

ΟΛΟΚΛΗΡΩΜΕΝΗ ΟΠΤΙΚΗ
(INTEGRATED OPTICS)

ΔΙΑΣΠΟΡΑ ΣΕ ΔΙΗΛΕΚΤΡΙΚΟΥΣ

ΟΠΤΙΚΟΥΣ ΚΥΜΑΤΟΔΗΓΟΥΣ

(Dispersion in Dielectric Optical Waveguides)

Σημειώσεις

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Dispersion Effects:

For optical waveguides the most severe limitation to the long distance communication is dispersion. The effect of dispersion is the spreading of an optical pulse in time. This occurs because the various frequency components of an optical pulse have differing propagation characteristics.

Optical waveguides exhibit three types of dispersion:

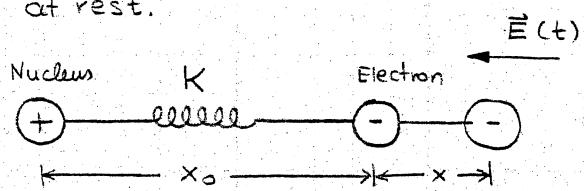
(a) Material Dispersion: In which lights of different wavelengths travel at different velocities within a given medium. This effect results in spreading in time of optical pulses.

(b) Modal Dispersion: This exists in waveguides where more than one mode can propagate and occurs because each mode propagates with a different velocity. The modal dispersion occurs even if the materials comprising the waveguide are dispersionless. The effect of the modal dispersion is again a spreading in time of optical pulses.

(c) Waveguide Dispersion: Again this exists in waveguides that can support even only one mode. Waveguide dispersion occurs because the mode propagation constant β depends on frequency (or wavelength). In this case a pulse of light will have a frequency content (even for monochromatic light) and the various wavelength components of the pulse will propagate at different velocities resulting in spreading of the pulse in time.

Material Dispersion:

A simple model to describe the interaction of matter with an electromagnetic field can be visualized by considering the electron and the nucleus tied together via a spring. When there is no external field the system of the electron and the nucleus is at rest.



When an electric field is applied then the electron/nucleus separation can change by x and for relatively small electric fields (as compared to the electron binding potential $\sim 10^{10}$ V/cm for ionization) the equation of motion of the electron is:

$$m_0 \frac{d^2x}{dt^2} + \gamma m_0 \frac{dx}{dt} + Kx = -e E(t) \quad (1)$$

where the $\gamma m_0 dx/dt$ terms represents friction (loss mechanisms), the Kx term represents the spring restoring force and $-e \vec{E}$ represents the force on the electron due to the applied electric field. Eq.(1) can also be written in the form: (where now \vec{x} is assumed a vector)

$$\frac{d^2\vec{x}}{dt^2} + \gamma \frac{d\vec{x}}{dt} + \left(\frac{K}{m_0} = \omega_0^2 \right) \vec{x} = -\frac{e}{m_0} \vec{E}(t) \quad (2)$$

where ω_0 is the resonance frequency of the oscillation. Using phasors the above equation yields:

$$\vec{x}(\omega) = \frac{-e/m_0}{(\omega_0^2 - \omega^2) + j\omega\gamma} \vec{E}(\omega) \quad (3)$$

where $\vec{x}(t) = \text{Re}\{\vec{x}(\omega)e^{j\omega t}\}$ and $\vec{E}(t) = \text{Re}\{\vec{E}(\omega)e^{j\omega t}\}$. Then the microscopic dipole moment \vec{p} due to the shift of the electron charge can be defined as $\vec{p}(\omega) = -e \vec{x}(\omega)$. If there are N microscopic

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dipoles per unit volume, then the macroscopic polarization of the material is:

$$\vec{P}(\omega) = N \vec{p} = + \frac{Ne^2/m_0}{\omega_0^2 - \omega^2 + j\gamma\omega} \vec{E}(\omega) \quad (4)$$

Then, the displacement (electric flux density) vector in the material can be written as:

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} = \left(\epsilon_0 + \frac{Ne^2/m_0}{\omega_0^2 - \omega^2 + j\gamma\omega} \right) \vec{E}(\omega) \quad (5)$$

The relative permittivity of the medium can be defined as

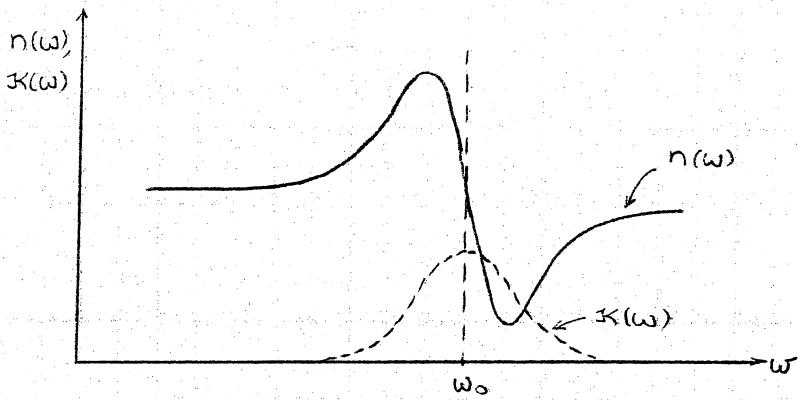
$$\begin{aligned} \epsilon_r(\omega) &= \frac{\epsilon(\omega)}{\epsilon_0} = 1 + \frac{Ne^2/m_0}{\epsilon_0 [(\omega_0^2 - \omega^2) + j\gamma\omega]} = \\ &= 1 + \frac{Ne^2}{\epsilon_0 m_0} \cdot \frac{(\omega_0^2 - \omega^2)}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2} - j \frac{Ne^2}{\epsilon_0 m_0} \frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2} \\ &= \epsilon'(\omega) - j\epsilon''(\omega) \end{aligned} \quad (6)$$

The refractive index is defined as the square root of $\epsilon_r(\omega)$

$$\tilde{n}(\omega) = n(\omega) - j\kappa(\omega) = \sqrt{\epsilon' - j\epsilon''} \quad (7)$$

where $n(\omega)$ is the real part of the refractive index and κ is the extinction coefficient.

The index of refraction $n(\omega)$ increases slowly as function of the frequency ω . However, as the frequency approaches ω_0 the refractive index decreases with increasing frequency. The region where the index decreases as the frequency increases is called anomalous dispersion. For example, $n(\omega)$ and $\kappa(\omega)$ could look like the curves shown in the next figure.



When there are more than a single resonance frequency $\epsilon_r(\omega)$ can be written in the form:

$$\epsilon_r(\omega) = 1 + \sum_i \frac{N_i^2 e^2}{m_b \epsilon_0} \frac{f_i}{\omega_i^2 - \omega^2 + j \gamma_i \omega} \quad (8)$$

where f_i is the oscillator strength of each resonance. In optics it is more common to use λ_0 instead of ω . If it is also assumed that γ_i very small then one can use the following phenomenological equation:

$$n^2(\lambda_0) - A = \sum_k \frac{G_k \lambda_0^2}{\lambda_0^2 - \lambda_k^2} \quad (9)$$

where A , G_k , λ_k are called the Sellmeir coefficients and represent resonant wavelengths and oscillator strengths.

Group Index and Group Delay:

For propagation of optical pulses that contain a spread of wavelengths in a dispersive material the group velocity instead of the phase velocity is of interest. The group velocity is defined as:

$$v_g = \frac{dw}{dk} \quad (10)$$

where $k = \frac{\omega}{c} n(\omega)$.

$$\text{Then } \frac{dk}{d\omega} = \frac{d}{d\omega} \left(\frac{\omega}{c} n(\omega) \right) = \frac{n(\omega)}{c} + \frac{\omega}{c} \frac{dn}{d\omega} = \frac{1}{c} \left(n + \omega \frac{dn}{d\omega} \right)$$

If λ instead of ω is used then: (λ = freespace wavelength)

$$\frac{dn}{d\lambda} = \frac{dn}{d\omega} \frac{d\omega}{d\lambda} = \frac{dn}{d\omega} \frac{d}{d\lambda} \left(\frac{2\pi c}{\lambda} \right) = \frac{dn}{d\omega} \left(-\frac{1}{\lambda^2} 2\pi c \right) \Rightarrow$$

$$\frac{dn}{d\omega} = -\frac{\lambda^2}{2\pi c} \frac{dn}{d\lambda} \rightarrow \frac{1}{c} \left(n + \omega \frac{dn}{d\omega} \right) = \frac{1}{c} \left(n + \frac{2\pi c}{\lambda} \left(-\frac{\lambda^2}{2\pi c} \frac{dn}{d\lambda} \right) \right) = \frac{1}{c} \left(n - \lambda \frac{dn}{d\lambda} \right).$$

Then the group velocity can be written as:

$$v_g = \frac{c}{n + \omega \frac{dn}{d\omega}} = \frac{c}{n - \lambda \frac{dn}{d\lambda}} = \frac{c}{N_g} \quad (11)$$

where N_g is the group index.

The group delay τ_g is defined as the time it takes for a pulse of light to travel a unit distance. Therefore,

$$\tau_g = \frac{1}{v_g} = \frac{dk}{d\omega} \quad (12)$$

Group Velocity Dispersion (GVD):

A narrow optical pulse that contains a spread of wavelengths $\Delta\lambda$ can propagate in a dispersive medium of length L . After propagation the pulse widens by $\Delta\tau$:

$$\Delta\tau = L \Delta\left(\frac{1}{v_g}\right) = L \frac{d}{d\lambda} \left(\frac{1}{v_g}\right) \Delta\lambda \quad (13)$$

The group velocity dispersion $D(\lambda)$ (or material dispersion) is defined as

$$D(\lambda) = \frac{\Delta\tau}{L \Delta\lambda} = \frac{d}{d\lambda} \left(\frac{1}{v_g}\right) \quad (14)$$

$$\text{However } v_g = \frac{c}{N_g} \Rightarrow \frac{1}{v_g} = \frac{1}{c} N_g \Rightarrow \frac{d}{d\lambda} \left(\frac{1}{v_g}\right) = \frac{1}{c} \frac{dN_g}{d\lambda} = \frac{1}{c} \frac{d}{d\lambda} \left(n - \lambda \frac{dn}{d\lambda}\right) \Rightarrow$$

$$\Rightarrow \frac{d}{d\lambda} \left(\frac{1}{v_g}\right) = \frac{1}{c} \left(\frac{dn}{d\lambda} - \frac{dn}{d\lambda} - \lambda \frac{d^2n}{d\lambda^2} \right) = -\frac{\lambda}{c} \frac{d^2n}{d\lambda^2}$$

An equivalent expression can be found if $v_g = \frac{dw}{dk}$ is used.

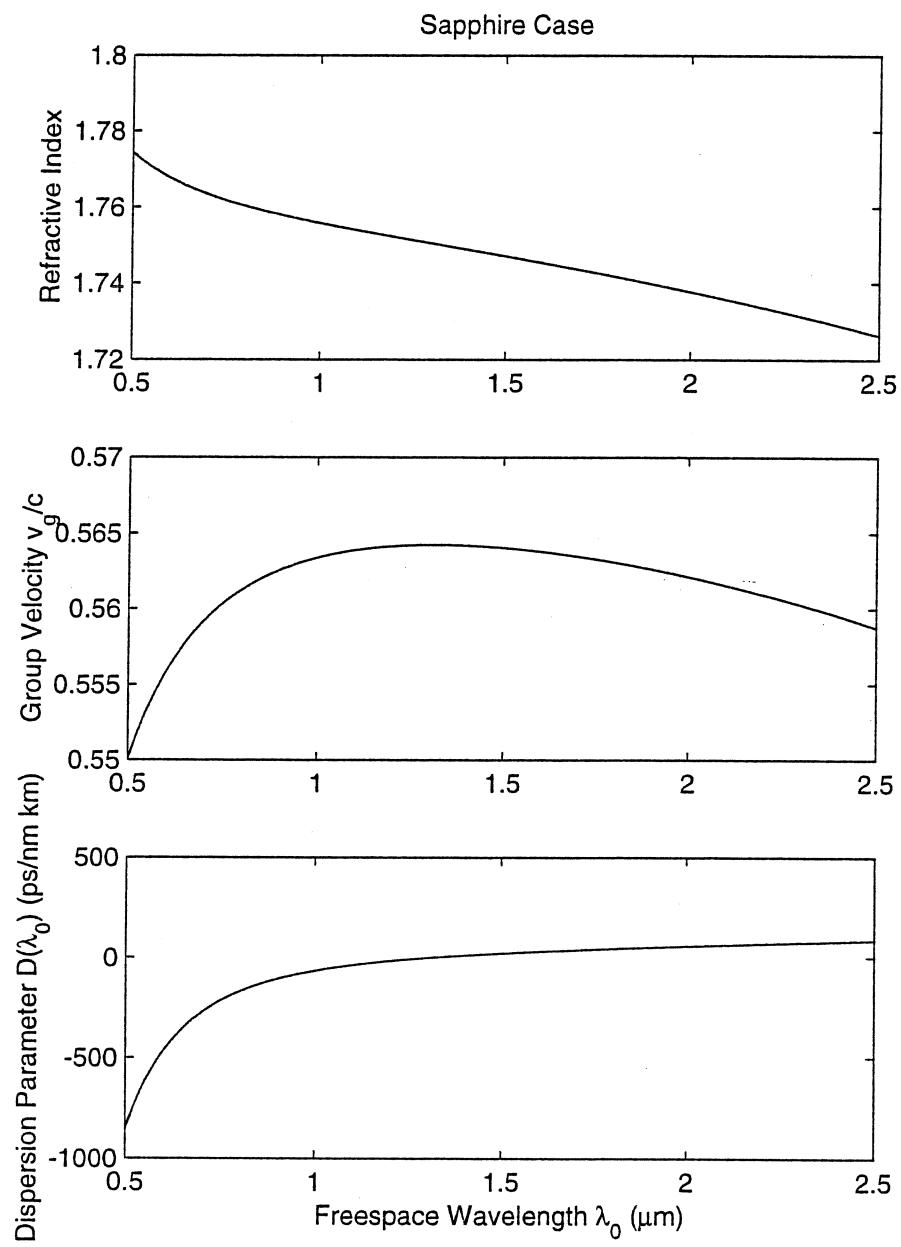
$$\frac{d}{d\lambda} \left(\frac{1}{v_g}\right) = \frac{d}{dw} \left(\frac{1}{v_g}\right) \frac{dw}{d\lambda} = \frac{d}{dw} \left(\frac{dk}{dw}\right) \frac{dw}{d\lambda} = \frac{d^2k}{dw^2} \left(-\frac{2\pi c}{\lambda^2}\right)$$

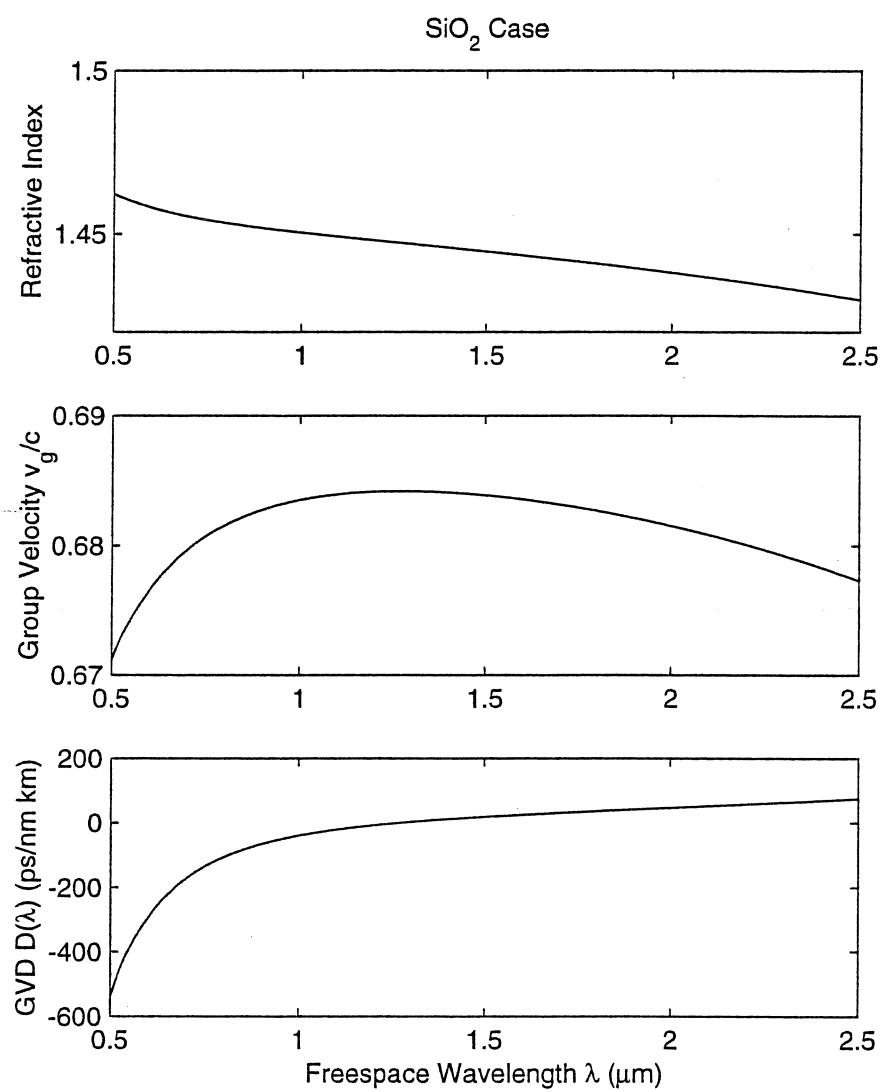
Therefore,

$$D(\lambda) = -\frac{\lambda}{c} \frac{d^2n}{d\lambda^2} = -\left(\frac{2\pi c}{\lambda^2}\right) \frac{d^2k}{dw^2} \quad (15)$$

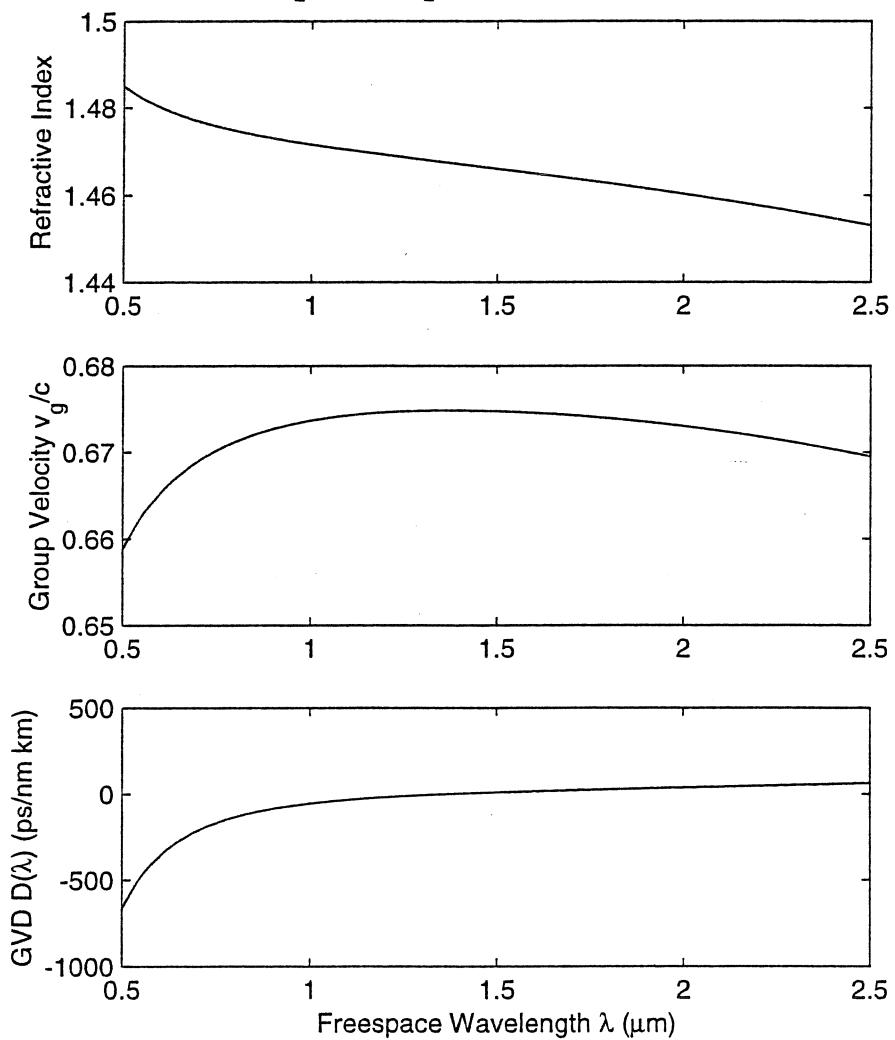
Later k can be replaced by β when the waveguide effects are also being considered.

Using the A and G_k parameters for the sapphire (Al_2O_3) we obtain the results shown on page 7. The same results for SiO_2 and SiO_2 doped with GeO_2 (fiber core) are shown on pages 8 and 9 respectively.





SiO_2 with GeO_2 Doping Case (Fiber Core)



The GVD (D) passes through zero at the zero material dispersion wavelength (let's define it as $\lambda_{D=0}$). It is straightforward to understand that $\lambda_{D=0}$ occurs at an extremum of v_g (in the graphs of pages 7-9 at the maximum). This is because $D = -\frac{2}{c} \frac{d^2n}{d\lambda^2} = 0 \Rightarrow \frac{d^2n}{d\lambda^2} = 0 \Rightarrow \frac{dv_g}{d\lambda} = 0$

$$\Rightarrow v_g(\lambda_{D=0}) \text{ an extremum} \Rightarrow v_g(\lambda_{D=0}) = \frac{c}{N_g(\lambda_{D=0})} \text{ at an extremum.}$$

Therefore, for $\lambda < \lambda_{D=0}$ $\frac{dv_g}{d\lambda} > 0 \sim$ Longer wavelengths travel faster than shorter wavelengths. But since $D = \frac{d}{d\lambda} \left(\frac{1}{v_g} \right) = -\frac{1}{v_g^2} \frac{dv_g}{d\lambda}$
 $D < 0$ when $\frac{dv_g}{d\lambda} > 0$.

Similarly, when $\lambda > \lambda_{D=0}$ $\frac{dv_g}{d\lambda} < 0 \sim$ shorter wavelengths travel faster than longer wavelengths. Again if $\frac{dv_g}{d\lambda} < 0 \sim D > 0$.

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Waveguide Dispersion:

Whenever light is guided in a waveguide the modal (or intermodal) and the waveguide (or intramodal) dispersion occur in addition to the material dispersion. In a single-mode waveguide the modal dispersion does not exist. However, the waveguide dispersion, even if it is not as large as the material dispersion, is still important since it can shift the dispersion characteristics of the waveguide.

It can be shown [A. Yariv, "Optical Electronics in Modern Communications] that the Group Velocity Dispersion D parameter can be written in the following form:

$$D = -\frac{2}{c} \left[\left(\frac{d^2 n}{d \lambda^2} \right)_m + \left(\frac{d^2 N_{eff}}{d \lambda^2} \right)_w \right] = -\frac{2\pi c}{\lambda^2} \left[\left(\frac{d^2 k}{d \omega^2} \right)_m + \left(\frac{d^2 \beta}{d \omega^2} \right)_w \right] = \frac{d}{d\lambda} \left(\frac{1}{v_g} \right) = -\frac{1}{v_g^2} \frac{dv_g}{d\lambda} \quad (16)$$

where $\beta = k_0 N_{eff}$, N_{eff} = effective index of the mode, and $n = n_1 \approx n_2$ (for a single mode fiber). It is assumed that $\frac{\partial n_1}{\partial \lambda} \approx \frac{\partial n_2}{\partial \lambda}$, i.e. both core and cladding have approximately the same dispersion.

The material term can be calculated using the Sellmeir coefficients as it was done previously. The waveguide term though requires the use of the transcendental eigenvalue equation. Let $f(\omega, \beta) = 0$ this eigenvalue equation. For example, for the HE₁₁ mode

$$f(\omega, \beta) = \frac{\alpha J_1(\kappa\alpha)}{J_0(\kappa\alpha)} - \gamma \alpha \frac{K_1(\gamma\alpha)}{K_0(\gamma\alpha)} = 0 \quad (17)$$

where $\kappa = \sqrt{\frac{\omega^2}{c^2} n_1^2 - \beta^2}$, $\gamma = \sqrt{\beta^2 - (\frac{\omega}{c} n_2)^2}$

From Eq. (17) we get:

$$\frac{\partial f}{\partial w} dw + \frac{\partial f}{\partial \beta} d\beta = 0 \Rightarrow \frac{dw}{d\beta} = -\frac{\partial f / \partial \beta}{\partial f / \partial w} = v_g \quad (18)$$

The term $\frac{d^2\beta}{dw^2}$ that is needed can be obtained from:

$$\begin{aligned} \frac{d^2\beta}{dw^2} &= \frac{d}{dw} \left(\frac{d\beta}{dw} \right) = \frac{d}{dw} \left(\frac{1}{v_g} \right) = \frac{d}{dw} \left(-\frac{\partial f / \partial \beta}{\partial f / \partial w} \right) = \\ &= -\frac{\frac{d}{dw}(\partial f / \partial \beta) \frac{\partial f}{\partial w} - \frac{d}{dw}(\partial f / \partial w) \frac{\partial f}{\partial \beta}}{(\partial f / \partial \beta)^2} \end{aligned} \quad (19)$$

where $\frac{d}{dw} \left(\frac{\partial f}{\partial w} \right) = \frac{\partial}{\partial \beta} \left(\frac{\partial f}{\partial w} \right) \frac{d\beta}{dw} + \frac{\partial}{\partial w} \left(\frac{\partial f}{\partial w} \right) = \frac{\partial^2 f}{\partial \beta \partial w} \frac{1}{v_g} + \frac{\partial^2 f}{\partial w^2}$ and

$$\frac{d}{dw} \left(\frac{\partial f}{\partial \beta} \right) = \frac{\partial}{\partial \beta} \left(\frac{\partial f}{\partial \beta} \right) \frac{d\beta}{dw} + \frac{\partial}{\partial w} \left(\frac{\partial f}{\partial \beta} \right) = \frac{\partial^2 f}{\partial \beta^2} \frac{1}{v_g} + \frac{\partial^2 f}{\partial w \partial \beta}$$

It is worth mentioning that in all the above expressions w, β should always satisfy $f(w, \beta) = 0$, i.e. their corresponding eigenvalue equation. Also it is obvious that waveguide dispersion calculations are tedious and can be performed strictly numerically.

For qualitative purposes the group velocity v_g as function of w is shown in the figure of page 13 for the lowest order mode HE_{11} . The slope $\frac{dv_g}{d\beta} = v_g$.

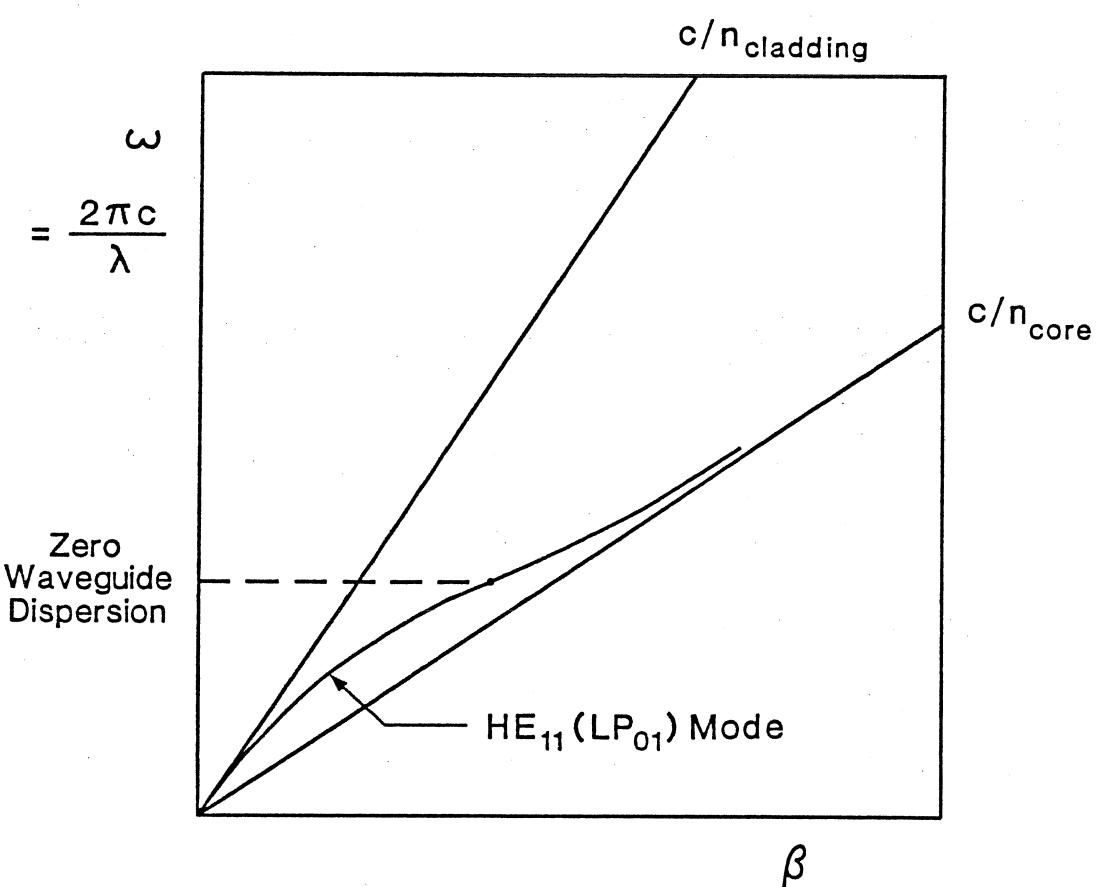
At the inflection point $\frac{dv_g}{dw} = 0$. But

$$\begin{aligned} D &= \frac{d}{d\lambda} \left(\frac{1}{v_g} \right) = \frac{d}{dw} \left(\frac{1}{v_g} \right) \frac{dw}{d\lambda} = \frac{d}{dw} \left(\frac{1}{v_g} \right) \left(-\frac{2\pi c}{\lambda^2} \right) \\ &= \frac{d}{dw} \left(\frac{d\beta}{dw} \right) \left(-\frac{2\pi c}{\lambda^2} \right) = \frac{d^2\beta}{dw^2} \left(-\frac{2\pi c}{\lambda^2} \right) \\ &= -\frac{1}{v_g^2} \frac{dv_g}{dw} \frac{dw}{d\lambda} = -\frac{1}{v_g^2} \frac{dv_g}{dw} \left(-\frac{2\pi c}{\lambda^2} \right) = +\frac{dv_g}{dw} \left(\dots \right) \end{aligned}$$

Therefore, $\frac{dv_g}{dw} = 0 \Rightarrow D = 0$.

When $\frac{dv_g}{d\lambda} > 0 \Rightarrow \frac{dv_g}{dw} < 0 \Rightarrow$ longer wavelengths travel faster than shorter wavelengths (frequencies below the dashed line in the figure of page 13). When $\frac{dv_g}{d\lambda} < 0 \Rightarrow \frac{dv_g}{dw} > 0 \Rightarrow$ shorter

DISPERSION CURVE FOR FUNDAMENTAL MODE
OF STEP-INDEX OPTICAL FIBER



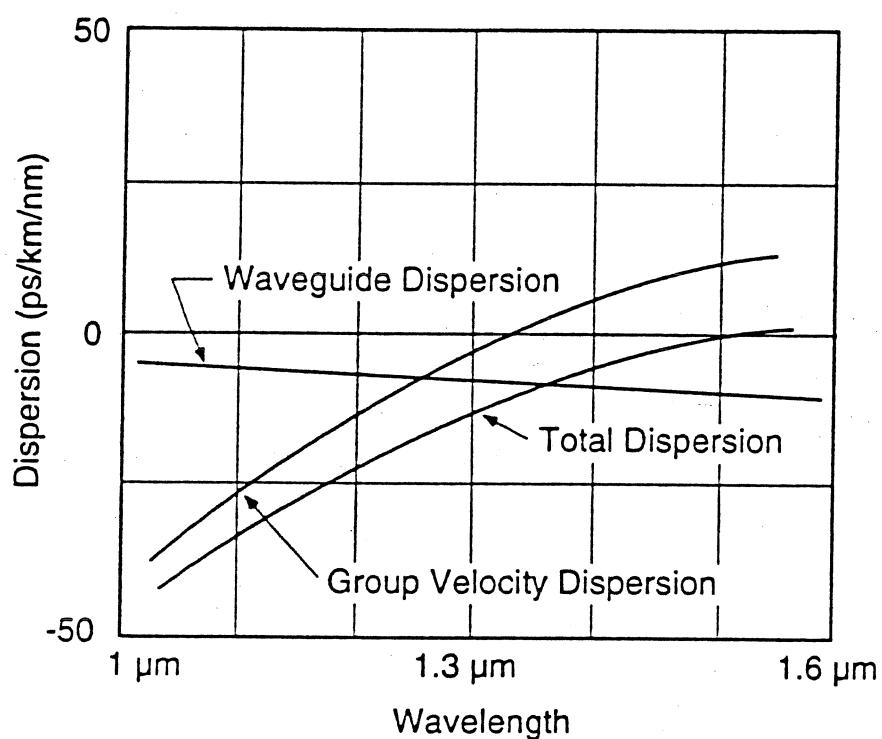
wavelengths travel faster than longer wavelengths. (frequencies above the dashed line in the figure of page 13).

When both material and waveguide dispersion are considered (since they exist simultaneously) they combine as is shown in the figure of page 15.

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TYPICAL DISPERSION CURVES FOR A GLASS OPTICAL FIBER



C. R. Pollock, Fundamentals of Optoelectronics. Chicago: Irwin, 1995.

Optical pulse Propagation in Waveguides:

Let's assume a Gaussian pulse (in time) at some point z ($z=0$) inside a waveguide (single-mode). The electric field of the pulse can be expressed as:

$$E(x, y, z=0, t) = E_0 \mathcal{E}(x, y) e^{-\frac{1}{2}(\frac{t}{\tau})^2} e^{j\omega_0 t} (e^{-j\beta(\omega_0)(z=0)}) \quad (20)$$

The frequency spectrum of $E(x, y, z=0, t)$ is given by

$$\begin{aligned} \tilde{E}(x, y, z=0, \omega) &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} E_0 \mathcal{E}(x, y) e^{-\frac{1}{2}(\frac{t}{\tau})^2} e^{j\omega_0 t} e^{-j\omega t} dt = \\ &= \frac{E_0 \mathcal{E}(x, y)}{\sqrt{2\pi}} \tau e^{-\frac{1}{2}(\omega - \omega_0)^2 \tau^2} \end{aligned} \quad (21)$$

In the above it is assumed that $\mathcal{E}(x, y)$ corresponds to one waveguide mode. Now each spectral component of \tilde{E} is propagated by distance z assuming a propagation constant $\beta(\omega)$:

$$E(x, y, z, t) = \int_{-\infty}^{+\infty} \tilde{E}(x, y, z=0, \omega) e^{-j\beta(\omega)z} e^{j\omega t} dw \quad (22)$$

usually it is assumed that $\beta(\omega)$ is a smooth function of ω and can be expanded in a Taylor series with only the first three terms:

$$\begin{aligned} \beta(\omega) &\simeq \underbrace{\beta(\omega_0)}_{\beta_1} + \underbrace{(\omega - \omega_0) \frac{d\beta}{dw}\Big|_{\omega_0}}_{\beta_1} + \underbrace{\frac{1}{2} (\omega - \omega_0)^2 \frac{d^2\beta}{dw^2}\Big|_{\omega_0}}_{\beta_2} = \\ &= \beta(\omega_0) + (\omega - \omega_0) \beta_1 + \frac{1}{2} (\omega - \omega_0)^2 \beta_2 \end{aligned} \quad (23)$$

where $\beta_1 = 1/(dw/d\beta)\Big|_{\omega_0} = 1/v_g(\omega_0)$ and β_2 is related to the dispersion parameter $D = -\frac{2\pi c}{\lambda^2} \left(\frac{d\beta}{dw^2} \right)$ at $\lambda_0 \leftrightarrow \omega_0$. Inserting Eq.(23) into (22) and evaluating the integral yields:

$$E(x, y, z, t) = \frac{E_0 \epsilon(x, y) [1 - j \Delta\tau/\tau]^{1/2}}{[1 + (\Delta\tau/\tau)^2]^{1/2}} \exp \left\{ -\frac{(t - \tau_{go})^2}{2(\tau^2 + (\Delta\tau)^2)} \right\} \\ \exp \left\{ \frac{j(\Delta\tau/\tau)(t - \tau_{go})^2}{2[\tau^2 + (\Delta\tau)^2]} \right\} \exp \left\{ j(\omega_0 t - \beta_0 z) \right\} \quad (24)$$

where $\Delta\tau = \beta_2 z / \tau$, $\beta_0 = \beta(\omega_0)$, $\tau_{go} = \beta_1 z$.

The character of the transmitted pulse can be understood by examining the various terms of Eq.(24). The first term corresponds to the pulse amplitude which has been decreased by $[1 + (\Delta\tau/\tau)^2]^{1/4}$ and depends on β_2 and the distance z traveled.

The second term corresponds to the pulse envelope. The peak of the pulse reaches z at time $\tau_{go} = \beta_1 z = z/v_g$. This reveals that the pulse peak (envelope) propagates with the group velocity. Furthermore, the pulse half width has been increased from τ to τ' given by

$$\tau' = [\tau^2 + \Delta\tau^2]^{1/2} = [\tau^2 + (\beta_2 z / \tau)^2]^{1/2} \quad (25)$$

Therefore, the pulse has been broadened.

The third term represents a frequency modulation. The instantaneous frequency of the pulse is

$$\omega' = \omega_0 + \frac{\Delta\tau}{\tau} \left(\frac{t - \tau_{go}}{\tau^2 + \Delta\tau^2} \right) \quad (26)$$

When $\beta_2 > 0$ ($D = -(\frac{2\pi c}{\lambda^2})\beta_2 < 0$, $d\omega_0/d\lambda > 0$) the frequency at a fixed z increases linearly with time (positive linear chirp). If $\beta_2 < 0$ ($D > 0$, $d\omega_0/d\lambda < 0$) the frequency decreases linearly with time. The last term of Eq.(24) represents the propagation of the center frequency, which represented the original field in Eq.(20).

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GAUSSIAN PULSE PROPAGATION IN A DISPERSIVE MEDIUM

(from J. A. Buck, "Fundamental of Fiber Optics")

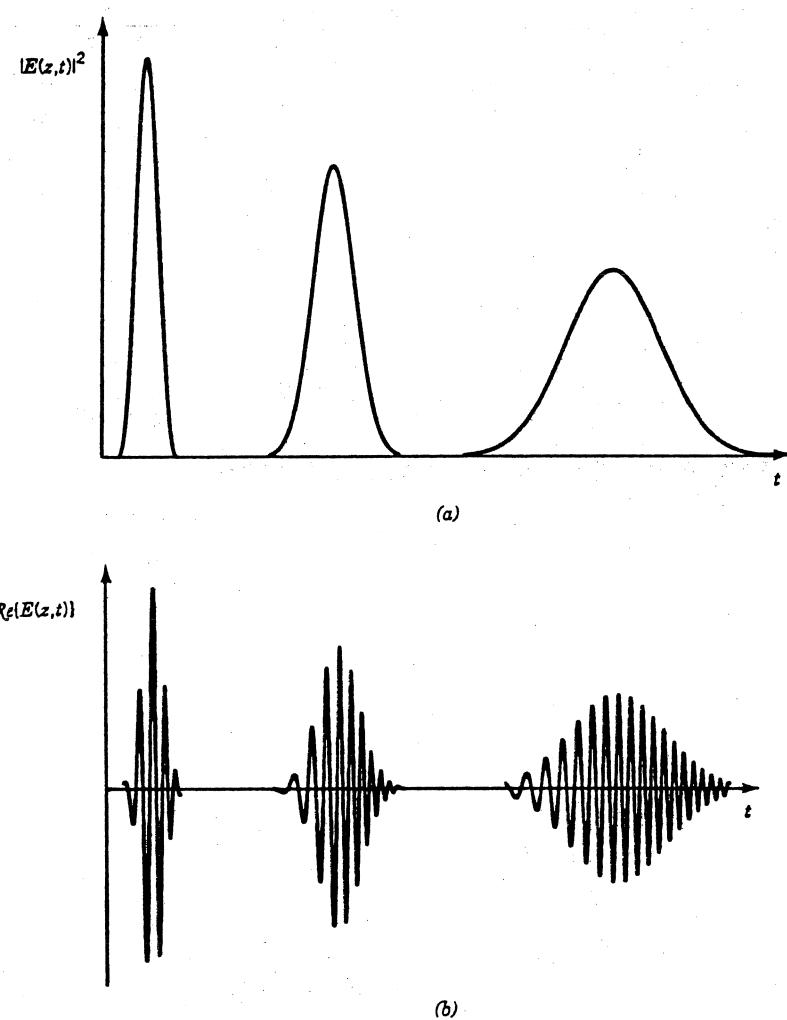


Figure 5.1. Plots of (a) the magnitude squared and (b) the real part of (5.5) for $\Delta\tau/T = 0.5, 2.0,$ and $5.0.$

REFRACTIVE INDEX AND DERIVATIVES

$$n^2 = A + \frac{G_1 \lambda^2}{\lambda^2 - \lambda_1^2} + \frac{G_2 \lambda^2}{\lambda^2 - \lambda_2^2} + \frac{G_3 \lambda^2}{\lambda^2 - \lambda_3^2}. \quad (1)$$

$$\frac{dn}{d\lambda} = -\frac{1}{n} \left[\frac{G_1 \lambda \lambda_1^2}{(\lambda^2 - \lambda_1^2)^2} + \frac{G_2 \lambda \lambda_2^2}{(\lambda^2 - \lambda_2^2)^2} + \frac{G_3 \lambda \lambda_3^2}{(\lambda^2 - \lambda_3^2)^2} \right]. \quad (2)$$

$$\frac{d^2 n}{d\lambda^2} = \frac{1}{n} \left\{ \left[\frac{G_1 \lambda_1^2 (3\lambda^2 + \lambda_1^2)}{(\lambda^2 - \lambda_1^2)^3} + \frac{G_2 \lambda_2^2 (3\lambda^2 + \lambda_2^2)}{(\lambda^2 - \lambda_2^2)^3} + \frac{G_3 \lambda_3^2 (3\lambda^2 + \lambda_3^2)}{(\lambda^2 - \lambda_3^2)^3} \right] - \left(\frac{dn}{d\lambda} \right)^2 \right\}. \quad (3)$$