Huygens Principle and Diffraction

Every point on a propagating wavefront becomes a secondary source of spherical wavelets. All these wavelets form the envelope of the next wavefront.

Diffraction – Bending of light into the shadow regions (wavelength, $\lambda$, of the order of obstacles)

https://www.usna.edu/Users/physics/ejtuchol/documents/SP212R/Chapter%20S37.ppt
Circular waves generated by diffraction from the narrow entrance of a flooded coastal quarry

Diffraction by a Razor

Diffraction by an Edge

Diffraction by CD


http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/grating.html

Gengage Learning, Chap. 38

Prof. Elias N. Glytsis, School of ECE, NTUA
Integral Theorem of Helmholtz and Kirchhoff

3D-Green’s Function

\[ \nabla^2 G(\vec{r}, \vec{r}') + k^2 G(\vec{r}, \vec{r}') = -\delta(\vec{r} - \vec{r}') \]

\[ G(\vec{r}, \vec{r}') = \frac{\exp(-jk|\vec{r} - \vec{r}'|)}{4\pi|\vec{r} - \vec{r}'|} \]

Scalar Field at a point as a function of the Fields on the Boundary \( S \)

\[ U(P_0) = \int \int_S \left( G(\vec{r}, \vec{r}') \frac{\partial U}{\partial n} - U \frac{\partial G(\vec{r}, \vec{r}')}{\partial n} \right) dS \]


Prof. Elias N. Glytsis, School of ECE, NTUA
Kirchhoff’s Formulation for a Planar Screen

\[ U(P_0) = \iint_{S_1} \left( G(\vec{r}, \vec{r}') \frac{\partial U}{\partial n} - U \frac{\partial G(\vec{r}, \vec{r}')}{\partial n} \right) dS \]

**Sommerfeld’s Radiation Condition**

\[ \lim_{R \to \infty} \left\{ R \left( \frac{\partial U}{\partial n} + jkU \right) \right\} = 0 \]

**Sommerfeld’s Finiteness Condition**

\[ \lim_{R \to \infty} \{ U \} = 0 \]

\[ \lim_{R \to \infty} \{ R|U| \} < \infty \]

**Green’s Function Selection**

\[ G_D(\vec{r}, \vec{r}') = 0 \quad \vec{r}' \in S_1 \quad \text{Dirichlet} \]

\[ \frac{\partial G_N(\vec{r}, \vec{r}')}{\partial n} = 0 \quad \vec{r}' \in S_1 \quad \text{Neumann} \]


Prof. Elias N. Glytsis, School of ECE, NTUA
Kirchhoff’s Formulation for a Planar Screen

\[ U(P_0) = \int \int_{S_1} \left( G \frac{\partial U}{\partial n} - U \frac{\partial G}{\partial n} \right) ds = \]
\[ = \int \int_{S_1} \left[ e^{-jkR} \frac{\partial U}{\partial n} - U \left( jk + \frac{1}{R} \right) \frac{e^{-jkR}}{4\pi R} (\hat{n}_R \cdot \hat{n}) \right] ds. \]
Rayleigh-Sommerfeld’s Formulations

Neumann Green’s Function

\[ G_N(\vec{r}, \vec{r}'; \vec{r}'') = \frac{\exp(-jk|\vec{r} - \vec{r}'|)}{4\pi|\vec{r} - \vec{r}'|} + \frac{\exp(-jk|\vec{r} - \vec{r}''|)}{4\pi|\vec{r} - \vec{r}''|} = \frac{\exp(-jkR_1)}{4\pi R_1} + \frac{\exp(-jkR_2)}{4\pi R_2}, \quad \text{where,} \]

\[ \begin{align*}
\vec{R}_1 &= (x_0 - x')\hat{i}_x + (y_0 - y')\hat{i}_y + (z_0 - z')\hat{i}_z, \quad R_1 = |\vec{R}_1|, \\
\vec{R}_2 &= (x_0 - x')\hat{i}_x + (y_0 - y')\hat{i}_y + (z_0 + z')\hat{i}_z, \quad R_2 = |\vec{R}_2|,
\end{align*} \]

\[ \frac{\partial G_N}{\partial n} = \nabla' G_N \cdot \hat{i}_n, \quad \hat{i}_n = -\hat{i}_z, \]

\[ \nabla' G_N = \left( jk + \frac{1}{R_1} \right) \frac{\exp(-jkR_1)}{4\pi R_1} \left( \frac{x_0 - x'}{R_1} \hat{i}_x + \frac{y_0 - y'}{R_1} \hat{i}_y + \frac{z_0 - z'}{R_1} \hat{i}_z \right) + \left( jk + \frac{1}{R_2} \right) \frac{\exp(-jkR_2)}{4\pi R_2} \left( \frac{x_0 - x'}{R_2} \hat{i}_x + \frac{y_0 - y'}{R_2} \hat{i}_y - \frac{z_0 + z'}{R_2} \hat{i}_z \right), \]

\[ \frac{\partial G_N}{\partial n} \bigg|_{z' \to 0} = 0, \quad \text{while,} \]

\[ G_N \bigg|_{z' \to 0} = \frac{\exp(-jkR)}{2\pi R}. \]
Rayleigh-Sommerfeld’s Formulations

Dirichlet Green’s Function

\[ G_D(\bar{r}, \bar{r}'; \bar{r}'') = \frac{\exp(-jkr')}{4\pi|\bar{r}' - \bar{r}'|} - \frac{\exp(-jk|\bar{r}' - \bar{r}''|)}{4\pi|\bar{r}'' - \bar{r}'|} = \]

\[ = \frac{\exp(-jkR_1)}{4\pi R_1} - \frac{\exp(-jkR_2)}{4\pi R_2}, \]

\[ \bar{R}_1 = (x_0 - x')\hat{i}_x + (y_0 - y')\hat{i}_y + (z_0 - z')\hat{i}_z, \quad R_1 = |\bar{R}_1|, \]

\[ \bar{R}_2 = (x_0 - x')\hat{i}_x + (y_0 - y')\hat{i}_y + (z_0 + z')\hat{i}_z, \quad R_2 = |\bar{R}_2|, \]

\[ \frac{\partial G_D}{\partial n} = \nabla' G_D \cdot \hat{n}, \quad \hat{n} = -\hat{i}_z, \]

\[ \nabla' G_D = \left( jk + \frac{1}{R_1} \right) \frac{\exp(-jkR_1)}{4\pi R_1} \left( \frac{x_0 - x'}{R_1}\hat{i}_x + \frac{y_0 - y'}{R_1}\hat{i}_y + \frac{z_0 - z'}{R_1}\hat{i}_z \right) - \]

\[ \left( jk + \frac{1}{R_2} \right) \frac{\exp(-jkR_2)}{4\pi R_2} \left( \frac{x_0 - x'}{R_2}\hat{i}_x + \frac{y_0 - y'}{R_2}\hat{i}_y - \frac{z_0 + z'}{R_2}\hat{i}_z \right), \]

\[ \frac{\partial G_D}{\partial n} \bigg|_{\bar{r}'=0} = -\left( jk + \frac{1}{R} \right) \frac{\exp(-jkR)}{4\pi R} \frac{2z_0}{R}, \]

where,

\[ R = [(x_0 - x')^2 + (y_0 - y')^2 + z_0^2]^{1/2}, \quad \text{while,} \]

\[ G_D|_{\bar{r}'=0} = 0. \]
Rayleigh-Sommerfeld’s 1 Formulation for a Planar Screen

\[ U(P_0) = \int \int_{S_1} \left( G(\vec{r}, \vec{r}') \frac{\partial U}{\partial n} - U \frac{\partial G(\vec{r}, \vec{r}')}{\partial n} \right) dS \]

\[ G_D(\vec{r}, \vec{r}') = 0 \]

\[ \frac{\partial G_D(\vec{r}, \vec{r}')}{\partial n} = - \left( jk + \frac{1}{R} \right) \frac{e^{-jkR}}{4\pi R} \frac{2z}{R} \]

\[ U(\vec{r}) = \int \int_{S_1} \frac{\exp(-jkR)}{2\pi R} \left( jk + \frac{1}{R} \right) \frac{z}{R} U_i(\vec{r}') dS' \]

\[ R = [(x - x')^2 + (y - y')^2 + z^2]^{1/2} \]

Rayleigh-Sommerfeld’s 2 Formulation for a Planar Screen

\[ U(P_0) = \iint_{S_1} \left( G(\vec{r}, \vec{r}') \frac{\partial U}{\partial n} - U \frac{\partial G(\vec{r}, \vec{r}')}{\partial n} \right) dS \]

\[ G_N(\vec{r}, \vec{r}') = \frac{e^{-jkR}}{2\pi R} \]

\[ \frac{\partial G_N(\vec{r}, \vec{r}')}{\partial n} = 0 \]

\[ U(\vec{r}) = \iint_{S_1} \frac{\exp(-jkR)}{2\pi R} \frac{\partial U_i(\vec{r}')}{\partial n} dS' \]

\[ R = [(x - x')^2 + (y - y')^2 + z^2]^{1/2} \]


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Integral Theorem for 2D Diffraction Problems

2D-Green’s Function

\[ \nabla^2 G^{2d}(\vec{r}, \vec{r}') + k^2 G^{2d}(\vec{r}, \vec{r}') = -\delta(\vec{r} - \vec{r}') \]

\[ G^{2d}(\vec{r}, \vec{r}') = \frac{j}{4} H_0^{(2)}(k|\vec{r} - \vec{r}'|) \]

\[ \begin{align*}
\vec{r} &= x\hat{x} + z\hat{z} \\
\vec{r}' &= x'\hat{x} + z'\hat{z}
\end{align*} \]

Scalar Field at a point as a function of the Fields on the Boundary Curve C

\[ U(P_0) = \int_C \left( G(\vec{r}, \vec{r}') \frac{\partial U}{\partial n} - U \frac{\partial G(\vec{r}, \vec{r})}{\partial n} \right) \, dl \]

Summary of Diffraction Formulas

Rayleigh-Sommerfeld 1

\[ U(x, y, z) = \int \int \frac{e^{-jkR}}{2\pi R} \left( jk + \frac{1}{R} \right) \frac{z}{R} U_i(x', y') dx' dy', \quad \text{3D Green’s Function} \]

\[ R = \sqrt{(x - x')^2 + (y - y')^2 + z^2}, \]

\[ U(x, z) = \int \frac{k}{2j} H_1^{(2)}(kR) \frac{z}{R} U_i(x') dx', \quad \text{2D Green’s Function} \]

\[ R = \sqrt{(x - x')^2 + z^2} \]

\[ H_1^{(2)}(z) = J_1(z) - jY_1(z), \quad \text{Hankel’s Function} \]

Rayleigh-Sommerfeld 2

\[ U(x, y, z) = \int \int \frac{e^{-jkR}}{2\pi R} \frac{\partial U_i(x', y')}{\partial n} dx' dy', \quad \text{3D Green’s Function} \]

\[ R = \sqrt{(x - x')^2 + (y - y')^2 + z^2}, \]

\[ U(x, z) = \int \frac{-j}{2} \frac{k}{2j} H_0^{(2)}(kR) \frac{\partial U_i(x')}{\partial n} dx', \quad \text{2D Green’s Function} \]

\[ R = \sqrt{(x - x')^2 + z^2} \]
Fresnel Approximation

\[ U(x_0, y_0, z_0) = \iint_{S_a} \left[ U_{inc} \left( jk + \frac{1}{R} \right) \frac{\exp(-jkR) z_0}{2\pi R R} \right]_{z' = 0} \ dx'dy', \quad \text{where}, \]

\[ R = \left[ (x_0 - x')^2 + (y_0 - y')^2 + z_0^2 \right]^{1/2} = z_0 \left[ 1 + \frac{(x_0 - x')^2}{2z_0^2} + \frac{(y_0 - y')^2}{2z_0^2} \right]^{1/2} \approx z_0 \left[ 1 + \frac{(x_0 - x')^2}{2z_0^2} + \frac{(y_0 - y')^2}{2z_0^2} \right] = z_0 + \frac{(x_0 - x')^2 + (y_0 - y')^2}{2z_0}. \]

\[(1 + X)^{1/2} \approx 1 + X/2 \text{ was used when } |X| \ll 1.\]

\[ U(x_0, y_0, z_0) \approx jk \frac{\exp(-jkz_0)}{2\pi z_0} \iint_{S_a} \left[ \exp \left\{ -jk \left[ \frac{(x_0 - x')^2}{2z_0} + \frac{(y_0 - y')^2}{2z_0} \right] \right\} U_{inc}(x', y') \right] \ dx'dy' \]

\[ z_0 \gg \left( \frac{\pi}{4\lambda_0} D^4 \right)^{1/3} \quad D = \max\{[(x_0 - x')^2 + (y_0 - y')^2]^{1/2}\} \]

Fresnel Number

\[ N_F = \frac{D^2}{\lambda_0 z_0} \]

\[ \frac{\pi}{4} N_F \theta_{max}^2 \ll 1 \]

\[ \theta_{max} = \frac{D}{z_0} \]

Fresnel diffraction \( N_F \gtrsim 1 \) while for Fraunhofer diffraction \( N_F \ll 1 \)
Fresnel Approximation

Fresnel Approximation in 2D

\[ U(x_0, z_0) = \int_{x'} \left[ U_{\text{inc}} \frac{k}{2j} H_1^{(2)}(kR_T) \frac{z_0}{R_T} \right]_{z'=0} \ dx' \]

\[ \approx \frac{k}{2j} e^{j3\pi/4} \sqrt{\frac{2}{\pi k z_0}} \int_{x'} \exp(-j k R_T) U_{\text{inc}}(x') \ dx' \]

\[ \approx j \sqrt{\frac{k}{2\pi z_0}} e^{-j k z_0} \int_{x'} \exp \left[ -j \frac{k}{2z_0} (x' - x_0)^2 \right] U_{\text{inc}}(x') \ dx' \]

with the condition \( z_0 \gg \left[ \left( \frac{\pi}{4\lambda} \right) (x_0 - x')^4 \right]^{1/3} \).
Fraunhofer Approximation

\[ k \left[ \frac{(x_0 - x')^2 + (y_0 - y')^2}{2z_0} \right] = k \left[ \frac{x_0^2 + y_0^2}{2z_0} + \frac{x'^2 + y'^2}{2z_0} - \frac{x_0x' + y_0y'}{z_0} \right] = k \left[ \frac{x_0^2 + y_0^2}{2z_0} - \frac{x_0x' + y_0y'}{z_0} \right], \quad \text{if}, \]

\[ k \left( \frac{x'^2 + y'^2}{2z_0} \right) \ll 1 \quad \Rightarrow \quad z \gg \frac{kD_{max}^2}{2} = \frac{\pi}{\lambda} D_{max}^2 \approx \frac{\text{Area}}{\lambda}, \]

\[
U_{ff}(x_0, y_0, z_0) = jk \frac{\exp(-jkz_0)}{2\pi z_0} \exp \left( -jk \frac{x_0^2 + y_0^2}{2z_0} \right) \iint_{S_{\alpha}} \left[ \exp \left\{ +jk \left( \frac{x_0x' + y_0y'}{z_0} \right) \right\} U_{inc}(x', y') \right] dx' dy'
\]

\[ = jk \frac{\exp(-jkz_0)}{2\pi z_0} \exp \left( -jk \frac{x_0^2 + y_0^2}{2z_0} \right) \int_{-\infty}^{\infty} \exp \left\{ +jk \left( \frac{x_0x' + y_0y'}{z_0} \right) \right\} \left[ t_{a}(x', y') U_{inc}(x', y') \right] dx' dy'
\]

\[ = jk \frac{\exp(-jkz_0)}{2\pi z_0} \exp \left( -jk \frac{x_0^2 + y_0^2}{2z_0} \right) \mathcal{F} \left\{ t_{a}(x', y') U_{inc}(x', y') \right\}_{k_x = kx_0/z_0, \ k_y = kx_0/y_0/z_0} \]
Fraunhofer Approximation in 2D

\[
U_{ff}(x_0, z_0) = j \sqrt{\frac{k}{2\pi z_0}} e^{-jkz_0} \int_{x'} \exp \left[ -j \frac{k}{2z_0} (x' - x_0)^2 \right] U_{inc}(x') dx'
\]

\[
\simeq j \sqrt{\frac{k}{2\pi z_0}} e^{-jkz_0} \exp \left( -j \frac{k}{2z_0} x_0^2 \right) \int_{x'} \exp \left[ +j \frac{kx_0}{z_0} x' \right] U_{inc}(x') dx'
\]

\[
\simeq j \sqrt{\frac{k}{2\pi z_0}} e^{-jkz_0} \exp \left( -j \frac{k}{2z_0} x_0^2 \right) \int_{-\infty}^{+\infty} \exp \left[ +j \frac{kx_0}{z_0} x' \right] \left[ t_a(x') U_{inc}(x') \right] dx'
\]

\[
= j \sqrt{\frac{k}{2\pi z_0}} e^{-jkz_0} \exp \left( -j \frac{k}{2z_0} x_0^2 \right) \mathcal{F} \left\{ t_a(x') U_{inc}(x') \right\} \bigg|_{k_x = kx_0/z_0}.
\]

with the condition \( z_0 \gg \frac{\pi}{\lambda} x'^2 \simeq \frac{\text{Length}^2}{\lambda} \).
Diffraction from a Single Slit


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Diffraction from a Single Slit

Rayleigh-Sommerfeld Formulations

\[
U_{RS1}(x_0, z_0) = \int_a^b \left[ U_{inc} \frac{k}{2j} H_1^{(2)}(kR_T) \frac{z_0}{R_T} \right]_{z'=0} \ dx', \quad \text{and}
\]

\[
U_{RS2}(x_0, z_0) = \int_a^b \left[ -\frac{j}{2} H_0^{(2)}(kR_T) \frac{\partial U_{inc}}{\partial n} \right]_{z'=0} \ dx', \quad \text{where,}
\]

\[
R_T = \left[ (x_0 - x')^2 + z_0^2 \right]^{1/2}.
\]

Incident Wave

\[
U_{inc}(x', z') = \begin{cases} 
U_0 \exp(-jkz'), & \text{(plane wave)} \\
U_0 \frac{\exp(-jkR_s)}{\sqrt{R_s}}, & R_s = |x'^2 + (z' + s_0)^2|^{1/2}, \quad \text{(diverging)}, \\
U_0 \frac{\exp(jkR_s)}{\sqrt{R_s}}, & R_s = |x'^2 + (z' - s_0)^2|^{1/2}, \quad \text{(converging)},
\end{cases}
\]
Diffraction from a Single Slit
\(d = 10\mu m, \lambda_0 = 1\mu m\)

**Fresnel Approximation**
\[\lambda_0 = 1 \mu m, \ q = 0.1 \mu m, \ z_1 = -5 \mu m, \ z_2 = 5 \mu m\]

**Rayleigh-Sommerfeld 1**
\[\lambda_0 = 1 \mu m, \ q = 0.1 \mu m, \ z_1 = -5 \mu m, \ z_2 = 5 \mu m\]

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Diffraction from a Single Slit
\( d = 10\mu m, \lambda_0 = 1\mu m \)

Comparison of Rayleigh-Sommerfeld (1) and Fresnel Approximation
Diffraction from a Single Slit
\( d = 10\mu m, \lambda_0 = 1\mu m \)
Diffraction from a Single Slit
\(d = 10\mu m, \lambda_0 = 1\mu m\)

Comparison of Rayleigh-Sommerfeld (1) with 2D and 3D Green’s Function

\[\lambda_0 = 1 \mu m, \ q = 1 \mu m, \ z_1 = -5 \mu m, \ z_2 = 5 \mu m\]
Diffraction from a Single Slit
\( d = 10 \mu m, \lambda_0 = 1 \mu m \)

**Fresnel Approximation**

- \( \lambda_0 = 1 \mu m, \ q = 5 \mu m, \ z_1 = -5 \mu m, \ z_2 = 5 \mu m \)

**Rayleigh-Sommerfeld 1**

- \( \lambda_0 = 1 \mu m, \ q = 5 \mu m, \ z_1 = -5 \mu m, \ z_2 = 5 \mu m \)

**Fresnel Approximation**

- \( \lambda_0 = 1 \mu m, \ q = 10 \mu m, \ z_1 = -5 \mu m, \ z_2 = 5 \mu m \)

**Rayleigh-Sommerfeld 1**

- \( \lambda_0 = 1 \mu m, \ q = 10 \mu m, \ z_1 = -5 \mu m, \ z_2 = 5 \mu m \)

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Diffraction from a Single Slit
\[ d = 10 \mu m, \lambda_0 = 1 \mu m \]

Comparison of Rayleigh-Sommerfeld (1) and Fresnel Approximation
Diffraction from a Single Slit
\( d = 10 \mu m, \lambda_0 = 1 \mu m \)

**Fresnel Approximation**

\( \lambda_0 = 1 \mu m, q = 20 \mu m, z_1 = 5 \mu m, z_2 = 5 \mu m \)

**Rayleigh-Sommerfeld 1**

\( \lambda_0 = 1 \mu m, q = 20 \mu m, z_1 = 5 \mu m, z_2 = 5 \mu m \)

\( \lambda_0 = 1 \mu m, q = 50 \mu m, z_1 = 5 \mu m, z_2 = 5 \mu m \)

\( \lambda_0 = 1 \mu m, q = 50 \mu m, z_1 = 5 \mu m, z_2 = 5 \mu m \)

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Diffraction from a Single Slit
\[ d = 10\mu m, \lambda_0 = 1\mu m \]

Comparison of Rayleigh-Sommerfeld (1) and Fresnel Approximation

\[ \lambda_0 = 1 \mu m, \ q = 50 \mu m, \ z_1 = -5 \mu m, \ z_2 = 5 \mu m \]
Diffraction from a Single Slit
\( d = 10 \mu m, \lambda_0 = 1 \mu m \)

**Fresnel Approximation**

\( \lambda_0 = 1 \mu m, \ q = 100 \mu m, \ z_1 = -5 \mu m, \ z_2 = 5 \mu m \)

**Rayleigh-Sommerfeld 1**

\( \lambda_0 = 1 \mu m, \ q = 100 \mu m, \ z_1 = -5 \mu m, \ z_2 = 5 \mu m \)

\( \lambda_0 = 1 \mu m, \ q = 500 \mu m, \ z_1 = -5 \mu m, \ z_2 = 5 \mu m \)

\( \lambda_0 = 1 \mu m, \ q = 500 \mu m, \ z_1 = -5 \mu m, \ z_2 = 5 \mu m \)
Interference from a Double Slit - Young’s Experiment

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Interference from a Double Slit - Young’s Experiment


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The combined effects of two-slit and single-slit interference. This is the pattern produced when 650-nm light waves pass through two 3.0μm slits that are 18μm apart. Notice how the diffraction pattern acts as an “envelope” and controls the intensity of the regularly spaced interference maxima.

The combined effects of two-slit and single-slit interference. This is the pattern produced when 650-nm light waves pass through two 3.0μm slits that are 18μm apart. Notice how the diffraction pattern acts as an “envelope” and controls the intensity of the regularly spaced interference maxima.
Diffraction from a Circular Aperture
Diffraction from a Circular Aperture

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Diffraction from a Circular Aperture

\[ U_{RS_1}(x_0, y_0, z_0) = \int_0^a \int_0^{2\pi} U_0 \left( \frac{jk + \frac{1}{R}}{2\pi R} \right) \frac{\exp(-jkR)\frac{z_0}{R}r_T' dr_T' d\phi',}{r_T} \]

\[ U_{RS_2}(x_0, y_0, z_0) = \int_0^a \int_0^{2\pi} \frac{\exp(-jkR)}{2\pi R} (jkU_0)r_T' dr_T' d\phi', \]

\[ U(x_0, y_0, z_0) = \frac{jk \exp(-jkz_0)}{2\pi z_0} \int_0^a \int_0^{2\pi} \exp \left\{ -jk \frac{r_T'^2 + r_T^2 - 2r_T r_T' \cos(\phi - \phi')}{2z_0} \right\} U_0 r_T' dr_T' d\phi' = \]

\[ = \frac{jk \exp(-jkz_0)}{z_0} \int_0^a \exp \left\{ -jk \frac{r_T'^2 + r_T^2}{2z_0} \right\} U_0 J_0 \left( \frac{kr_T r_T'}{z_0} \right) r_T' dr_T' , \]

where

\[ R = \left[ r_T'^2 + r_T^2 - 2r_T r_T' \cos(\phi - \phi') + z_0^2 \right]^{1/2}, \quad \text{and,} \]

\[ r_T = \left( x_0^2 + y_0^2 \right)^{1/2}, \quad \phi = \tan^{-1} \left( \frac{y_0}{x_0} \right). \]
Diffraction from a Circular Aperture

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Diffraction from a Circular Aperture

**Fraunhofer Approximation**

\[
U_{ff} = \frac{j k}{2 \pi z_0} \exp(-jkz_0) \exp\left(-j k \frac{r_T^2}{2z_0}\right) \int_0^a \int_0^{2\pi} \exp\left\{+j k \left(\frac{r_T' r_T' \cos(\phi - \phi')}{z_0}\right)\right\} U_0 \, r_T' \, dr_T' \, d\phi' =
\]

\[
= j k \frac{\exp(-jkz_0)}{2 \pi z_0} \exp\left(-j k \frac{r_T^2}{2z_0}\right) U_0 \int_0^a 2\pi J_0 \left(\frac{kr_T'r_T'}{z_0}\right) r_T' \, dr_T' =
\]

\[
= j k \frac{\exp(-jkz_0)}{2 \pi z_0} \exp\left(-j k \frac{r_T^2}{2z_0}\right) U_0 2\pi \frac{aJ_1(ka r_T/z_0)}{kr_T/z_0} \Rightarrow
\]

\[
U_{ff} = j U_0 \exp(-jkz_0) \exp\left[-j k \frac{r_T^2}{2z_0}\right] \frac{aJ_1(ka r_T/z_0)}{r_T}, \quad \text{and the intensity is}
\]

\[
I_{ff} = \frac{|U_0|^2 k^2 a^4}{4Z z_0^2} \left(\frac{J_1(ka r_T/z_0)}{ka r_T/z_0}\right)^2 = I_0 \left(\frac{2J_1(ka r_T/z_0)}{ka r_T/z_0}\right)^2,
\]
Diffraction from a Circular Aperture

**Fraunhofer Regime**

\[ \lambda_0 = 1 \mu m, \quad z = 1000 \mu m, \quad \rho_1 = 0 \mu m, \quad \rho_2 = 10 \mu m \]

**Airy Pattern**
Individual diffraction patterns of two point sources (solid curves) and the resultant patterns (dashed curves) for various angular separations of the sources. In each case, the dashed curve is the sum of the two solid curves. (a) The sources are far apart, and the patterns are well resolved. (b) The sources are closer together such that the angular separation just satisfies Rayleigh’s criterion, and the patterns are just resolved. (c) The sources are so close together that the patterns are not resolved.

https://en.wikipedia.org/wiki/Angular_resolution
Fresnel Diffraction – Cornu Spiral

\[ U(x, y, q; P) = \frac{e^{-jkq}}{2\pi q} \int \int \exp \left[ -jk \frac{(x - x')^2 + (y - y')^2}{2q} \right] U_i(x', y', z' = 0) dx' dy' \]

\[ U_i(x', y', z' = 0) = E_0 \frac{e^{-jk(x'^2 + y'^2 + p^2)^{1/2}}}{(x'^2 + y'^2 + p^2)^{1/2}} \approx E_0 \frac{e^{-j kp}}{p} e^{-jk \frac{x^2 + y^2}{2p}} \]

\[ U(0, 0, q; P) \approx E_0 \frac{e^{-j k(p+q)}}{2\pi pq} \int \int \exp \left[ -jk \frac{x'^2 + y'^2}{2} \left( \frac{1}{p} + \frac{1}{q} \right) \right] dx' dy' \]

\[ u^2 = s^2 \left[ \frac{2}{\lambda_0} \left( \frac{1}{p} + \frac{1}{q} \right) \right] \]

\[ U(q; P) = E_0 \frac{e^{-j k(p+q)}}{2\pi pq} \left[ \frac{w_y}{\lambda_0} \left( \frac{1}{p} + \frac{1}{q} \right) \right]^{1/2} \int_{u_1}^{u_2} \exp \left[ -\frac{\pi}{2} u^2 \right] du \]
Fresnel Diffraction – Cornu Spiral

\[ C(u) = \int_0^u \cos \left( \frac{\pi}{2} v^2 \right) dv \]
\[ S(u) = \int_0^u \sin \left( \frac{\pi}{2} v^2 \right) dv \]
\[ C(-u) = -C(u) \]
\[ S(-u) = -S(u) \]

\[ u^2 = (x' - x_0)^2 \left[ \frac{2}{\lambda_0} \left( \frac{1}{p} + \frac{1}{q} \right) \right] \]

\[ U(q; P) = E_0 e^{-jk(p+q)} \frac{jk}{2\pi pq} \frac{w_y}{\left[ \frac{2}{\lambda_0} \left( \frac{1}{p} + \frac{1}{q} \right) \right]^{1/2}} \int_{u_1}^{u_2} \exp \left[ -\frac{\pi}{2} u^2 \right] du \]
Cornu Spiral

\[
C(u) = \int_0^u \cos \left( \frac{\pi}{2} v^2 \right) dv \\
S(u) = \int_0^u \sin \left( \frac{\pi}{2} v^2 \right) dv
\]

\[
E(P) = E_0 \left[ C(u_2) - C(u_1) \right] \pm j \left[ (S(u_2) - S(u_1)) \right]
\]

\[
I(P) = |E_0|^2 \left[ [C(u_2) - C(u_1)]^2 + [(S(u_2) - S(u_1))^2 \right]
\]

\[
E(P) = E_{unobstructed} = E_u = E_0 [1 \pm j]
\]

\[
I(P) = |E_{unobstructed}|^2 = |E_u|^2 = 2 |E_0|^2
\]
Light from a small source passes by the edge of an opaque object and continues on to a screen. A diffraction pattern consisting of bright and dark fringes appears on the screen in the region above the edge of the object.

Diffraction by an Edge

\[
\lambda_0 = 1 \, \mu m, \quad p = 10000000 \, \mu m, \quad q = 10 \, \mu m, \quad z_1 = 0 \, \mu m, \quad z_2 = 1000000 \, \mu m
\]

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Diffraction by an Edge
**Cornu Spiral: 1D Slit**

**Fresnel Approximation - Incident Plane Wave**

\[ U(x_0, z_0) \approx \int_a^b \left[ U_{inc} \frac{k}{2j} \sqrt{\frac{2}{\pi k R_T}} \exp \left[ -j \left( k R_T - \frac{3\pi}{4} \right) \frac{z_0}{R_T} \right] \right]_{z'=0} \, dx' \]

\[ \approx U_0 \frac{k}{2j} \sqrt{\frac{2}{\pi k z_0}} e^{j3\pi/4} e^{-j k z_0} \int_a^b \exp \left( -j \frac{k}{2z_0} (x_0 - x')^2 \right) \, dx'. \]

\[ \frac{\pi}{2} u^2 = \frac{k}{2z_0} (x_0 - x')^2, \quad \text{resulting in} \]

\[ \int_a^b \exp \left( -j \frac{k}{2z_0} (x_0 - x')^2 \right) \, dx' = \sqrt{\frac{\lambda z_0}{2}} \int_{u_1}^{u_2} \exp \left( -j \frac{\pi}{2} u^2 \right) \, du, \quad \text{where,} \]

\[ u_1 = \sqrt{\frac{2}{\lambda z_0}} (x_0 - b), \quad u_2 = \sqrt{\frac{2}{\lambda z_0}} (x_0 - a). \]
Cornu Spiral: 1D Slit

\[
\int_0^w \exp \left( -j \frac{\pi}{2} u^2 \right) du = C(w) - j S(w), \quad \text{where}
\]
\[
C(w) = \int_0^w \cos \left( \frac{\pi}{2} u^2 \right) du, \quad S(w) = \int_0^w \sin \left( \frac{\pi}{2} u^2 \right) du.
\]

\[
U(x_0, z_0) = U_0 e^{j\pi/4} \sqrt{\frac{k}{2\pi z_0}} \sqrt{\frac{\lambda z_0}{2}} e^{-jkz_0} \left\{ [C(u_2) - C(u_1)] - j [S(u_2) - S(u_1)] \right\} =
\]
\[
= \frac{U_0}{\sqrt{2}} e^{j\pi/4} e^{-jkz_0} \left\{ [C(u_2) - C(u_1)] - j [S(u_2) - S(u_1)] \right\}.
\]

\[
I(x_0, z_0) = \frac{|U_0|^2}{4Z} \frac{1}{2} \left\{ [C(u_2) - C(u_1)]^2 + [S(u_2) - S(u_1)]^2 \right\} =
\]
\[
= \frac{I_u}{2} \left\{ [C(u_2) - C(u_1)]^2 + [S(u_2) - S(u_1)]^2 \right\},
\]

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Cornu Spiral: Example of Diffraction by an Edge

\[ a = 0, \quad b = +\infty, \quad \lambda_0 = 1 \mu m, \quad z_0 = 10 \mu m \]

\[ x_0 = 0 \]

\[ u_1 = \sqrt{\frac{2}{\lambda_0 z_0}} (x_0 - b) = -\infty \]

\[ u_2 = \sqrt{\frac{2}{\lambda_0 z_0}} (x_0 - a) = 0 \]

\[ C(u_1) = -0.50 \]

\[ S(u_1) = -0.50 \]

\[ C(u_2) = 0 \]

\[ S(u_2) = 0 \]

\[ U(x_0, z_0) = \frac{U_0}{\sqrt{2}} e^{i\pi/4} e^{-jkz_0} [0.5 - j0.5] \]

\[ |U(x_0, z_0)| = 0.5U_0 = 0.5E_u \]

\[ I(x_0, z_0) = \frac{I_u}{2} [0.5^2 + 0.5^2] = \frac{I_u}{4} \]
Cornu Spiral with $exp(-jkr)$ convention

- $u = -\infty$
  - $I/I_u = 0.25$
  - $E/E_u = 0.50$
- $u = 0$
- $u = 1.21$
  - $I/I_u = 1.37$
  - $E/E_u = 1.17$

**Mathematical Expressions**

- $S(u) = \int_0^u \sin(\pi x^2/2) \, dx$
- $C(u) = \int_0^u \cos(\pi x^2/2) \, dx$

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Cornu Spiral: Example of Diffraction by an Edge

\[ a = 0, \quad b = +\infty, \quad \lambda_0 = 1 \mu m, \quad z_0 = 10 \mu m \]

\[ x_0 = 2.7056 \]

\[ u_1 = \sqrt{\frac{2}{\lambda_0 z_0}} (x_0 - b) = -\infty \]

\[ u_2 = \sqrt{\frac{2}{\lambda_0 z_0}} (x_0 - a) = 1.21 \]

\[ C(u_1) = -0.50 \]

\[ S(u_1) = -0.50 \]

\[ C(u_2) = 0.7089 \]

\[ S(u_2) = 0.6310 \]

\[ U(x_0, z_0) = \frac{U_0}{\sqrt{2}} e^{j\pi/4} e^{-jkz_0} \left[ (0.7089 + 0.5) - j(0.6310 + 0.5) \right] \]

\[ |U(x_0, z_0)| = 1.1706 U_0 = 1.1706 U_u \]

\[ I(x_0, z_0) = \frac{I_u}{2} \left[ 1.2089^2 + 1.1310^2 \right] = 1.3703 I_u \]
Diffraction by an Edge

Comparison of Rayleigh-Sommerfeld (1) and Fresnel Approximation

\[ \lambda_0 = 1 \, \mu m, \; q = 10 \, \mu m, \; z_1 = 0 \, \mu m, \; z_2 = 10000000 \, \mu m \]

\[ \lambda_0 = 1 \, \mu m, \; q = 20 \, \mu m, \; z_1 = 0 \, \mu m, \; z_2 = 10000000 \, \mu m \]

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Diffraction by an Edge

Comparison of Rayleigh-Sommerfeld (1) with 2D and 3D Green’s Function

\[ \lambda_0 = 1 \mu m, \quad q = 10 \mu m, \quad z_1 = 0 \mu m, \quad z_2 = 100000 \mu m \]

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Babinet’s Principle

Aperture in Infinite Opaque Screen

Obstacle in Infinite Transparent Screen

\[ U_1(x_0, y_0, z_0) + U_2(x_0, y_0, z_0) = U_{inc}(x_0, y_0, z_0). \]
**Diffraction from an Opaque Disk**

\[ \lambda_0 = 1 \mu m, \quad \rho = 0 \mu m, \quad \rho_1 = 10 \mu m, \quad \rho_2 = 500000 \mu m \]

- Analytical Sommerfeld Lemma
- Numerical Integration

\[ U(\vec{r}) = j \frac{2\pi z}{\lambda} \int_{\rho_1}^{\rho_2} \frac{e^{-jkR_0}}{R_0^2} J_0 \left( \frac{k \rho \rho'}{R_0} \right) \rho' d\rho', \quad I = |U|^2 \]

\[ R_0 = \sqrt{\rho^2 + \rho'^2 + z^2} \]

**Sommerfeld’s Lemma**

\[ E_1 = E_u \left[ e^{-jkz} - \frac{e^{-jkz\sqrt{1+(\rho_1/z)^2}}}{\sqrt{1+(\rho_1/z)^2}} \right] \quad E_2 = E_u \frac{e^{-jkz\sqrt{1+(\rho_1/z)^2}}}{\sqrt{1+(\rho_1/z)^2}} \]


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Diffraction from an Opaque Disk

Diffraction pattern created by the illumination of an opaque disk (U.S. penny), with the disk positioned midway between screen and light source. Note the bright spot at the center (Poisson Spot)


http://en.wikipedia.org/wiki/Arago_spot
In 1818, Augustin Fresnel submitted a paper on the theory of diffraction for a competition sponsored by the French Academy. Simeon D. Poisson, a member of the judging committee for the competition, was very critical of the wave theory of light. Using Fresnel's theory, Poisson deduced the seemingly absurd prediction that a bright spot should appear behind a circular obstruction, a prediction he felt was the last nail in the coffin for Fresnel's theory.

However, Dominique Arago, another member of the judging committee, almost immediately verified the spot experimentally. Fresnel won the competition, and, although it may be more appropriate to call it "the Spot of Arago," the spot goes down in history with the name "Poisson's bright spot".
Diffraction from an Opaque Disk

Circular Obstacle: $a = 10\mu m, \lambda_0 = 0.5145\mu m$
Diffraction from an Opaque Disk
Diffraction from an Opaque Disk with Randomness at its Edge

- 0.0% radius variation
- 2.5% radius variation

Polygons

- $\lambda = 1 \, \mu m$, $y_0 = 0 \, \mu m$, $z_0 = 1000 \, \mu m$

Normalized Intensity, $U^2/U_0^2$

Distance along x, (\(\mu m\))
Diffraction from an Opaque Disk with Randomness at its Edge

5.0% radius variation

10.0% radius variation

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Babinet’s Principle

\[ E_1 + E_2 = E_u \]

\[ \lambda_0 = 1 \, \mu m, \quad \rho = 0 \, \mu m, \quad \rho_1 = 0 \, \mu m, \quad \rho_2 = 10 \, \mu m \]

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Babinet’s Principle

\[
E_1 = E_u \left[ e^{-j k z} - \frac{e^{-j k z \sqrt{1+(\rho_1/z)^2}}}{\sqrt{1 + (\rho_1/z)^2}} \right]
\]

\[
E_2 = E_u \frac{e^{-j k z \sqrt{1+(\rho_1/z)^2}}}{\sqrt{1 + (\rho_1/z)^2}}
\]
Babinet’s Principle

\[ E_1 = E_u \left[ e^{-jkz} - \frac{e^{-jkz\sqrt{1+(\rho_1/z)^2}}}{\sqrt{1 + (\rho_1/z)^2}} \right] \]

\[ E_2 = E_u \frac{e^{-jkz\sqrt{1+(\rho_1/z)^2}}}{\sqrt{1 + (\rho_1/z)^2}} \]

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Babinet’s Principle

$E_1 + E_2 = E_u$

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Babinet’s Principle

\[ E_1 + E_2 = E_u \]
Babinet’s Principle

$$E_1 + E_2 = E_u$$

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Babinet’s Principle
Babinet’s Principle

Complementary Screen Diffraction Patterns in Fraunhofer Regime

Diffraction by a Rectangular Aperture

\[ U_{RS1}(P_0) = \int_{-a}^{+a} \int_{-b}^{+b} \left[ U_{inc} \left( jk + \frac{1}{R} \right) \frac{\exp(-jkR)}{2\pi R} \frac{z_0}{R} \right]_{z'=0} \, dx' \, dy' , \]

\[ U_{RS2}(P_0) = \int_{-a}^{+a} \int_{-b}^{+b} \left[ \frac{\exp(-jkR)}{2\pi R} \frac{\partial U_{inc}}{\partial n} \right]_{z'=0} \, dx' \, dy' , \]

with \[ R = \left[ (x_0 - x')^2 + (y_0 - y')^2 + z_0^2 \right]^{1/2} . \]
Diffraction by a Rectangular Aperture

Fresnel Approximation

\[
U(x_0, y_0, z_0) = jk \frac{\exp(-jkz_0)}{2\pi z_0} \int_{-a}^{+a} \int_{-b}^{+b} \left[ \exp \left\{ -jk \left[ \frac{(x_0 - x')^2}{2z_0} + \frac{(y_0 - y')^2}{2z_0} \right] \right\} U_0 \right] dx' dy' =
\]
\[
= jk \frac{\exp(-jkz_0)}{2\pi z_0} U_0 \int_{-a}^{+a} \exp \left[ -jk \frac{(x_0 - x')^2}{2z_0} \right] dx' \int_{-b}^{+b} \exp \left[ -jk \frac{(y_0 - y')^2}{2z_0} \right] dy'.
\]

\[
U_x = U_x(x_0, z_0) = \sqrt{\frac{\lambda z_0}{2}} \left\{ C(u_2) - C(u_1) - j [S(u_2) - S(u_2)] \right\},
\]
with \( u_2 = \sqrt{\frac{2}{\lambda z_0}} (x_0 + a), \quad u_1 = \sqrt{\frac{2}{\lambda z_0}} (x_0 - a), \)

\[
V_y = V_y(y_0, z_0) = \sqrt{\frac{\lambda z_0}{2}} \left\{ C(v_2) - C(v_1) - j [S(v_2) - S(v_2)] \right\},
\]
with \( v_2 = \sqrt{\frac{2}{\lambda z_0}} (y_0 + b), \quad v_1 = \sqrt{\frac{2}{\lambda z_0}} (y_0 - b). \)

\[
U(x_0, y_0, z_0) = j \frac{U_0}{2} \exp(-jkz_0) F(u_2, u_1) F(v_2, v_1),
\]
where \( F(u_2, u_1) = \left\{ C(u_2) - C(u_1) - j [S(u_2) - S(u_2)] \right\}, \)
and \( F(v_2, v_1) = \left\{ C(v_2) - C(v_1) - j [S(v_2) - S(v_2)] \right\}. \)
Diffraction by a Rectangular Aperture

Fraunhofer Approximation

\[ U_{ff}(x_0, y_0, z_0) = jkU_0 \frac{e^{-jkz_0}}{2\pi z_0} \exp \left[ -j \frac{k}{2z_0} (x_0^2 + y_0^2) \right] \left\{ \begin{array}{c} \sin \left( \frac{a}{z_0} kx_0 \right) \\ \frac{a}{z_0} kx_0 \end{array} \right\} \left\{ \begin{array}{c} \sin \left( \frac{b}{z_0} ky_0 \right) \\ \frac{b}{z_0} ky_0 \end{array} \right\} \]

\[ I_{ff}(x_0, y_0, z_0) = \left( \frac{|U_0|^2}{4Z} \frac{4ab}{\lambda z_0} \right)^2 \left\{ \begin{array}{c} \sin \left( \frac{a}{z_0} kx_0 \right) \\ \frac{a}{z_0} kx_0 \end{array} \right\}^2 \left\{ \begin{array}{c} \sin \left( \frac{b}{z_0} ky_0 \right) \\ \frac{b}{z_0} ky_0 \end{array} \right\}^2 = I_0(z_0) \sin^2 \left( 2a \frac{x_0}{\lambda z_0} \right) \sin^2 \left( 2b \frac{y_0}{\lambda z_0} \right) \]

\[ \text{sinc}(u) = \sin(\pi u) / (\pi u) \]
Diffraction by a Rectangular Aperture
Diffraction by a Rectangular Aperture
Fresnel Zone Plate

\[ R_m = f + m \frac{\lambda}{2} \]
\[ R_{m-1} = f + (m - 1) \frac{\lambda}{2} \]

\[
\rho_m^2 + f^2 = R_m^2 = \left( f + m \frac{\lambda}{2} \right)^2 \implies \\
\rho_m = \left( m \lambda f + m^2 \frac{\lambda^2}{4} \right)^{1/2}, \quad m = 0, 1, 2, \cdots
\]
Fresnel Zone Plate

Kirchhoff’s Formulation for a Planar Screen

\[
U(P_0) = \iint_{S_1} \left( G \frac{\partial U}{\partial n} - U \frac{\partial G}{\partial n} \right) ds = \\
= \iint_{S_1} \left[ e^{-\frac{jkr}{R}} \frac{\partial U}{\partial n} - U \left( jk + \frac{1}{R} \right) \frac{e^{-\frac{jkr}{R}}}{4\pi R} \left( \hat{i}_R \cdot \hat{i}_n \right) \right] ds.
\]

\[
U_K(P_0) = U(P_0) = \iint_{S_a} \left[ e^{-\frac{jkr}{R}} \frac{\partial U_{inc}}{\partial n} - U_{inc} \left( jk + \frac{1}{R} \right) \frac{e^{-\frac{jkr}{R}}}{4\pi R} \left( \hat{i}_R \cdot \hat{i}_n \right) \right] ds = \\
= \iint_{S_a} \frac{e^{-\frac{jkr}{R}}}{4\pi R} \left[ \frac{\partial U_{inc}(x', y', 0)}{\partial n} + U_{inc}(x', y', 0) \left( jk + \frac{1}{R} \right) \cos \theta \right] dx' dy',
\]

where \( \cos \theta = \frac{z_0}{R} \).
\[ U_K(f) = U_0 jk \int \int_{S_a} \frac{e^{-jkR}}{4\pi R} \left( 1 + \frac{f}{R} \right) dS \]

\[ = \frac{U_0 jk}{2} \sum_{m=1}^{N} \int_{\rho_{m-1}}^{\rho_m} \frac{e^{-jkR}}{R} \left( 1 + \frac{f}{R} \right) r_T dr_T = \frac{U_0 jk}{2} \sum_{m=1}^{N} \int_{\rho_{m-1}}^{\rho_m} \frac{e^{-jkR}}{R} \left( 1 + \frac{f}{R} \right) RdR \]

\[ = \frac{U_0 jk}{2} \sum_{m=1}^{N} \left\{ \frac{\exp(-jkR_{m-1}) - \exp(-jkR_m)}{jk} + f [E_1(kR_{m-1}) - E_1(kR_m)] \right\}, \]

where \[ E_1(w) = \int_{w}^{+\infty} \frac{e^{-t}}{t} dt. \]
Fresnel Zone Plate

\[ E_1(w) \simeq e^{-w/w} \]

\[ kR_{m-1} = kf + (m - 1)\pi \]

\[ kR_m = kf + m\pi \]

\[
U_K(f) \simeq \frac{U_0 e^{-jkf}}{2} \sum_{m=1}^{N} \left\{\exp(-j(m-1)\pi) - \exp(-jm\pi)\right\} + \left\{\exp(-j(m-1)\pi) - \exp(-jm\pi)\right\}
\]

\[
= U_0 e^{-jkf} \sum_{m=1}^{N} \left[ \exp(-j(m-1)\pi) - \exp(-jm\pi) \right]
\]

\[
= U_0 e^{-jkf} \left\{ \frac{1 - (-1)}{V_0^1} + \frac{(-1) - 1}{V_1^2} + \frac{1 - (-1)}{V_2^3} + \cdots \right\} = \sum_{m=1}^{N} [U_K]^m_{m-1} \implies
\]

\[
U_K(f) \simeq \begin{cases} 
2U_0 e^{-jkf}, & \text{for odd } N, \\
0, & \text{for even } N,
\end{cases}
\]
Fresnel Zone Plate
Fresnel Zone Plate

Each zone is subdivided into 15 subzones

\[ A_n = a_1 + a_2 e^{i\pi} + a_3 e^{i2\pi} + a_4 e^{i3\pi} + \ldots + a_n e^{i(n-1)\pi} \]

\[ A_n = a_1 - a_2 + a_3 - a_4 + \ldots + e^{i(n-1)\pi} a_n \]
Adding up the zones

For large $N$, resultant amplitude = half that of zone 1

Fresnel Zone Plate

\[ R_m = f + m \frac{\lambda}{2} \]

\[ R_{m-1} = f + (m - 1) \frac{\lambda}{2} \]

\[ U_{FK}^{\text{odd}}(f) \approx \begin{cases} 
2U_0 e^{-jkf} \frac{N}{2} = NU_0 e^{-jkf}, & \text{for even } N, \\
2U_0 e^{-jkf} \frac{N+1}{2} = (N+1)U_0 e^{-jkf}, & \text{for odd } N.
\end{cases} \]

\[ U_{FK}^{\text{even}}(f) \approx \begin{cases} 
2U_0 e^{-jkf} \frac{N}{2} = NU_0 e^{-jkf}, & \text{for even } N, \\
2U_0 e^{-jkf} \frac{N-1}{2} = (N-1)U_0 e^{-jkf}, & \text{for odd } N.
\end{cases} \]

\[ I_{FK}^{\text{odd}}(f) \approx \begin{cases} 
\frac{|U_0|^2}{4Z} N^2 = N^2 I_0, & \text{for even } N, \\
\frac{|U_0|^2}{4Z} (N+1)^2 = (N+1)^2 I_0, & \text{for odd } N.
\end{cases} \]

\[ I_{FK}^{\text{even}}(f) \approx \begin{cases} 
\frac{|U_0|^2}{4Z} N^2 = N^2 I_0, & \text{for even } N, \\
\frac{|U_0|^2}{4Z} (N-1)^2 = (N-1)^2 I_0, & \text{for odd } N.
\end{cases} \]
Fresnel Zone Plate

\[ E_{tot} = |E_1| + |E_3| + \ldots + |E_{2m-1}| \approx m|E_i| \quad E_{tot} = -|E_2| - |E_4| - \ldots - |E_{2m}| = -m|E_i| \]

Odd (positive) Fresnel Plate

Even (negative) Fresnel Plate
Fresnel Zone Plate

(a) Fresnel Zones (odd shaded): Number of zones, $N = 11$
$\lambda_0 = 0.6328 \mu\text{m}, f = 25 \mu\text{m}$

(b) Normalized intensity, $|U|^2|U_0|^2$
Distance along $z$, ($\mu\text{m}$)

(c) Fresnel Zones (even shaded): Number of zones, $N = 11$
$\lambda_0 = 0.6328 \mu\text{m}, f = 25 \mu\text{m}$

(d) Normalized intensity, $|U|^2|U_0|^2$
Distance along $z$, ($\mu\text{m}$)
Fresnel Zone Plate

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Fresnel Zone Plate

(a) Fresnel Zones (odd shaded): Number of zones, N = 50
\( \lambda = 0.6328 \mu m, f = 16000 \mu m \)

Normalized Intensity, \( |U|^2 \mu^2 \)
Distance along z, (\( \mu m \))

(b) Fresnel Zones (odd shaded): Number of zones, N = 50
\( \lambda = 0.6328 \mu m, f = 20000 \mu m \)

(c) Fresnel Zones (even shaded): Number of zones, N = 50
\( \lambda = 0.6328 \mu m, f = 16000 \mu m \)

Normalized Intensity, \( |U|^2 \mu^2 \)
Distance along z, (\( \mu m \))

(d) Fresnel Zones (even shaded): Number of zones, N = 50
\( \lambda = 0.6328 \mu m, f = 20000 \mu m \)

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Fresnel Lenses


http://www.fresneloptic.com/images/solar_fresnel_lens.jpg
Diffractive Lens Design (Non-parabolic)

Zone Boundaries

\[ x_m = \sqrt{2fm\lambda_2 + m^2\lambda_2^2} \quad \lambda_2 = \frac{\lambda_0}{n_2} \]

\[ n_1[d - s(x)] + n_2s(x) + n_2\sqrt{f^2 + x^2} = n_1d + f + m\lambda_2 \]

**s(x) Determination**

\[ s(x) = \frac{\lambda_0}{n_1 - n_2} \left[ \frac{\sqrt{f^2 + x^2} - f}{\lambda_2} - m \right] \quad x_m \leq x < x_{m+1} \]
Diffractive Lens Design Example

\[ \lambda_0 = 1\,\mu m, \, n_1 = 1.5, \, n_2 = 1.0, \, f = 100\,\mu m, \]

Number of Zones = 10

Number of Zones = 15
Diffractive Lens Performance

\[ U(x, z) = \int_{-D/2}^{D/2} \frac{\exp(-jkR)}{2\pi R} \left( jk + \frac{1}{R} \right) \frac{z}{R} U_i(x', z' = 0) \, dx' \]

\[ R = \left[ (x - x')^2 + z^2 \right]^{1/2} \]

\[ U_i(x', z' = 0) = E_0 e^{-j\Phi(x')} \]

\[ \Phi(x') = -k_0(n_1 - n_2)s(x') \]
Diffractive Lens Performance

Number of Zones = 10

Number of Zones = 15

$n_1 = 1.5, n_2 = 1, \lambda_0 = 1 \mu m, f = 100 \mu m$
Diffractive Lens Performance

Number of Zones = 10

Number of Zones = 15
Diffractive Lens Performance

Number of Zones = 25

$n_1 = 1.5, n_2 = 1, \lambda_0 = 1 \, \mu m, f = 100 \, \mu m$

Normalized Intensity vs. z distance (microns)

$n_1 = 1.5, n_2 = 1, \lambda_0 = 1 \, \mu m, f = 100 \, \mu m$

z distance (microns) vs. x distance (microns)
Diffractive Lenses


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N-Level Diffractive Lens Design Example

$$\lambda_0 = 1\mu m, \ n_1 = 1.5, \ n_2 = 1.0, \ f = 100\mu m,$$

Refractive Design

Diffractive Design (Non-Parabolic)
N-Level Diffractive Lens Design Example

\[ \lambda_0 = 1 \mu m, \ n_1 = 1.5, \ n_2 = 1.0, \ f = 100 \mu m, \]

Diffractive Design (Parabolic)

Diffractive Design (Non-Parabolic/Thickness)
Fresnel Lighthouse Lens

other applications: overhead projectors
automobile headlights
solar collectors
traffic lights

Prof. F.A. van Goor, Twente University
http://edu.tnw.utwente.nl/inlopt/overhead_sheets/

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