20/02/2025

Spatial & Temporal Coherence

Optical Engineering

Prof. Elias N. Glytsis

School of Electrical & Computer Engineering
National Technical University of Athens

Coherence Concept

Coherence

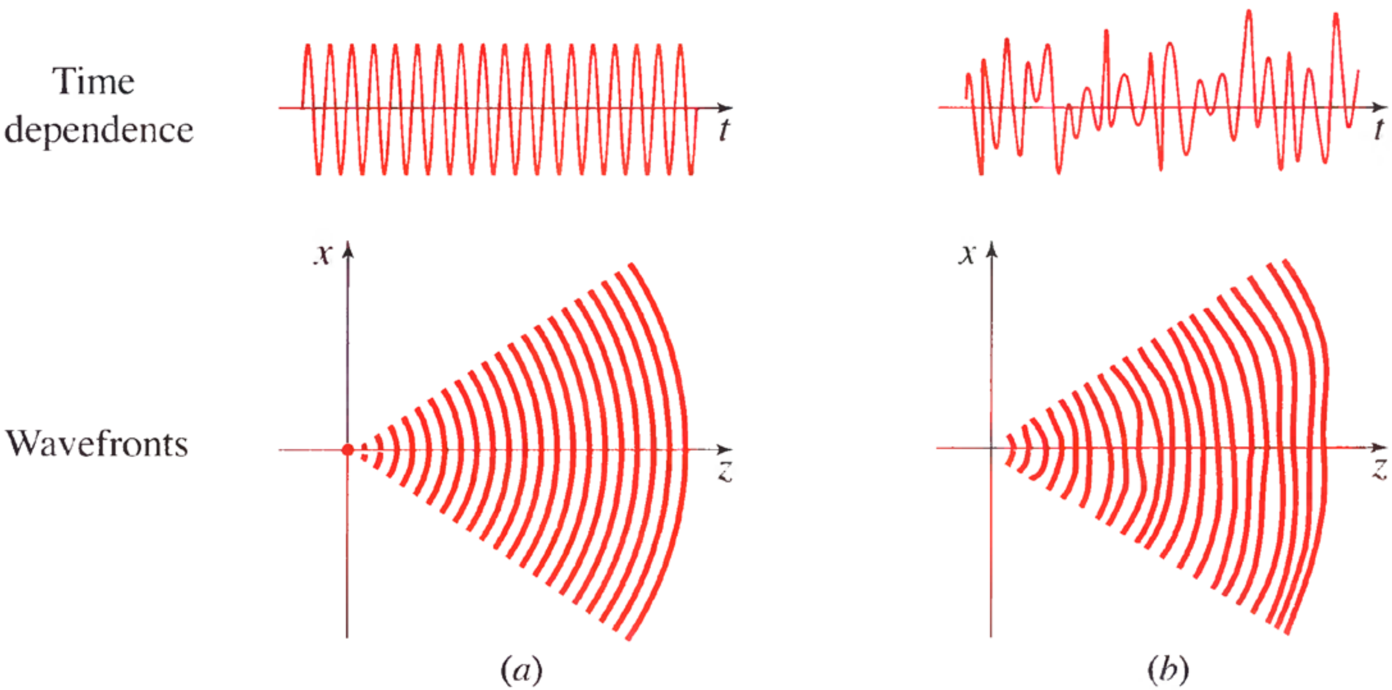
Incoherence

Partial Coherence

Spatial and Temporal Coherence

Coherence is a measure of the correlation between the phases measured at different (temporal and spatial) points on a wave.

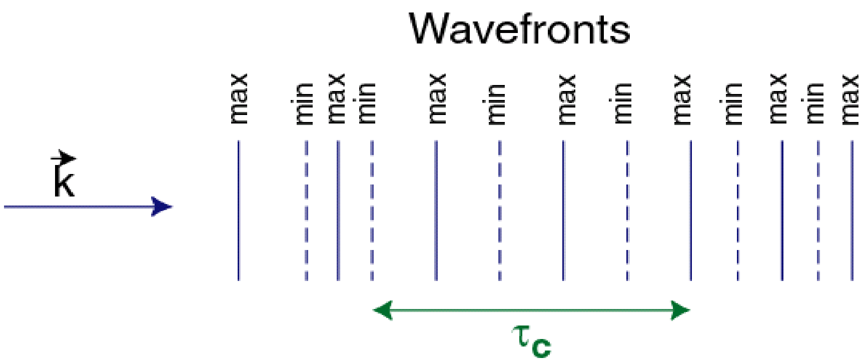
- Temporal Coherence is a measure of the correlation of light wave's phase at different points along the direction of propagation – it tells us how monochromatic a source is.
- Spatial Coherence is a measure of the correlation of a light wave's phase at different points transverse to the direction of propagation - it tells us how uniform the phase of a wavefront is.



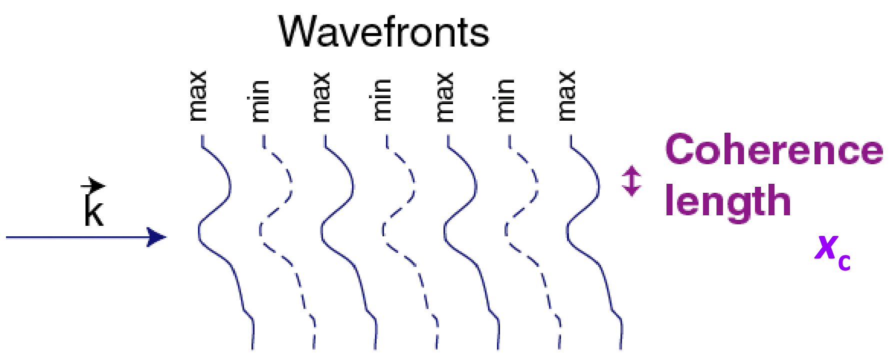
B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2nd Ed., J. Wiley 2007

Spatial and Temporal Coherence

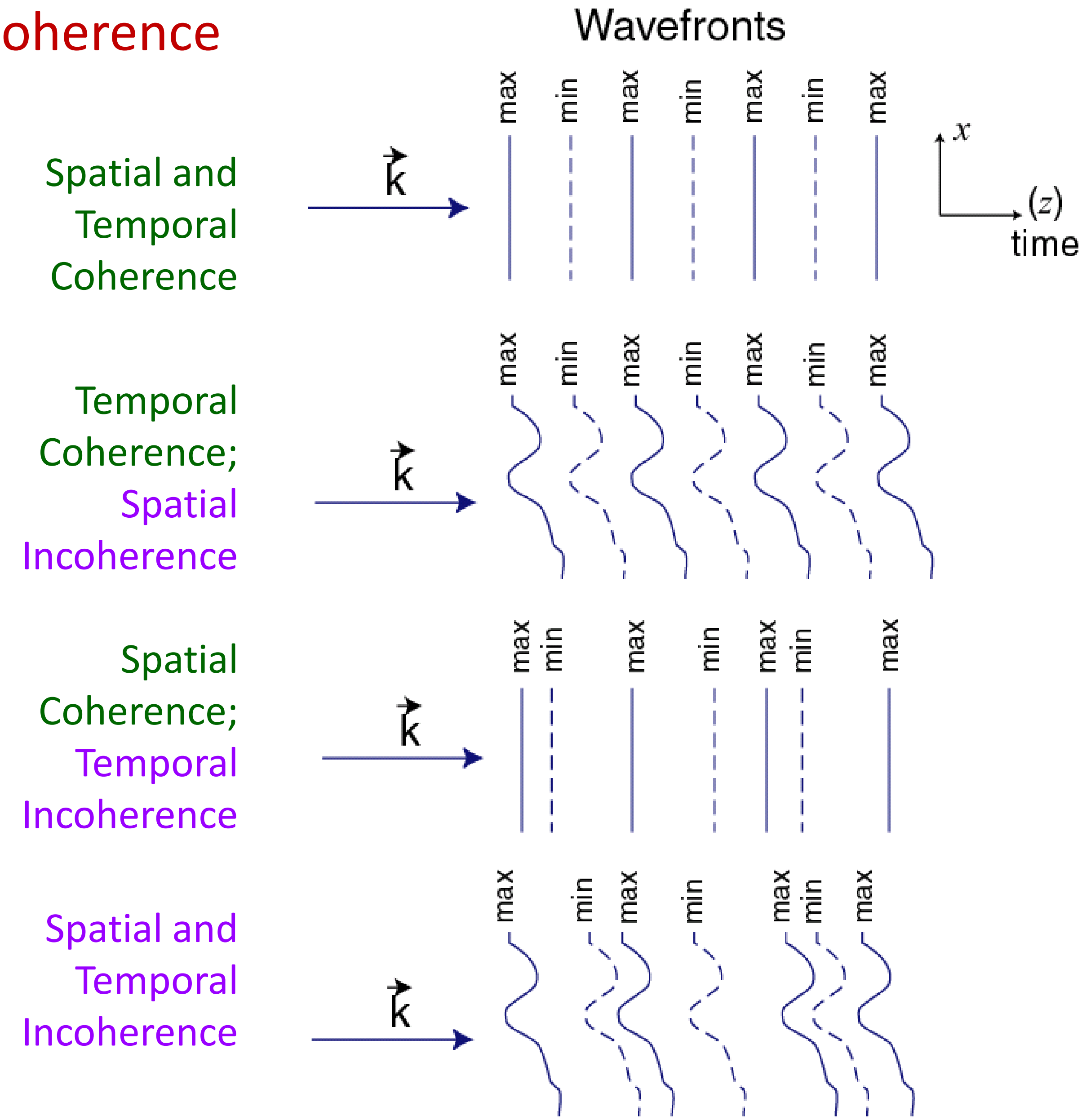
Temporal Coherence Time, τ_c Temporal Coherence Length, $\ell_c = c\,\tau_c$



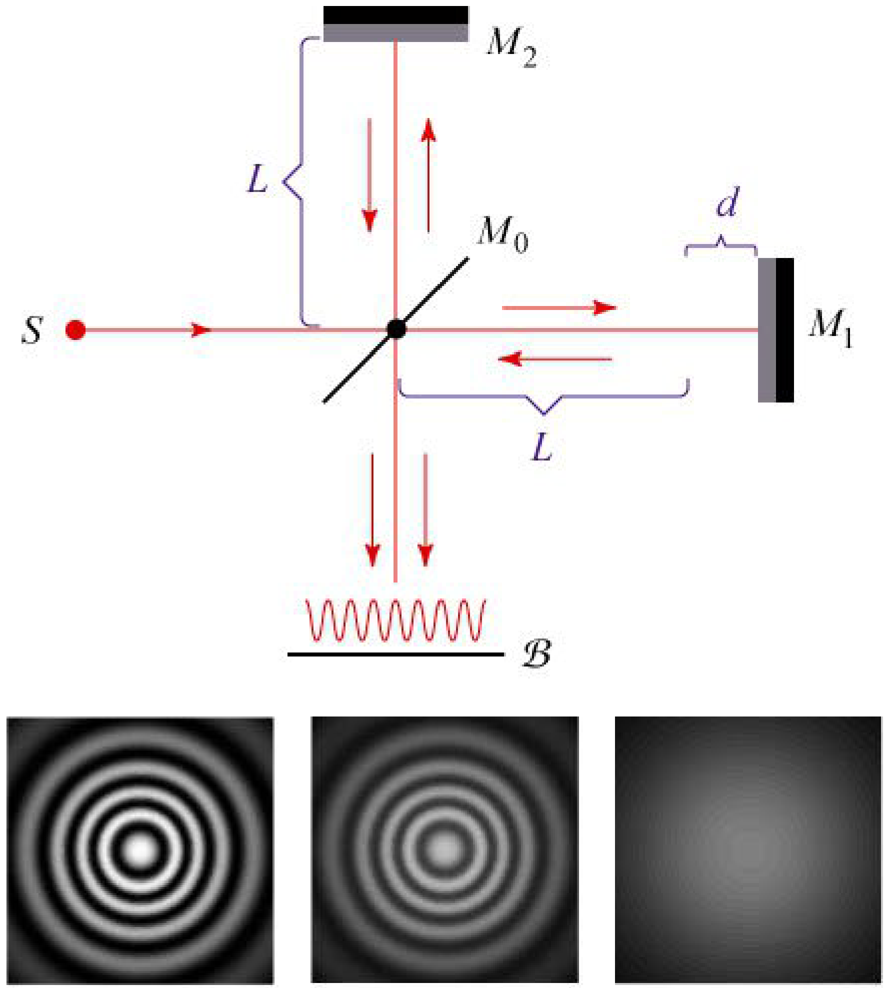
Spatial Coherence Length, x_c



Spatial and Temporal Coherence



Temporal Coherence



$d = 0$ d larger d very large

Temporal Coherence Time, τ_c Temporal Coherence Length, $\ell_c = 2d = c\,\tau_c$

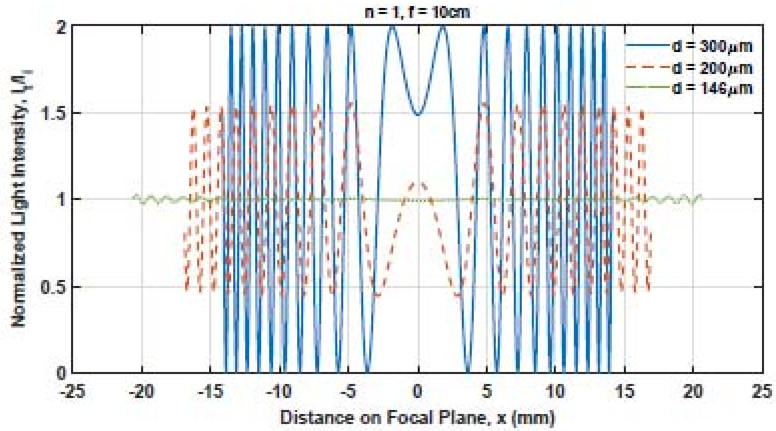
Temporal Coherence (doublet source)

$$I = I(\lambda_1) + I(\lambda_2) = I_0 \cos^2\left(\frac{2\pi}{\lambda_1} d\cos\theta + \frac{\pi}{2}\right) + I_0 \cos^2\left(\frac{2\pi}{\lambda_2} d\cos\theta + \frac{\pi}{2}\right)$$

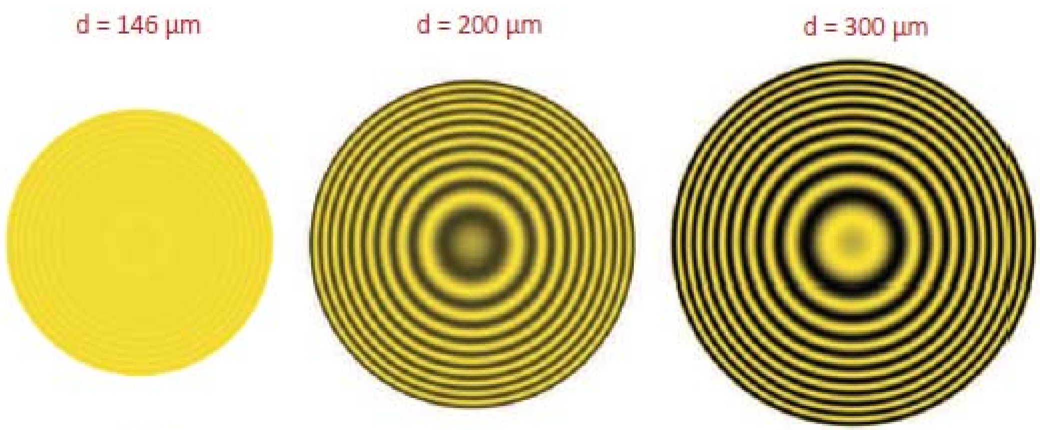
Sodium Doublet

$\lambda_1 = 588.995\text{nm}$

$\lambda_2 = 589.592\text{nm}$



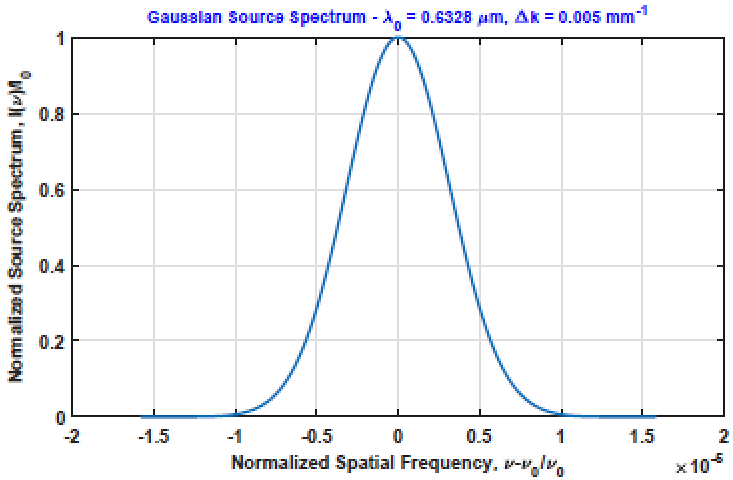
(a)

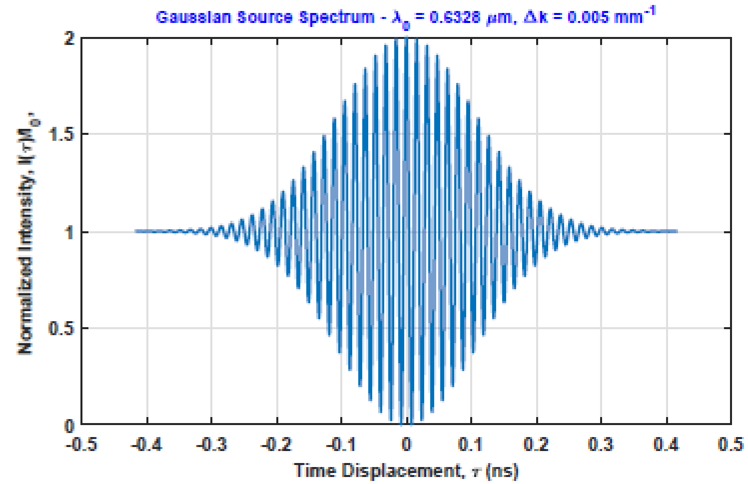


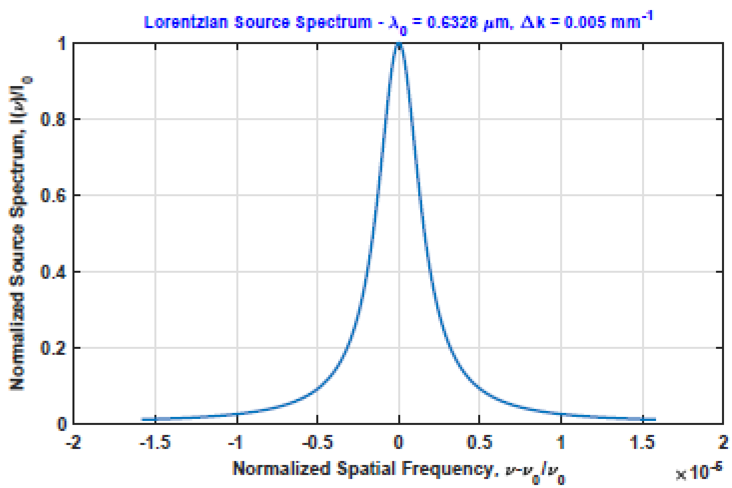
(a) (b) (c)

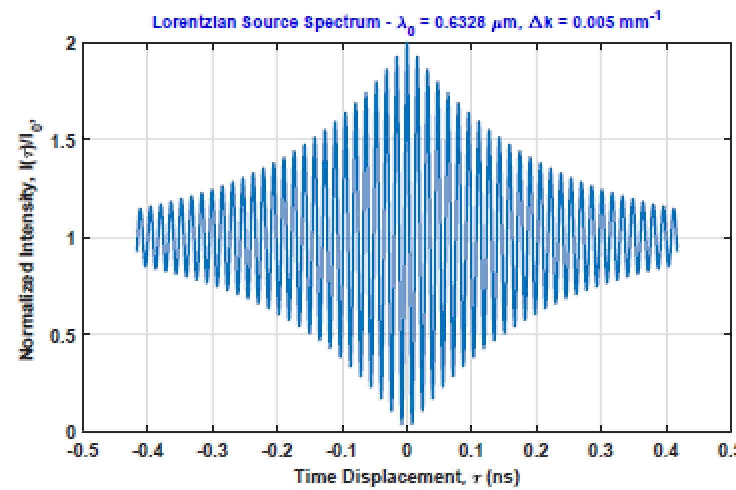
Temporal Coherence (multichromatic source)

$$I(\tau) = \frac{1}{2}\int_{-\infty}^{+\infty} I(\nu)\left[1+\cos\left(2\pi\nu\frac{2d\cos\theta}{c}\right)\right]d\nu \qquad \tau = 2d\cos\theta/c$$

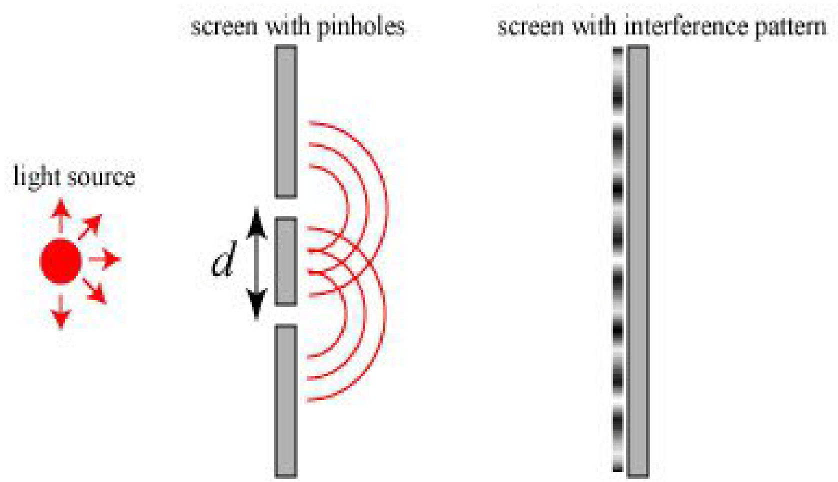




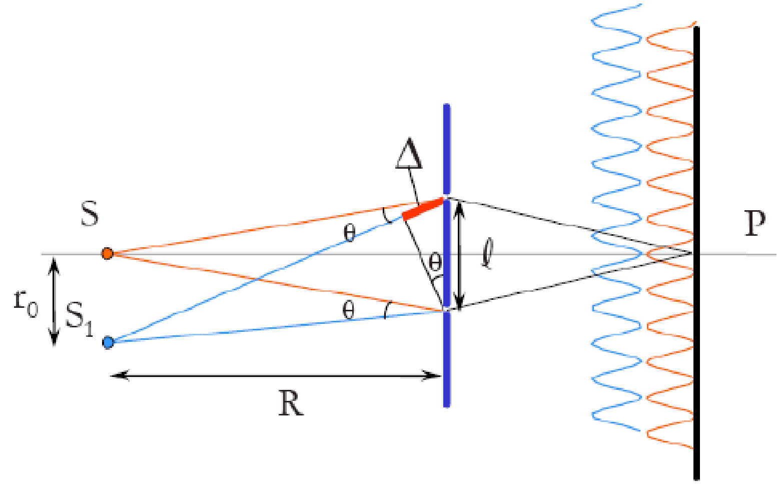




Spatial Coherence



Spatial Coherence Area, $A_c = \pi d^2$



Temporal Coherence

Temporal Coherence Function (Autocorrelation)

$$
\begin{aligned}
G(\tau) = \int_{-\infty}^{+\infty} U^*(t)U(t+\tau)dt \;&=\; \int_{-\infty}^{+\infty} U(t)U^*(t-\tau)dt \\
G(-\tau) \;&=\; G^*(\tau)
\end{aligned}
$$

Degree of Temporal Coherence

$$
g(\tau) = \frac{G(\tau)}{G(0)} \qquad 0 \leq |g(\tau)| \leq 1
$$

Coherence Time and Coherence Length

$$
\begin{aligned}
\tau_c &= \int_{-\infty}^{+\infty} |g(\tau)|^2 d\tau \\
\ell_c &= c\tau_c
\end{aligned}
$$

B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2nd Ed., J. Wiley 2007

Temporal Coherence

Power Spectral Density

$$S(\nu) = \int_{-\infty}^{+\infty} G(\tau)\exp(-j2\pi\nu\tau)d\tau$$

$$\Delta\nu_c = \frac{\left|\int_{-\infty}^{+\infty} S(\nu)d\nu\right|^2}{\int_{-\infty}^{+\infty} |S(\nu)|^2 d\nu} = \frac{1}{\tau_c}$$

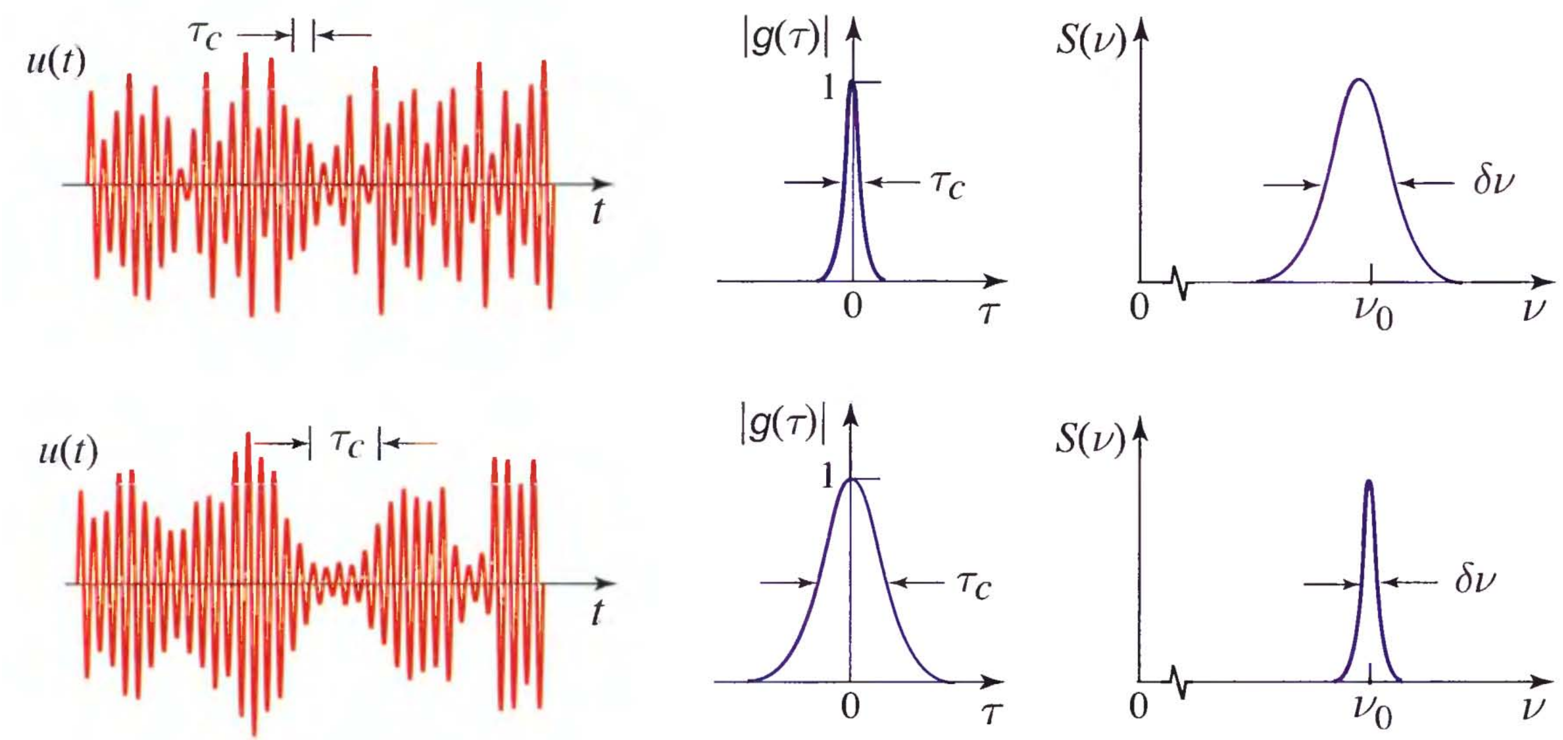
$$\nu = \frac{c}{\lambda_0} \Longrightarrow \Delta\nu = -\frac{c}{\lambda_0^2}\Delta\lambda_0$$

Table 11.1-2 Spectral widths of a number of light sources together with their coherence times and coherence lengths in free space.

Source	$\Delta\nu_c$ (Hz)	$\tau_c = 1/\Delta\nu_c$	$l_c = c\tau_c$
Filtered sunlight ($\lambda_o = 0.4$–$0.8\ \mu$m)	3.74×10^{14}	2.67 fs	800 nm
Light-emitting diode ($\lambda_o = 1\ \mu$m, $\Delta\lambda_o = 50$ nm)	1.5×10^{13}	67 fs	20 μm
Low-pressure sodium lamp	5×10^{11}	2 ps	600 μm
Multimode He–Ne laser ($\lambda_o = 633$ nm)	1.5×10^{9}	0.67 ns	20 cm
Single-mode He–Ne laser ($\lambda_o = 633$ nm)	1×10^{6}	1 μs	300 m

B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2nd Ed., J. Wiley 2007

Temporal Coherence



B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2^{nd} Ed., J. Wiley 2007

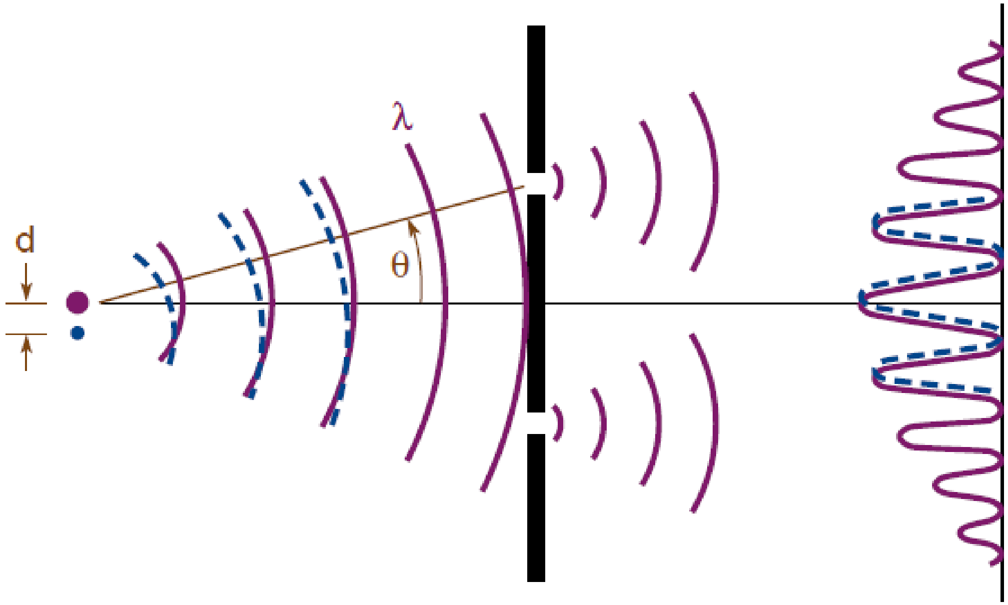
Temporal Coherence

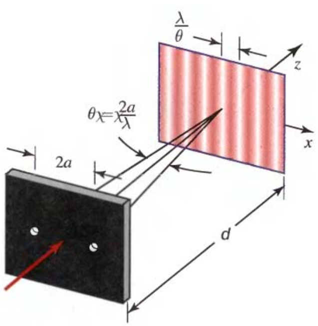
Power Spectral Density

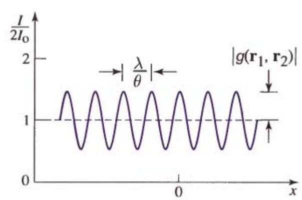
B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2nd Ed., J. Wiley 2007

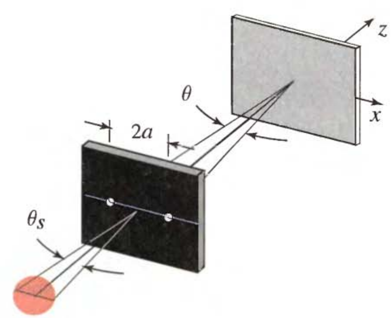
Young's Experiment to Demonstrate Spatial Coherence

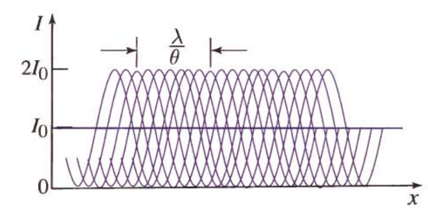
Persistence of fringes as the source grows
from a point source to finite size.











Spatial Coherence

Mutual Coherence Function

$$
\begin{aligned}
G(\vec{r}_1,\vec{r}_2,\tau) &= G_{12}(\tau) = \langle U^*(\vec{r}_1,t)U(\vec{r}_2,t+\tau)\rangle \\
G_{11}(\tau) &= \text{Self Coherence Function at } \vec{r}_1 \\
G_{22}(\tau) &= \text{Self Coherence Function at } \vec{r}_2 \\
G_{12}(0) &= \text{Spatial Coherence Function} \\
G_{11}(0) &= \text{Intensity at } \vec{r}_1,\ I(\vec{r}_1) \\
G_{22}(0) &= \text{Intensity at } \vec{r}_2,\ I(\vec{r}_2)
\end{aligned}
$$

Mutual Degree of Coherence

$$
g(\vec{r}_1,\vec{r}_2,\tau) = \frac{G(\vec{r}_1,\vec{r}_2,\tau)}{\sqrt{I(\vec{r}_1)I(\vec{r}_2)}}
$$

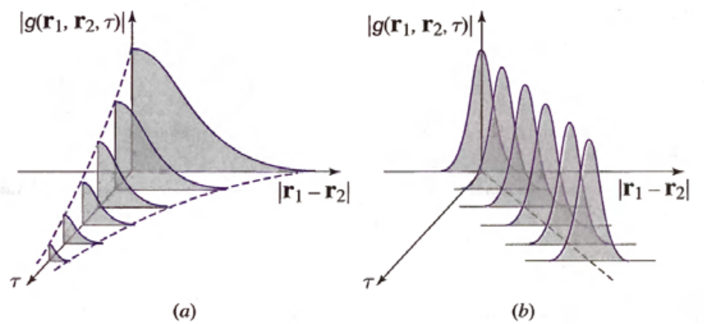


Figure 11.1-7 Two examples of $|g(\mathbf{r}_1, \mathbf{r}_2, \tau)|$ as a function of the separation $|\mathbf{r}_1 - \mathbf{r}_2|$ and the time delay τ. In (*a*) the maximum correlation for a given $|\mathbf{r}_1 - \mathbf{r}_2|$ occurs at $\tau = 0$. In (*b*) the maximum correlation occurs at $|\mathbf{r}_1 - \mathbf{r}_2| = c\tau$.

B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2nd Ed., J. Wiley 2007

Spatial Coherence Function

$$G(\vec{r}_1, \vec{r}_2, 0) = G_{12}(0) = \langle U(\vec{r}_1, t) U(\vec{r}_2, t) \rangle$$

$$g(\vec{r}_1, \vec{r}_2) = \frac{G(\vec{r}_1, \vec{r}_2, 0)}{\sqrt{I(\vec{r}_1) I(\vec{r}_2)}}$$

Spatial Coherence Area, A_c

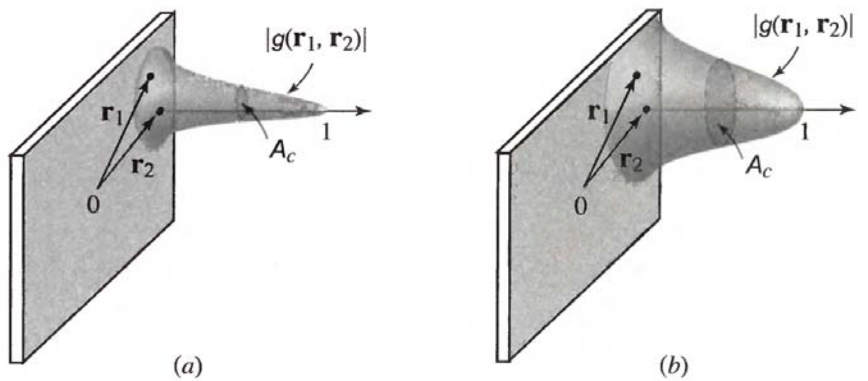


Figure 11.1-8 Two illustrative examples of the magnitude of the normalized mutual intensity as a function of $\mathbf{r}_1$ in the vicinity of a fixed point $\mathbf{r}_2$. The coherence area in (a) is smaller than that in (b).

B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2nd Ed., J. Wiley 2007

Coherence area	Thermal source	Sun (500nm filter)	Betelgeuse (500nm filter)
A	$1 mm^2$	3.68×10^{-3} mm^2	$6 m^2$

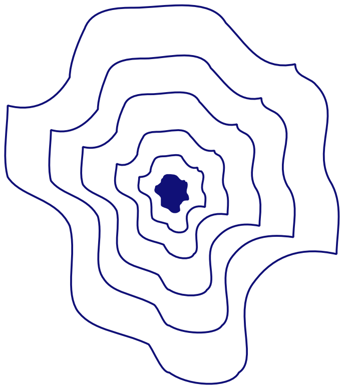
Spatial Coherence

The <u>van Cittert-Zernike Theorem</u> states that the spatial coherence area A_c is given by:

$$\boxed{A_c = \frac{D^2 \lambda^2}{\pi d^2}}$$

where *d* is the radius of the light source and *D* is the distance away.

Basically, wave-fronts smooth out as they propagate away from the source.



Spatial Coherence

$$A_c = \frac{D^2 \lambda^2}{\pi d^2}$$

Example 1: Spatial Coherence of the **Sun**

$\lambda = 0.5\mu m$

$D = 150 \times 10^6$ km $= 1.5 \times 10^{11}$ m

$d = 0.696 \times 10^6$ km $= 0.696 \times 10^9$ m

$A_c \approx 3.69 \times 10^{-3}$ mm^2

Example 2: Spatial Coherence of the **Betelguese**

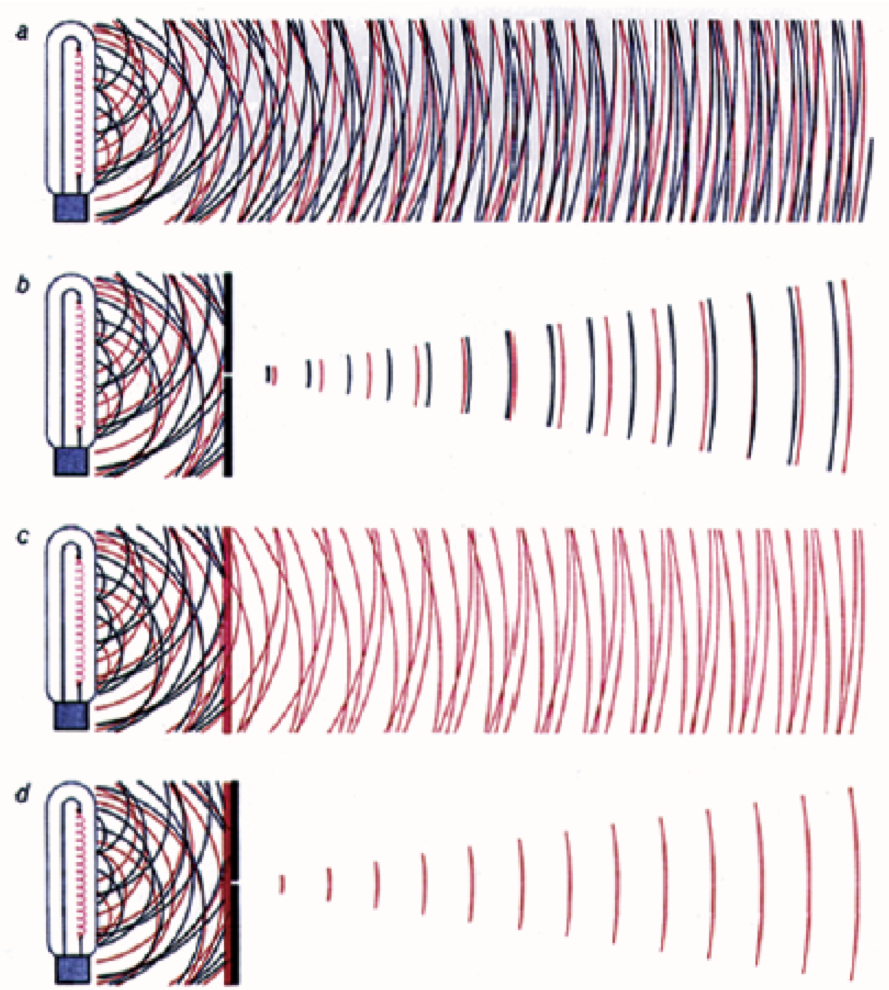
$\lambda = 0.8\mu m$ (blackbody at 3500 Kelvin)

$D \approx 548$ light years $= 548 \times 9.46 \times 10^{12}$ km $= 5.184 \times 10^{18}$ m

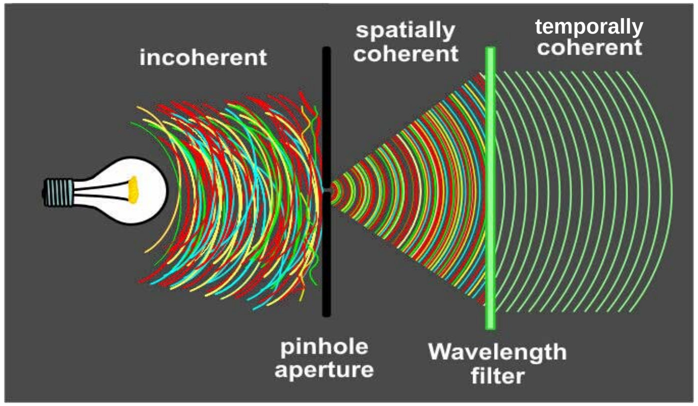
$d \approx 764\, d_{sun} = 764 \times 0.696 \times 10^6$ km $= 5.312 \times 10^{11}$ m

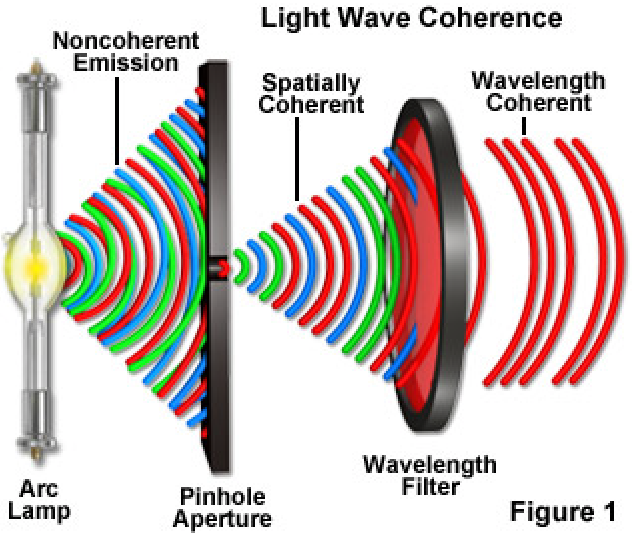
$A_c \approx 19.36$ m^2

Spatial and Spectral Filtering to Produce Coherence Radiation



Courtesy of A. Schawlow, Stanford.





http://zeiss-campus.magnet.fsu.edu/tutorials/coherence/indexflash.html