21/12/2022

Diffraction Gratings Fundamentals

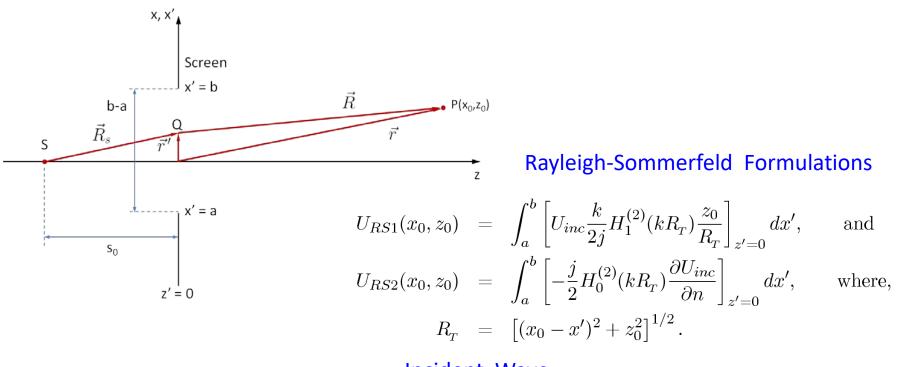
Integrated Optics

Prof. Elias N. Glytsis



School of Electrical & Computer Engineering National Technical University of Athens

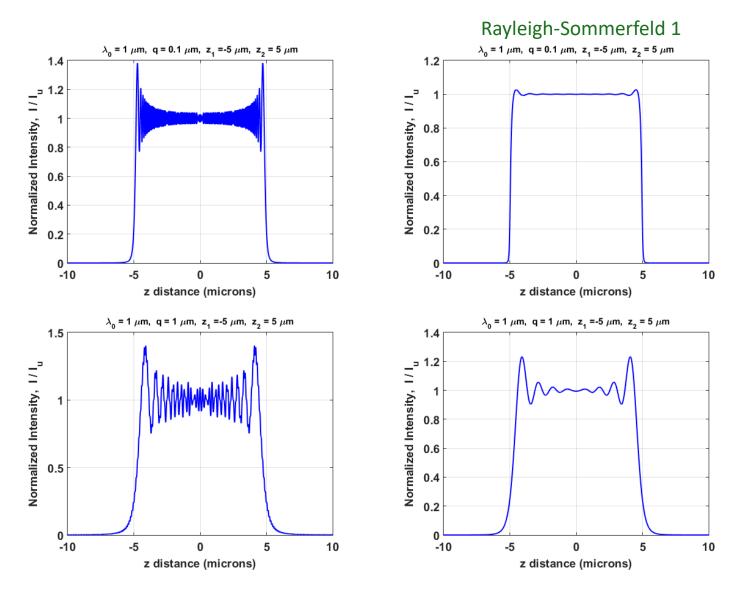
Diffraction from a Single Slit



Incident Wave

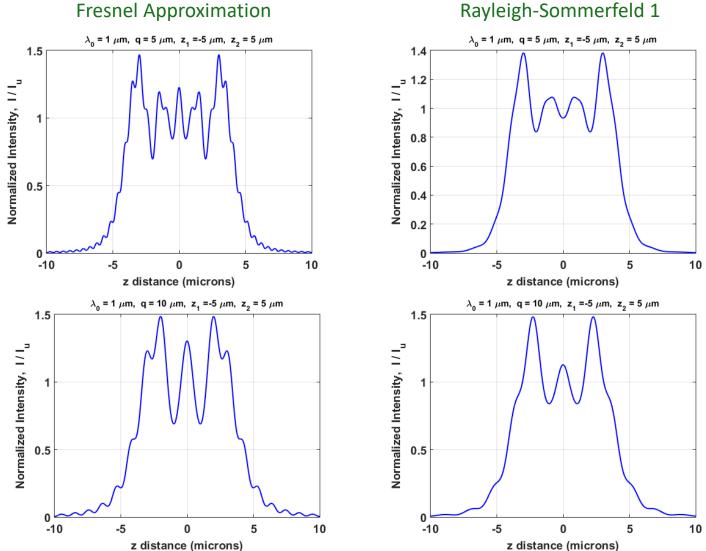
$$U_{inc}(x',z') = \begin{cases} U_0 \exp(-jkz'), & \text{(plane wave)} \\ U_0 \frac{\exp(-jkR_s)}{\sqrt{R_s}}, & R_s = [x'^2 + (z'+s_0)^2]^{1/2}, & \text{(diverging)}, \end{cases}$$
$$U_0 \frac{\exp(+jkR_s)}{\sqrt{R_s}}, & R_s = [x'^2 + (z'-s_0)^2]^{1/2}, & \text{(converging)}, \end{cases}$$

Diffraction from a Single Slit d = $10\mu m$, $\lambda_0 = 1\mu m$



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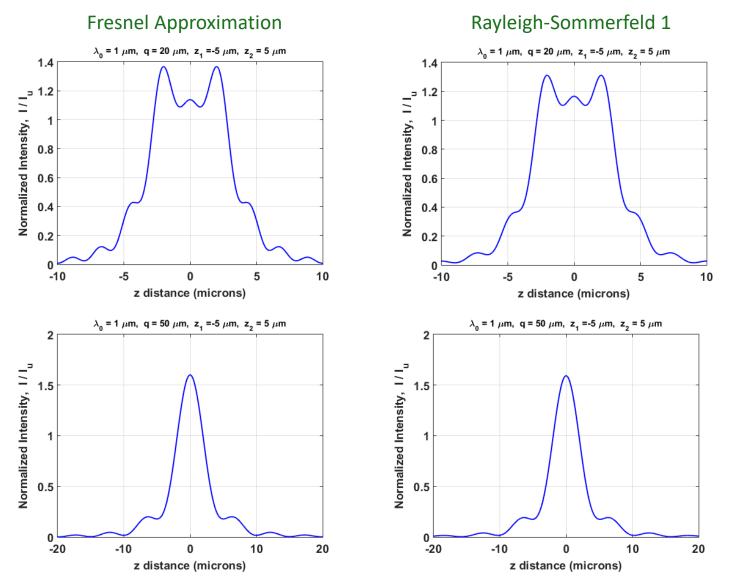
Diffraction from a Single Slit d = 10 μ m, λ_0 = 1 μ m



Rayleigh-Sommerfeld 1

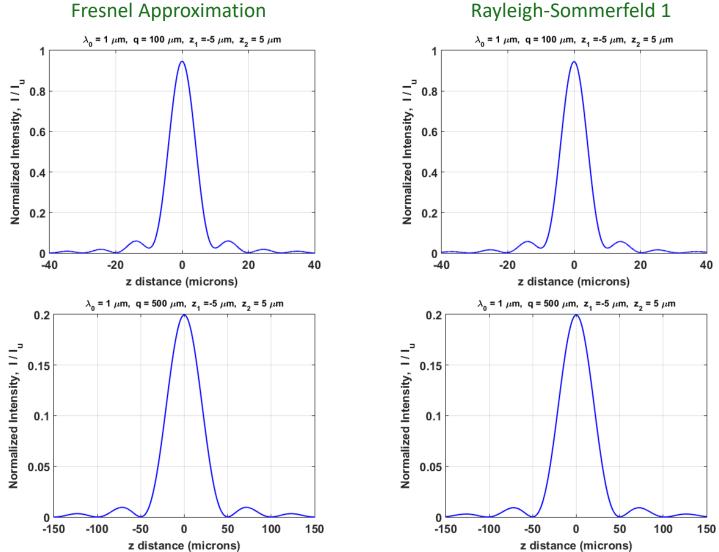
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Diffraction from a Single Slit d = $10\mu m$, $\lambda_0 = 1\mu m$



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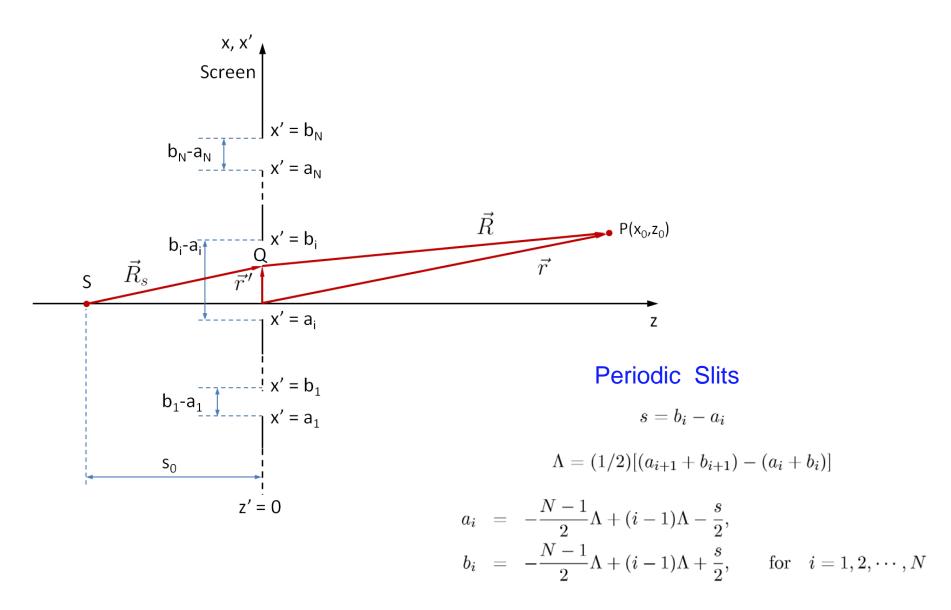
Diffraction from a Single Slit d = 10 μ m, λ_0 = 1 μ m



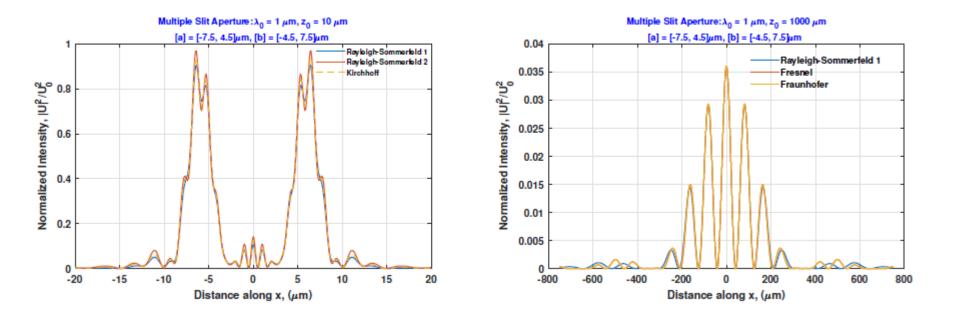
Rayleigh-Sommerfeld 1

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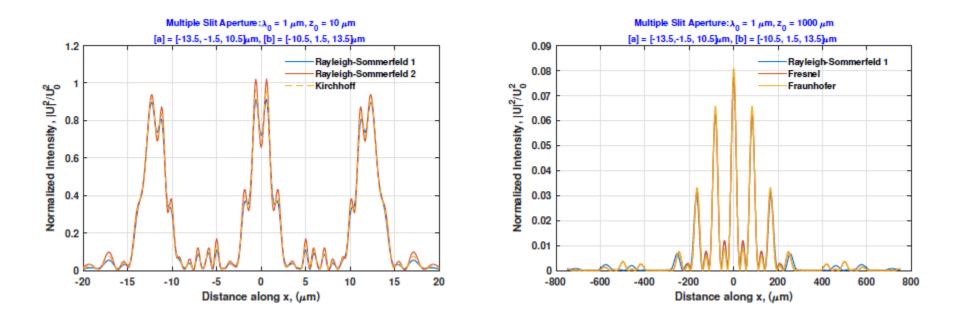
Multiple 1D-Slit Diffraction (General Case)



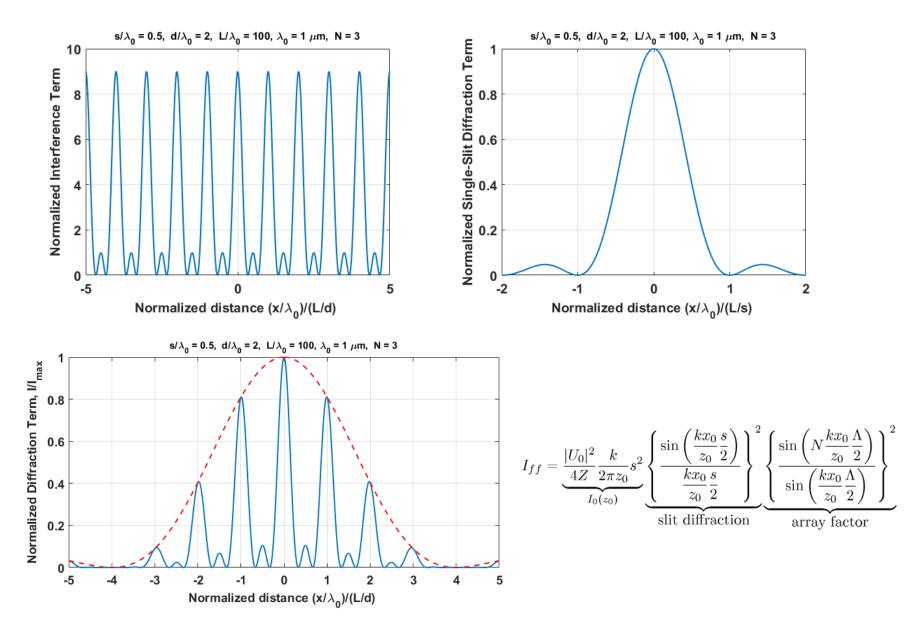
Multiple Slit Diffraction (2 slits)



Multiple Slit Diffraction (3 slits)

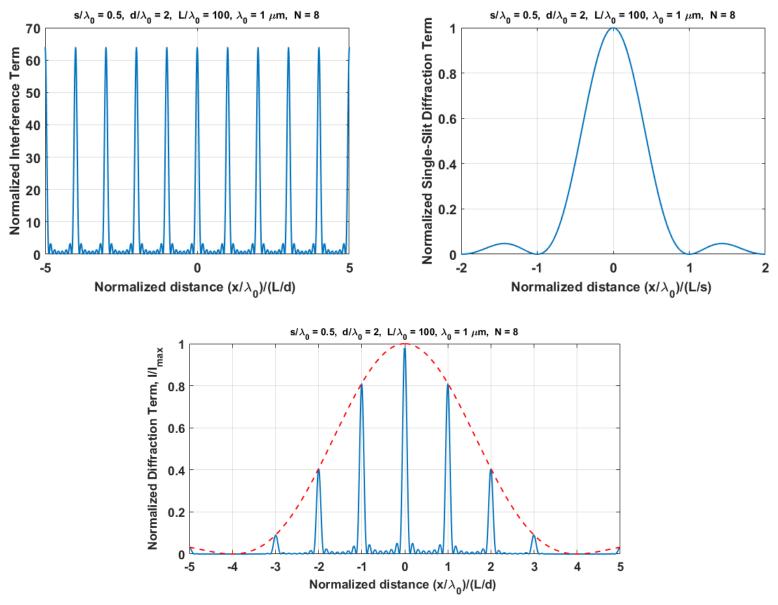


Multiple Slit Diffraction (N=3) – Fraunhofer Approximation



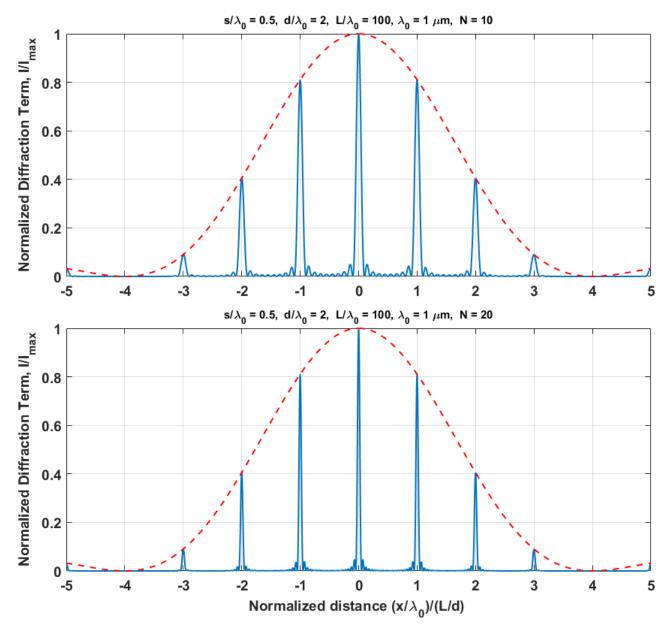
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Multiple Slit Diffraction (N=8) – Fraunhofer Approximation



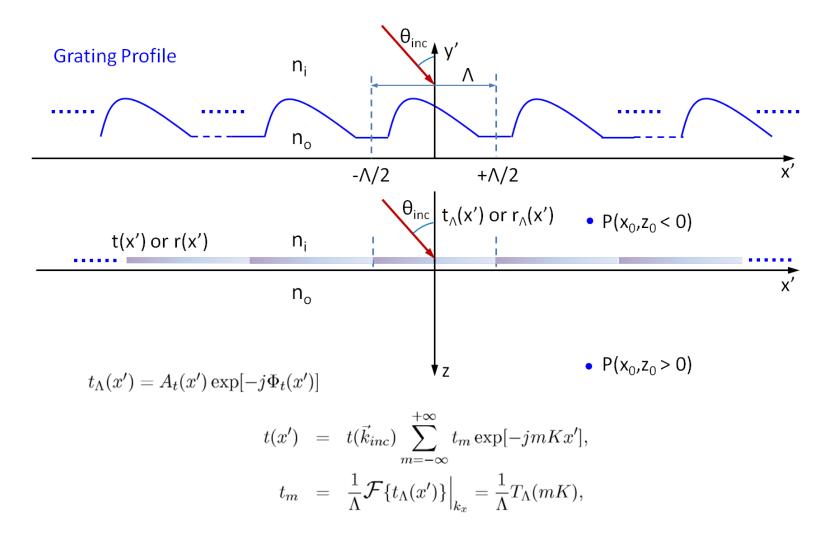
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Multiple Slit Diffraction (N=10,20) – Fraunhofer Approximation



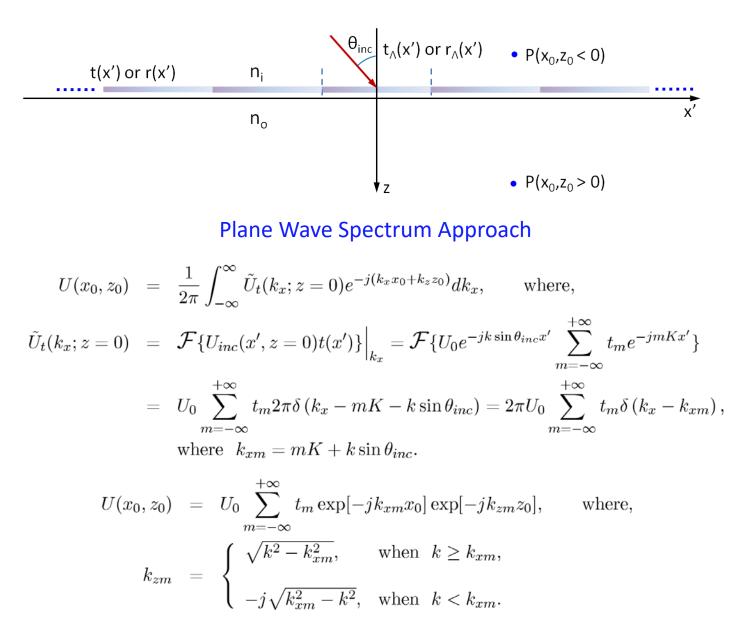
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Scalar Theory of Grating Diffraction – Transmittance Approach



 $U_{inc}(x',z) = U_0 \exp[-jk\sin\theta_{inc}x'] \exp[-jk\cos\theta_{inc}z].$

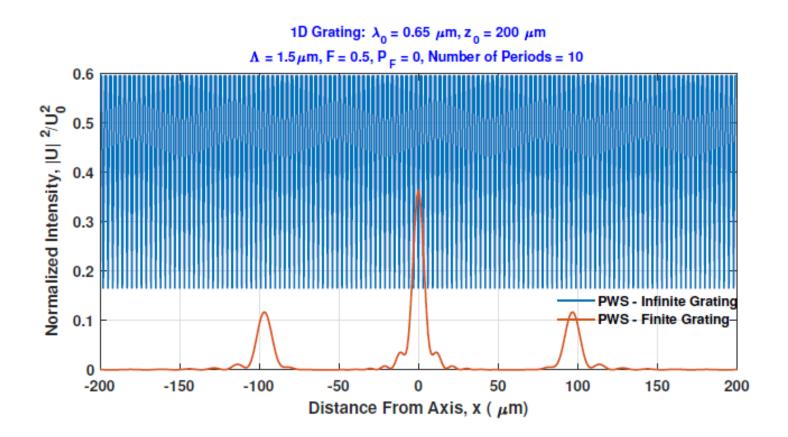
Scalar Theory of Grating Diffraction – Transmittance Approach



Scalar Theory of Grating Diffraction – Transmittance Approach

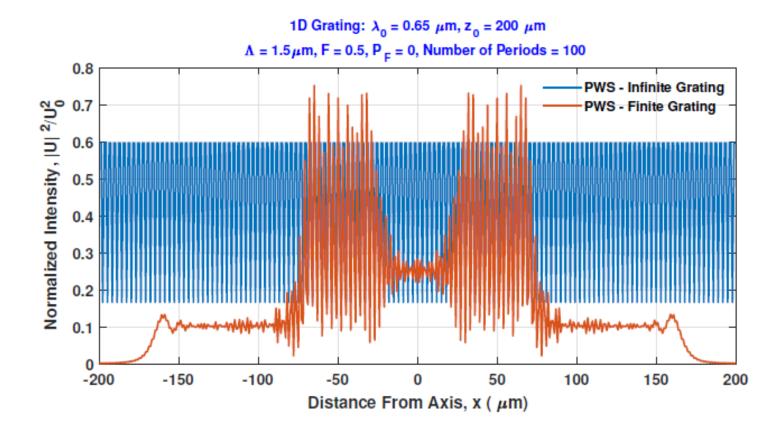
$$\begin{array}{c|c} t(x') \text{ or } r(x') & \mathbf{n}_{\mathbf{i}} & \mathbf{h}_{\mathbf{i}}(x') \text{ or } r_{\mathbf{i}}(x') & \mathbf{P}(\mathbf{x}_{0}, \mathbf{z}_{0} < \mathbf{0}) \\ \hline \mathbf{n}_{\mathbf{0}} & \mathbf{x}' \\ \hline \mathbf{comb}(x; \Lambda) = \sum_{m=-\infty}^{+\infty} \delta(x - m\Lambda) & \mathbf{x}' \\ \hline \mathbf{comb}(x; \Lambda) = \sum_{m=-\infty}^{+\infty} \delta(x - m\Lambda) & \mathbf{x}' \\ \hline \mathbf{comb}(x; \Lambda) = \sum_{m=-\infty}^{+\infty} \delta(x - m\Lambda) & \mathbf{x}' \\ \hline \mathbf{comb}(x; \Lambda) = \sum_{m=-\infty}^{+\infty} \delta(x - m\Lambda) \\ \hline \mathbf{comb}(x; \Lambda) = \left[t_{\Lambda}(x') * \operatorname{comb}(x'; \Lambda) \right] \operatorname{rect} \left(\frac{x'}{T} \right) \\ \hline \mathcal{F}\{t(x')\}\Big|_{k_{x}} &= \frac{1}{2\pi} \left[T_{\Lambda}(k_{x}) \frac{2\pi}{\Lambda} \sum_{m=-\infty}^{+\infty} \delta(k_{x} - mK) \right] * \left[L \operatorname{sinc} \left(\frac{k_{x}L}{2\pi} \right) \right] \\ &= L \sum_{m=-\infty}^{+\infty} \frac{1}{\Lambda} T_{\Lambda}(mK) \operatorname{sinc} \left(\frac{(k_{x} - mK)L}{2\pi} \right) . \\ U(x_{0}, z_{0}) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{U}_{t}(k_{x}; z = 0) e^{-j(k_{x}x_{0} + k_{z}z_{0})} dk_{x}, \quad \text{where,} \\ \hline \tilde{U}_{t}(k_{x}; z = 0) &= \mathcal{F}\{U_{inc}(x', z = 0)t(x')\}\Big|_{k_{x}} = U_{0}L \sum_{m=-\infty}^{+\infty} \frac{1}{\Lambda} T_{\Lambda}(k_{xm}) \operatorname{sinc} \left(\frac{(k_{x} - k_{xm})L}{2\pi} \right) \\ &= U_{0}L \sum_{m=-\infty}^{+\infty} t_{m} \operatorname{sinc} \left(\frac{(k_{x} - k_{xm})L}{2\pi} \right), \quad \text{where } k_{xm} = mK + k \sin \theta_{inc}. \end{array}$$

Scalar Theory of Grating Diffraction – Transmittance Approach Multiple-Slit Grating



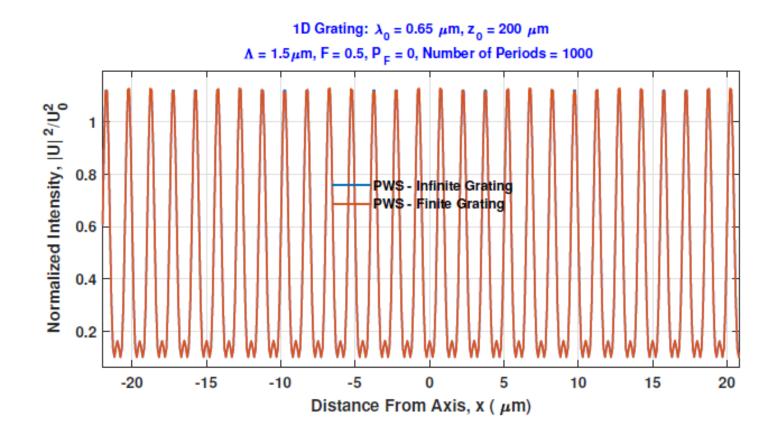
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Scalar Theory of Grating Diffraction – Transmittance Approach Multiple-Slit Grating



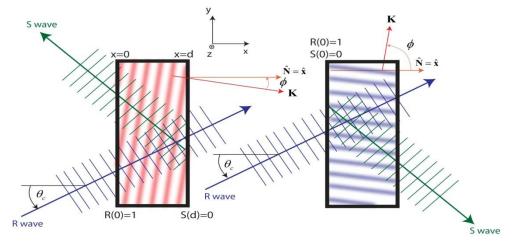
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Scalar Theory of Grating Diffraction – Transmittance Approach Multiple-Slit Grating



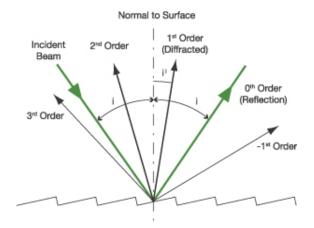
Diffraction Gratings

Example of a reflecting and transmitting holographic (volume) gratings



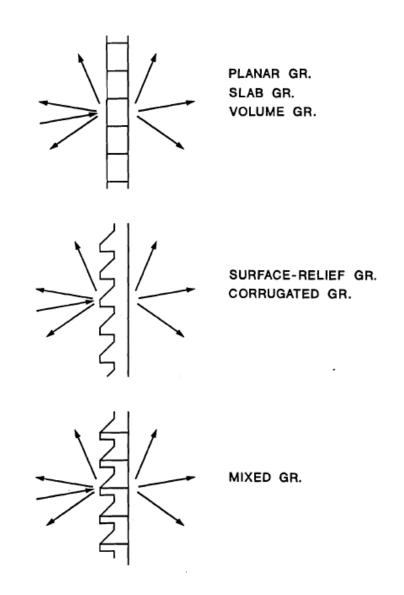
http://www.intechopen.com/books/holography-basic-principles-and-contemporary-applications/understanding-diffraction-in-volume-gratings-and-holograms

Example of a reflecting surface-relief grating

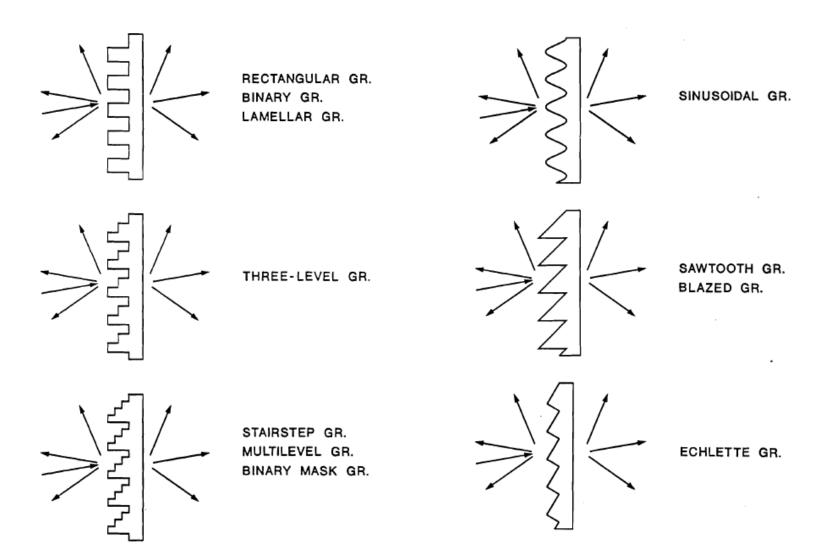


http://www.andor.com/learning-academy/diffraction-gratings-understanding-diffraction-gratings-and-the-grating-equation

Grating Classification



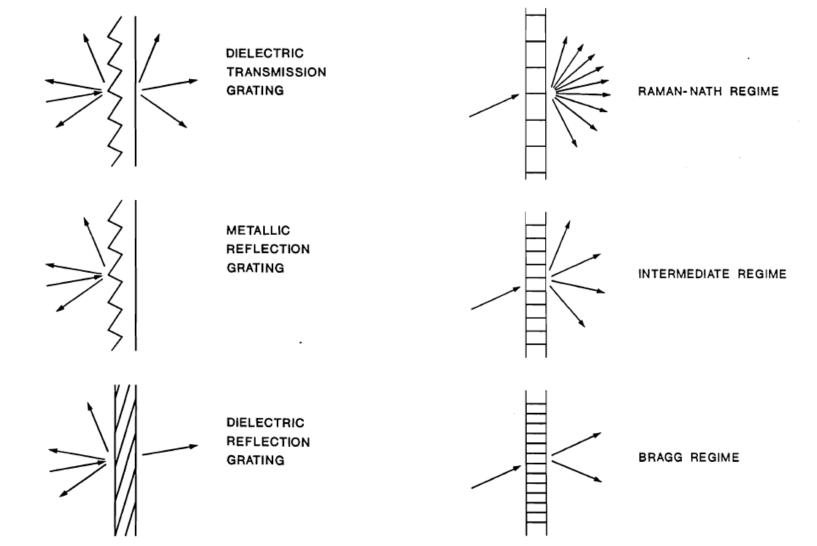
Surface-Relief Grating Types



Grating Classification

Transmission or Reflection

Classification based on Regime



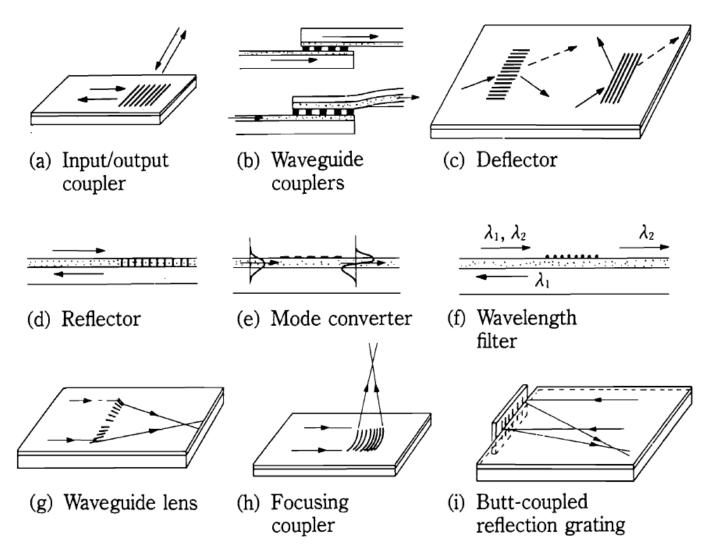
Diffraction By Gratings

- Acousto-Optics
- Diffractive Optics
- Integrated Optics
- Holography
- Optical Computing
- Optical Signal Processing
- Spectroscopy

Grating Applications

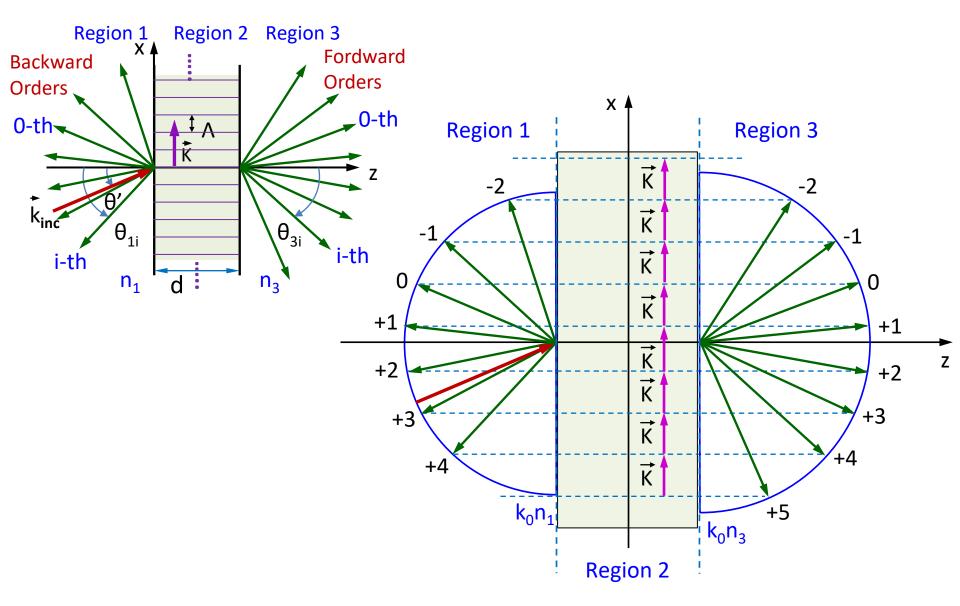
- Acoustic-Wave Generation
- Antireflection Surfaces
- Beam Coding, Coupling, Detection, etc.
- Grating Lenses
- Grating Scanners
- Head-Up Displays
- Holographic Optical Elements
- Interferometry
- Instrumentation
- Mode Conversion
- Multiplexing / Demultiplexing
- Modulation / Switching
- Optical Interconnections
- Photonic Crystal Devices
- Spectral Analysis

Grating Applications In Integrated Optics

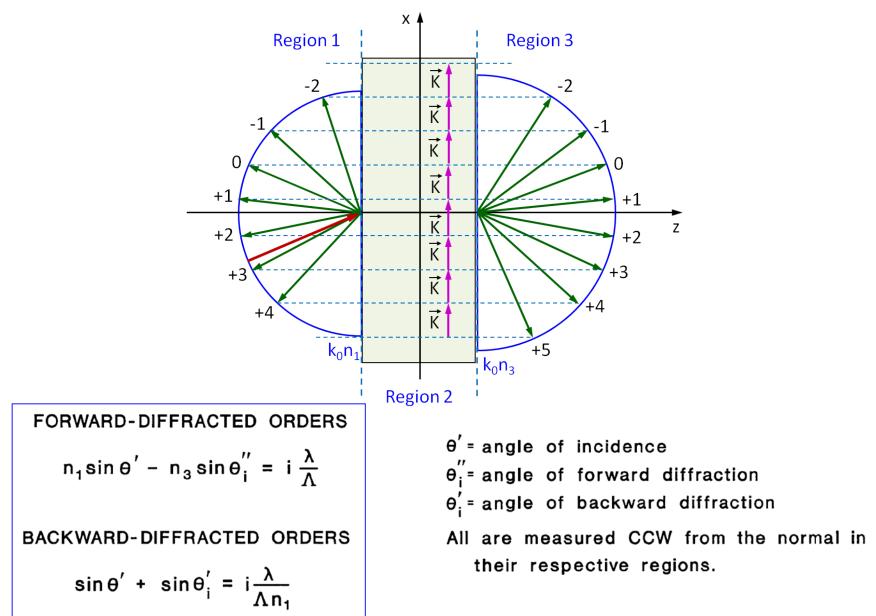


From "Optical Integrated Circuits", Nishihara, Haruna, and Suhara, McGraw-Hill 1989

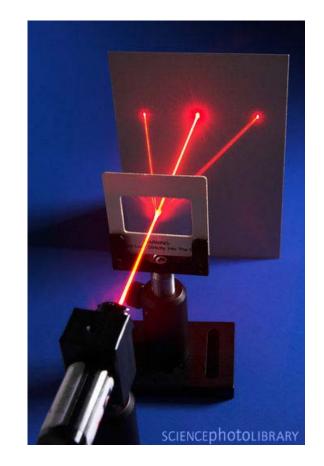
Floquet Condition

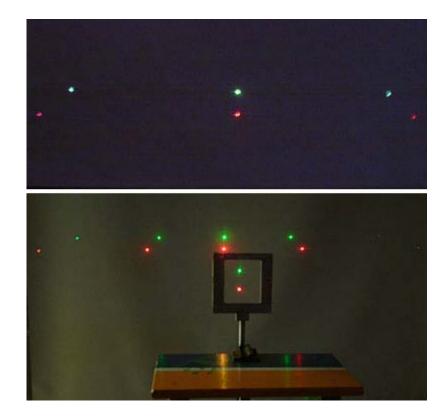


Grating Equation



Grating Equation





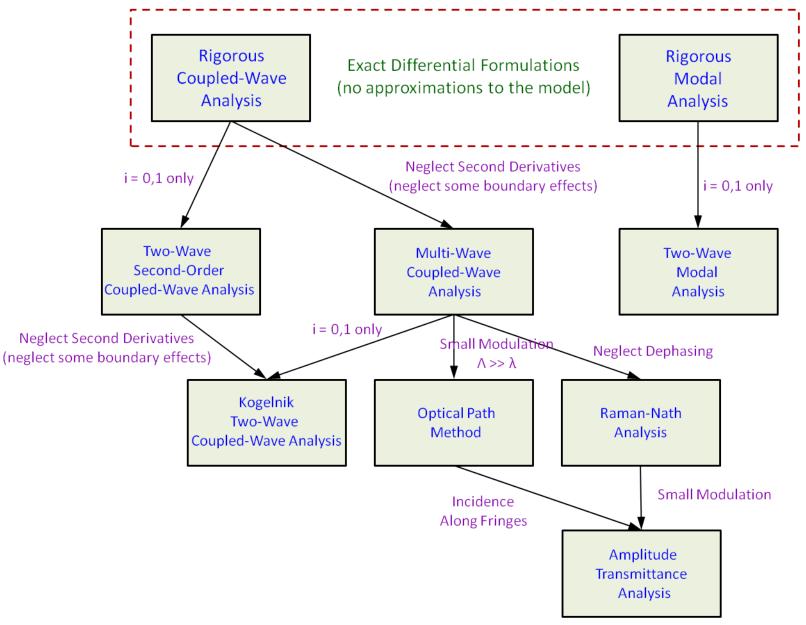
Red laser beam split by a diffraction grating. Transmission diffraction gratings consist of many thin lines of either absorptive material or thin grooves on an otherwise transparent substrate. Light transmission through a diffraction grating occurs along discrete directions, called diffraction orders. Here a diode laser beam (635 nm) is split into three diffraction orders (+1, 0, -1). This grating's groove density is 500

lines/mm.sciencephoto.com/media/92635/view

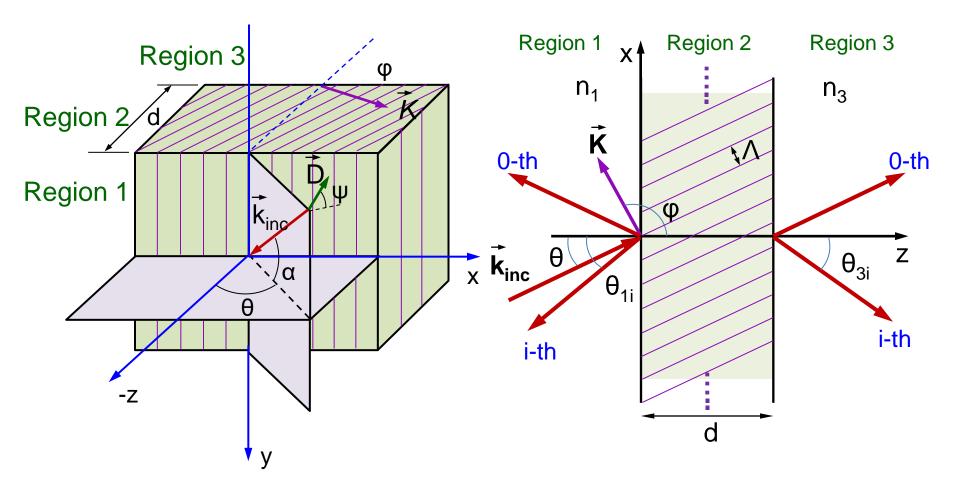
Methods Of Analysis Of Gratings

- Integral Methods
 - Finite Elements Boundary Elements
- Differential Methods
 - **Exact Methods**
 - Rigorous Coupled Wave Analysis (RCWA)
 - Modal Analysis
 - **Approximate Methods**
 - Two-Wave Coupled-Wave Analysis (Kogelnik's)
 - Raman-Nath Analysis
 - Others

Differential Grating Diffraction Analysis Hierarchy



Holographic Grating Diffraction Geometry



Electromagnetic Problem Formulation

Maxwell Equations

$$\vec{\nabla} \times \vec{E} = -j\omega \vec{B}$$

$$\vec{\nabla} \times \vec{H} = +j\omega\vec{D} + \vec{J}$$

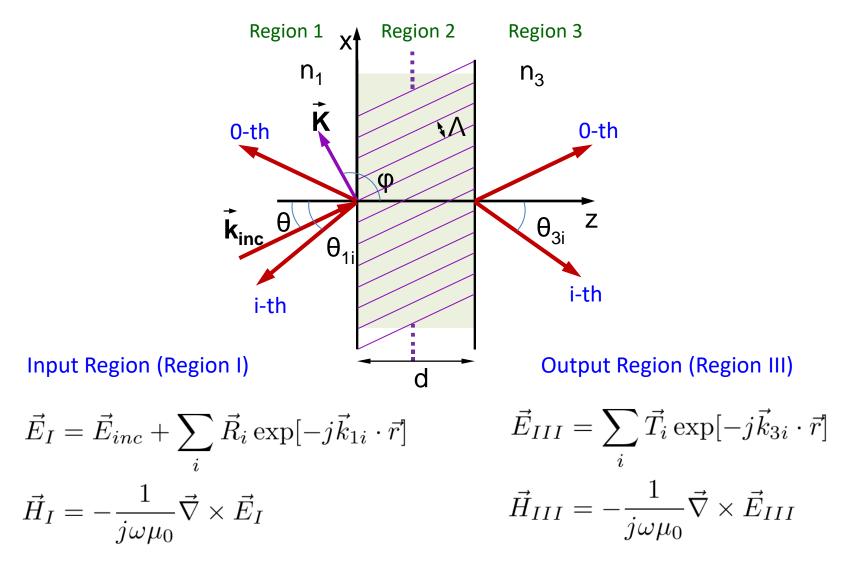
Constitutive Relations

$$egin{aligned} ec{D} &= \epsilon_0 \widetilde{arepsilon} ec{E} \ ec{B} &= \mu_0 ec{H} \ ec{J} &= \widetilde{\sigma} ec{E} \end{aligned}$$

Medium Properties: Permittivity, Conductivity Tensors are Periodic

Electromagnetic Boundary Conditions: Continuity of Tangential Electric and Magnetic Field Components

Electromagnetic Field Expansions Rigorous Coupled Wave Analysis (RCWA)



Electromagnetic Field Expansions Rigorous Coupled Wave Analysis (RCWA)

Grating Region (Region II)

$$\vec{E}_{II} = \sum_{i} \vec{S}_{i}(z) \exp[-j\vec{\sigma}_{i} \cdot \vec{r}] = \sum_{i} \vec{S}_{i}(z) \exp[-j(\vec{k}_{inc} - i\vec{K}) \cdot \vec{r}]$$
$$\vec{H}_{II} = \left(\frac{\epsilon_{0}}{\mu_{0}}\right)^{1/2} \sum_{i} \vec{U}_{i}(z) \exp[-j\vec{\sigma}_{i} \cdot \vec{r}] = \left(\frac{\epsilon_{0}}{\mu_{0}}\right)^{1/2} \sum_{i} \vec{U}_{i}(z) \exp[-j(\vec{k}_{inc} - i\vec{K}) \cdot \vec{r}]$$

Complex Permittivity Tensor Expansions (Region II)

$$\tilde{\varepsilon} = \sum_{h} \tilde{\varepsilon}_{h} \exp[jh\vec{K}\cdot\vec{r}]$$
$$\tilde{\varepsilon}_{h} = [\tilde{\varepsilon} - j\tilde{\sigma}/\omega\epsilon_{0}]_{h} = \text{Fourier Tensor Component}$$

Rigorous Coupled Wave Analysis (RCWA) Numerical Implementation

Truncation to Arbitrary Number of Diffraction Orders: M = 2m+1

Grating Region Equations

$$\frac{d\tilde{V}}{dz} = j\tilde{A}\tilde{V}$$

$$\tilde{V}^T = [\tilde{S}_x^T, \tilde{S}_y^T, \tilde{U}_x^T, \tilde{U}_y^T] \quad (4M \times 1)$$
$$\tilde{A} = \text{Coupling Matrix} \quad (4M \times 4M)$$

Standard Eigenvector/Eigenvalue Analysis

 $\tilde{V}(z) = \tilde{W} \exp[\tilde{\Lambda} z] \tilde{C}$

 $\widetilde{W} = \text{Matrix of Eigenvectors of } \widetilde{A} \quad (4M \times 4M)$ $\widetilde{\Lambda} = \text{Matrix of Eigenvalues (diagonal) of } \widetilde{A} \quad (4M \times 4M)$ $\widetilde{C} = \text{Vector of Unknown Coefficients} \quad (4M \times 1)$

Boundary Conditions: Input and Output Regions Boundaries

Rigorous Coupled Wave Analysis (RCWA) Numerical Implementation

Number of Unknowns = 10M

$ec{R_i}$	(Region I)	3 <i>M</i>
$\vec{T_i}$	(Region III)	3 <i>M</i>
\widetilde{C}	(Region III)	4M

Number of Equations = 10M

Boundary Conditions (Regions I-II and II-III) 4M+4M = 8M

$$ec{k}_{1i} ullet ec{R}_i = 0$$
 Region I Plane Waves *M*
 $ec{k}_{3i} ullet ec{T}_i = 0$ Region III Plane Waves *M*

Rigorous Coupled Wave Analysis (RCWA) Numerical Implementation

System of Linear Equations (10M x 10M)

$$\tilde{L}\tilde{x}=\tilde{b}$$

$$\begin{split} \tilde{L} &= \text{Matrix of Coefficients of Linear Equations} \quad (10M \times 10M) \\ \tilde{x}^T &= [\vec{R}_i^T, \vec{T}_i^T, \tilde{C}^T] \quad (10M \times 1) \\ \tilde{b} &= \text{Excitation Vector (depends on Incident Wave)} \quad (10M \times 1) \end{split}$$

Size of Linear System can be Reduced

Rigorous Coupled Wave Analysis (RCWA) Diffraction Efficiencies

Efficiencies of Backward-Diffracted Waves

$$DE_{1i} = -\frac{\mathcal{R}e\{k_{1iz}^*\}}{k_{inc,z}} |\vec{R}_i|^2$$

Efficiencies of Forward-Diffracted Waves

$$DE_{3i} = +\frac{\mathcal{R}e\{k_{3iz}^*\}}{k_{inc,z}} |\vec{T_i}|^2$$

For Lossless Gratings

$$\sum_{i} \{ DE_{1i} + DE_{3i} \} = 1$$

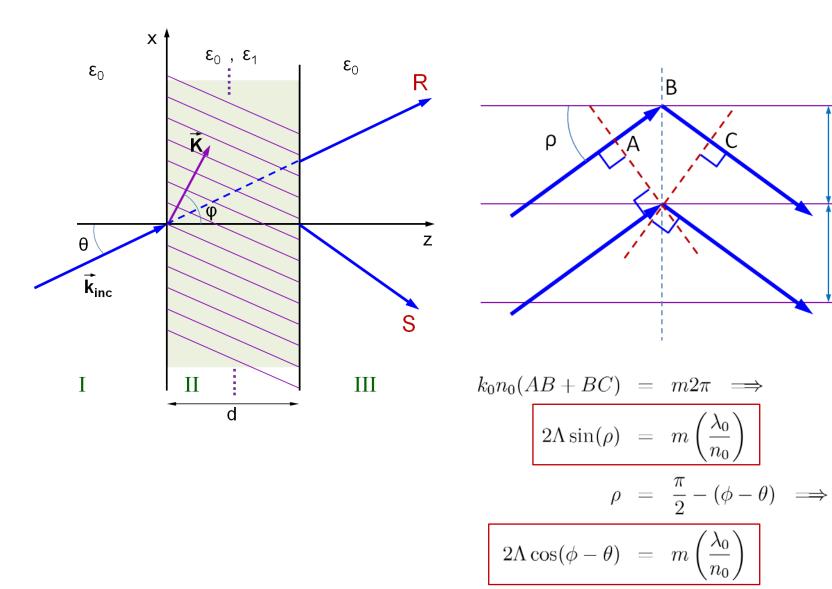
Rigorous Coupled Wave Analysis (RCWA) Generalizations

Generalized Media (in terms of constitutive equations)

 $\vec{D} = \epsilon_0 \tilde{\varepsilon} \vec{E} + \tilde{g} \vec{H}$ $\vec{B} = \tilde{h} \vec{E} + \mu_0 \tilde{\mu} \vec{H}$

- Multiple Cascaded Gratings
- Surface-Relief Gratings
- Varying Modulation Gratings
- Multiplexed Gratings
- Biaxial Input and/or Output Regions

Bragg Condition

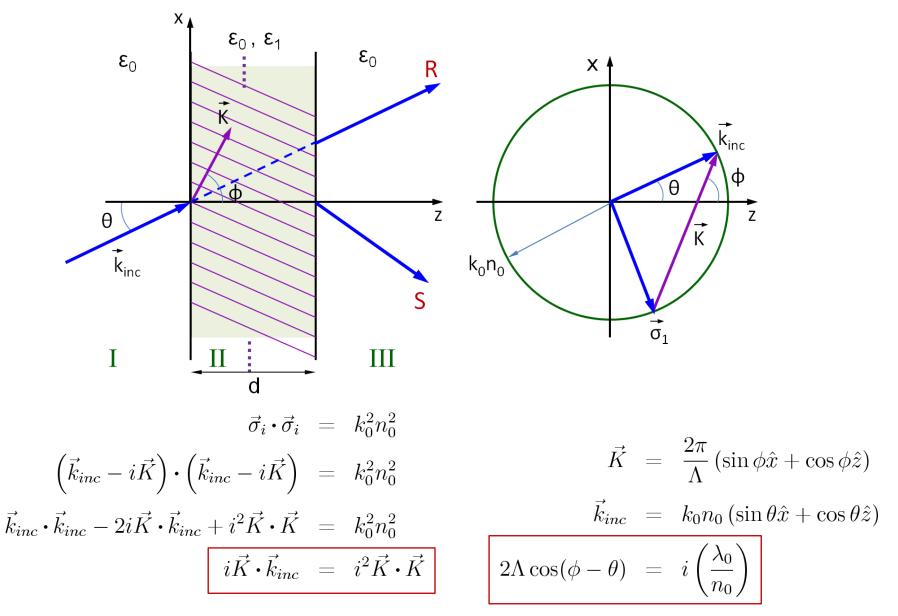


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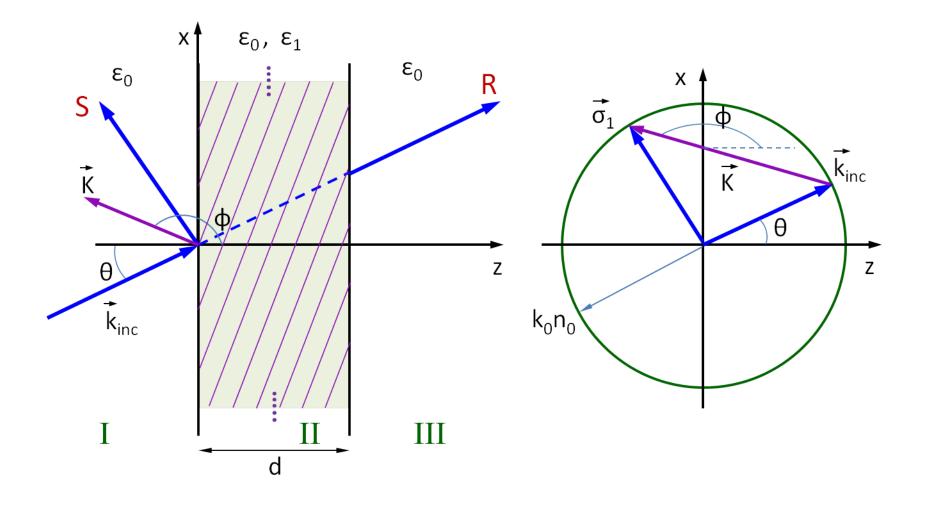
Λ

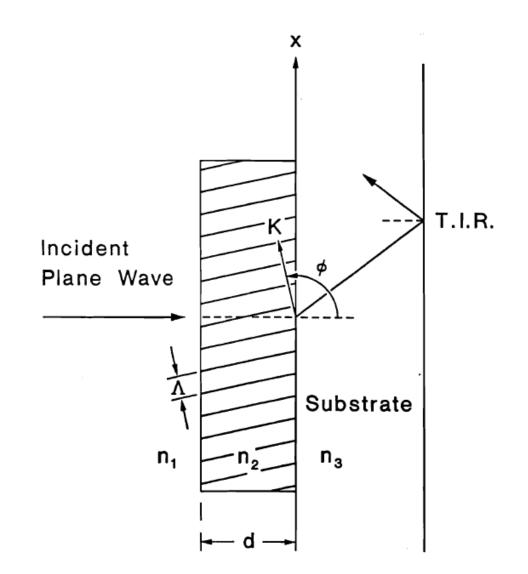
Bragg Condition

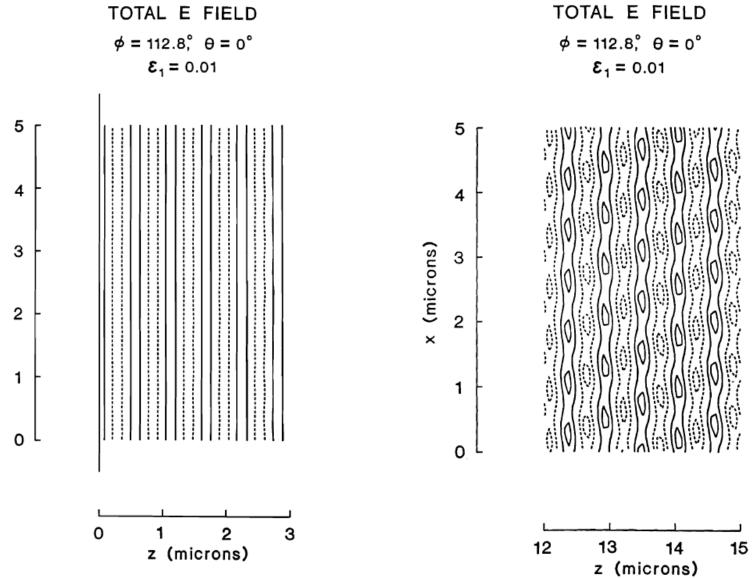


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Bragg Condition (Reflection Grating)





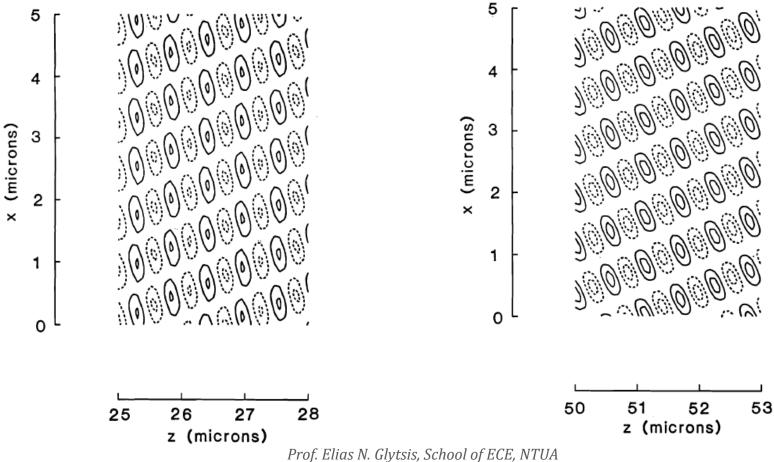


x (microns)

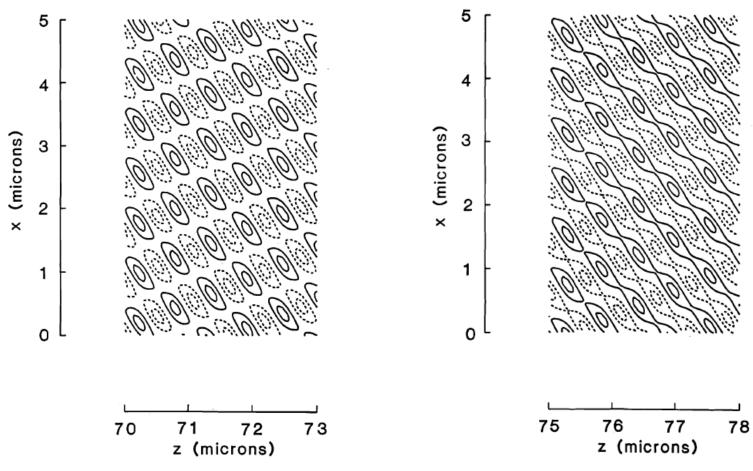
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TOTAL E FIELD $\phi = 112.8^{\circ}, \ \theta = 0^{\circ}$ $\varepsilon_1 = 0.01$

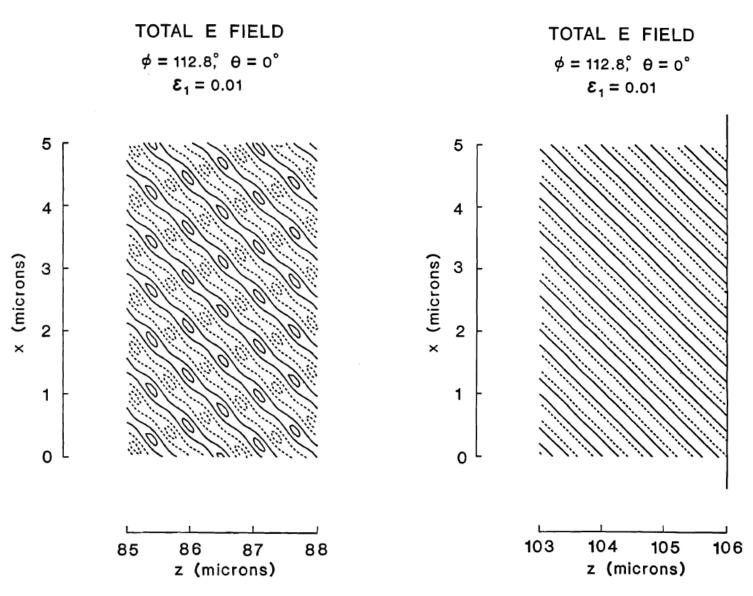
TOTAL E FIELD $\phi = 112.8$, $\theta = 0^{\circ}$ ε₁ = 0.01



TOTAL E FIELD $\phi = 112.8$, $\theta = 0^{\circ}$ $\varepsilon_1 = 0.01$ TOTAL E FIELD $\phi = 112.8$, $\theta = 0^{\circ}$ $\varepsilon_1 = 0.01$

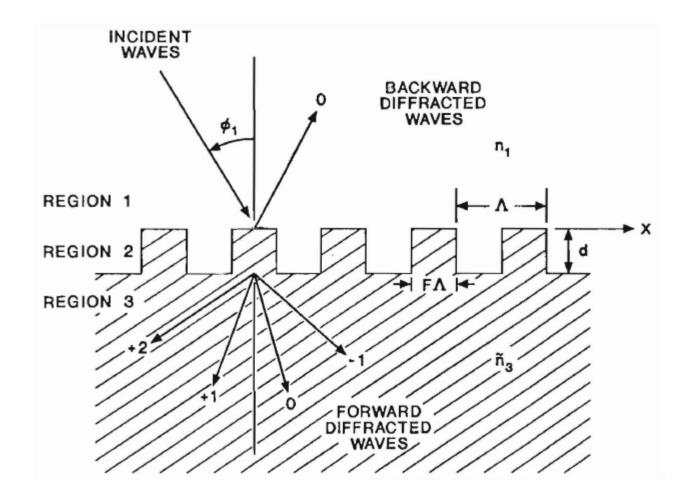


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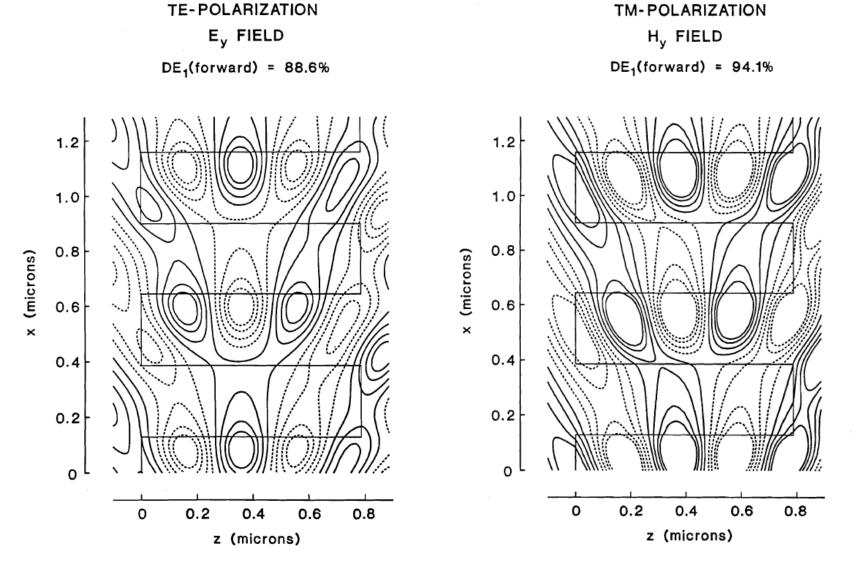


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Rigorous Coupled Wave Analysis (RCWA) Surface-Relief Grating

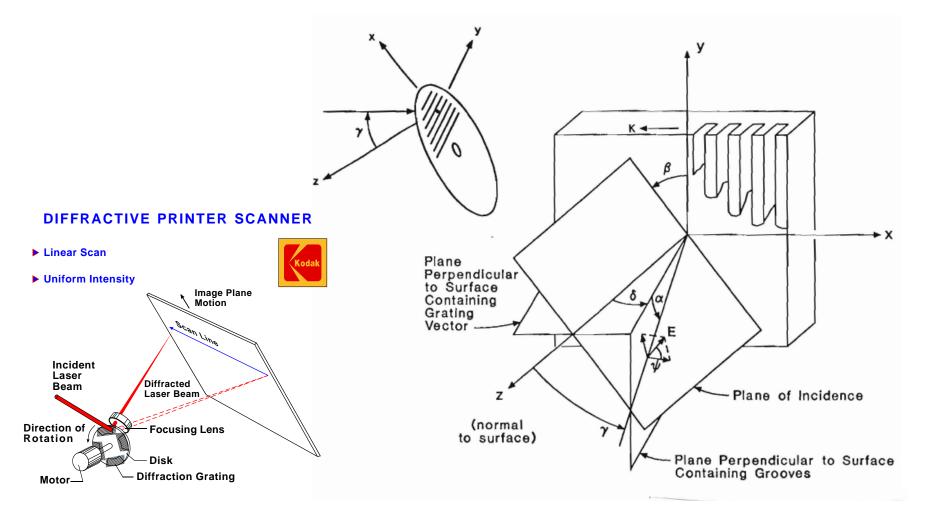


Rigorous Coupled Wave Analysis (RCWA) Surface-Relief Grating

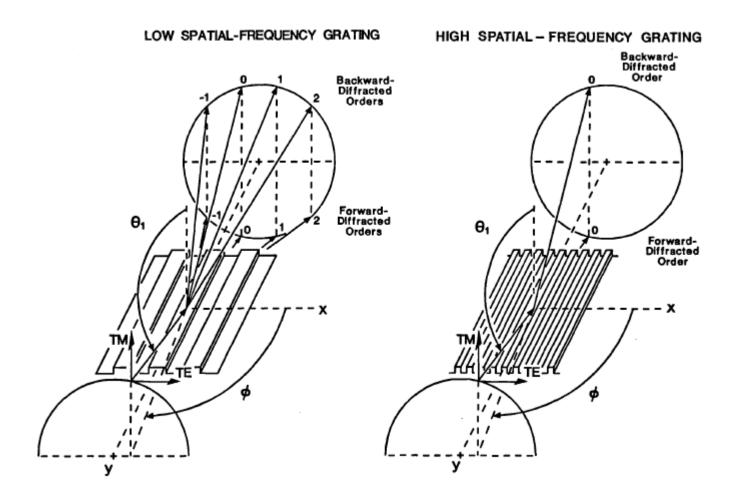


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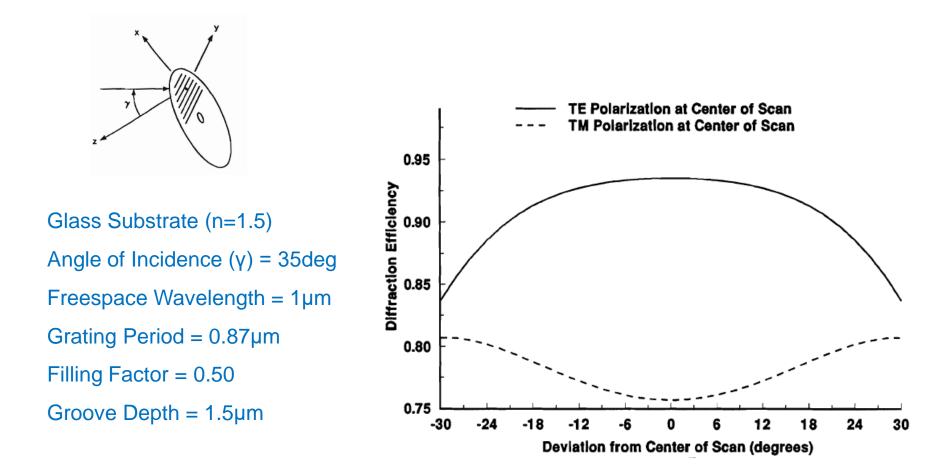
Rigorous Coupled Wave Analysis (RCWA) Holographic Grating Scanner Example <u>3D-Diffraction Problem (Conical Diffraction)</u>



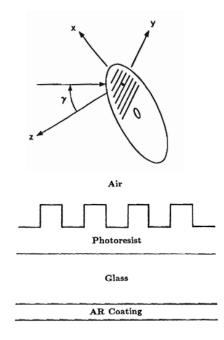
Conical Diffraction for Low- and High-Spatial Frequency Gratings



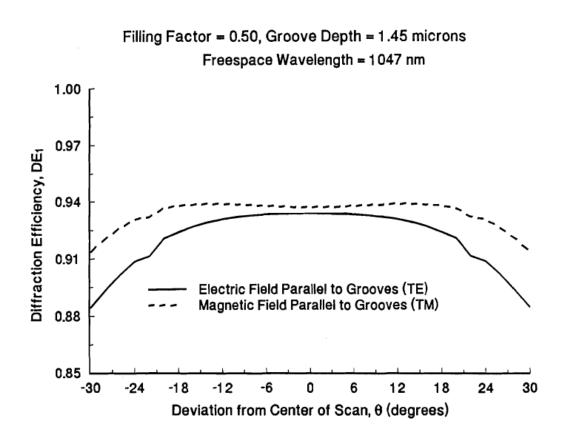
Rigorous Coupled Wave Analysis (RCWA) Holographic Grating Scanner Example <u>3D-Diffraction Problem (Conical Diffraction)</u>



Rigorous Coupled Wave Analysis (RCWA) Holographic Grating Scanner Example <u>3D-Diffraction Problem (Conical Diffraction)</u>

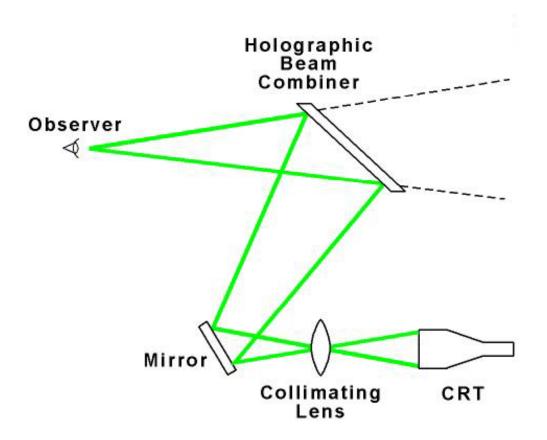


Glass Substrate (n=1.531) Angle of Incidence (γ) = 33 deg Freespace Wavelength = 1.047 μ m Photoresist Grating Photoresist Thickness = 2.4 μ m Grating Period = 0.96 μ m Filling Factor = 0.50 Groove Depth = 1.45 μ m

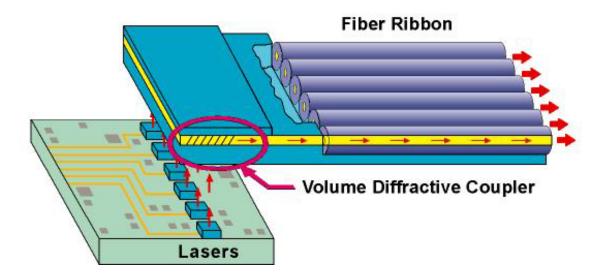


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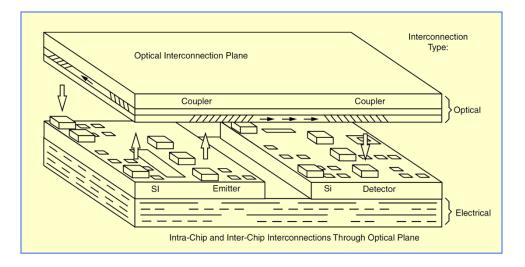
HOLOGRAPHIC HEAD-UP DISPLAYS

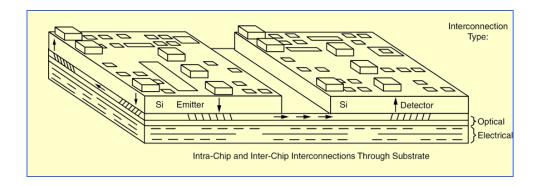


OPTICAL INTERCONNECTION

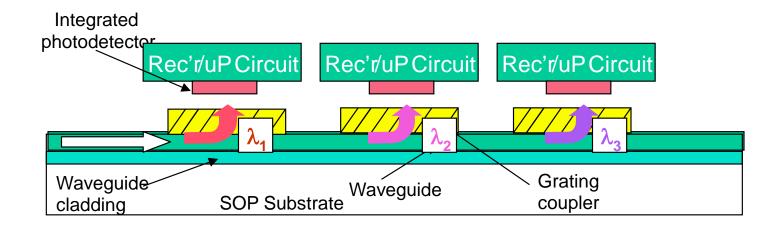


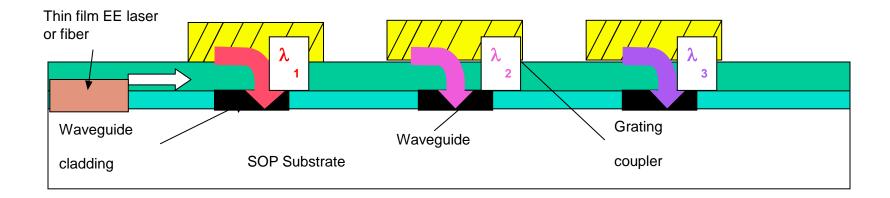
Optical Interconnect Architectures



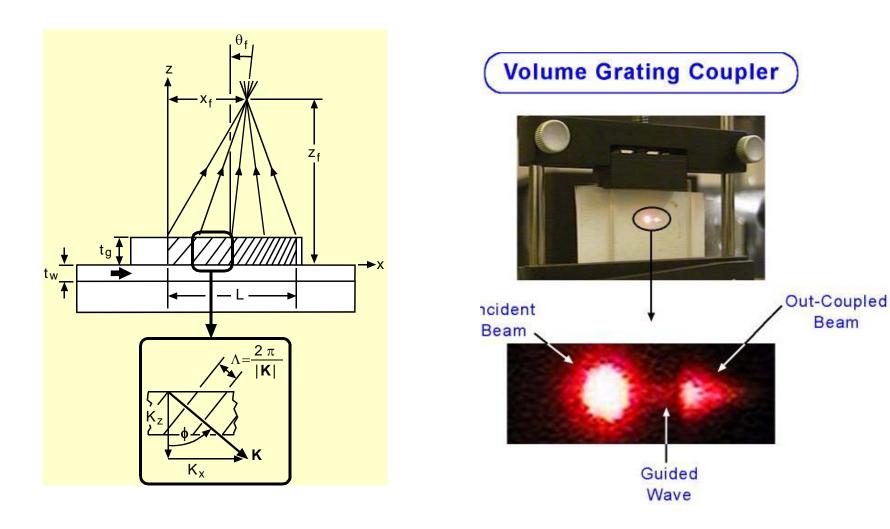


OPTOELECTRONICS PACKAGING WAVELENGTH DIVISION DEMULTIPLEXING

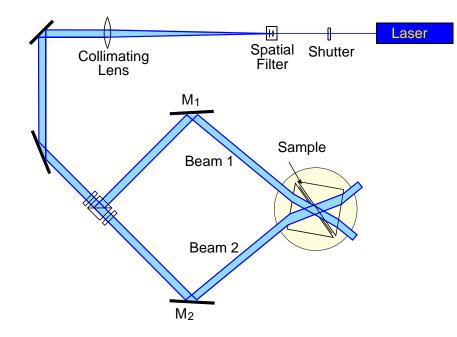




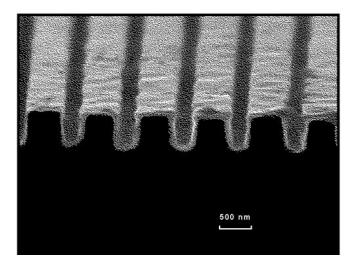
Holographic Grating Coupler



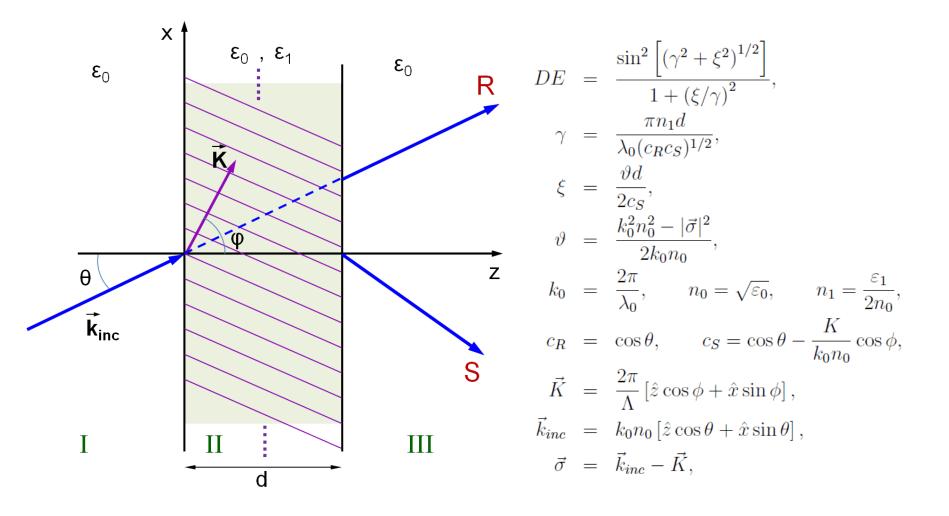
INTERFEROMETRIC GRATING FABRICATION FACILITY



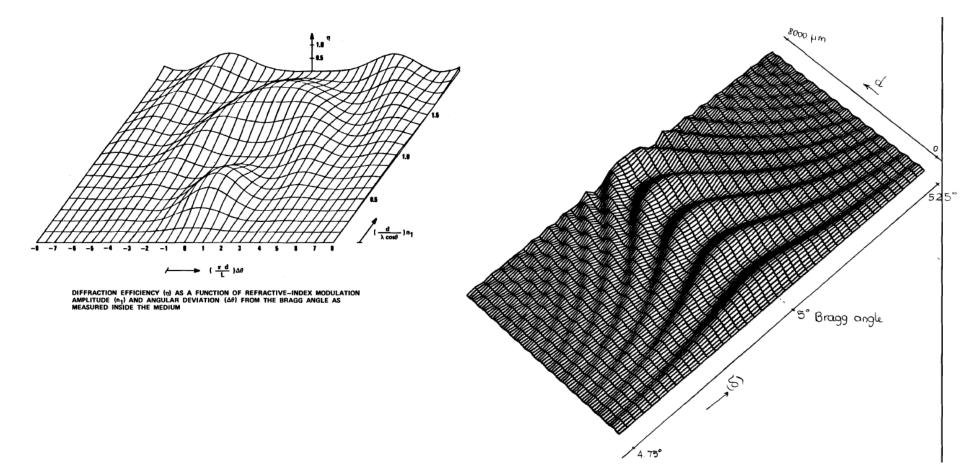
823 nm PERIOD GRATING



Kogelnik's Two-wave Coupled-wave Theory (Transmission Grating Case)



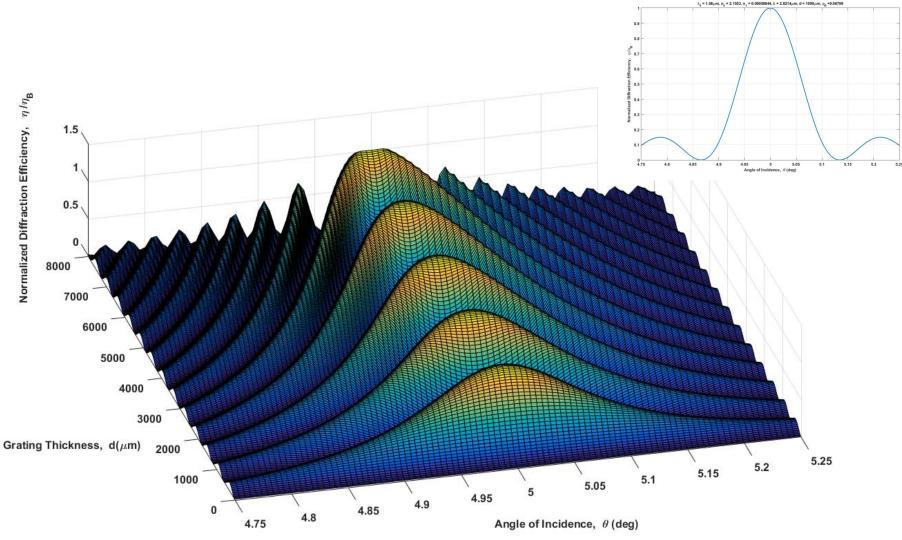
Angular Sensitivity of "Thick" Gratings



Angular Sensitivity of "Thick" Gratings

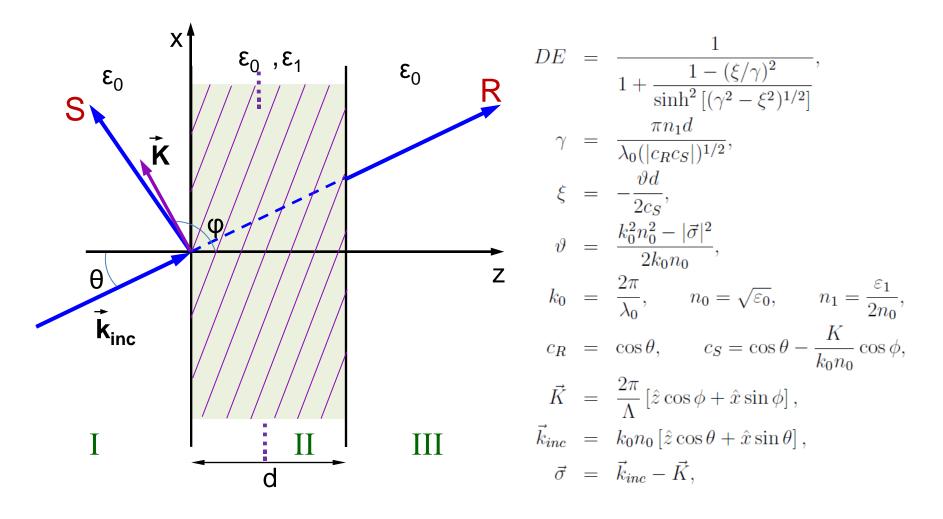
Case Parameters (Transmission Grating)

 $\lambda_0 = 1.06 \mu m$, $n_0 = 2.155313$, $n_1 = 0.588444 \times 10^{-3}$, $\Lambda = 2.821431514 \mu m$, $\phi = 90^{\circ}$



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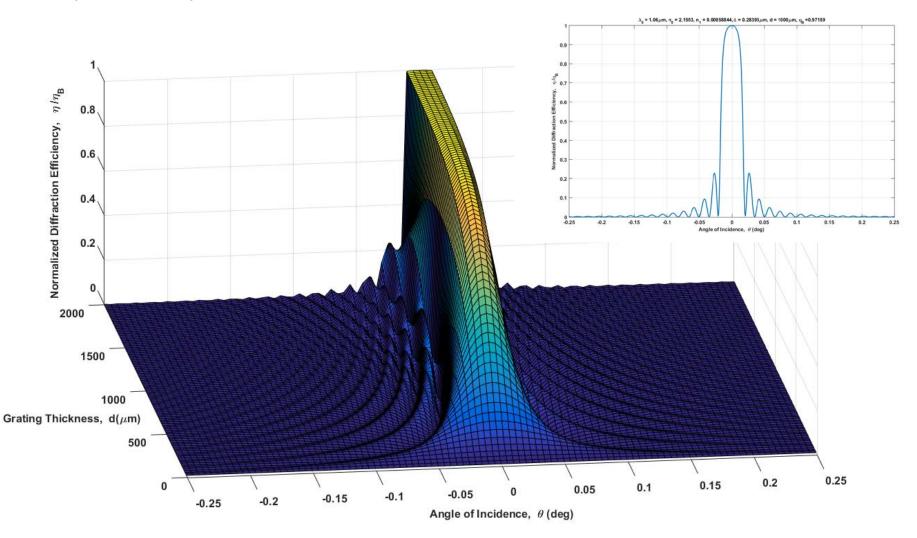
KOGELNIK's TWO-WAVE COUPLED-WAVE THEORY (Reflection Grating Case)



Angular Sensitivity of "Thick" Gratings

Case Parameters (Reflection Grating)

 $\lambda_0 = 1.06 \mu m$, $n_0 = 2.155313$, $n_1 = 0.588444 \times 10^{-3}$, $\Lambda = 0.283945434 \mu m$, $\phi = 150^{\circ}$

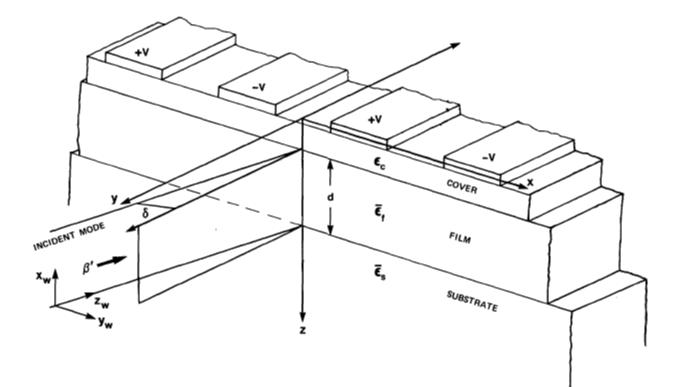


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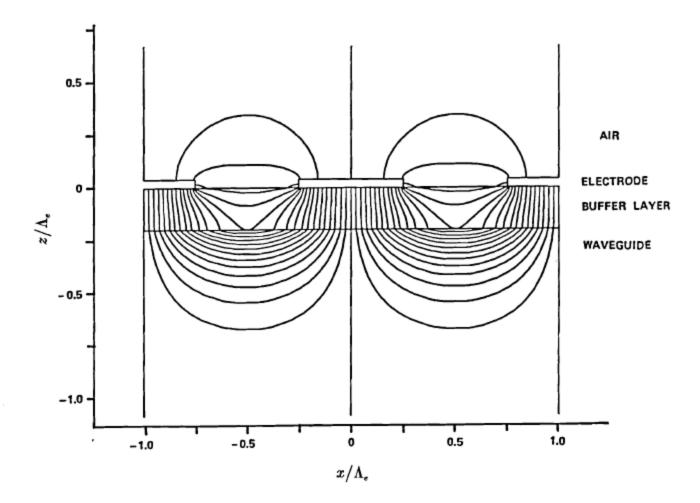
iigulai	Sensitivity Of	Thick Gradings
でき	n/n_B	A = 25/11
->0	0.99	0.1105
н	0.98	0.1566
61	0.95	0.2491
**	0.90	0.3561
34	0.80 (1dB)	0.5694
••	0.70	0.6467
**	0.60	0.7678
14	0.50 (3dB)	0.8859
0.5	0.99	0.1082
· •	0.98	0.1532
	0.95	0.2438
	0.90	0.3484
	0.80 (1 d B)	0.5039
••	0.70	0.6326
	0.60	0.1509
••	0.50 (3dB)	0.8660
1.0	0.99	0.1002
	0.98	0.1420
**	0.95	0.2258
•	0.90	0.3226
. ••	0.80 (1 d B)	0.4662
**	0.70	0.5848
	0.60	0.6933
"	0.50 (31B)	0.7987

Angular Sensitivity of "Thick" Gratings

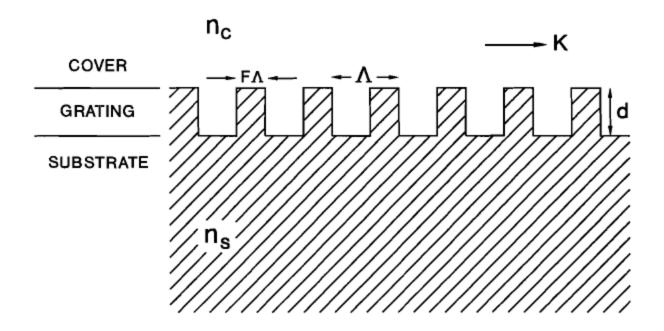
Interdigitated Electrodes Electro-optic Grating



Interdigitated Electrodes Electro-optic Grating D-Field Lines

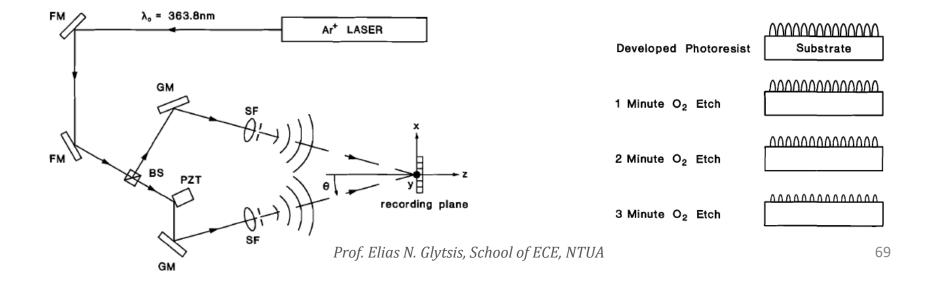


Rigorous Coupled Wave Analysis (RCWA) Antireflecting Surface-Relief Grating Example Rectangular-Groove Grating

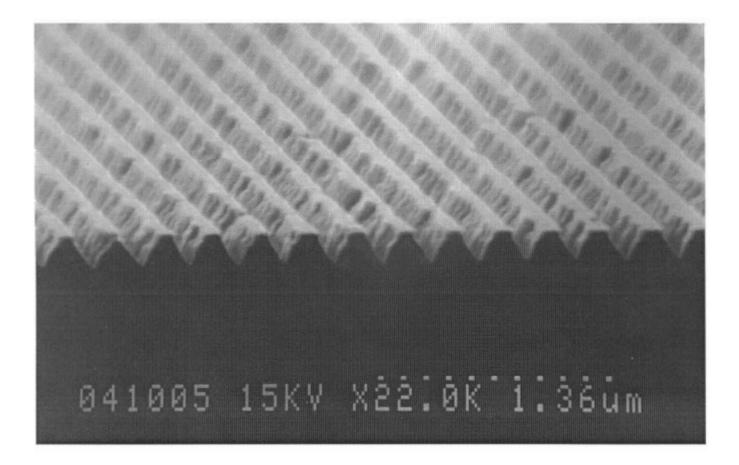


Rigorous Coupled Wave Analysis (RCWA) Antireflecting Surface-Relief Grating Example Rectangular-Groove Grating Fabrication Process

Clean Substrate	Etch Photoresist	
Spin On Photoresist	Deposit Chrome	
	Lift Off Photoresist	
Expose Photoresist	Reactive ion Etch	
Develop Photoresist	Remove Chrome	

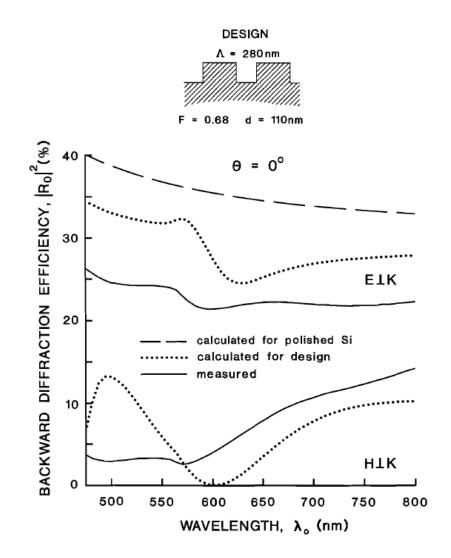


Rigorous Coupled Wave Analysis (RCWA) Antireflecting Surface-Relief Grating Example Electron-Microscope Picture of Fabricated Grating

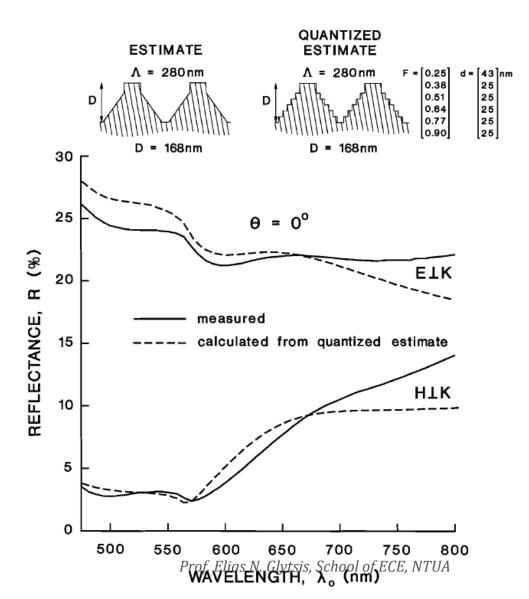


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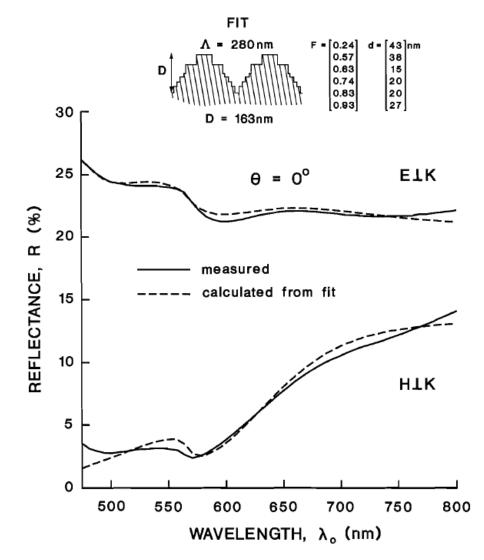
Rigorous Coupled Wave Analysis (RCWA) Antireflecting Surface-Relief Grating Example Spectral Response of Grating



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Rigorous Coupled Wave Analysis (RCWA) Antireflecting Surface-Relief Grating Example Spectral Response of Grating



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