Problem Set # 1

1. REFLECTION AND TRANSMISSION AT A DIELECTRIC BOUNDARY

An electromagnetic plane wave of freespace wavelength of 0.5μm exists in titanium in-diffused lithium niobate of refractive index 2.234. This plane wave is incident upon a plane boundary with lithium niobate (undoped) on other side of the boundary with refractive index 2.214.

(a) What is the critical angle in degrees ?

(b) What is the Brewster angle for this incident wave in degrees ?

For an angle of incidence of 60° and 85° for both TE and TM polarizations:

(c) What fraction of the amplitude is transmitted ?

(d) What fraction of the power is transmitted ?

(e) What fraction of the amplitude is reflected ?

(f) What fraction of the power is reflected ?

(g) What is the phase shift upon reflection ?

(h) Make plots of the reflected and transmitted powers as functions of the angle of incidence in the range between 0 and 90 degrees.

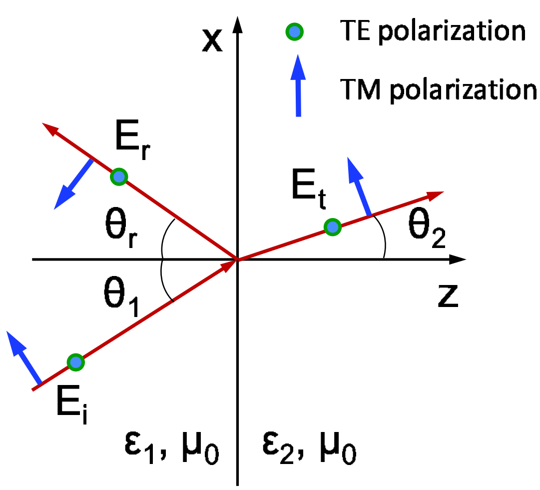

Figure 1: (Problem 1) A planar boundary between two isotropic, nonmagnetic, dielectrics with permittivities ϵ_1 and ϵ_2. The green electric field direction (along the y-axis) corresponds to the TE polarization, while the blue electric field direction (in the xz-plane) corresponds to the TM polarization. The angle of incidence is θ_1, the angle of reflection is $\theta_r = \theta_1$, and the angle of refraction is θ_2.

2. GOOS-HÄNCHEN SHIFT

In class we discussed the case of a finite beam being incident at an angle greater than the critical angle at a planar interface between two media of refractive indices n_1 and n_2 ($n_1 > n_2$). The Goos-Hänchen shift is shown in the figure below. In class we proved what $2z_s$ and x_s are for the case of TE polarization of the beam. In this problem the corresponding expressions of $2z_s = d_{GH}$ and x_s in the case of TM polarization should be determined. Show that these are given by:

$$\begin{aligned} 2z_s &= 2\frac{(n_1/n_2)^2(n_1^2 - n_2^2)}{n_1^2[\cos^2\theta + (n_1/n_2)^4\sin^2\theta] - (n_1/n_2)^4 n_2^2} \; \frac{\lambda_0 \tan\theta}{2\pi(n_1^2\sin^2\theta - n_2^2)^{1/2}} \\ x_s &= \frac{(n_1/n_2)^2(n_1^2 - n_2^2)}{n_1^2[\cos^2\theta + (n_1/n_2)^4\sin^2\theta] - (n_1/n_2)^4 n_2^2} \; \frac{\lambda_0}{2\pi(n_1^2\sin^2\theta - n_2^2)^{1/2}} \end{aligned}$$

Plot both $2z_s$ and x_s as a function of the angle of incidence θ for both TE and TM polarizations if $n_1 = 1.50$ and $n_2 = 1.00$.

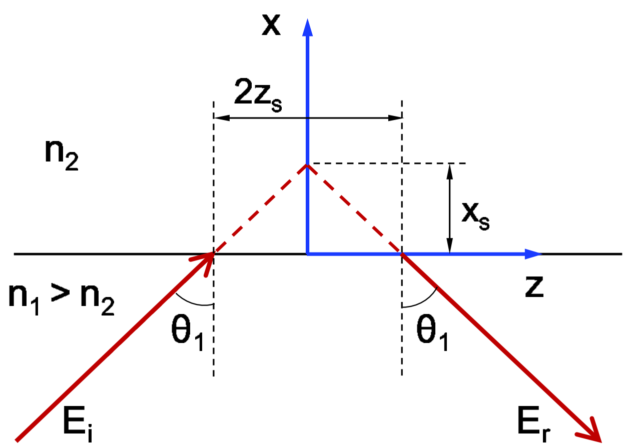

Figure 2: (Problem 2) A planar boundary between two isotropic dielectrics with $n_1 > n_2$ and at an angle of incidence $\theta_1 > \theta_{cr}$. In case that the incident field, E_i, represents a beam of finite size the reflected field, E_r, (beam) experience a Goos-Hänchen shift of $2z_s$.

3. PLANE WAVES IN ANISOTROPIC MEDIA

The plane-wave (algebraic) form of Maxwell's equations for a linear, homogeneous, non-magnetic, and anisotropic medium is given by

$$\begin{aligned}\vec{k} \times \vec{H} &= -\omega \vec{D}, \\ \vec{k} \times \vec{E} &= \omega \vec{B}, \\ \vec{k} \cdot \vec{D} &= 0, \\ \vec{k} \cdot \vec{B} &= 0,\end{aligned}$$

where $\vec{B} = \mu_0 \vec{H}$, and $\vec{D} = [\tilde{\epsilon}]\vec{E}$.

(a) Show that the wavevector $\vec{k}$ always points in the direction of $\vec{D} \times \vec{B}$.

(b) Show that the magnitude of the wavevector is given by $\vec{k} \cdot \vec{k} = \omega^2 \mu_0 [(\vec{D} \cdot \vec{D})/(\vec{D} \cdot \vec{E})]$.

(c) Show that the complex Poynting vector, $\vec{S} = (1/2)\vec{E} \times \vec{H}^*$, can point in a direction other than that of the wavevector $\vec{k}$.