## 1. EFFECTIVE-INDEX METHOD FOR CHANNEL WAVEGUIDES

A raised-strip type channel waveguide has the following parameters (figure below): $n_{c}=1.90$, $n_{f}=2.20, n_{s}=2.10, h=1.0 \mu m$, and $w=1.0 \mu m$. Laser light of freespace wavelength $\lambda_{0}=1.0 \mu \mathrm{~m}$ is coupled into the channel waveguide. The effective-index method is going to be used for the approximate calculation of the effective indices of the guided modes. (a) Assume that only the effective indices of the $T E$-like modes are needed. For a $T E$-like mode the electric field has the largest component along the $y$ direction, i.e. $E_{x} \simeq 0$ and $E_{y} \neq 0$. A $T E$-like mode is denoted by $E_{y}^{\nu \mu}$ where $\nu$ is the mode number that correspond to the $x$ direction and $\mu$ is the mode number that correspond to the $y$ direction. Find how many and which $T E$-like guided modes, $E_{y}^{\nu \mu}$, can be supported by this channel waveguide, and draw qualitatively transverse field intensity patterns in the $x y$ plane for each guided mode. (b) Find the effective indices $N_{\nu \mu}$ of the $E_{y}^{\nu \mu}$ guided modes supported by this channel waveguide.


Figure 1: Views of a rib waveguide.

## 2. COUPLING COEFFICIENT BETWEEN TWO SLAB WAVEGUIDES (TE MODE)

The two waveguides shown in the figure are designed to support a single TE mode. The electric fields (for the fundamental TE mode) for each waveguide are

$$
\vec{E}= \begin{cases}\hat{y} 2 E_{f i} \cos \left(k_{f x i} h_{i}-\phi_{s i}\right) \exp \left[-\gamma_{s i}\left(x_{i}-h_{i}\right)\right] \exp \left(-j \beta_{i} z\right) & \text { for } h_{i}<x_{i}<\infty \\ \hat{y} 2 E_{f i} \cos \left(k_{f x i} x_{i}-\phi_{s i}\right) \exp \left(-j \beta_{i} z\right) & \text { for } 0<x_{i}<h_{i} \\ \hat{y} 2 E_{f i} \cos \phi_{s i} \exp \left(\gamma_{s i} x_{i}\right) \exp \left(-j \beta_{i} z\right) & \text { for }-\infty<x_{i}<0\end{cases}
$$

where $i=A, B$ ( $A$ corresponds to waveguide A and $B$ correpsonds to waveguide B in the figure) and $\phi_{s i}=\tan ^{-1}\left(\gamma_{s i} / k_{f x i}\right)$. (a) Calculate coefficients $2 E_{f i}(i=A, B)$ to normalize the modes of the two waveguides. In order to avoid calculations use the equations that express the time-average power that were given in class. (b) Calculate the coupling coefficient $C_{A B}$ when the two waveguides are brought together (at a distance $s$ as shown in the figure).


Figure 2: A simple directional coupler.

