NTUA Integrated Optics **Not to be returned**

Problem Set # 2

1. MODES IN A GLASS SLAB WAVEGUIDE

A 1.5μm thick ZnS (of refractive index $n = 2.2899$) film is deposited on glass substrate (of refractive index $n = 1.5040$). This slab waveguide is excited in air with a Nd:YAG laser of free-space wavelength $\lambda_0 = 1.06\mu$m. For TE and TM modes, determine (a) the allowable mode numbers, (b) the corresponding zig-zag angles, (c) the effective guide indices, (d) the phase velocities, and (e) the spatial period in the direction of propagation of the guided modes (z direction).

Express angles in degrees to nearest 0.0001°. Express velocities as a fraction of the speed of light in free-space to nearest 0.0001c. Express distances in μm to nearest 0.01μm.

2. SINGLE TE MODE SLAB WAVEGUIDE

A general slab waveguide is characterized by n_s, n_f, n_c and h. (a) Calculate, showing all work, the range of thicknesses, h, at a fixed wavelength for which only a single TE mode will propagate. (b) Calculate, showing all work, the range of free-space wavelengths at a fixed thickness for which only a single TE mode will propagate.

Express your answers in terms of n_s, n_f, n_c, h, λ_0 and constants only.

3. CUTOFF WAVELENGTH COMPARISON

For a titanium in-diffused slab waveguide in air, calculate, showing all work, the cutoff wavelength of the TE_0 mode as a percentage of the cutoff wavelength of the TM_0 mode. Is the TE_0 cutoff wavelength greater or lesser than the TM_0 cutoff wavelength ?

4. NORMALIZED DISPERSION EQUATION

(a) Using the normalized frequency $V = (2\pi/\lambda_0)h\sqrt{n_f^2 - n_s^2}$ and the normalized effective index $b_{TE} = (N^2 - n_s^2)/(n_f^2 - n_s^2)$, (it is implied that $n_f > n_s > n_c$) where $N = n_f \sin\theta$, prove that the dispersion equation for the TE modes can be written in the following normalized form:

$$V\sqrt{1 - b_{TE}} - \tan^{-1}\left(\sqrt{\frac{b_{TE}}{1 - b_{TE}}}\right) - \tan^{-1}\left(\sqrt{\frac{b_{TE} + \text{a}_{TE}}{1 - b_{TE}}}\right) = \nu\pi,$$

where "a" is the asymmetry parameter defined as $\text{a}_{TE} = (n_s^2 - n_c^2)/(n_f^2 - n_s^2)$.

(b) Similar normalization can not be done for the TM modes. It can be proved that by defining $b_{TM} = (n_f^2/qn_s^2)[(N^2 - n_s^2)/(n_f^2 - n_s^2)]$, where $q = (N^2/n_f^2) + (N^2/n_s^2) - 1$, and $\text{a}_{TM} = (n_f/n_c)^4[(n_s^2 - n_c^2)/(n_f^2 - n_s^2)]$, the resulting normalized dispersion equation for the TM modes is given by

$$V\sqrt{q}\frac{n_f}{n_s}\sqrt{1-b_{TM}} - \tan^{-1}\sqrt{\frac{b_{TM}}{1-b_{TM}}} - \tan^{-1}\sqrt{\frac{b_{TM}+\mathrm{a}_{TM}(1-b_{TM}d)}{1-b_{TM}}} = \nu\pi,$$

where $d = [1-(n_s/n_f)^2][1-(n_c/n_f)^2]$. It is obvious that the above normalized equation is not exactly normalized completely since still the ratios n_s/n_f and n_c/n_f and n_s, n_f are necessary. In addition, it is not possible to solve directly for the normalized effective index b_{TM} as it was possible for the TE mode case. However, if $n_f - n_s \simeq 0$ then the normalized dispersion equation for the TM modes becomes similar to the one for the TE modes if the corresponding to the TM modes asymmetry factor, a_{TM}, is used.