

# *Electro-Optic & Acousto-Optic Effects: Light Modulation*

**Integrated Optics**

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# Constitutive Relations

$$\vec{D}(\vec{r}, t) = \epsilon_0 \vec{\mathcal{E}}(\vec{r}, t) + \vec{\mathcal{P}}(\vec{r}, t), \quad \vec{\mathcal{P}} = \text{Material Polarization}$$

$$\vec{\mathcal{H}}(\vec{r}, t) = \frac{1}{\mu_0} \vec{\mathcal{B}}(\vec{r}, t) - \vec{\mathcal{M}}(\vec{r}, t), \quad \vec{\mathcal{M}} = \text{Material Magnetization}$$

For Isotropic, Homogeneous, Linear, and Causal media:

$$\vec{\mathcal{P}}(\vec{r}, t) = \epsilon_0 \int_0^{\infty} G_e(\tau) \vec{\mathcal{E}}(\vec{r}, t - \tau) d\tau$$

$G_e$  and  $G_m$  represent the material «memory» to previous times of electromagnetic field

$$\vec{\mathcal{M}}(\vec{r}, t) = \int_0^{\infty} G_m(\tau) \vec{\mathcal{H}}(\vec{r}, t - \tau) d\tau$$

Fourier Transform:

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{+\infty} f(t) e^{-j\omega t} dt$$

$$\vec{D}(\vec{r}, \omega) = \epsilon_0 \vec{E}(\vec{r}, \omega) + \vec{P}(\vec{r}, \omega),$$

$$\vec{P}(\vec{r}, \omega) = \epsilon_0 \mathcal{F}\{G_e\} \vec{E}(\vec{r}, \omega) = \epsilon_0 \chi_e(\omega) \vec{E}(\vec{r}, \omega)$$

$$\vec{H}(\vec{r}, \omega) = \frac{1}{\mu_0} \vec{B}(\vec{r}, \omega) - \vec{M}(\vec{r}, \omega),$$

$$\vec{M}(\vec{r}, \omega) = \mathcal{F}\{G_m\} \vec{H}(\vec{r}, \omega) = \chi_m(\omega) \vec{H}(\vec{r}, \omega)$$

$\chi_e$  = electric susceptibility

$\chi_m$  = magnetic susceptibility

# Constitutive Relations

Therefore:

$$\vec{D}(\vec{r}, \omega) = \epsilon_0 \vec{E}(\vec{r}, \omega) + \epsilon_0 \chi_e(\omega) \vec{E}(\vec{r}, \omega) = \epsilon_0 \underbrace{[1 + \chi_e(\omega)]}_{\epsilon_r(\omega)} \vec{E}(\vec{r}, \omega) = \epsilon(\omega) \vec{E}(\vec{r}, \omega)$$

$$\vec{H}(\vec{r}, \omega) = \frac{1}{\mu_0} \vec{B}(\vec{r}, \omega) - \chi_m(\omega) \vec{H}(\vec{r}, \omega) \implies \vec{B}(\vec{r}, \omega) = \mu_0 \underbrace{[1 + \chi_m(\omega)]}_{\mu_r(\omega)} \vec{H}(\vec{r}, \omega) = \mu(\omega) \vec{H}(\vec{r}, \omega)$$

# Constitutive Relations for Anisotropic Media

For Anisotropic Media:

$$\vec{\mathcal{P}}(\vec{r}, t) = \epsilon_0 \int_0^\infty \tilde{G}_e^{(1)}(\tau) \vec{\mathcal{E}}(\vec{r}, t - \tau) d\tau \xrightarrow{\mathcal{F}} \vec{P}(\vec{r}, \omega) = \epsilon_0 \tilde{\chi}_e^{(1)}(\omega) \vec{E}(\vec{r}, \omega)$$

↑  
↑ tensors  
↓

$$\vec{D}(\vec{r}, \omega) = \epsilon_0 [1 + \tilde{\chi}_e(\omega)] \vec{E}(\vec{r}, \omega) \implies \vec{D}(\vec{r}, \omega) = \tilde{\epsilon}(\omega) \vec{E}(\vec{r}, \omega)$$

$$\tilde{\epsilon}(\omega) = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}$$

Permittivity Matrix is symmetric  $\epsilon_{ij} = \epsilon_{ji}$  ( $i, j = x, y, z$ )

$$\tilde{\epsilon}(\omega) = \begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix}$$

- (a) If  $\epsilon_{xx} \neq \epsilon_{yy} \neq \epsilon_{zz} \neq \epsilon_{xx}$  (biaxial medium)  
(two optic axes)
- (b) If  $\epsilon_{xx} = \epsilon_{yy} \neq \epsilon_{zz}$  (uniaxial medium)  
(one optic axis)
- (c) If  $\epsilon_{xx} = \epsilon_{yy} = \epsilon_{zz}$  (isotropic medium)

# Constitutive Relations for Nonlinear Media

$$\begin{aligned}\vec{P}(\vec{r}, t) &= \epsilon_0 \int_0^\infty \tilde{G}_e^{(1)}(\tau) \vec{\mathcal{E}}(\vec{r}, t - \tau) d\tau + \epsilon_0 \int_0^\infty \int_0^\infty \tilde{G}_e^{(2)}(\tau_1, \tau_2) \vec{\mathcal{E}}(\vec{r}, t - \tau_1) \vec{\mathcal{E}}(\vec{r}, t - \tau_2) d\tau_1 d\tau_2 + \\ &\quad \epsilon_0 \int_0^\infty \int_0^\infty \int_0^\infty \tilde{G}_e^{(3)}(\tau_1, \tau_2, \tau_3) \vec{\mathcal{E}}(\vec{r}, t - \tau_1) \vec{\mathcal{E}}(\vec{r}, t - \tau_2) \vec{\mathcal{E}}(\vec{r}, t - \tau_3) d\tau_1 d\tau_2 d\tau_3 + \dots\end{aligned}$$

$$\begin{aligned}\vec{P}(\vec{r}, \omega) &= \epsilon_0 \tilde{\chi}_e^{(1)}(\omega) \vec{E}(\vec{r}, \omega) + \epsilon_0 \tilde{\chi}_e^{(2)}(\omega_1, \omega_2) \vec{E}(\vec{r}, \omega_1) \vec{E}(\vec{r}, \omega_2) \Big|_{\omega=\omega_1+\omega_2} + \\ &\quad \epsilon_0 \tilde{\chi}_e^{(3)}(\omega_1, \omega_2, \omega_3) \vec{E}(\vec{r}, \omega_1) \vec{E}(\vec{r}, \omega_2) \vec{E}(\vec{r}, \omega_3) \Big|_{\omega=\omega_1+\omega_2+\omega_3} + \dots\end{aligned}$$

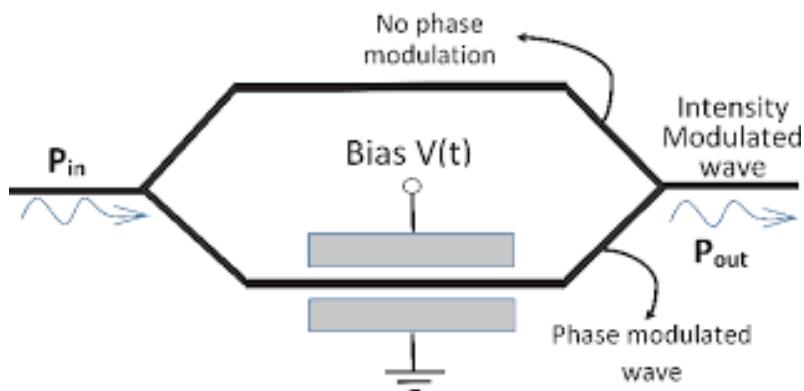
$\tilde{\chi}_e^{(m)}$  = tensors of order m+1

Example: Isotropic, Hmogeneous Medium with Nonlinearity (Pockels effect)

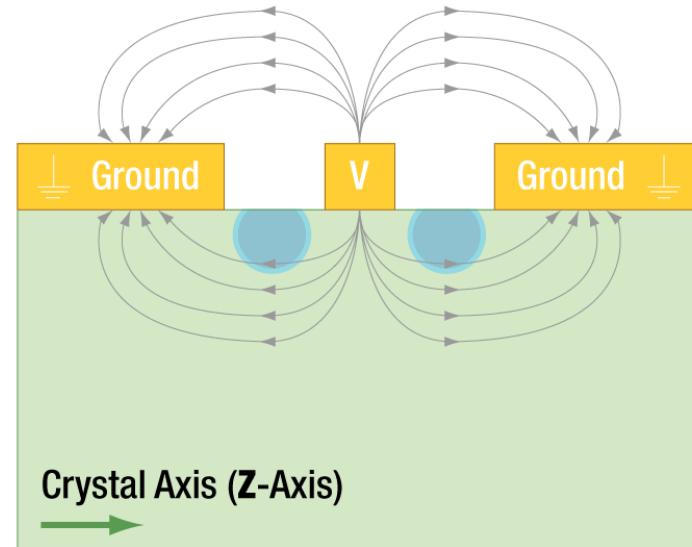
$$\begin{aligned}\vec{P}(\vec{r}, \omega) &= \epsilon_0 \tilde{\chi}_e^{(1)}(\omega) \vec{E}(\vec{r}, \omega) + \epsilon_0 \left[ \tilde{\chi}_e^{(2)}(\omega, 0) \vec{E}(\vec{r}, 0) \right] \vec{E}(\vec{r}, \omega) \\ &= \epsilon_0 \left[ \tilde{\chi}_e^{(1)} + \tilde{\chi}_e^{(2)}(\omega, 0) \vec{E}(\vec{r}, 0) \right] \vec{E}(\vec{r}, \omega) \implies\end{aligned}$$

$$\vec{D}(\vec{r}, \omega) = \epsilon(\omega) \vec{E}(\vec{r}, \omega) = \underbrace{\epsilon_0 \left[ 1 + \tilde{\chi}_e^{(1)} + \tilde{\chi}_e^{(2)}(\omega, 0) \vec{E}(\vec{r}, 0) \right]}_{\tilde{\epsilon}_r(\omega)} \vec{E}(\vec{r}, \omega)$$

# Electro-Optic Modulator



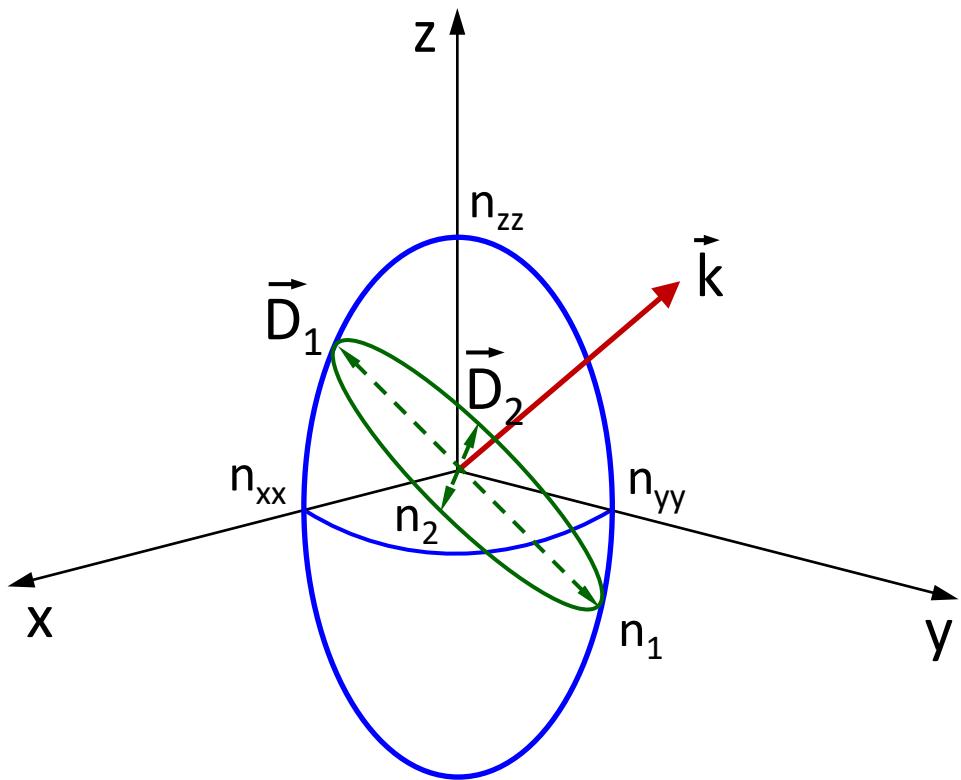
[https://encrypted-tbn0.gstatic.com/images?q=tbn%3AANd9GcTffV\\_gO0qOYUmFR2xVStdKjY9mmXhiK3xFVmqPb0Apn7dLA7n](https://encrypted-tbn0.gstatic.com/images?q=tbn%3AANd9GcTffV_gO0qOYUmFR2xVStdKjY9mmXhiK3xFVmqPb0Apn7dLA7n)



● **Ti-Indiffused Waveguide**      ■ **Electrode**  
→ **Electric Field Line**      ■  **$\text{LiNbO}_3$  Substrate**

[https://www.thorlabs.com/images/tabcimages/LiNbO3\\_Modulator\\_CrossSectionX\\_D1-780.gif](https://www.thorlabs.com/images/tabcimages/LiNbO3_Modulator_CrossSectionX_D1-780.gif)

## Index Ellipsoid



$$\frac{x^2}{n_{xx}^2} + \frac{y^2}{n_{yy}^2} + \frac{z^2}{n_{zz}^2} = 1$$

$$\begin{bmatrix} x & y & z \end{bmatrix}^T [\mathcal{A}] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix}^T \begin{bmatrix} \frac{1}{n_{xx}^2} & \frac{1}{n_{xy}^2} & \frac{1}{n_{xz}^2} \\ \frac{1}{n_{yx}^2} & \frac{1}{n_{yy}^2} & \frac{1}{n_{yz}^2} \\ \frac{1}{n_{zx}^2} & \frac{1}{n_{zy}^2} & \frac{1}{n_{zz}^2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1$$

# Index Ellipsoid

$$\vec{x}^T [\mathcal{A}] \vec{x} = 1, \quad [\mathcal{A}] = [\varepsilon]^{-1}$$

Index Ellipsoid In Principal Axes System  
(before applying electric field)

$[\mathcal{A}]$  = Impermeability Matrix

$$\vec{x}^T \underbrace{\left\{ [\mathcal{A}] + [\Delta(1/n^2)] \right\}}_{[\mathcal{A}_{new}]} \vec{x} = 1$$

Index Ellipsoid In Principal Axes System  
(after applying electric field)

## Linear Electro-optic Effect (Pockels Effect)

$$[\Delta(1/n^2)]_{ij} = \sum_k r_{ijk} E_k, \quad i, j, k = x, y, z$$

## Quadratic Electro-optic Effect (Kerr Effect)

$$[\Delta(1/n^2)]_{ij} = \sum_k \sum_\ell g_{ijk\ell} E_k E_\ell, \quad i, j, k, \ell = x, y, z$$

# Linear Electro-Optic Effect (Pockels Effect)

Convention

$$[\Delta(1/n^2)]_{ij} = \sum_k r_{ijk} E_k, \quad i, j, k = x, y, z$$

$$xx \implies 1$$

$$yy \implies 2$$

$$zz \implies 3$$

$$yz \implies 4$$

$$xz \implies 5$$

$$xy \implies 6$$

Matrix Form

$$\begin{bmatrix} \Delta(1/n^2)_{xx} \\ \Delta(1/n^2)_{yy} \\ \Delta(1/n^2)_{zz} \\ \Delta(1/n^2)_{yz} \\ \Delta(1/n^2)_{xz} \\ \Delta(1/n^2)_{xy} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix} \begin{bmatrix} E_{0x} \\ E_{0y} \\ E_{0z} \end{bmatrix}$$

# Quadratic Electro-Optic Effect (Kerr Effect)

**Convention**

$$xx \implies 1$$

$$yy \implies 2$$

$$zz \implies 3$$

$$yz \implies 4$$

$$xz \implies 5$$

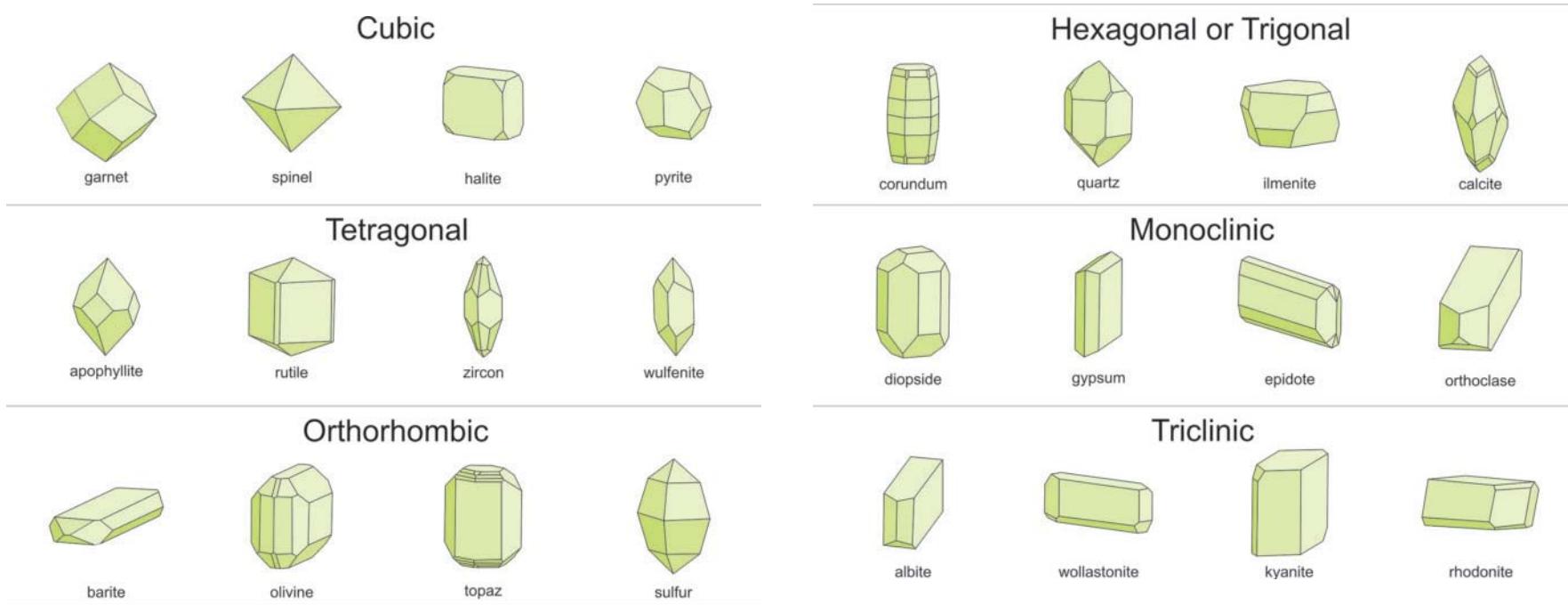
$$xy \implies 6$$

$$[\Delta(1/n^2)]_{ij} = \sum_k \sum_\ell g_{ijk\ell} E_k E_\ell, \quad i, j, k, \ell = x, y, z$$

**Matrix Form**

$$\begin{bmatrix}
 \Delta(1/n^2)_{xx} \\
 \Delta(1/n^2)_{yy} \\
 \Delta(1/n^2)_{zz} \\
 \Delta(1/n^2)_{yz} \\
 \Delta(1/n^2)_{xz} \\
 \Delta(1/n^2)_{xy}
 \end{bmatrix} = 
 \begin{bmatrix}
 g_{11} & g_{12} & g_{13} & g_{14} & g_{15} & g_{16} \\
 g_{21} & g_{22} & g_{23} & g_{24} & g_{25} & g_{26} \\
 g_{31} & g_{32} & g_{33} & g_{34} & g_{35} & g_{36} \\
 g_{41} & g_{42} & g_{43} & g_{44} & g_{45} & g_{46} \\
 g_{51} & g_{52} & g_{53} & g_{54} & g_{55} & g_{56} \\
 g_{61} & g_{62} & g_{63} & g_{64} & g_{65} & g_{66}
 \end{bmatrix} \begin{bmatrix}
 E_{0x}^2 \\
 E_{0y}^2 \\
 E_{0z}^2 \\
 E_{0y} E_{0z} \\
 E_{0x} E_{0z} \\
 E_{0x} E_{0y}
 \end{bmatrix}$$

# Crystal Classes



crystal system	possible first symbol	possible second symbol	possible third symbol	example of a point group
cubic	$4, 4/m, 4, 2, 2/m$	$3, 3$	$2, 2/m, m$	$43m$
hexagonal	$6, 6/m, 6$	$2, 2/m, m$	$2, 2/m, m$	$6/m\bar{2}/m\bar{2}/m$
trigonal	$3, 3$	$2, 2/m, m$	$2, 2/m, m$	$32$
tetragonal	$4, 4/m, 4$	$2, 2/m, m$	$2, 2/m, m$	$422$
orthorhombic	$2, m\bar{2}/m$	$2, 2/m, m$	$2, 2/m$	$mm2$
monoclinic	$2, 2/m, m$			$2/m$
triclinic	$1, 1$			$1$

# Crystal Classes

Crystal System	Crystal Class	Symmetry	Name of Class
Triclinic	1	none	Pedial
	$\bar{1}$	i	Pinacoidal
Monoclinic	2	$1A_2$	Sphenoidal
	m	$1m$	Domatic
	$2/m$	i, $1A_2$ , $1m$	Prismatic
Orthorhombic	222	$3A_2$	Rhombic-disphenoidal
	mm2 (2mm)	$1A_2$ , $2m$	Rhombic-pyramidal
	$2/m2/m2/m$	i, $3A_2$ , $3m$	Rhombic-dipyramidal
Tetragonal	4	$1A_4$	Tetragonal- Pyramidal
	$\bar{4}$	$\bar{A}_4$	Tetragonal-disphenoidal
	$4/m$	i, $1A_4$ , $1m$	Tetragonal-dipyramidal
	422	$1A_4$ , $4A_2$	Tetragonal-trapezohedral
	4mm	$1A_4$ , $4m$	Ditetragonal-pyramidal
	$\bar{4}2m$	$1\bar{A}_4$ , $2A_2$ , $2m$	Tetragonal-scalenohedral
	$4/m2/m2/m$	i, $1A_4$ , $4A_2$ , $5m$	Ditetragonal-dipyramidal
Hexagonal	3	$1A_3$	Trigonal-pyramidal
	$\bar{3}$	$1\bar{A}_3$	Rhombohedral
	32	$1A_3$ , $3A_2$	Trigonal-trapezohedral
	3m	$1A_3$ , $3m$	Ditrigonal-pyramidal
	$\bar{3}2/m$	$1\bar{A}_3$ , $3A_2$ , $3m$	Hexagonal-scalenohedral
	6	$1A_6$	Hexagonal-pyramidal
	$\bar{6}$	$1\bar{A}_6$	Trigonal-dipyramidal
	$6/m$	i, $1A_6$ , $1m$	Hexagonal-dipyramidal
	622	$1A_6$ , $6A_2$	Hexagonal-trapezohedral
	6mm	$1A_6$ , $6m$	Dihexagonal-pyramidal
Isometric	$\bar{6}m2$	$1\bar{A}_6$ , $3A_2$ , $3m$	Ditrigonal-dipyramidal
	$6/m2/m2/m$	i, $1A_6$ , $6A_2$ , $7m$	Dihexagonal-dipyramidal
	23	$3A_2$ , $4A_3$	Tetaroidal
	$2/m\bar{3}$	$3A_2$ , $3m$ , $4\bar{A}_3$	Diploidal
	432	$3A_4$ , $4A_3$ , $6A_2$	Gyroidal
	$\bar{4}3m$	$3\bar{A}_4$ , $4A_3$ , $6m$	Hextetrahedral
	$4/m\bar{3}2/m$	$3A_4$ , $4\bar{A}_3$ , $6A_2$ , $9m$	Hexoctahedral

# Linear Electro-Optic Effect (Pockels Effect)

Centrosymmetric

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Triclinic

$$\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{pmatrix}$$

Monoclinic

$$\begin{pmatrix} 2 & (2 \parallel x_2) \\ 0 & r_{12} & 0 \\ 0 & r_{22} & 0 \\ 0 & r_{32} & 0 \\ r_{41} & 0 & r_{43} \\ 0 & r_{52} & 0 \\ r_{61} & 0 & r_{63} \end{pmatrix} \quad \begin{pmatrix} 2 & (2 \parallel x_3) \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{23} \\ 0 & 0 & r_{33} \\ r_{41} & r_{42} & 0 \\ r_{51} & r_{52} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

$\mathbf{m}$  ( $\mathbf{m} \perp x_2$ )

$$\begin{pmatrix} r_{11} & 0 & r_{13} \\ r_{21} & 0 & r_{23} \\ r_{31} & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{51} & 0 & r_{53} \\ 0 & r_{62} & 0 \end{pmatrix} \quad \begin{pmatrix} r_{11} & r_{12} & 0 \\ r_{21} & r_{22} & 0 \\ r_{31} & r_{32} & 0 \\ 0 & 0 & r_{43} \\ 0 & 0 & r_{53} \\ r_{61} & r_{62} & 0 \end{pmatrix}$$

A. Yariv and P. Yeh, "Optical Waves in Crystals", Wiley & Sons, 2007 (Table 7.2)

# Linear Electro-Optic Effect (Pockels Effect)

Orthorombic

222

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{52} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

2mm

$$\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{23} \\ 0 & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

4

$$\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Tetragonal

$\bar{4}$

$$\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & -r_{13} \\ 0 & 0 & 0 \\ r_{41} & -r_{51} & 0 \\ r_{51} & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

422

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

4mm

$$\begin{pmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$\bar{4}2m$  ( $2 \parallel x_1$ )

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix}$$

A. Yariv and P. Yeh, "Optical Waves in Crystals", Wiley & Sons, 2007 (Table 7.2)

# Linear Electro-Optic Effect (Pockels Effect)

Trigonal

$$\begin{array}{c} 3 \\ \left( \begin{array}{ccc} r_{11} & -r_{22} & r_{13} \\ -r_{11} & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ -r_{22} & -r_{11} & 0 \end{array} \right) \end{array}$$

$$\begin{array}{c} 32 \\ \left( \begin{array}{ccc} r_{11} & 0 & 0 \\ -r_{11} & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & -r_{11} & 0 \end{array} \right) \end{array}$$

Hexagonal

$$\begin{array}{c} 6 \\ \left( \begin{array}{ccc} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ 0 & 0 & 0 \end{array} \right) \end{array}$$

$$\begin{array}{c} 6\text{mm} \\ \left( \begin{array}{ccc} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \end{array}$$

$$\begin{array}{c} 622 \\ \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & 0 & 0 \end{array} \right) \end{array}$$
  

$$\begin{array}{c} 3\text{m } (\mathbf{m} \perp x_1) \\ \left( \begin{array}{ccc} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{array} \right) \end{array}$$

$$\begin{array}{c} 3\text{m } (\mathbf{m} \perp x_2) \\ \left( \begin{array}{ccc} r_{11} & 0 & r_{13} \\ -r_{11} & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & -r_{11} & 0 \end{array} \right) \end{array}$$

$$\begin{array}{c} \bar{6} \\ \left( \begin{array}{ccc} r_{11} & -r_{22} & 0 \\ -r_{11} & r_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -r_{22} & -r_{11} & 0 \end{array} \right) \end{array}$$

$$\begin{array}{c} \bar{6}\text{m}2 \quad (\mathbf{m} \perp x_1) \\ \left( \begin{array}{ccc} 0 & -r_{22} & 0 \\ 0 & r_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -r_{22} & 0 & 0 \end{array} \right) \end{array}$$

$$\begin{array}{c} \bar{6}\text{m}2 \quad (\mathbf{m} \perp x_2) \\ \left( \begin{array}{ccc} r_{11} & 0 & 0 \\ -r_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -r_{11} & 0 \end{array} \right) \end{array}$$

Cubic

$$\begin{array}{c} \bar{43}\text{m}, 23 \\ \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{array} \right) \end{array}$$

$$\begin{array}{c} 432 \\ \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) \end{array}$$

A. Yariv and P. Yeh, "Optical Waves in Crystals", Wiley & Sons, 2007 (Table 7.2)

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# Quadratic Electro-Optic Effect (Kerr Effect)

Triclinic

$$1, \bar{1}$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & s_{16} \\ s_{21} & s_{22} & s_{23} & s_{24} & s_{25} & s_{26} \\ s_{31} & s_{32} & s_{33} & s_{34} & s_{35} & s_{36} \\ s_{41} & s_{42} & s_{43} & s_{44} & s_{45} & s_{46} \\ s_{51} & s_{52} & s_{53} & s_{54} & s_{55} & s_{56} \\ s_{61} & s_{62} & s_{63} & s_{64} & s_{65} & s_{66} \end{pmatrix}$$

Monoclinic

$$2, m, 2/m$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & s_{15} & 0 \\ s_{21} & s_{22} & s_{23} & 0 & s_{25} & 0 \\ s_{31} & s_{32} & s_{33} & 0 & s_{35} & 0 \\ 0 & 0 & 0 & s_{44} & 0 & s_{46} \\ s_{51} & s_{52} & s_{53} & 0 & s_{55} & 0 \\ 0 & 0 & 0 & s_{64} & 0 & s_{66} \end{pmatrix}$$

Orthorombic

$$2mm, 222, mmm$$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{21} & s_{22} & s_{23} & 0 & 0 & 0 \\ s_{31} & s_{32} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{pmatrix}$$

Tetragonal

$4, \bar{4}, 4/m$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & s_{16} \\ s_{12} & s_{11} & s_{13} & 0 & 0 & -s_{16} \\ s_{31} & s_{31} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & s_{45} & 0 \\ 0 & 0 & 0 & -s_{45} & s_{44} & 0 \\ s_{61} & -s_{61} & 0 & 0 & 0 & s_{66} \end{pmatrix}$$

$422, 4mm, \bar{4}2m, 4/mm$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{12} & s_{11} & s_{13} & 0 & 0 & 0 \\ s_{31} & s_{31} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66} \end{pmatrix}$$

Trigonal

$3, \bar{3}$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} & -s_{61} \\ s_{12} & s_{11} & s_{13} & -s_{14} & -s_{15} & s_{61} \\ s_{31} & s_{31} & s_{33} & 0 & 0 & 0 \\ s_{41} & -s_{41} & 0 & s_{44} & s_{45} & -s_{51} \\ s_{51} & -s_{51} & 0 & -s_{45} & s_{44} & s_{41} \\ s_{61} & -s_{61} & 0 & -s_{15} & s_{14} & \frac{1}{2}(s_{11} - s_{12}) \end{pmatrix}$$

$32, 3m, \bar{3}m$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} & 0 & 0 \\ s_{12} & s_{11} & s_{13} & -s_{14} & 0 & 0 \\ s_{13} & s_{13} & s_{33} & 0 & 0 & 0 \\ s_{41} & -s_{41} & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44} & s_{41} \\ 0 & 0 & 0 & 0 & s_{14} & \frac{1}{2}(s_{11} - s_{12}) \end{pmatrix}$$

Hexagonal

$6, \bar{6}, 6/m$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & -s_{61} \\ s_{12} & s_{11} & s_{13} & 0 & 0 & s_{61} \\ s_{31} & s_{31} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & s_{45} & 0 \\ 0 & 0 & 0 & -s_{45} & s_{44} & 0 \\ s_{61} & -s_{61} & 0 & 0 & 0 & \frac{1}{2}(s_{11} - s_{12}) \end{pmatrix}$$

$622, 6mm, \bar{6}m2, 6/mmm$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{12} & s_{11} & s_{13} & 0 & 0 & 0 \\ s_{31} & s_{31} & s_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(s_{11} - s_{12}) \end{pmatrix}$$

# Quadratic Electro-Optic Effect (Kerr Effect)

Cubic

$23, m3$

$$\begin{pmatrix} s_{11} & s_{12} & s_{13} & 0 & 0 & 0 \\ s_{13} & s_{11} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{13} & s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{44} \end{pmatrix}$$

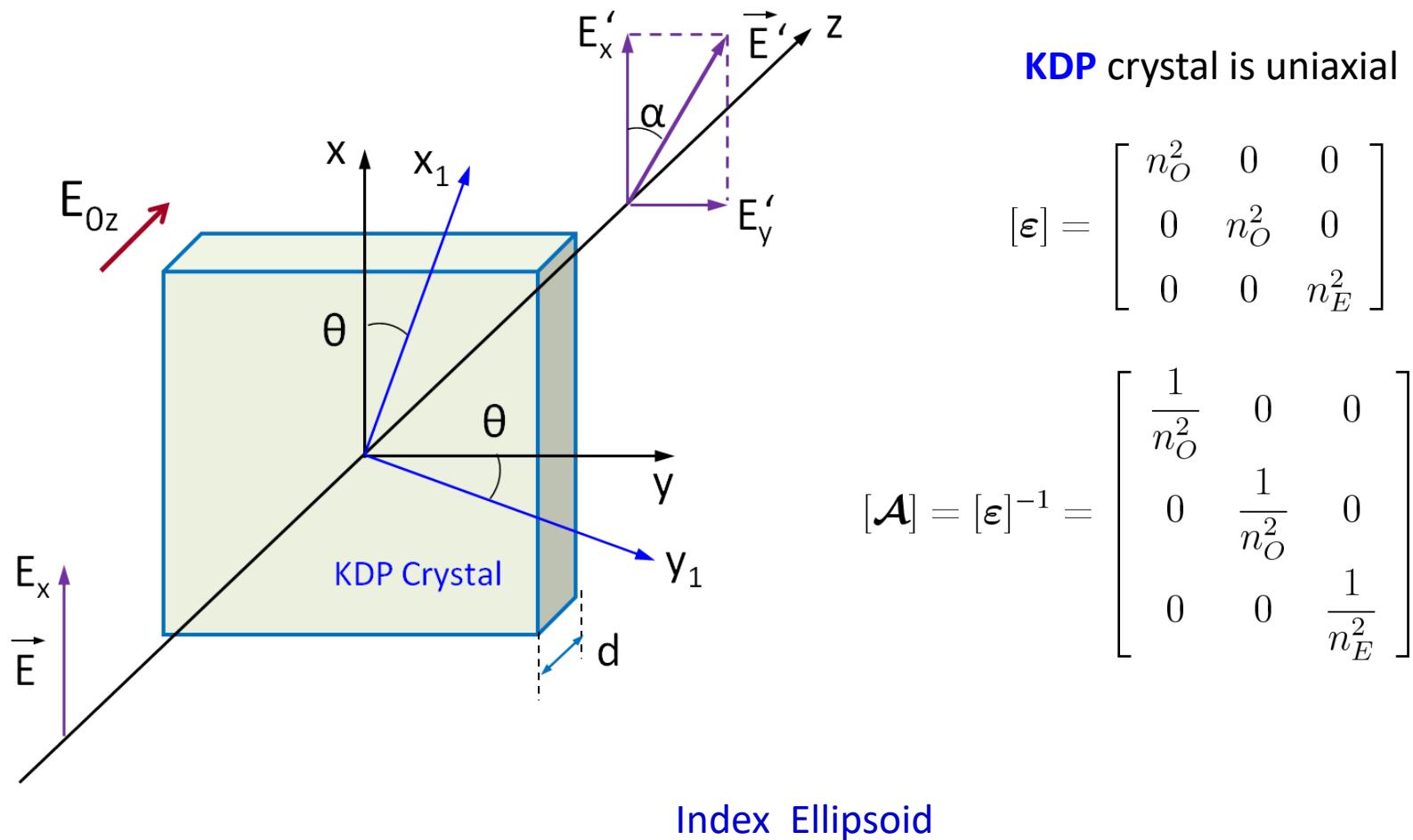
$432, m3m, \bar{4}3m$

$$\begin{pmatrix} s_{11} & s_{12} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{11} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{12} & s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{44} \end{pmatrix}$$

Isotropic

$$\begin{pmatrix} s_{11} & s_{12} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{11} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{12} & s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(s_{11} - s_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(s_{11} - s_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(s_{11} - s_{12}) \end{pmatrix}$$

## Example: Linear Electro-Optic Effect (Pockels Effect)



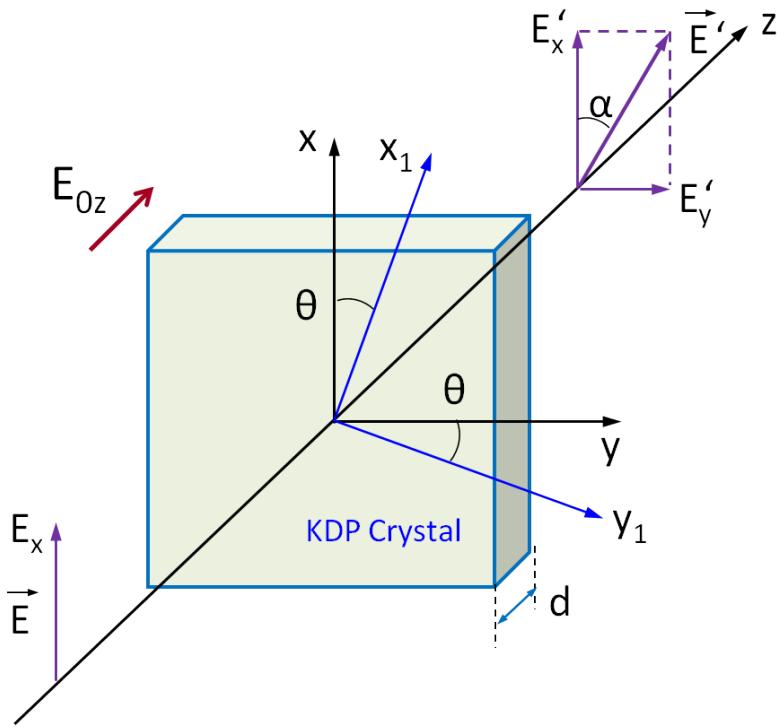
$$\vec{x}^T [\mathcal{A}] \vec{x} = 1 \implies \frac{x^2}{n_O^2} + \frac{y^2}{n_O^2} + \frac{z^2}{n_E^2} = 1$$

## Example: Linear Electro-Optic Effect (Pockels Effect)

$$\begin{bmatrix} \Delta(1/n^2)_{xx} \\ \Delta(1/n^2)_{yy} \\ \Delta(1/n^2)_{zz} \\ \Delta(1/n^2)_{yz} \\ \Delta(1/n^2)_{xz} \\ \Delta(1/n^2)_{xy} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ E_{0z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ r_{63}E_{0z} \end{bmatrix}$$

$$[\mathcal{A}_{new}] = \begin{bmatrix} \frac{1}{n_O^2} & r_{63}E_{0z} & 0 \\ r_{63}E_{0z} & \frac{1}{n_O^2} & 0 \\ 0 & 0 & \frac{1}{n_E^2} \end{bmatrix}$$

## Example: Linear Electro-Optic Effect (Pockels Effect)



Coordinate System Transformation

$$\vec{x}_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [\mathbf{Q}] \vec{x}$$

Impermeability Matrix Transformation

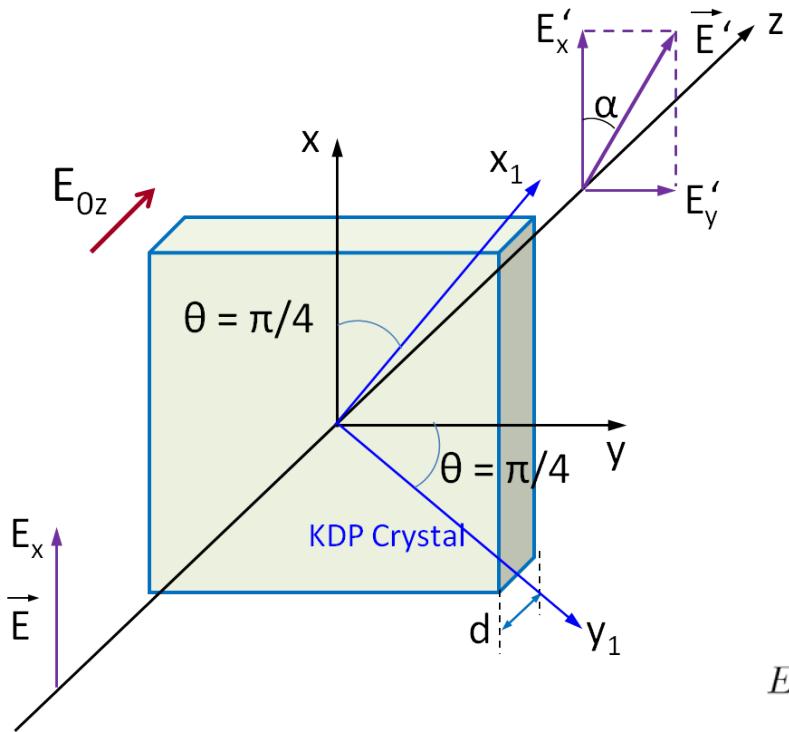
$$\vec{x}^T [\mathcal{A}_{new}] \vec{x} = 1 \implies \vec{x}_1^T \underbrace{[\mathbf{Q}] [\mathcal{A}_{new}] [\mathbf{Q}]^T}_{[\mathcal{A}_1]} \vec{x}_1 = 1$$

For  $\theta = \pi/4$

$$[\mathcal{A}_1] = \begin{bmatrix} \frac{1}{n_O^2} + r_{63}E_{0z} \sin(2\theta) & r_{63}E_{0z} \cos(2\theta) & 0 \\ r_{63}E_{0z} \cos(2\theta) & \frac{1}{n_O^2} - r_{63}E_{0z} \sin(2\theta) & 0 \\ 0 & 0 & \frac{1}{n_E^2} \end{bmatrix}$$

$$[\mathcal{A}_1] = \begin{bmatrix} \frac{1}{n_O^2} + r_{63}E_{0z} & 0 & 0 \\ 0 & \frac{1}{n_O^2} - r_{63}E_{0z} & 0 \\ 0 & 0 & \frac{1}{n_E^2} \end{bmatrix}$$

# Example: Linear Electro-Optic Effect (Pockels Effect)



New Principal Indices

$$n_{x_1 x_1} = n_O - \frac{1}{2} n_O^3 r_{63} E_{0z}$$

$$n_{y_1 y_1} = n_O + \frac{1}{2} n_O^3 r_{63} E_{0z}$$

$$n_{z_1 z_1} = n_E$$

Input/Output Fields

$$\vec{E} = E_0 \hat{x} = \frac{E_0}{\sqrt{2}} \hat{x}_1 - \frac{E_0}{\sqrt{2}} \hat{y}_1$$

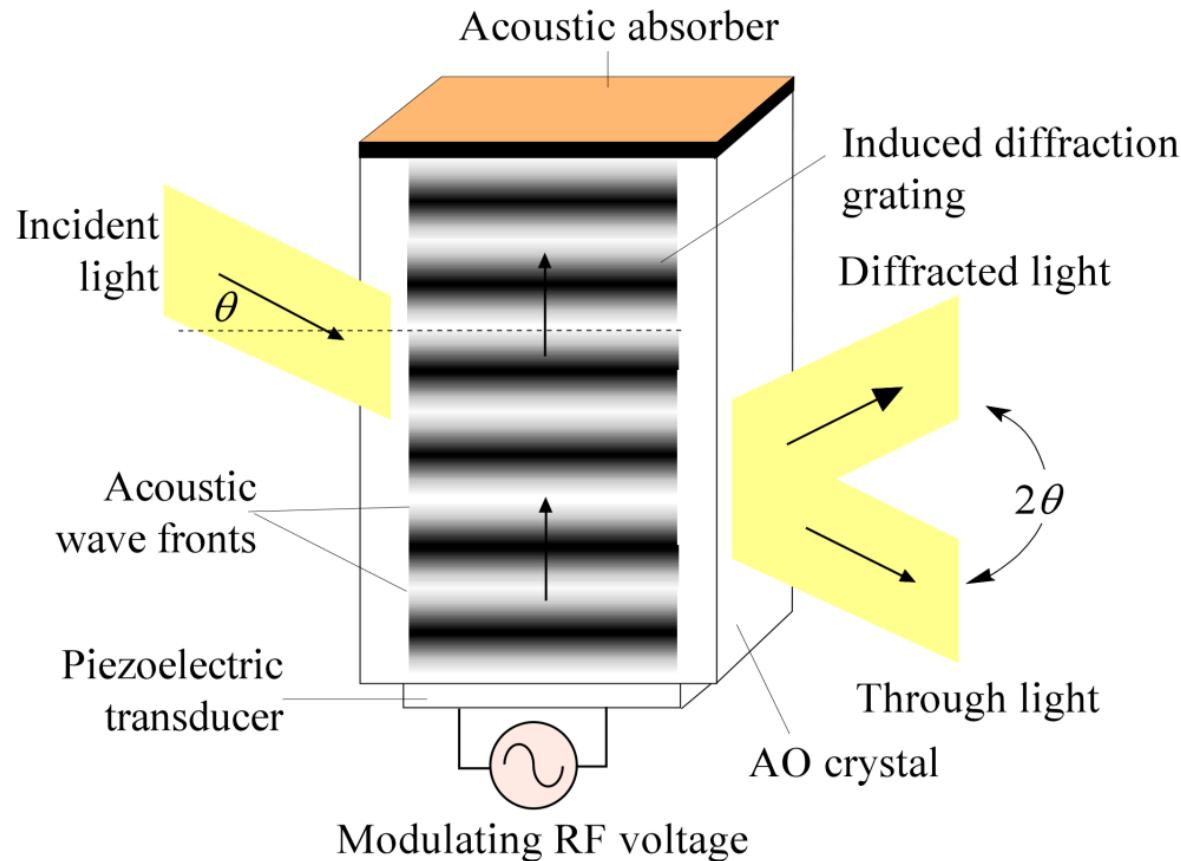
$$E_{x_1}(z = d) = \frac{E_0}{\sqrt{2}} \exp \left[ -jk_0 \left( n_O - \frac{1}{2} n_O^3 r_{63} E_{0z} \right) d \right]$$

$$E_{y_1}(z = d) = -\frac{E_0}{\sqrt{2}} \exp \left[ -jk_0 \left( n_O + \frac{1}{2} n_O^3 r_{63} E_{0z} \right) d \right]$$

Acquired Phase Shift

$$\phi = \phi_{y_1} - \phi_{x_1} = \pi - k_0 r_{63} n_O^3 E_{0z} d$$

# Basic Acousto-Optic Transducer

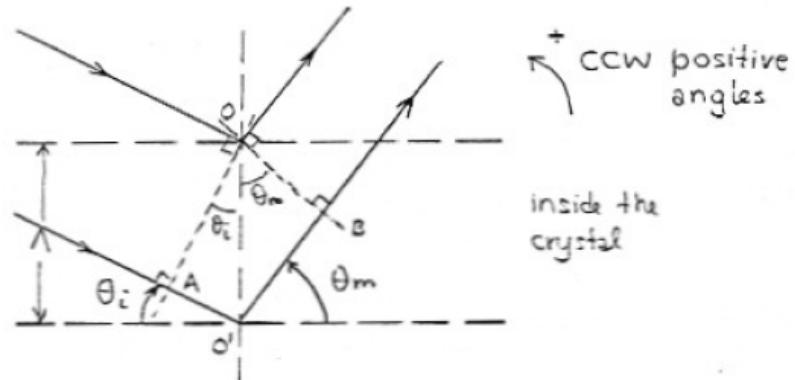
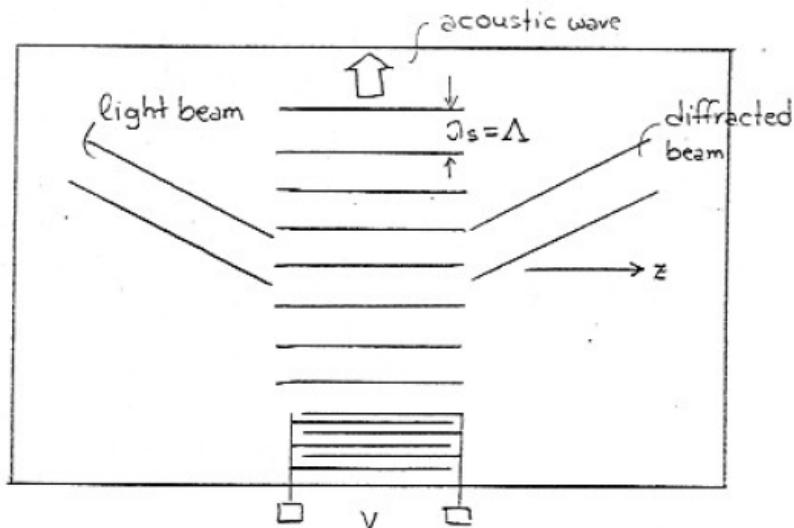


S. O. Kasap, "Optoelectronics & Photonics" 2<sup>nd</sup> Ed., Pearson 2013, (Instructor's Powerpoint Slides)

# Basic Acousto-Optic Transducer

$$\Delta n(z,t) = \Delta n \sin [\omega_s t - \vec{k}_s \cdot \vec{r}]$$

$$v_s = \omega_s / k_s \quad k_s = \frac{2\pi}{\lambda_s} = \frac{2\pi}{\Lambda}$$



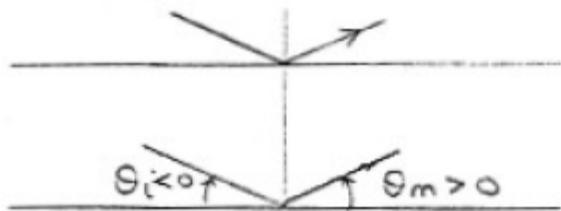
In order for the diffracted wave to build up, it is necessary that the optical path difference  $AO' + O'B$  is multiple of wavelength (inside the crystal). This can be written as

$\Lambda (\sin \theta_i + \sin \theta_m) = m \frac{\lambda_0}{n}$  where  $n$  is the average refractive index of the crystal. If we make the convention that all CCW angles are positive (as measured from the horizontal) and all CW angles are negative, then the above equation can be written as

$$\Lambda (\sin \theta_i - \sin \theta_m) = m \frac{\lambda_0}{n} \quad m = 0, \pm 1, \pm 2, \dots$$

## Basic Acousto-Optic Transducer

The previous equation is also called the grating equation. For  $m=0 \Rightarrow \sin\theta_i = \sin\theta_m \Rightarrow \theta_m = \theta_i < 0$  and it corresponds to the zero diffracted order or the transmitted beam. If  $-\theta_i = \theta_m = \theta$ , as shown in the following diagram,

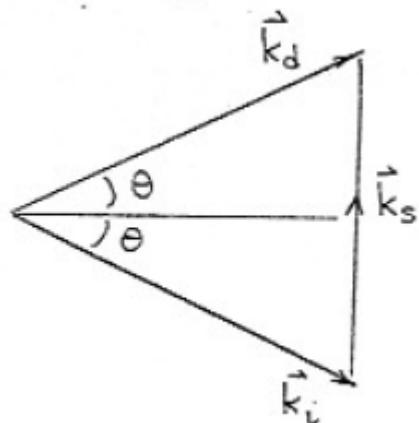


then the above condition can be written as follows;

$$2 \Delta \sin\theta = m \frac{2\theta_0}{n},$$

and is called the Bragg condition.

# Particle Picture of Bragg Diffraction of Light by Sound



$$\vec{k}_d = \vec{k}_i + \vec{k}_s \quad (\text{momentum conservation})$$

$$\omega_d = \omega_i + \omega_s \quad (\text{energy conservation})$$

Usually  $\omega_s \ll \omega_i \Rightarrow \omega_d \approx \omega_i$  and  $|\vec{k}_i| = |\vec{k}_d|$

From the diagram if  $k_d = k_i = k$

$$k_s = 2k \sin\theta \Rightarrow$$

$$\frac{2\pi}{\Lambda} = 2 \cdot \frac{2\pi}{\lambda_0/n} \cdot \sin\theta \Rightarrow$$

$2\Lambda \sin\theta = \lambda_0/n$  which is the Bragg

condition for the first ( $m=1$ ) diffracted order. In general we can have multiple diffracted waves with wavevectors and frequencies given by

$$\vec{k}_m = \vec{k}_i + m \vec{k}_s \quad m = 0, \pm 1, \pm 2, \dots$$

$$\omega_m = \omega_i + m\omega_s$$

## Analysis of Bragg Diffraction of Light by Sound

Let's start from the Maxwell's equations:

$$\vec{\nabla} \times \vec{e} = -\mu_0 \frac{\partial \vec{h}}{\partial t} \quad \vec{\nabla} \cdot \epsilon \vec{e} = 0 \approx \vec{\nabla} \cdot \vec{e} \approx 0$$

$$\vec{\nabla} \times \vec{h} = \frac{\partial}{\partial t} (\epsilon_0 \vec{e} + \vec{p}) \quad \vec{\nabla} \cdot \vec{h} = 0$$

Now let's decompose  $\vec{p} = \vec{p}_0 + \Delta \vec{p}$ , where  $\vec{p}_0$  is the induced polarization without any acoustic wave and  $\Delta \vec{p}$  is the change of the polarization due to the acoustic wave. The  $\Delta \vec{p}$  will be treated as a perturbation. The relation between  $\Delta \vec{p}$  and the index change  $\Delta n$  due to the acoustic wave can be found as follows:

$$\epsilon \vec{e} = \epsilon_0 \vec{e} + \vec{p} \Rightarrow \vec{p} = \epsilon_0 (n^2 - 1) \vec{e} \Rightarrow$$

$$\Delta \vec{p} = \epsilon_0 2n \Delta n \vec{e} = 2 \epsilon_0 \sqrt{\epsilon_r} \Delta n \vec{e} = 2 \sqrt{\epsilon_0 \epsilon} \Delta n \vec{e}$$

and  $\Delta n = \Delta n(\vec{r}, t) = \Delta n_0 \cos [\omega_{st} t - \vec{k}_s \cdot \vec{r}]$

## Analysis of Bragg Diffraction of Light by Sound

From Maxwell's equations we get:

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{e}) = -\mu_0 \frac{\partial}{\partial t} \left( \frac{\partial}{\partial t} (\epsilon_0 \vec{e} + \vec{p} + \Delta \vec{p}) \right) = -\mu_0 \frac{\partial^2}{\partial t^2} (\epsilon \vec{e} + \Delta \vec{p}) \Rightarrow$$
$$\Rightarrow \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{e}) - \vec{\nabla}^2 \vec{e} = -\mu_0 \epsilon \frac{\partial^2}{\partial t^2} \vec{e} - \mu_0 \frac{\partial^2}{\partial t^2} \Delta \vec{p} \Rightarrow$$
$$\vec{\nabla}^2 \vec{e} - \mu_0 \epsilon \frac{\partial^2 \vec{e}}{\partial t^2} = \mu_0 \frac{\partial^2}{\partial t^2} \Delta \vec{p}$$

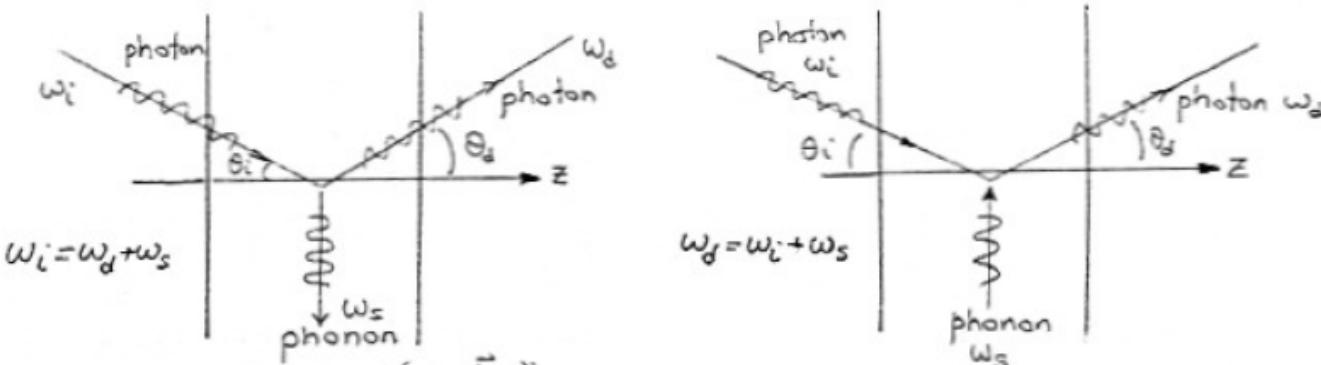
The total electric field  $\vec{e}$  is the sum of the incident and the diffracted fields.

$$\vec{e} = e \hat{e} = \left\{ \frac{1}{2} \left( A_i(z) e^{+j(\omega_i t - \vec{k}_i \cdot \vec{r})} + c.c. \right) + \frac{1}{2} \left( A_d(z) e^{+j(\omega_d t - \vec{k}_d \cdot \vec{r})} + c.c. \right) \right\} \hat{e}$$

where c.c. denotes complex conjugate and  $A_i(z)$ ,  $A_d(z)$  are slowly varying amplitudes.  $\omega_i$ ,  $\omega_d$  are the angular frequencies of the incident and diffracted waves and  $\vec{k}_i$ ,  $\vec{k}_d$  their corresponding wavevectors.

$\hat{e}$  is the polarization unit vector that remains constant.

# Analysis of Bragg Diffraction of Light by Sound



If  $e = \frac{1}{2} (A(z) e^{j(\omega t - \vec{k} \cdot \vec{r})} + c.c.)$  it is straightforward to show that

$$\nabla^2 e = -\frac{1}{2} e^{j(\omega t - \vec{k} \cdot \vec{r})} \left[ \vec{k} \cdot \vec{k} A + j 2 k_z \frac{dA}{dz} + \frac{d^2 A}{dz^2} \right] + c.c.$$

$$\mu_0 \epsilon \frac{\partial^2}{\partial t^2} e = -\omega^2 \mu_0 \epsilon \frac{1}{2} A(z) e^{j(\omega t - \vec{k} \cdot \vec{r})} + c.c.$$

# Analysis of Bragg Diffraction of Light by Sound

Therefore,

$$\nabla^2 e - \mu_0 \epsilon \frac{\partial^2 e}{\partial t^2} = -\frac{1}{2} \left[ \vec{k}_i \cdot \vec{k}_i A_i + 2j k_{iz} \frac{dA_i}{dz} + \frac{d^2 A_i}{dz^2} - \mu_0 \epsilon \omega_i^2 A_i \right] e^{j(\omega_i t - \vec{k}_i \cdot \vec{r})} + c.c.$$
$$-\frac{1}{2} \left[ \vec{k}_d \cdot \vec{k}_d A_d + 2j k_{dz} \frac{dA_d}{dz} + \frac{d^2 A_d}{dz^2} - \mu_0 \epsilon \omega_d^2 A_d \right] e^{j(\omega_d t - \vec{k}_d \cdot \vec{r})} + c.c.$$

But  $\vec{k}_i \cdot \vec{k}_i = \omega_i^2 \mu \epsilon$  and  $\vec{k}_d \cdot \vec{k}_d = \omega_d^2 \mu \epsilon$  (for plane waves) and since  $A_i, A_d$  are slowly varying functions of  $z$   $\frac{d^2 A_i}{dz^2} \approx 0, \frac{d^2 A_d}{dz^2} \approx 0$

as compared to the  $\frac{dA_i}{dz} k_{iz}$  and  $\frac{dA_d}{dz} k_{dz}$  terms. Consequently,

$$\nabla^2 e - \mu_0 \epsilon \frac{\partial^2 e}{\partial t^2} \approx -\frac{1}{2} \left[ +2j k_{iz} \frac{dA_i}{dz} \right] e^{j(\omega_i t - \vec{k}_i \cdot \vec{r})} + c.c.$$
$$-\frac{1}{2} \left[ +2j k_{dz} \frac{dA_d}{dz} \right] e^{j(\omega_d t - \vec{k}_d \cdot \vec{r})} + c.c.$$

# Analysis of Bragg Diffraction of Light by Sound

Now let's examine the  $\mu_0 \frac{\partial^2}{\partial t^2} (\Delta p)$  term:

$$\begin{aligned}
 \mu_0 \frac{\partial^2}{\partial t^2} (\Delta p) &= \mu_0 \frac{\partial^2}{\partial t^2} [ 2 \sqrt{\epsilon_0 \epsilon} \Delta n \left( \frac{1}{2} A_i e^{j(\omega_i t - \vec{k}_i \cdot \vec{r})} + c.c. + \frac{1}{2} A_d e^{j(\omega_d t - \vec{k}_d \cdot \vec{r})} + c.c. \right) ] \\
 &= \mu_0 2 \sqrt{\epsilon_0 \epsilon} \Delta n_0 \frac{\partial^2}{\partial t^2} \left[ \left( \frac{1}{2} e^{j(\omega_s t - \vec{k}_s \cdot \vec{r})} + c.c. \right) \left( \frac{1}{2} A_i e^{j(\omega_i t - \vec{k}_i \cdot \vec{r})} + \frac{1}{2} A_d e^{j(\omega_d t - \vec{k}_d \cdot \vec{r})} + c.c. \right) \right] = \\
 &= \mu_0 2 \sqrt{\epsilon_0 \epsilon} \Delta n_0 \frac{\partial^2}{\partial t^2} \left[ \frac{1}{4} A_i e^{j[(\omega_s + \omega_i)t - (\vec{k}_s + \vec{k}_i) \cdot \vec{r}]} + \right. \\
 &\quad \left. \frac{1}{4} A_i^* e^{j[(\omega_s - \omega_i)t - (\vec{k}_s - \vec{k}_i) \cdot \vec{r}]} + \right. \\
 &\quad \left. \frac{1}{4} A_d e^{j[(\omega_s + \omega_d)t - (\vec{k}_s + \vec{k}_d) \cdot \vec{r}]} + \right. \\
 &\quad \left. \frac{1}{4} A_d^* e^{j[(\omega_s - \omega_d)t - (\vec{k}_s - \vec{k}_d) \cdot \vec{r}]} + \right. \\
 &\quad \left. \frac{1}{4} A_i e^{j[-(\omega_s + \omega_i)t - (-\vec{k}_s + \vec{k}_i) \cdot \vec{r}]} + \right. \\
 &\quad \left. \frac{1}{4} A_i^* e^{j[-(\omega_s + \omega_i)t + (\vec{k}_s + \vec{k}_i) \cdot \vec{r}]} + \right. \\
 &\quad \left. \frac{1}{4} A_d e^{j[-(\omega_s + \omega_d)t - (-\vec{k}_s + \vec{k}_d) \cdot \vec{r}]} + \right. \\
 &\quad \left. \frac{1}{4} A_d^* e^{j[-(\omega_s + \omega_d)t + (\vec{k}_s + \vec{k}_d) \cdot \vec{r}]} \right]
 \end{aligned}$$

## Analysis of Bragg Diffraction of Light by Sound

Now we have to keep only the in-phase terms because only these are going to contribute at steady state. Assume that

$$\omega_d = \omega_i + \omega_s \quad \sim \quad \omega_i = \omega_d - \omega_s$$

Keeping only the in-phase terms we get:

$$\begin{aligned} & -\frac{1}{2} \left[ +2j k_{iz} \frac{dA_i}{dz} \right] e^{j(\omega_i t - \vec{k}_i \cdot \vec{r})} + c.c. - \frac{1}{2} \left[ +2j k_{dz} \frac{dA_d}{dz} \right] e^{j(\omega_d t - \vec{k}_d \cdot \vec{r})} + c.c. = \\ & = \mu_0 2 \sqrt{\epsilon_0 \epsilon} \Delta n_0 \left\{ \left[ \frac{1}{4} A_d e^{j[(\tilde{\omega}_d - \tilde{\omega}_s)t - (\vec{k}_d - \vec{k}_s) \cdot \vec{r}]} + c.c. \right] [-(\omega_d - \omega_s)^2] + \right. \\ & \quad \left. - \frac{1}{4} A_i e^{j[(\tilde{\omega}_i + \tilde{\omega}_s)t - (\vec{k}_i + \vec{k}_s) \cdot \vec{r}]} + c.c. \right] [-(\omega_i + \omega_s)^2] \right\} \end{aligned}$$

while terms like  $\omega_d + \omega_s$ ,  $\omega_i - \omega_s$  and their conjugates have been neglected. The above equation should be satisfied for all times. Therefore,

$$j k_{iz} \frac{dA_i}{dz} e^{-j \vec{k}_i \cdot \vec{r}} = \frac{1}{2} \mu_0 \sqrt{\epsilon_0 \epsilon} \Delta n_0 \frac{\tilde{\omega}_i^2}{(\tilde{\omega}_d - \tilde{\omega}_s)^2} A_d e^{-j (\vec{k}_d - \vec{k}_s) \cdot \vec{r}}$$

$$j k_{dz} \frac{dA_d}{dz} e^{-j \vec{k}_d \cdot \vec{r}} = \frac{1}{2} \mu_0 \sqrt{\epsilon_0 \epsilon} \Delta n_0 \frac{\tilde{\omega}_d^2}{(\tilde{\omega}_i + \tilde{\omega}_s)^2} A_i e^{-j (\vec{k}_i + \vec{k}_s) \cdot \vec{r}}$$

## Analysis of Bragg Diffraction of Light by Sound

Define  $\eta_i = \frac{1}{2} \mu_0 \sqrt{\epsilon_0 \epsilon} \frac{\Delta n_0}{k_{iz}} \cdot \omega_i^2 =$   
 $= \frac{1}{2} \mu_0 \sqrt{\epsilon_0 \epsilon} \frac{\Delta n_0}{\omega_i \sqrt{\mu_0 \epsilon} \cos \theta_i} \omega_i^2 = \frac{1}{2} \frac{1}{c} \cdot \frac{\omega_i \Delta n_0}{\cos \theta_i}$

Similarly,  $\eta_d = \frac{1}{2} \frac{1}{c} \frac{\omega_d \Delta n_0}{\cos \theta_d}$ . Using these definitions the above equations become:

$$\frac{dA_i}{dz} = -j \eta_i A_d e^{+j [\vec{k}_i + \vec{k}_s - \vec{k}_d] \cdot \vec{r}}$$

$$\frac{dA_d}{dz} = -j \eta_d A_i e^{-j [\vec{k}_i + \vec{k}_s - \vec{k}_d] \cdot \vec{r}}$$

For strong interaction within the volume of the acoustic wave disturbance the exponential factors need to be such that :

$$\vec{k}_i + \vec{k}_s - \vec{k}_d = 0 \quad \text{Bragg Condition} .$$

## Analysis of Bragg Diffraction of Light by Sound

If the Bragg condition is satisfied then  $\theta_i = \theta_d$  and  $\omega_d \approx \omega_i$  since  $\omega_s \ll \omega_i$  and the above equations become: ( $\eta = \eta_i = \eta_d = \frac{\Delta n_0 \omega_i}{2c \cos \theta_i}$ )

$$\frac{dA_i}{dz} = -j\eta A_d$$

$$\frac{dA_d}{dz} = -j\eta A_i$$

# Analysis of Bragg Diffraction of Light by Sound

Solution:

It is straightforward to show that:

$$A_d(z) = A_d(0) \cos(\eta z) - j A_i(0) \sin(\eta z)$$

$$A_i(z) = A_i(0) \cos(\eta z) - j A_d(0) \sin(\eta z)$$

and assuming that at  $z=0$  there is no diffracted field the result is

$$A_d(z) = -j A_i(0) \sin(\eta z),$$

$$A_i(z) = A_i(0) \cos(\eta z).$$

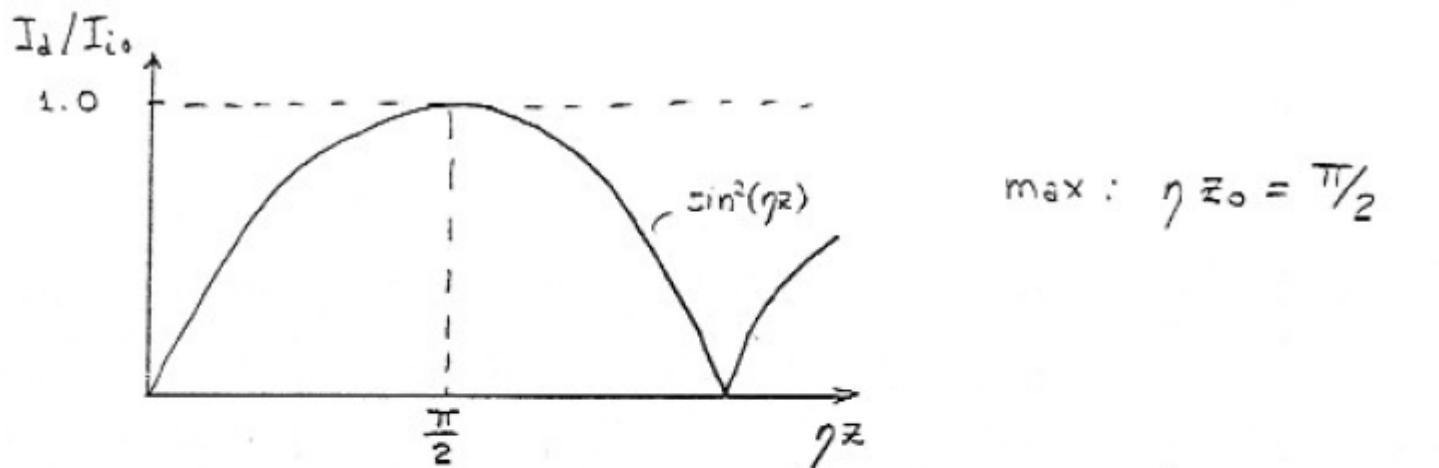
Then the diffracted and transmitted power are:

$$\frac{I_d(z)}{I_i(z=0)} = \frac{|A_d(z)|^2}{|A_i(0)|^2} = \sin^2(\eta z)$$

$$\frac{I_i(z)}{I_i(z=0)} = \frac{|A_i(z)|^2}{|A_i(0)|^2} = \cos^2(\eta z)$$

which satisfy the conservation of energy since  $I_d(z) + I_i(z) = I_i(0)$  for any  $z$ .

# Analysis of Bragg Diffraction of Light by Sound



$$\begin{aligned}\frac{I_d(z)}{I_i(0)} &= \sin^2[\eta z] = \sin^2\left[\frac{\omega \Delta n_0}{2c \cos \theta_i} z\right] = \sin^2\left[\frac{2\pi \Delta n_0}{Z \lambda_0 \cos \theta_i} z\right] = \\ &= \sin^2\left[\frac{\pi \Delta n_0}{\lambda_0 \cos \theta_i} z\right]\end{aligned}$$

## Analysis of Bragg Diffraction of Light by Sound

Now let's relate  $\Delta n_0$  with the acoustic wave intensity.

$\Delta n_0$  is related to the strain via

$$\Delta n_0 = -\frac{n^3 p}{2} s_0$$

where  $p$  the photoelastic constant of the medium and  $s_0$  is the amplitude of the induced strain.  $s_0$  is related to  $I_{\text{acoust}}$  (acoustic wave intensity in  $\text{W/m}^2$ ) by

$$s_0 = \sqrt{\frac{2 I_{\text{acoust}}}{\rho v_s^3}} \quad \text{where } \rho \text{ is the mass density } (\text{kg/m}^3) \text{ and}$$

$v_s$  is the velocity of sound in the medium. Then,

$$\frac{I_d}{I_i(0)} = \sin^2 \left[ \frac{\pi z}{\lambda_0 \cos \theta_i} \cdot \sqrt{\frac{M I_{\text{acoust}}}{2}} \right]$$

where  $M = \frac{n^6 p}{\rho v_s^3}$  is the figure of merit of the acousto-optic interaction.

## Deflection of Light by Sound

One advantage of the acousto-optic devices over the electro-optic devices is that by changing the frequency of the sound wave we can change the direction of the diffracted optical beam. This can be easily understood if we observe the grating equation:

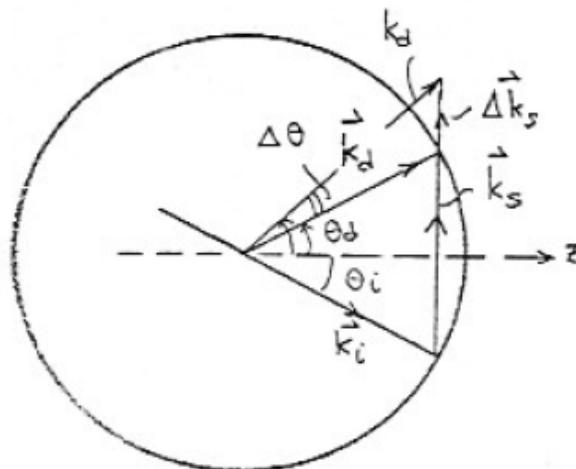
$$\Delta [\sin \theta_i + \sin \theta_m] = m \frac{\lambda_0}{n} \Rightarrow$$

$$\sin \theta_m = -\sin \theta_i + m \frac{\lambda_0/n}{\Delta} = -\sin \theta_i + m \frac{\lambda_0/n}{v_s} v_s$$

where  $\Delta = \lambda s = v_s / \nu_s$  and  $v_s$  is the sound frequency.

# Deflection of Light by Sound

Now let's assume that the Bragg condition is satisfied. This is shown in the following diagram.



At the Bragg condition

$$\theta_i = \theta_d = \theta \text{ and}$$

$$2\Delta \sin \theta = m \left( \frac{\lambda_0}{n} \right)$$

For  $m=1$  (1st. order)

$$2\Delta \sin \theta = \lambda_0/n$$

By changing  $v_s$  by  $\Delta v_s$ ,  $\vec{k}_s$  is also changing by  $\Delta \vec{k}_s$ .

From the grating equation we have

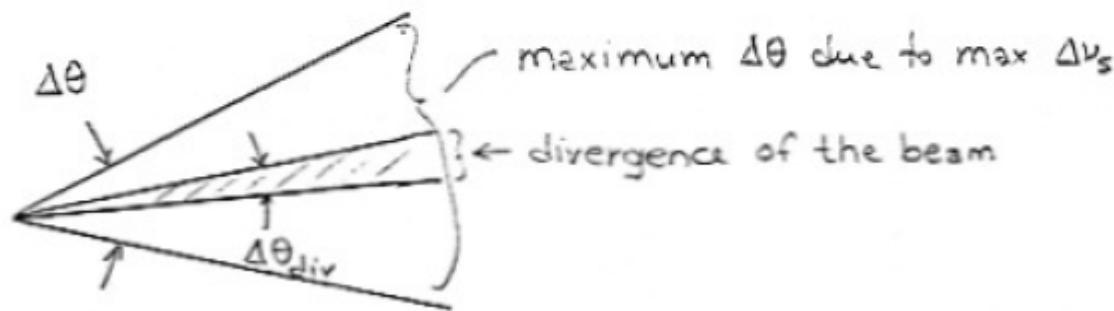
$$\sin \theta_d = -\sin \theta_i + \frac{\lambda_0/n}{v_s} v_s \Rightarrow \cos \theta_d \cdot \Delta \theta = \frac{\lambda_0/n}{v_s} \Delta v_s$$

and since  $\theta_i, \theta_d$  are small close to the Bragg condition

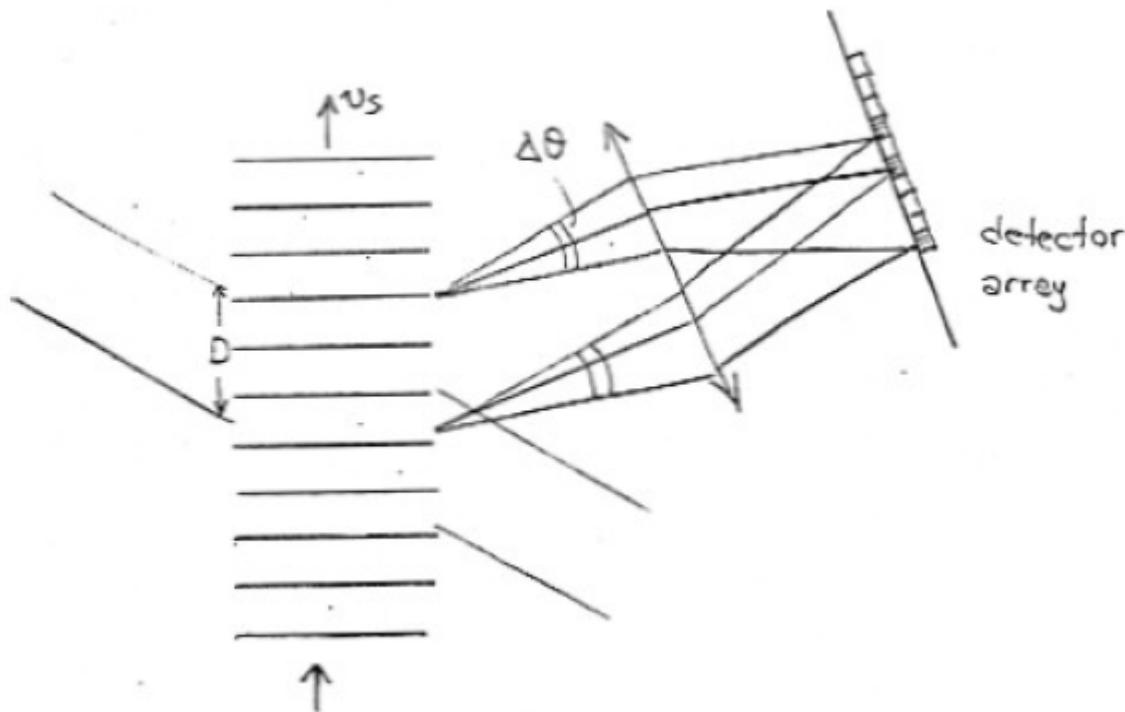
$$\Delta \theta \approx \frac{\lambda_0/n}{v_s} \cdot \Delta v_s$$

## Deflection of Light by Sound

The important parameter is not the magnitude of the actual  $\Delta\theta$  due to a change  $\Delta\nu_s$  in frequency but how this angle compares to the divergence of a beam. Recall that the TEM<sub>00</sub> mode has a beam divergence of  $\Delta\theta_{div} \approx \frac{4}{\pi} \frac{\lambda_0/n}{D}$  where D is the beam diameter.



# Deflection of Light by Sound



The number of resolvable spots is:

$$N = \frac{\Delta\theta}{\Delta\theta_{\text{dir}}} = \frac{\frac{2n}{v_s} \Delta v_s}{\left(\frac{\pi}{D}\right) \frac{2n}{v_s}} = \left(\frac{\pi}{2}\right) \cdot \frac{D}{v_s} \cdot \Delta v_s \leftarrow T \cdot B$$

where  $T = \frac{D}{v_s}$  = transit time for the sound to cross the light beam  
and  $B$  is the acoustic signal bandwidth.

$T \cdot B$  is also called the time bandwidth product.

# Deflection of Light by Sound

Example:

Assume an acousto-optic cell on LiNbO<sub>3</sub>:

$$v_s = 6.57 \text{ km/sec}, \rho = 4.7 \cdot 10^3 \text{ kg/m}^3, n = 2.214 \text{ (at } \lambda_0 = 0.6328 \mu\text{m})$$

$$p = 0.15, I_{\text{acoust}} = 10^6 \text{ W/m}^2, \nu_s = 100 \text{ MHz}$$

$$M = \frac{n^6 p^2}{\rho v_s^3} = \frac{2.214^6 \cdot 0.15^2}{4.7 \cdot 10^3 (6.57 \cdot 10^3)^3} \cdot \frac{\text{sec}^3}{\text{kg}} = 1.99 \cdot 10^{-15} \frac{\text{sec}^3}{\text{kg}} = 1.99 \cdot 10^{-15} \frac{\text{m}^2}{\text{W}}$$

$$\Delta n_0 = \left( \frac{1}{2} M I_{\text{acoust}} \right)^{1/2} = 3.15 \cdot 10^{-5}$$

$$\Delta = \lambda_s = \frac{v_s}{\nu_s} = \frac{6.57 \cdot 10^3}{10^8} = 65.7 \mu\text{m}$$

Bragg condition:

$$\frac{\lambda_0}{n_0} = 2 \Delta \sin \theta \Rightarrow \sin \theta = \frac{\lambda_0 / n_0}{2 \Delta} \Rightarrow \theta = 0.125^\circ$$

$$\Delta \theta \approx \frac{\lambda_0 / n_0}{v_s} \cdot \Delta f_s = 0.249^\circ$$

for  $\Delta f_s = 100 \text{ MHz}$