Spatial and Temporal Coherence

Time dependence

Wavefronts

(a)

(b)

B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2nd Ed., J. Wiley 2007
Spatial and Temporal Coherence

Temporal Coherence Time, $\tau_c$ Temporal Coherence Length, $\ell_c = c \tau_c$

Spatial Coherence Length, $\chi_c$

Coherence length $\chi_c$
Spatial and Temporal Coherence

Wavefronts

Spatial and Temporal Coherence

Temporal Coherence; Spatial Incoherence

Spatial Coherence; Temporal Incoherence

Spatial and Temporal Incoherence
Temporal Coherence

Temporal Coherence Time, $\tau_c$ Temporal Coherence Length, $\ell_c = 2d = c \tau_c$
Spatial Coherence

Spatial Coherence Area, $A_c = \pi d^2$
Temporal Coherence

Temporal Coherence Function (Autocorrelation)

\[ G(\tau) = \int_{-\infty}^{+\infty} U(t)U(t - \tau)dt = \int_{-\infty}^{+\infty} U(t)U(t + \tau)dt \]

\[ G(\tau) = G(-\tau) \]

Degree of Temporal Coherence

\[ g(\tau) = \frac{G(\tau)}{G(0)} \quad 0 \leq |g(\tau)| \leq 1 \]

Coherence Time and Coherence Length

\[ \tau_c = \int_{-\infty}^{+\infty} |g(\tau)|^2 d\tau \]

\[ \ell_c = c\tau_c \]

B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2nd Ed., J. Wiley 2007
## Temporal Coherence

### Power Spectral Density

\[ S(\nu) = \int_{-\infty}^{+\infty} G(\tau) \exp(-j2\pi\nu\tau) d\tau \]

\[ \Delta \nu_c = \frac{\left( \int_{0}^{\infty} S(\nu) d\nu \right)^2}{\int_{0}^{\infty} S(\nu)^2 d\nu} = \frac{1}{\tau_c} \]

### Table 11.1-2

Spectral widths of a number of light sources together with their coherence times and coherence lengths in free space.

<table>
<thead>
<tr>
<th>Source</th>
<th>( \Delta \nu_c ) (Hz)</th>
<th>( \tau_c = 1/\Delta \nu_c )</th>
<th>( l_c = c\tau_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filtered sunlight ((\lambda_o = 0.4–0.8 , \mu m))</td>
<td>(3.74 \times 10^{14})</td>
<td>2.67 fs</td>
<td>800 nm</td>
</tr>
<tr>
<td>Light-emitting diode ((\lambda_o = 1 , \mu m, \Delta \lambda_o = 50 , nm))</td>
<td>(1.5 \times 10^{13})</td>
<td>67 fs</td>
<td>20 , \mu m</td>
</tr>
<tr>
<td>Low-pressure sodium lamp</td>
<td>(5 \times 10^{11})</td>
<td>2 ps</td>
<td>600 , \mu m</td>
</tr>
<tr>
<td>Multimode He–Ne laser ((\lambda_o = 633 , nm))</td>
<td>(1.5 \times 10^9)</td>
<td>0.67 ns</td>
<td>20 cm</td>
</tr>
<tr>
<td>Single-mode He–Ne laser ((\lambda_o = 633 , nm))</td>
<td>(1 \times 10^6)</td>
<td>1 , \mu s</td>
<td>300 m</td>
</tr>
</tbody>
</table>
Temporal Coherence

B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2nd Ed., J. Wiley 2007
Young’s Experiment to Demonstrate Spatial Coherence

Persistence of fringes as the source grows from a point source to finite size.
Spatial Coherence

Mutual Coherence Function

\[ G(\vec{r}_1, \vec{r}_2, \tau) = G_{12}(\tau) = \langle U(\vec{r}_1, t)U(\vec{r}_2, t + \tau) \rangle \]

\[ G_{11}(\tau) = \text{Self Coherence Function at } \vec{r}_1 \]

\[ G_{22}(\tau) = \text{Self Coherence Function at } \vec{r}_2 \]

\[ G_{12}(0) = \text{Spatial Coherence Function} \]

\[ G_{11}(0) = \text{Intensity at } \vec{r}_1, I(\vec{r}_1) \]

\[ G_{22}(0) = \text{Intensity at } \vec{r}_2, I(\vec{r}_2) \]

Mutual Degree of Coherence

\[ g(\vec{r}_1, \vec{r}_2, \tau) = \frac{G(\vec{r}_1, \vec{r}_2, \tau)}{\sqrt{I(\vec{r}_1)I(\vec{r}_2)}} \]

**Figure 11.1-7** Two examples of \(|g(\vec{r}_1, \vec{r}_2, \tau)|\) as a function of the separation \(|\vec{r}_1 - \vec{r}_2|\) and the time delay \(\tau\). In (a) the maximum correlation for a given \(|\vec{r}_1 - \vec{r}_2|\) occurs at \(\tau = 0\). In (b) the maximum correlation occurs at \(|\vec{r}_1 - \vec{r}_2| = c\tau\).
Spatial Coherence Function

\[ G(\vec{r}_1, \vec{r}_2, 0) = G_{12}(0) = \langle U(\vec{r}_1, t)U(\vec{r}_2, t) \rangle \]

\[ g(\vec{r}_1, \vec{r}_2) = \frac{G(\vec{r}_1, \vec{r}_2, 0)}{\sqrt{I(\vec{r}_1)I(\vec{r}_2)}} \]

Spatial Coherence Area, \( A_c \)

Figure 11.1-8 Two illustrative examples of the magnitude of the normalized mutual intensity as a function of \( r_1 \) in the vicinity of a fixed point \( r_2 \). The coherence area in (a) is smaller than that in (b).
The van Cittert-Zernike Theorem states that the spatial coherence area $A_c$ is given by:

$$A_c = \frac{D^2 \lambda^2}{\pi d^2}$$

where $d$ is the diameter of the light source and $D$ is the distance away.
Spatial and Spectral Filtering to Produce Coherence Radiation

http://zeiss-campus.magnet.fsu.edu/tutorials/coherence/indexflash.html