Spatial & Temporal Coherence

Optical Engineering
Prof. Elias N. Glytsis

School of Electrical & Computer Engineering
National Technical University of Athens
Coherence Concept

Coherence

Incoherence

Partial Coherence
Coherence is a measure of the correlation between the phases measured at different (temporal and spatial) points on a wave.

- **Temporal Coherence** is a measure of the correlation of light wave’s phase at different points along the direction of propagation – it tells us how monochromatic a source is.
- **Spatial Coherence** is a measure of the correlation of a light wave’s phase at different points transverse to the direction of propagation - it tells us how uniform the phase of a wavefront is.

B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2nd Ed., J. Wiley 2007
Spatial and Temporal Coherence

Temporal Coherence Time, $\tau_c$ Temporal Coherence Length, $\ell_c = c \tau_c$

Wavefronts

Spatial Coherence Length, $x_c$

Coherence length $x_c$
Spatial and Temporal Coherence

Wavefronts

Spatial and Temporal Coherence

Temporal Coherence; Spatial Incoherence

Spatial Coherence; Temporal Incoherence

Spatial and Temporal Incoherence
Temporal Coherence

Temporal Coherence Time, $\tau_c$

Temporal Coherence Length, $\ell_c = 2d = c \tau_c$

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Spatial Coherence

Spatial Coherence Area, $A_c = \pi d^2$

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Temporal Coherence

Temporal Coherence Function (Autocorrelation)

\[ G(\tau) = \int_{-\infty}^{+\infty} U^*(t)U(t+\tau)dt = \int_{-\infty}^{+\infty} U(t)U^*(t-\tau)dt \]

\[ G(-\tau) = G^*(\tau) \]

Degree of Temporal Coherence

\[ g(\tau) = \frac{G(\tau)}{G(0)} \quad 0 \leq |g(\tau)| \leq 1 \]

Coherence Time and Coherence Length

\[ \tau_c = \int_{-\infty}^{+\infty} |g(\tau)|^2 d\tau \]

\[ \ell_c = c\tau_c \]

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Temporal Coherence

Power Spectral Density

\[
S(\nu) = \int_{-\infty}^{+\infty} G(\tau) \exp(-j2\pi \nu \tau) d\tau
\]

\[
\Delta \nu_c = \frac{\left| \int_{-\infty}^{+\infty} S(\nu) d\nu \right|^2}{\int_{-\infty}^{+\infty} |S(\nu)|^2 d\nu} = \frac{1}{\tau_c}
\]

\[
\nu = \frac{c}{\lambda_0} \implies \Delta \nu = -\frac{c}{\lambda_0^2} \Delta \lambda_0
\]

**Table 11.1-2**  Spectral widths of a number of light sources together with their coherence times and coherence lengths in free space.

<table>
<thead>
<tr>
<th>Source</th>
<th>(\Delta \nu_c) (Hz)</th>
<th>(\tau_c = 1/\Delta \nu_c)</th>
<th>(l_c = c\tau_c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filtered sunlight ((\lambda_o = 0.4–0.8 \mu m))</td>
<td>(3.74 \times 10^{14})</td>
<td>2.67 fs</td>
<td>800 nm</td>
</tr>
<tr>
<td>Light-emitting diode ((\lambda_o = 1 \mu m, \Delta \lambda_o = 50 \text{ nm}))</td>
<td>(1.5 \times 10^{13})</td>
<td>67 fs</td>
<td>20 \mu m</td>
</tr>
<tr>
<td>Low-pressure sodium lamp</td>
<td>(5 \times 10^{11})</td>
<td>2 ps</td>
<td>600 \mu m</td>
</tr>
<tr>
<td>Multimode He–Ne laser ((\lambda_o = 633 \text{ nm}))</td>
<td>(1.5 \times 10^9)</td>
<td>0.67 ns</td>
<td>20 cm</td>
</tr>
<tr>
<td>Single-mode He–Ne laser ((\lambda_o = 633 \text{ nm}))</td>
<td>(1 \times 10^6)</td>
<td>1 \mu s</td>
<td>300 m</td>
</tr>
</tbody>
</table>

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Temporal Coherence

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Young’s Experiment to Demonstrate Spatial Coherence

Persistence of fringes as the source grows from a point source to finite size.
Spatial Coherence

**Mutual Coherence Function**

\[ G(\vec{r}_1, \vec{r}_2, \tau) = G_{12}(\tau) = (U^*(\vec{r}_1, t)U(\vec{r}_2, t + \tau)) \]

- \[ G_{11}(\tau) = \text{Self Coherence Function at } \vec{r}_1 \]
- \[ G_{22}(\tau) = \text{Self Coherence Function at } \vec{r}_2 \]
- \[ G_{12}(0) = \text{Spatial Coherence Function} \]
- \[ G_{11}(0) = \text{Intensity at } \vec{r}_1, I(\vec{r}_1) \]
- \[ G_{22}(0) = \text{Intensity at } \vec{r}_2, I(\vec{r}_1) \]

**Mutual Degree of Coherence**

\[ g(\vec{r}_1, \vec{r}_2, \tau) = \frac{G(\vec{r}_1, \vec{r}_2, \tau)}{\sqrt{I(\vec{r}_1)I(\vec{r}_2)}} \]

*Figure 11.1-7* Two examples of \(|g(\vec{r}_1, \vec{r}_2, \tau)|\) as a function of the separation \(|\vec{r}_1 - \vec{r}_2|\) and the time delay \(\tau\). In (a) the maximum correlation for a given \(|\vec{r}_1 - \vec{r}_2|\) occurs at \(\tau = 0\). In (b) the maximum correlation occurs at \(|\vec{r}_1 - \vec{r}_2| = c\tau\).

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Spatial Coherence Function

\[
G(\vec{r}_1, \vec{r}_2, 0) = G_{12}(0) = \langle U(\vec{r}_1, t)U(\vec{r}_2, t) \rangle
\]

\[
g(\vec{r}_1, \vec{r}_2) = \frac{G(\vec{r}_1, \vec{r}_2, 0)}{\sqrt{I(\vec{r}_1)I(\vec{r}_2)}}
\]

Spatial Coherence Area, \( A_c \)

**Figure 11.1-8** Two illustrative examples of the magnitude of the normalized mutual intensity as a function of \( r_1 \) in the vicinity of a fixed point \( r_2 \). The coherence area in (a) is smaller than that in (b).

B. E. A. Saleh and M. C. Teich, Fundamentals of Photonics, 2nd Ed., J. Wiley 2007

<table>
<thead>
<tr>
<th>Coherence area</th>
<th>Thermal source</th>
<th>Sun (500nm filter)</th>
<th>Betelgeuse (500nm filter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1mm(^2)</td>
<td>3.68 \times 10^{-3} \text{ mm}^2</td>
<td>6m(^2)</td>
</tr>
</tbody>
</table>
The van Cittert-Zernike Theorem states that the spatial coherence area $A_c$ is given by:

$$A_c = \frac{D^2 \lambda^2}{\pi d^2}$$

where $d$ is the diameter of the light source and $D$ is the distance away.

Basically, wave-fronts smooth out as they propagate away from the source.
Spatial and Spectral Filtering to Produce Coherence Radiation

http://zeiss-campus.magnet.fsu.edu/tutorials/coherence/indexflash.html

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