Spatial & Temporal Coherence

Optical Engineering
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Coherence Concept

Coherence

Incoherence

Partial Coherence
**Spatial and Temporal Coherence**

**Coherence** is a measure of the correlation between the phases measured at different (temporal and spatial) points on a wave.  
**Temporal Coherence** is a measure of the correlation of light wave’s phase at different points along the direction of propagation – it tells us how monochromatic a source is.  
**Spatial Coherence** is a measure of the correlation of a light wave’s phase at different points transverse to the direction of propagation - it tells us how uniform the phase of a wavefront is.

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**B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2nd Ed., J. Wiley 2007**
Spatial and Temporal Coherence

Temporal Coherence Time, $\tau_c$

Temporal Coherence Length, $\ell_c = c \tau_c$

Spatial Coherence Length, $x_c$

Coherence length $x_c$
Spatial and Temporal Coherence

Spatial and Temporal Coherence

Temporal Coherence; Spatial Incoherence

Spatial Coherence; Temporal Incoherence

Spatial and Temporal Incoherence

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Temporal Coherence

Temporal Coherence Time, $\tau_c$
Temporal Coherence Length, $\ell_c = 2d = c \tau_c$

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Spatial Coherence

Spatial Coherence Area, $A_c = \pi d^2$
Temporal Coherence

Temporal Coherence Function (Autocorrelation)

\[ G(\tau) = \int_{-\infty}^{+\infty} U^*(t)U(t + \tau)dt = \int_{-\infty}^{+\infty} U(t)U^*(t - \tau)dt \]

\[ G(-\tau) = G^*(\tau) \]

Degree of Temporal Coherence

\[ g(\tau) = \frac{G(\tau)}{G(0)} \quad 0 \leq |g(\tau)| \leq 1 \]

Coherence Time and Coherence Length

\[ \tau_c = \int_{-\infty}^{+\infty} |g(\tau)|^2 d\tau \]

\[ l_c = c\tau_c \]

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Temporal Coherence

Power Spectral Density

\[
S(\nu) = \int_{-\infty}^{+\infty} G(\tau) \exp(-j2\pi \nu \tau) d\tau \\
\Delta \nu_c = \frac{\left| \int_{-\infty}^{+\infty} S(\nu) d\nu \right|^2}{\int_{-\infty}^{+\infty} |S(\nu)|^2 d\nu} = \frac{1}{\tau_c}
\]

\[
\nu = \frac{c}{\lambda_0} \implies \Delta \nu = -\frac{c}{\lambda_0^2} \Delta \lambda_0
\]

Table 11.1-2  Spectral widths of a number of light sources together with their coherence times and coherence lengths in free space.

<table>
<thead>
<tr>
<th>Source</th>
<th>( \Delta \nu_c ) (Hz)</th>
<th>( \tau_c = 1/\Delta \nu_c )</th>
<th>( l_c = c\tau_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filtered sunlight ((\lambda_o = 0.4–0.8 \ \mu m))</td>
<td>(3.74 \times 10^{14})</td>
<td>2.67 fs</td>
<td>800 nm</td>
</tr>
<tr>
<td>Light-emitting diode ((\lambda_o = 1 \ \mu m, \Delta \lambda_o = 50 \ \text{nm}))</td>
<td>(1.5 \times 10^{13})</td>
<td>67 fs</td>
<td>20 \ \mu m</td>
</tr>
<tr>
<td>Low-pressure sodium lamp</td>
<td>(5 \times 10^{11})</td>
<td>2 ps</td>
<td>600 \ \mu m</td>
</tr>
<tr>
<td>Multimode He–Ne laser ((\lambda_o = 633 \ \text{nm}))</td>
<td>(1.5 \times 10^{9})</td>
<td>0.67 ns</td>
<td>20 cm</td>
</tr>
<tr>
<td>Single-mode He–Ne laser ((\lambda_o = 633 \ \text{nm}))</td>
<td>(1 \times 10^{6})</td>
<td>1 \ \mu s</td>
<td>300 m</td>
</tr>
</tbody>
</table>

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Temporal Coherence

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Temporal Coherence

Power Spectral Density

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Young’s Experiment to Demonstrate Spatial Coherence

Persistence of fringes as the source grows from a point source to finite size.

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Spatial Coherence

Mutual Coherence Function

\[ G(r_1, r_2, \tau) = G_{12}(\tau) = \langle U^*(\vec{r}_1, t) U(\vec{r}_2, t + \tau) \rangle \]

\[ G_{11}(\tau) = \text{Self Coherence Function at } \vec{r}_1 \]

\[ G_{22}(\tau) = \text{Self Coherence Function at } \vec{r}_2 \]

\[ G_{12}(0) = \text{Spatial Coherence Function} \]

\[ G_{11}(0) = \text{Intensity at } \vec{r}_1, I(\vec{r}_1) \]

\[ G_{22}(0) = \text{Intensity at } \vec{r}_2, I(\vec{r}_1) \]

Mutual Degree of Coherence

\[ g(r_1, r_2, \tau) = \frac{G(r_1, r_2, \tau)}{\sqrt{I(r_1)I(r_2)}} \]

**Figure 11.1-7** Two examples of $|g(r_1, r_2, \tau)|$ as a function of the separation $|r_1 - r_2|$ and the time delay $\tau$. In (a) the maximum correlation for a given $|r_1 - r_2|$ occurs at $\tau = 0$. In (b) the maximum correlation occurs at $|r_1 - r_2| = c\tau$.

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Spatial Coherence Function

\[ G(\vec{r}_1, \vec{r}_2, 0) = G_{12}(0) = \langle U(\vec{r}_1, t)U(\vec{r}_2, t) \rangle \]

\[ g(\vec{r}_1, \vec{r}_2) = \frac{G(\vec{r}_1, \vec{r}_2, 0)}{\sqrt{I(\vec{r}_1)I(\vec{r}_2)}} \]

Spatial Coherence Area, \( A_c \)

**Figure 11.1-8** Two illustrative examples of the magnitude of the normalized mutual intensity as a function of \( r_1 \) in the vicinity of a fixed point \( r_2 \). The coherence area in (a) is smaller than that in (b).

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<table>
<thead>
<tr>
<th>Coherence area</th>
<th>Thermal source</th>
<th>Sun (500nm filter)</th>
<th>Betelgeuse (500nm filter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1mm²</td>
<td>3.68 \times 10^{-3} \text{ mm}²</td>
<td>6m²</td>
</tr>
</tbody>
</table>
Spatial Coherence

The van Cittert-Zernike Theorem states that the spatial coherence area $A_c$ is given by:

$$A_c = \frac{D^2 \lambda^2}{\pi d^2}$$

where $d$ is the diameter of the light source and $D$ is the distance away.

Basically, wave-fronts smooth out as they propagate away from the source.
Spatial and Spectral Filtering to Produce Coherence Radiation

http://zeiss-campus.magnet.fsu.edu/tutorials/coherence/indexflash.html

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