Physical & Electromagnetic Optics: Basic Principles of Diffraction

Optical Engineering
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Huygens Principle and Diffraction

Every point on a propagating wavefront becomes a secondary source of spherical wavelets. All these wavelets form the envelope of the next wavefront.

Diffraction – Bending of light into the shadow regions (wavelength, $\lambda$, of the order of obstacles)

https://www.usna.edu/Users/physics/ejtuchol/documents/SP212R/Chapter%20S37.ppt
Diffraction Examples

Diffraction by a Razor

Diffraction by an Edge

Gengage Learning, Chap. 38
Integral Theorem of Helmholtz and Kirchhoff

3D-Green’s Function

\[ \nabla^2 G(\vec{r}, \vec{r}') + k^2 G(\vec{r}, \vec{r}') = -\delta(\vec{r} - \vec{r}') \]

\[ G(\vec{r}, \vec{r}') = \frac{\exp(-jk|\vec{r} - \vec{r}'|)}{4\pi|\vec{r} - \vec{r}'|} \]

Scalar Field at a point as a function of the Fields on the Boundary \( S \)

\[ U(P_0) = \iint_S \left( G(\vec{r}, \vec{r}') \frac{\partial U}{\partial n} - U \frac{\partial G(\vec{r}, \vec{r}')}{\partial n} \right) dS \]

Kirchhoff’s Formulation for a Planar Screen

\[ U(P_0) = \iint_{S_1} \left( G(\vec{r}, \vec{r}') \frac{\partial U}{\partial n} - U \frac{\partial G(\vec{r}, \vec{r}')}{\partial n} \right) dS \]

Sommerfeld’s Radiation Condition

\[ \lim_{R \to \infty} \left\{ R \left( \frac{\partial U}{\partial n} + jkU \right) \right\} = 0 \]

Green’s Function Selection

\[ G_D(\vec{r}, \vec{r}') = 0 \quad \vec{r}' \in S_1 \quad \text{Dirichlet} \]

\[ \frac{\partial G_N(\vec{r}, \vec{r}')}{\partial n} = 0 \quad \vec{r}' \in S_1 \quad \text{Neumann} \]

Rayleigh-Sommerfeld’s Formulations

Neumann Green’s Function

\[
G_N(\vec{r}, \vec{r}') = \frac{e^{-jkr - r'}}{4\pi|\vec{r} - \vec{r}'|} + \frac{e^{-jkr - r''}}{4\pi|\vec{r} - \vec{r}''|}
\]

\[
G_N|_{z'=0} = \frac{e^{-jkR}}{2\pi R}
\]

\[
\frac{\partial G_N}{\partial n}|_{z'=0} = \vec{n}' G_N \cdot \hat{n} = \vec{n}' G_N \cdot (-\hat{z}) = -\frac{\partial G_N}{\partial z'}|_{z'=0} = 0
\]

\[
R = \left[ (x - x')^2 + (y - y')^2 + z^2 \right]^{1/2}
\]

Dirichlet Green’s Function

\[
G_D(\vec{r}, \vec{r}') = \frac{e^{-jk|r - r'|}}{4\pi|\vec{r} - \vec{r}'|} - \frac{e^{-jk|r - r''|}}{4\pi|\vec{r} - \vec{r}''|}
\]

\[
G_D|_{z'=0} = 0
\]

\[
\frac{\partial G_D}{\partial n}|_{z'=0} = \vec{n}' G_D \cdot \hat{n} = \vec{n}' G_D \cdot (-\hat{z}) = -\frac{\partial G_D}{\partial z'}|_{z'=0} = -\frac{e^{-jkR}}{2\pi R} \left( jk + \frac{1}{R} \right) \frac{z}{R}
\]
Rayleigh-Sommerfeld’s 1 Formulation for a Planar Screen

\[ U(\vec{r}) = \int \int_{S_1} \frac{\exp(-jkR)}{2\pi R} \left( jk + \frac{1}{R} \right) \frac{z}{R} U_i(\vec{r}') dS' \]

\[ R = \left[ (x-x')^2 + (y-y')^2 + z^2 \right]^{1/2} \]

Rayleigh-Sommerfeld’s 2 Formulation for a Planar Screen

\[ U(\vec{r}) = \int \int_{S_1} \frac{\exp(-jkR) \partial U_i(\vec{r}')}{2\pi R} \frac{\partial}{\partial n} dS' \]

\[ R = \left[ (x-x')^2 + (y-y')^2 + z^2 \right]^{1/2} \]

Integral Theorem in 2D Diffraction Problems

2D-Green’s Function

\[ \nabla^2 G(\vec{r}, \vec{r}') + k^2 G(\vec{r}, \vec{r}') = -\delta(\vec{r} - \vec{r}') \]

\[ G(\vec{r}, \vec{r}') = \frac{-j}{4} H_0^{(2)}(k|\vec{r} - \vec{r}'|) \]

\[ \vec{r} = x\hat{x} + z\hat{z} \]

\[ \vec{r}' = x'\hat{x} + z'\hat{z} \]

Scalar Field at a point as a function of the Fields on the Boundary Curve C

\[ U(P_0) = \int_C \left( G(\vec{r}, \vec{r}') \frac{\partial U}{\partial n} - U \frac{\partial G(\vec{r}, \vec{r}')}{\partial n} \right) d\ell \]


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Summary of Diffraction Formulas

Rayleigh-Sommerfeld 1

\[ U(x, y, z) = \int \int \frac{e^{-jkR}}{2\pi R} \left( jk + \frac{1}{R} \right) \frac{z}{R} U_i(x', y') dx' dy', \quad 3D \text{ Green's Function} \]

\[ R = \sqrt{(x-x')^2 + (y-y')^2 + z^2}, \]

\[ U(x, z) = \int \frac{k}{2j} H_1^{(2)}(kR) \frac{z}{R} U_i(x') dx', \quad 2D \text{ Green's Function} \]

\[ R = \sqrt{(x-x')^2 + z^2} \]

\[ H_1^{(2)}(z) = J_1(z) - jY_1(z), \quad \text{Hankel's Function} \]

Fresnel Approximation

\[ U(x, y, z) = \frac{e^{-jkz}}{2\pi z} jk \int \int \exp \left[ -jk \frac{(x-x')^2 + (y-y')^2}{2z} \right] U_i(x', y') dx' dy', \]

\[ z \gg \left( \frac{\pi}{4\lambda} \right)^{1/3} \left[ (x-x')^2 + (y-y')^2 \right]^{2/3} \approx \left( \frac{\pi}{4\lambda} \right)^{1/3} D^{4/3} \]
Summary of Diffraction Formulas (continued)

**Fraunhofer Approximation**

\[
U(x, y, z) = \frac{e^{-j kz}}{2\pi z} \int \int \exp \left[-j k \frac{x'^2 + y'^2}{2z} \right] \int \exp \left[j k \frac{xx' + yy'}{z} \right] U_i(x', y') dx' dy' = \\
= \frac{e^{-j kz}}{2\pi z} j k \exp \left[-j k \frac{x'^2 + y'^2}{2z} \right] \mathcal{F} \{ U_i(x', y') \} \bigg|_{\omega_x=kx/z, \omega_y=ky/z} \quad \text{for} \quad z \gg \frac{D^2}{\lambda}
\]
Diffraction from a Single Slit


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Diffraction from a Single Slit
\( d = 10 \mu \text{m}, \lambda_0 = 1 \mu \text{m} \)

Fresnel Approximation
\( \lambda_0 = 1 \mu \text{m}, q = 0.1 \mu \text{m}, z_1 = 5 \mu \text{m}, z_2 = 5 \mu \text{m} \)

Rayleigh-Sommerfeld 1
\( \lambda_0 = 1 \mu \text{m}, q = 0.1 \mu \text{m}, z_1 = 5 \mu \text{m}, z_2 = 5 \mu \text{m} \)
Diffraction from a Single Slit
d = 10μm, λ₀ = 1μm

Comparison of Rayleigh-Sommerfeld and Fresnel Approximation

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Diffraction from a Single Slit
\[ d = 10\mu m, \lambda_0 = 1\mu m \]

Comparison of Rayleigh-Sommerfeld with 2D and 3D Green’s Function

\[ \lambda_0 = 1\mu m, \ q = 1\mu m, \ z_1 = -5\mu m, \ z_2 = 5\mu m \]

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Diffraction from a Single Slit
\[ d = 10 \mu m, \lambda_0 = 1 \mu m \]

Fresnel Approximation

Rayleigh-Sommerfeld 1

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Diffraction from a Single Slit
\( d = 10\mu m, \lambda_0 = 1\mu m \)

Comparison of Rayleigh-Sommerfeld and Fresnel Approximation

\( \lambda_0 = 1 \mu m, \ q = 10 \mu m, \ z_1 = -5 \mu m, \ z_2 = 5 \mu m \)

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Diffraction from a Single Slit
\[ d = 10\mu m, \lambda_0 = 1\mu m \]

Fresnel Approximation

Rayleigh-Sommerfeld 1

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Diffraction from a Single Slit
\[ d = 10 \mu m, \lambda_0 = 1 \mu m \]

Comparison of Rayleigh-Sommerfeld and Fresnel Approximation
Diffraction from a Single Slit
d = 10\mu m, \lambda_0 = 1\mu m

Fresnel Approximation

Rayleigh-Sommerfeld 1

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Interference from a Double Slit
Young’s Experiment


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Interference from a Double Slit - Young’s Experiment


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The combined effects of two-slit and single-slit interference. This is the pattern produced when 650-nm light waves pass through two 3.0μm slits that are 18μm apart. Notice how the diffraction pattern acts as an “envelope” and controls the intensity of the regularly spaced interference maxima.

Diffraction from a Double Slit

The combined effects of two-slit and single-slit interference. This is the pattern produced when 650-nm light waves pass through two 3.0μm slits that are 18μm apart. Notice how the diffraction pattern acts as an “envelope” and controls the intensity of the regularly spaced interference maxima.
Diffraction from a Circular Aperture

Rayleigh-Sommerfeld

\[
U(\vec{r}) = j \frac{2\pi z}{\lambda} \int_{\rho_1}^{\rho_2} \frac{e^{-jkR_0}}{R_0^2} J_0 \left( \frac{k\rho\rho'}{R_0} \right) \rho' d\rho', \quad I = |U|^2
\]

\[
R_0 = \sqrt{\rho^2 + \rho'^2 + z^2}
\]

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Diffraction from a Circular Aperture

Fraunhofer Regime

\[ \lambda_0 = 1 \, \mu m, \quad z = 1000 \, \mu m, \quad \rho_1 = 0 \, \mu m, \quad \rho_2 = 10 \, \mu m \]

Airy Pattern

\[
I = I_0 \frac{J_1^2 \left( \frac{k \frac{d}{2} \rho}{z} \right)}{\left( \frac{k \frac{d}{2} \rho}{2z} \right)^2}
\]
Individual diffraction patterns of two point sources (solid curves) and the resultant patterns (dashed curves) for various angular separations of the sources. In each case, the dashed curve is the sum of the two solid curves. (a) The sources are far apart, and the patterns are well resolved. (b) The sources are closer together such that the angular separation just satisfies Rayleigh’s criterion, and the patterns are just resolved. (c) The sources are so close together that the patterns are not resolved.
Fresnel Diffraction – Cornu Spiral

\[ U(x, y, q; P) = \frac{e^{-jkq}}{2\pi q} jk \iint \exp \left[ -jk \frac{(x-x^\prime)^2 + (y-y^\prime)^2}{2q} \right] U_i(x^\prime, y^\prime, z^\prime = 0) dx^\prime dy^\prime \]

\[ U_i(x^\prime, y^\prime, z^\prime = 0) = E_0 \frac{e^{-jk(x^2+y^2+p^2)^{1/2}}}{(x^2+y^2+p^2)^{1/2}} \simeq E_0 \frac{e^{-jkp}}{p} e^{-jk \left( \frac{s^2+y^2}{2p} \right)} \]

\[ U(0, 0; q; P) \simeq E_0 \frac{e^{-jk(p+q)}}{2\pi pq} jk \iint \exp \left[ -jk \frac{x^\prime y^\prime}{2} \left( \frac{1}{p} + \frac{1}{q} \right) \right] dx^\prime dy^\prime \]

\[ w^2 = s^2 \left[ \frac{2}{\lambda_0 \left( \frac{1}{p} + \frac{1}{q} \right)} \right] \]

\[ U(q; P) = E_0 \frac{e^{-jk(p+q)}}{2\pi pq} jk \frac{u_y}{\left[ \frac{2}{\lambda_0 \left( \frac{1}{p} + \frac{1}{q} \right)} \right]^{1/2}} \int_{u_1}^{u_2} \exp \left[ -\frac{\pi}{2} \frac{u^2}{u_y} \right] du \]
Fresnel Diffraction – Cornu Spiral

\[
C(u) = \int_0^u \cos \left( \frac{\pi}{2} v^2 \right) dv \\
S(u) = \int_0^u \sin \left( \frac{\pi}{2} v^2 \right) dv \\
C(-u) = -C(u) \\
S(-u) = -S(u)
\]

\[
u^2 = (x' - x_0)^2 \left[ \frac{2}{\lambda_0} \left( \frac{1}{p} + \frac{1}{q} \right) \right]
\]

\[
U(q; P) = E_0 \frac{e^{-jk(p+q)}}{2\pi pq} \frac{jk}{\left[ \frac{2}{\lambda_0} \left( \frac{1}{p} + \frac{1}{q} \right) \right]^{1/2}} \int_{u_1}^{u_2} \exp \left[ -\frac{\pi}{2} u^2 \right] du
\]
Cornu Spiral

\[ C(u) = \int_0^u \cos \left( \frac{\pi}{2} v^2 \right) dv \]
\[ S(u) = \int_0^u \sin \left( \frac{\pi}{2} v^2 \right) dv \]

\[ E(P) = E_0 \left[ (C(u_2) - C(u_1)) \pm j [(S(u_2) - S(u_1))] \right] \]
\[ I(P) = |E_0|^2 \left[ (C(u_2) - C(u_1))^2 + (S(u_2) - S(u_1))^2 \right] \]

\[ \text{E}_{\text{unobstructed}} = E_u = E_0 [1 \pm j] \]
\[ I(P) = |\text{E}_{\text{unobstructed}}|^2 = |E_u|^2 = 2 |E_0|^2 \]
Light from a small source passes by the edge of an opaque object and continues on to a screen. A diffraction pattern consisting of bright and dark fringes appears on the screen in the region above the edge of the object.

Diffraction by an Edge

\( \lambda_0 = 1 \, \mu m, \ p = 1000000 \, \mu m, \ q = 10 \, \mu m, \ z_1 = 0 \, \mu m, \ z_2 = 1000000 \, \mu m \)

\( \lambda_0 = 1 \, \mu m, \ q = 10 \, \mu m, \ z_1 = 0 \, \mu m, \ z_2 = 1000000 \, \mu m \)

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Diffraction by an Edge

Comparison of Rayleigh-Sommerfeld and Fresnel Approximation

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Diffraction by an Edge

Comparison of Rayleigh-Sommerfeld with 2D and 3D Green’s Function

\[ \lambda_0 = 1 \mu m, \quad q = 10 \mu m, \quad z_1 = 0 \mu m, \quad z_2 = 100000 \mu m \]

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Diffraction from an Opaque Disk

Diffraction pattern created by the illumination of an opaque disk (U.S. penny), with the disk positioned midway between screen and light source. Note the bright spot at the center (Poisson Spot)


http://en.wikipedia.org/wiki/Arago_spot
In 1818, Augustin Fresnel submitted a paper on the theory of diffraction for a competition sponsored by the French Academy. Simeon D. Poisson, a member of the judging committee for the competition, was very critical of the wave theory of light. Using Fresnel's theory, Poisson deduced the seemingly absurd prediction that a bright spot should appear behind a circular obstruction, a prediction he felt was the last nail in the coffin for Fresnel's theory.

However, Dominique Arago, another member of the judging committee, almost immediately verified the spot experimentally. Fresnel won the competition, and, although it may be more appropriate to call it "the Spot of Arago," the spot goes down in history with the name "Poisson's bright spot".
Diffraction from a Circular Aperture

$\lambda_0 = 1 \, \mu m, \quad \rho = 0 \, \mu m, \quad \rho_1 = 0 \, \mu m, \quad \rho_2 = 10 \, \mu m$

$U(\vec{r}) = \frac{j2\pi z}{\lambda} \int_{\rho_1}^{\rho_2} \frac{e^{-jkR_0}}{R_0^2} J_0 \left( \frac{kr}{R_0} \right) \rho' d\rho'$, \quad $I = |U|^2$

$R_0 = \sqrt{\rho^2 + \rho'^2 + z^2}$

Sommerfeld’s Lemma*

$E_1 = E_u \left[ e^{-jkz} - \frac{e^{-jkz\sqrt{1 + (\rho_1^2/z)^2}}}{\sqrt{1 + (\rho_1^2/z)^2}} \right]$

$E_2 = E_u \frac{e^{-jkz\sqrt{1 + (\rho_1^2/z)^2}}}{\sqrt{1 + (\rho_1^2/z)^2}}$


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Babinet’s Principle

\[ E_1 + E_2 = E_u \]

\[ \lambda_0 = 1 \, \mu m, \quad \rho = 0 \, \mu m, \quad \rho_1 = 0 \, \mu m, \quad \rho_2 = 10 \, \mu m \]

Normalized Intensity, \( I / I_u \)

\( z \) distance

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Babinet’s Principle

Using Sommerfeld’s Lemma

\[ E_1 = E_u \left[ e^{-jkz} - \frac{e^{-jkz\sqrt{1+(\rho_1^2/z)^2}}}{\sqrt{1 + (\rho_1^2/z)^2}} \right] \]

\[ E_2 = E_u \frac{e^{-jkz\sqrt{1+(\rho_1^2/z)^2}}}{\sqrt{1 + (\rho_1^2/z)^2}} \]

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Babinet’s Principle

Using Sommerfeld’s Lemma

\[ E_1 = E_u \left[ e^{-jkz} - e^{-jkz} \frac{\sqrt{1+(\rho_1^2/z)^2}}{\sqrt{1 + (\rho_1^2/z)^2}} \right] \]

\[ E_2 = E_u \frac{e^{-jkz} \sqrt{1+(\rho_1^2/z)^2}}{\sqrt{1 + (\rho_1^2/z)^2}} \]

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Babinet’s Principle

\[ E_1 + E_2 = E_u \]
Babinet’s Principle

\[ E_1 + E_2 = E_u \]

\[ \lambda_0 = 1 \mu m, \quad \rho = 0 \mu m, \quad \rho_1 = 0 \mu m, \quad \rho_2 = 10 \mu m \]
Babinet’s Principle

\[ \lambda_0 = 1 \mu m, \quad \rho = 0 \mu m, \quad \rho_1 = 5 \mu m, \quad \rho_2 = 15 \mu m \]

\[ E_1 + E_2 = E_u \]

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Babinet’s Principle

Complementary Screen Diffraction Patterns in Fraunhofer Regime

Fresnel Zone Plate

\[ f^2 + r_n^2 = \left( f + \frac{n\lambda}{2} \right)^2 \]

\[ r_n^2 = n\lambda f + \frac{n^2\lambda^2}{4} \]

\[ r_n \approx \sqrt{n\lambda f} \]

Fresnel Zone Plate

Each zone is subdivided into 15 subzones

\[ A_n = a_1 + a_2 e^{i\pi} + a_3 e^{i2\pi} + a_4 e^{i3\pi} + \ldots + a_n e^{i(n-1)\pi} \]

\[ A_n = a_1 - a_2 + a_3 - a_4 + \ldots + e^{i(n-1)\pi} a_n \]

Prof. F.A. van Goor, Twente University

http://edu.tnw.utwente.nl/inlopt/overhead_sheets/


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Adding up the zones

For large $N$, resultant amplitude = half that of zone 1

$A_R = A_N = \frac{A_1}{2}$
Fresnel Zone Plate

\[ E_{\text{tot}} = |E_1| + |E_3| + \ldots + |E_{2m-1}| \approx m|E_i| \]

\[ E_{\text{tot}} = -|E_2| - |E_4| - \ldots - |E_{2m}| = -m|E_i| \]
Fresnel Lenses


http://www.fresneloptic.com/images/solar_fresnel_lens.jpg

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Diffractive Lens Design (Non-parabolic)

Zone Boundaries

\[ x_m = \sqrt{2f m \lambda_2 + m^2 \lambda_2^2} \]
\[ \lambda_2 = \frac{\lambda_0}{n_2} \]

s(x) Determination

\[ s(x) = \frac{\lambda_0}{n_1 - n_2} \left[ \frac{\sqrt{f^2 + x^2} - f}{\lambda_2} - m \right] \]
\[ x_m \leq x \leq x_{m+1} \]
Diffractive Lens Design Example

\[ \lambda_0 = 1 \mu m, n_1 = 1.5, n_2 = 1.0, f = 100 \mu m, \]

Number of Zones = 10

Number of Zones = 15
Diffractive Lens Performance

\[ U(x, z) = \int_{-D/2}^{D/2} \frac{\exp(-jkR)}{2\pi R} \left( jk + \frac{1}{R} \right) \frac{z}{R} U_i(x', z' = 0) dx' \]

\[ R = \left[ (x - x')^2 + z^2 \right]^{1/2} \]

\[ U_i(x', z' = 0) = E_0 e^{-j\Phi(x')} \]

\[ \Phi(x') = -k_0(n_1 - n_2)s(x') \]
Diffractive Lens Performance

Number of Zones = 10

\[ n_1 = 1.5, \ n_2 = 1, \ \lambda_0 = 1 \ \mu m, \ f = 100 \ \mu m \]

Number of Zones = 15

\[ n_1 = 1.5, \ n_2 = 1, \ \lambda_0 = 1 \ \mu m, \ f = 100 \ \mu m \]
Diffractive Lens Performance

Number of Zones = 10

Number of Zones = 15

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Diffractive Lens Performance

Number of Zones = 25

Graph showing normalized intensity vs. z distance (microns) with parameters $n_1 = 1.5$, $n_2 = 1$, $\lambda_0 = 1 \mu m$, $f = 100 \mu m$.
Diffractive Lenses


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N-Level Diffractive Lens Design Example

\[ \lambda_0 = 1\mu m, \ n_1 = 1.5, \ n_2 = 1.0, \ f = 100\mu m, \]

Refractive Design

Diffractive Design (Non-Parabolic)
N-Level Diffractive Lens Design Example

\[ \lambda_0 = 1 \mu m, \ n_1 = 1.5, \ n_2 = 1.0, \ f = 100 \mu m, \]

Diffractive Design (Parabolic)

Diffractive Design (Non-Parabolic/Thickness)
Fresnel Lighthouse Lens

other applications:
  overhead projectors
  automobile headlights
  solar collectors
  traffic lights

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http://edu tnw utwente nl/inlopt/overhead_sheets/

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