NTUA Integrated Optics **<u>Not to be returned</u>**

Problem Set # 5

1. OPTICAL FIBER

A step index fiber has a core refractive index of 1.51 and a cladding refractive index of 1.49. The core diameter is 16μm. A free-space wavelength of 0.9μm is used to excite the fiber. (a) Can the EH_{22} mode exist in this fiber? Explain your answer. (b) What is the numerical aperture of this fiber and what is the maximum acceptance angle in degrees?

2. LP-MODES

Given the following modes – EH_{51}, TE_{04}, TE_{05}, HE_{17}, TM_{07}, EH_{37}, TM_{04}, HE_{71}, and HE_{27} – group the modes into degenerate sets and indicate the LP designation of each set. Also find the LP designation for any of the above modes that are not degenerate with others shown. Describe the intensity patterns for all LP modes that are found.

3. MATERIAL DISPERSION

The material dispersion can be exressed analytically via the Sellmeir formula as:

$$n^2(\lambda) = A + \frac{G_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{G_2\lambda^2}{\lambda^2 - \lambda_2^2} + \frac{G_3\lambda^2}{\lambda^2 - \lambda_3^2},$$

where A is a constant (usually 1) and G_i, λ_i ($i = 1, 2, 3$) are the Sellmeir's coefficients. Show the following expressions for the first and second derivatives of $n(\lambda)$:

$$\begin{aligned}
\frac{dn}{d\lambda} &= -\frac{\lambda}{n}\left[\frac{G_1\lambda_1^2}{(\lambda^2-\lambda_1^2)^2} + \frac{G_2\lambda_2^2}{(\lambda^2-\lambda_2^2)^2} + \frac{G_3\lambda_3^2}{(\lambda^2-\lambda_3^2)^2}\right], \\
\frac{d^2n}{d\lambda^2} &= [-\frac{1}{n} + \frac{\lambda}{n^2}\frac{dn}{d\lambda}]\sum_{i=1}^{3}\frac{G_i\lambda_i^2}{(\lambda^2-\lambda_i^2)^2} + \frac{4\lambda^2}{n}\sum_{i=1}^{3}\frac{G_i\lambda_i^2}{(\lambda^2-\lambda_i^2)^3}, \\
\frac{d^2n}{d\lambda^2} &= -\frac{1}{n}(\frac{dn}{d\lambda})^2 + \frac{1}{\lambda}\frac{dn}{d\lambda} + \frac{4\lambda^2}{n}\sum_{i=1}^{3}\frac{G_i\lambda_i^2}{(\lambda^2-\lambda_i^2)^3}, \\
\frac{d^2n}{d\lambda^2} &= \frac{N_g}{n\lambda}\frac{dn}{d\lambda} + \frac{4\lambda^2}{n}\sum_{i=1}^{3}\frac{G_i\lambda_i^2}{(\lambda^2-\lambda_i^2)^3}, \\
\frac{d^2n}{d\lambda^2} &= -\frac{1}{n}(\frac{dn}{d\lambda})^2 + \frac{1}{n}\sum_{i=1}^{3}\frac{G_i\lambda_i^2(3\lambda^2+\lambda_i^2)}{(\lambda^2-\lambda_i^2)^3},
\end{aligned}$$

where $N_g = n - \lambda(dn/d\lambda)$ is the group index of the material. For a SiO_2 GeO_2-doped single-mode fiber core the Sellmeir's coefficients are $G_1 = 0.7088876$, $\lambda_1 = 0.0609053\,\mu$m, $G_2 = 0.4206803$, $\lambda_2 = 0.1254514\,\mu$m, $G_3 = 0.8956551$, and $\lambda_3 = 9.896162\,\mu$m. Calculate using the above equations the refractive index $n(\lambda)$, the normalized group velocity v_g/c, and the GVD parameter $D(\lambda)$ for the range of freespace wavelength between 1.2 to 1.65 microns.