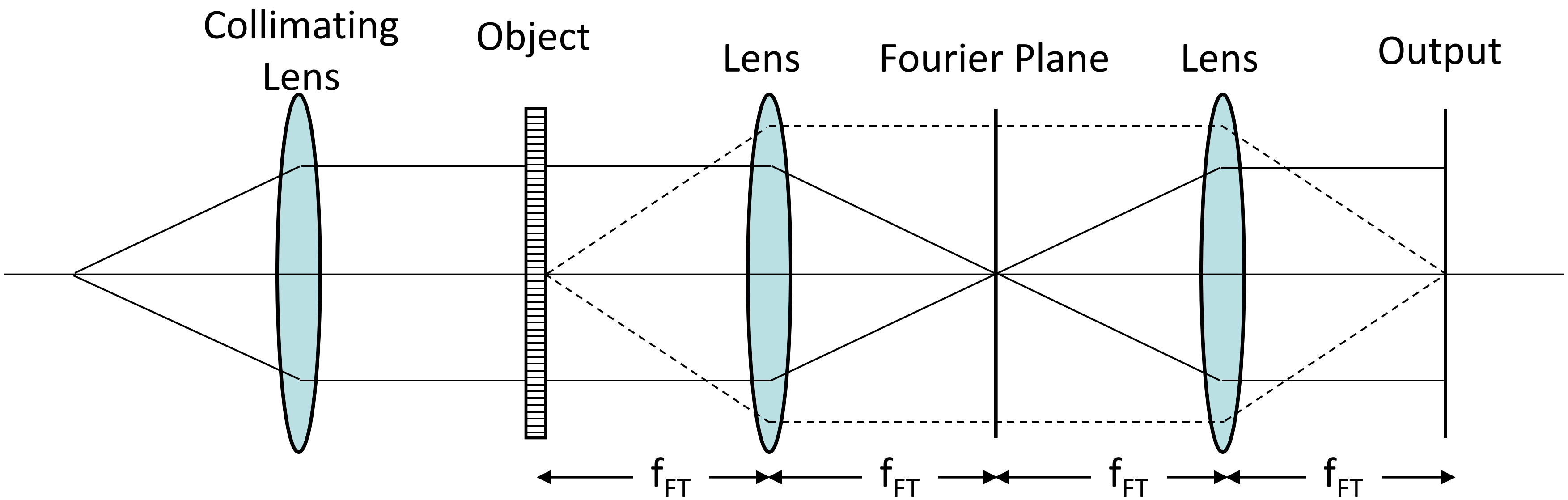


$$U_f(x_f, y_f) = jk_0 \frac{e^{-jk_0 f}}{2\pi f} e^{-jk_0 n_2 b_0} e^{-jk_0 \frac{x_f^2 + y_f^2}{2f}} \mathcal{F}\left\{U_L(x', y')\right\}\Big|_{u = x_f/\lambda f, v = y_f/\lambda f}$$

$$U_L(x_L, y_L) = jk_0 \frac{e^{-jk_0 d}}{2\pi d} \iint \exp\left[-jk_0 \frac{(x_L - x_i)^2 + (y_L - y_i)^2}{2d}\right] U_i(x_i, y_i) dx_i dy_i$$

$$U_f(x_f, y_f) = jk_0^2 \frac{e^{-jk_0(f+d)}}{4\pi^2 f} e^{-jk_0 n_2 b_0} \exp\left[-jk_0 \left(1 - \frac{d}{f}\right) \frac{x_f^2 + y_f^2}{2f}\right] \mathcal{F}\left\{U_i(x_i, y_i)\right\}\Big|_{u = x_f/\lambda f, v = y_f/\lambda f}$$

4-f Optical System



4-f Optical System

Spatial Filtering in the Fourier Plane

Spatial filtering with a cross-grating as the object

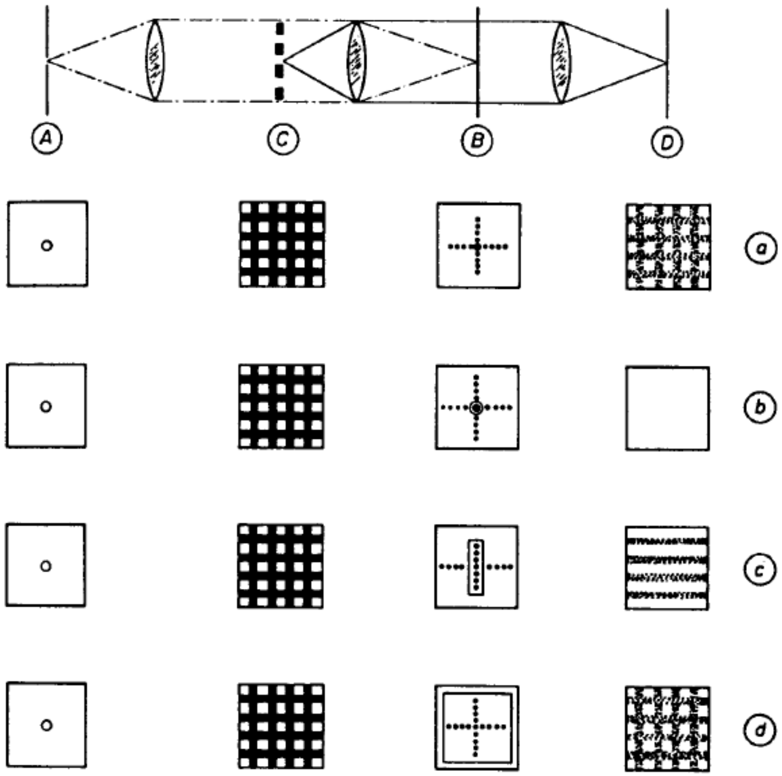
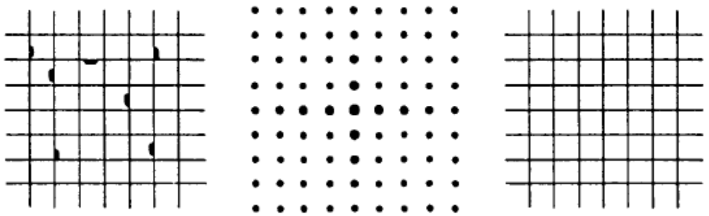


Image of original wire grid with dust (left)
Diffraction pattern of grid (center)
Image of wire grid; dust particles removed by spatial filtering (right)



4-f Optical System

Spatial Filtering in the Fourier Plane

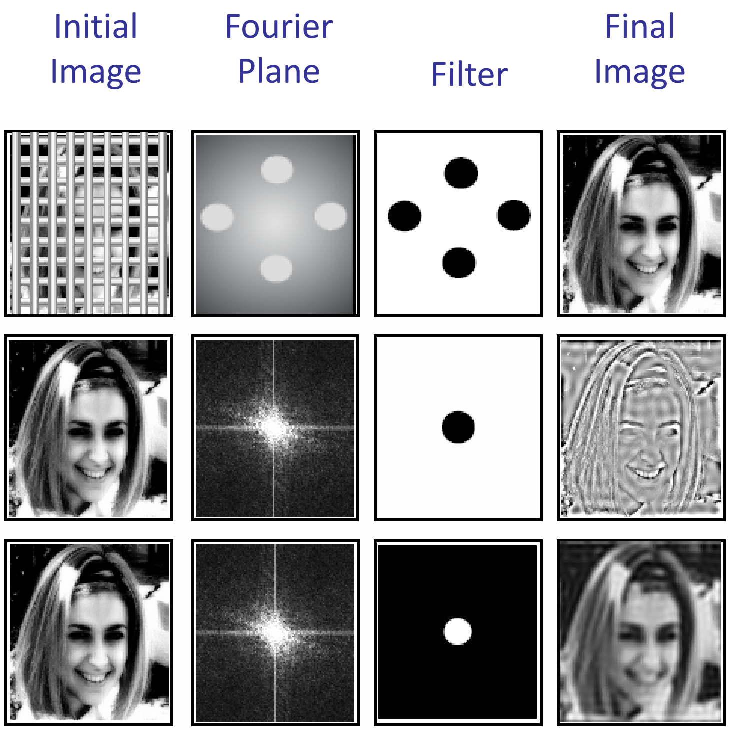


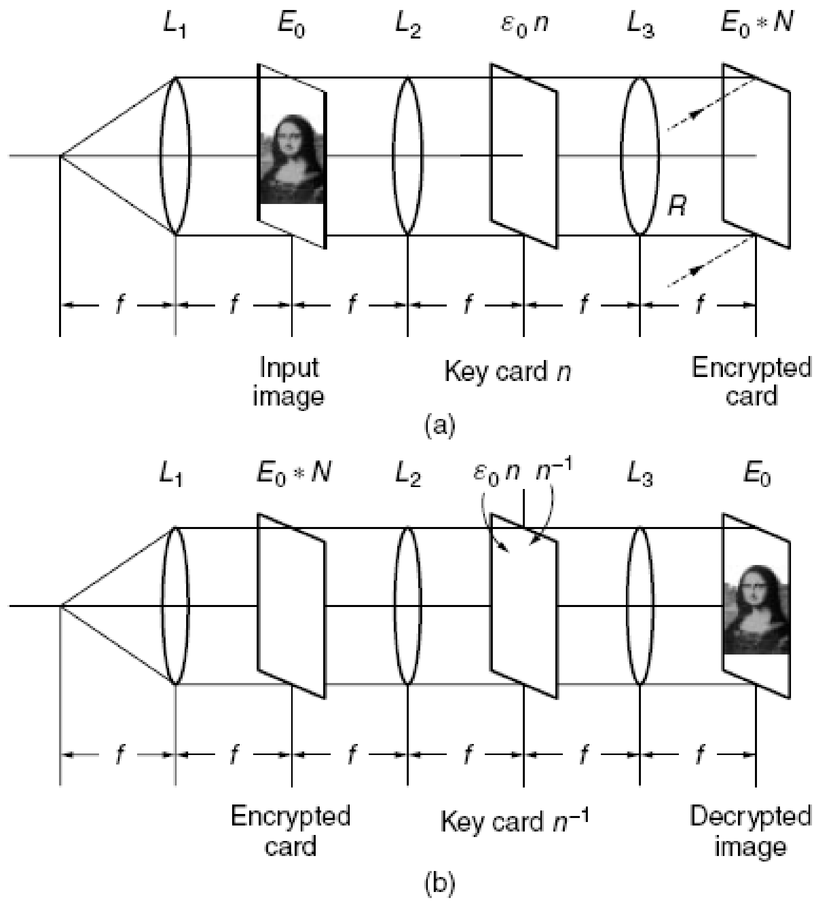
Image is filtered from a mask that has opaque regions in the places where the grid effect is

Image is filtered from a mask that permits only high spatial frequencies – thus shows enhances edges

Image is filtered from a mask that permits only low spatial frequencies – thus edges become blurred

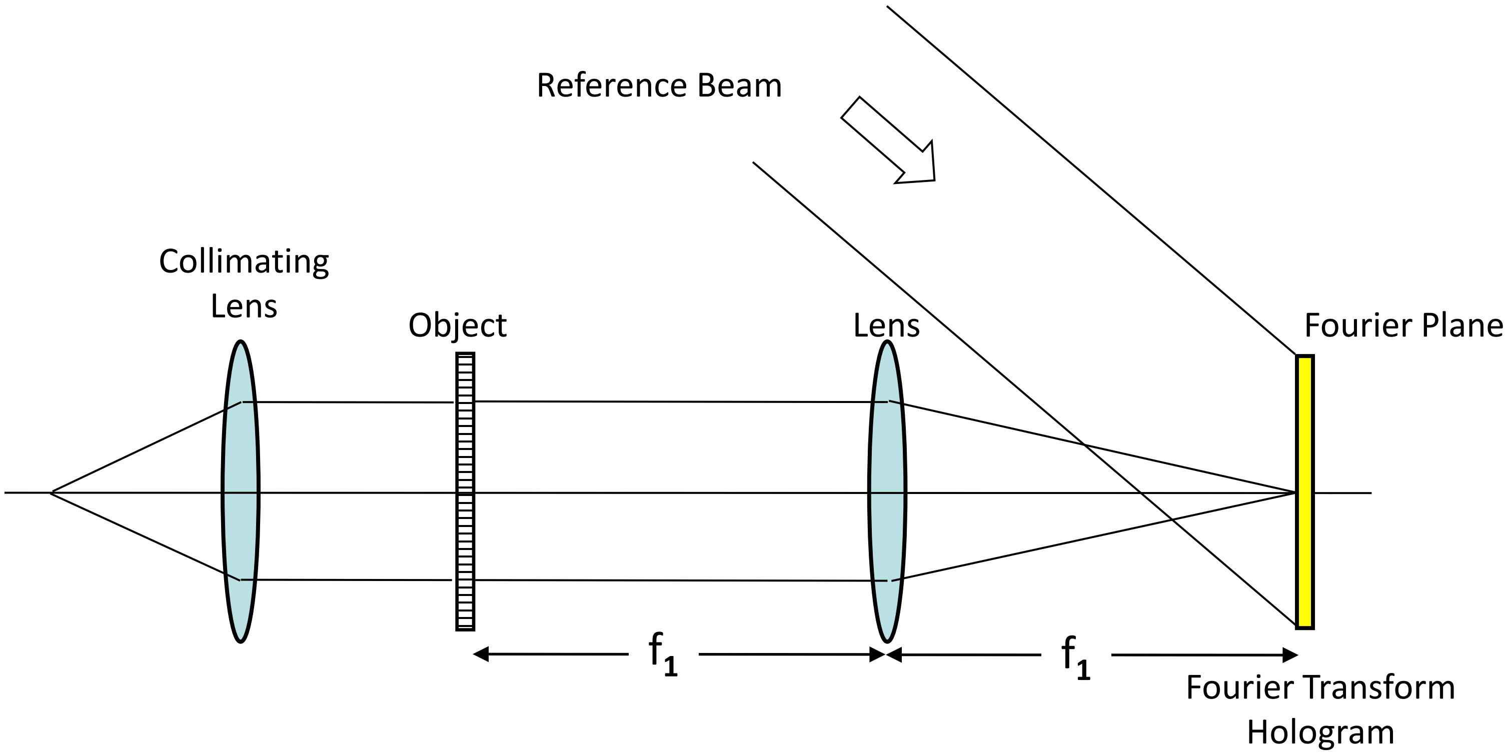
4-f Optical System

Cryptographic Spatial Filters

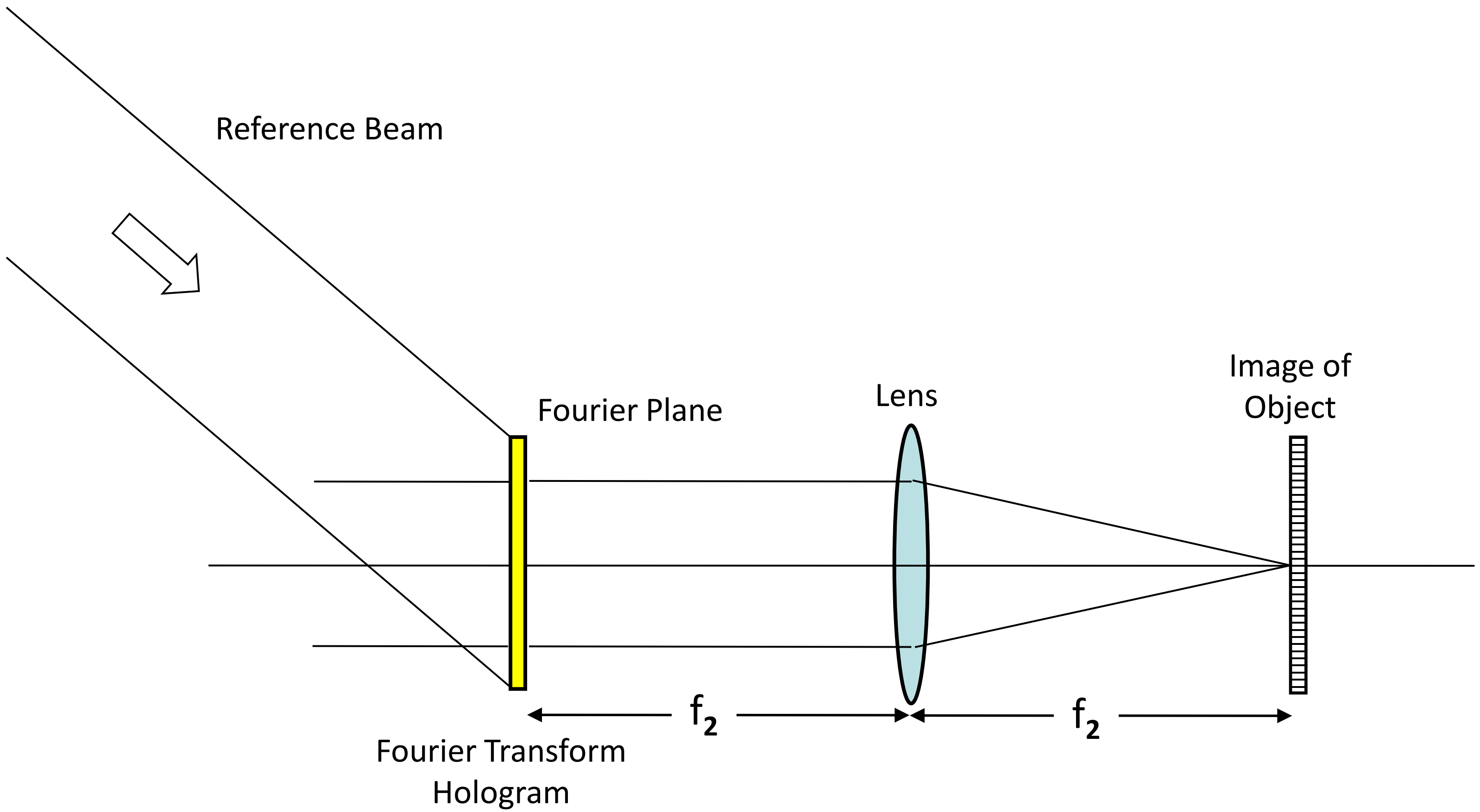


(a) Encryption, (b) Decryption.

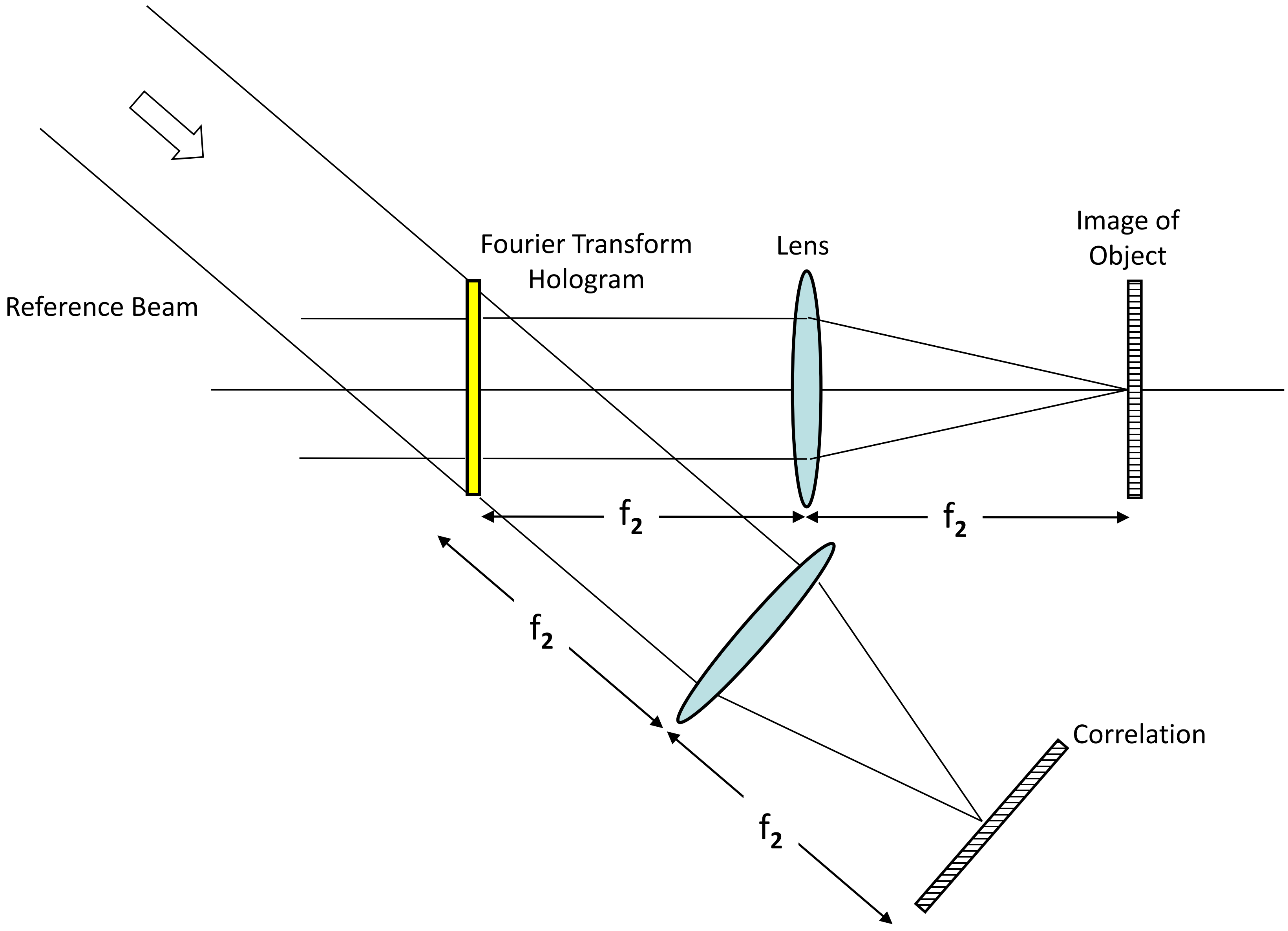
Fourier Transform Hologram



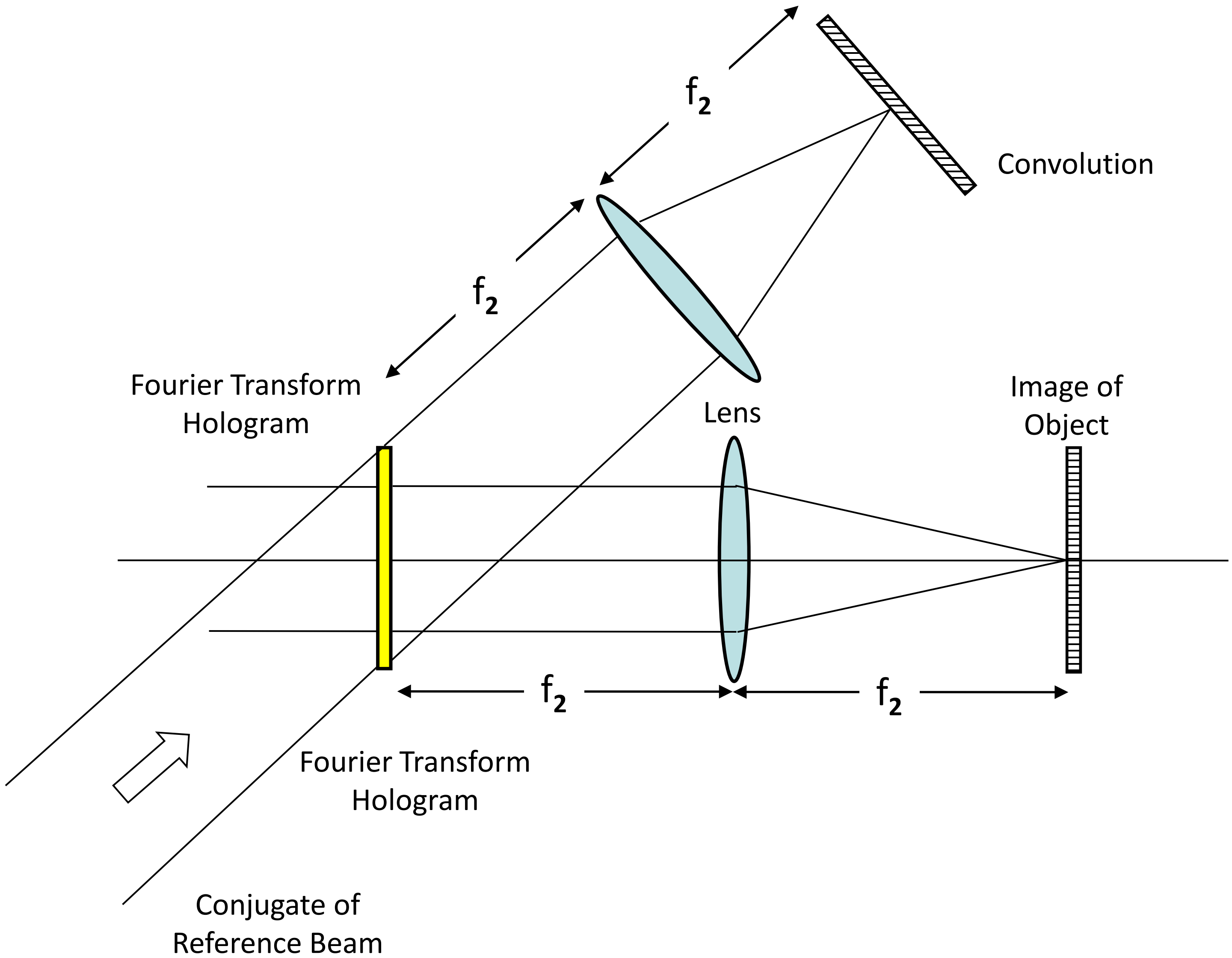
Fourier Transform Hologram - Playback



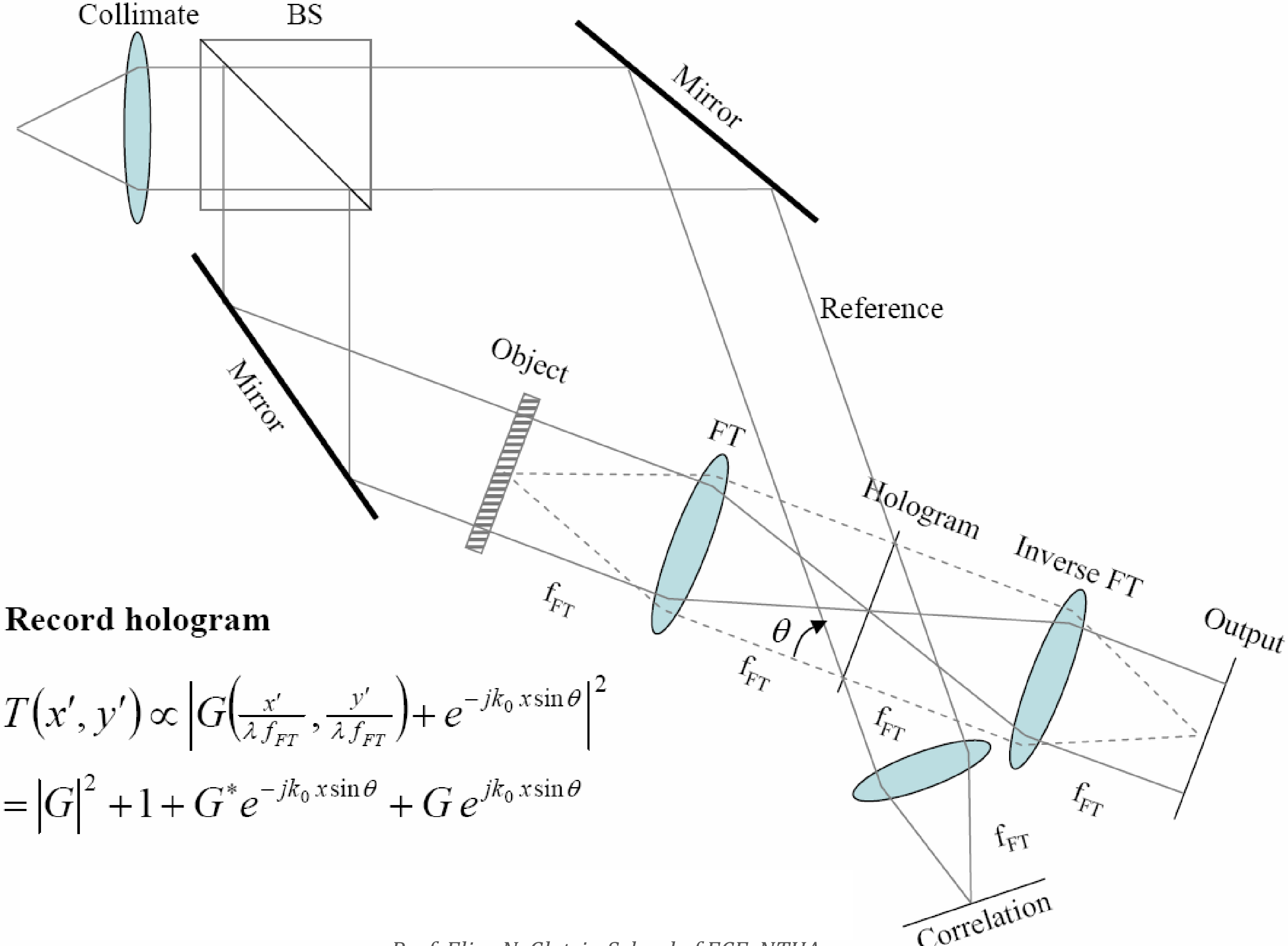
Van der Lugt Correlator



Van der Lugt Convolver



Van der Lugt Correlator



Record hologram

$$T(x',y') \propto \left| G\left(\frac{x'}{\lambda f_{FT}}, \frac{y'}{\lambda f_{FT}}\right) + e^{-jk_0 x \sin\theta} \right|^2$$

$$= |G|^2 + 1 + G^* e^{-jk_0 x \sin\theta} + G\, e^{jk_0 x \sin\theta}$$