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Problem Set No. 3

(Due Date: 27/11/2024)

Problem 1 (*Optical Resonator Design*) [30%]

Design an optical resonator with $R_1 = 30\,\text{cm}$, $R_2 = 60\,\text{cm}$ (according to the convention for stability) and cavity length L. The medium between the mirrors is air ($n = 1.0$, corresponding to a gas laser). The freespace wavelength for the resonator modes is $\lambda_0 = 0.5145\,\mu\text{m}$.

(a) [15%] Determine the ranges of cavity length L for which this resonator is stable. Make graphs of the minimum spot size w_0, Rayleigh distance z_0, stability factor g_{12}, and spherical mirror positions z_1 and z_2 as functions of the cavity length L. For which cavity length is the minimum spot size maximum?

(b) [15%] Now select the cavity length to be the one that maximizes the minimum spot size. For these cavity lengths determine the Gaussian beam characteristics, i.e. determine the spot sizes w_1 and w_2 at the two mirrors, the positions z_1 and z_2 of the two mirrors, the Rayleigh distance z_0 and the minimum spot size w_0 and its location, and the stability factor g_{12}. Draw a diagram of the envelope of the Gaussian beam inside the cavity.

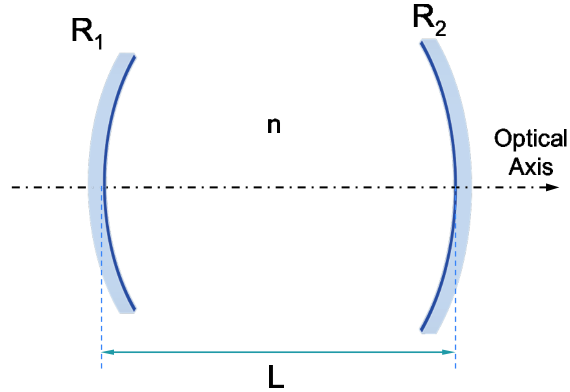

Figure 1: A stable cavity of a spherical left mirror of radius $R_1 = 30\,\text{cm}$ and a spherical right mirror of radius $R_2 = 60\,\text{cm}$, at a distance L apart. The refractive index of the medium inside the resonator is $n = 1.0$.

Problem 2 (*Fabry-Perot Etalon*) [40%]

Consider a diverging monochromatic beam in air that is incident on a symmetric plane-parallel Fabry-Perot etalon of thickness ℓ, refractive index n, etalon finesse $\mathcal{F}$. Every set of parallel rays of the diverging beam passes through the Fabry-Perot etalon if it satisfies the resonance condition for the etalon. Then

with the help of a thin lens, of focal length f, the propagating rays are focused on the focal plane as it is shown in Fig. 2. Depending on the angle θ' concentric bright rings will appear on the focal plane.
(a) [6%] Obtain an expression for the various angles θ' for which the resonance condition will be satisfied and therefore will propagate through the system and get focused on the focal plane of the thin lens.
(b) [7%] As it was explained previously the energy distribution in the focal plane of the lens consists of a series of bright circles, each corresponding to a different value of m. Obtain an expression for the radii of these circles. Use the paraxial beam approximation (i.e., small angles).
(c) [7%] Consider the effect of having simultaneously two freespace wavelengths λ_1 and λ_2 present in the diverging incident beam. Derive an expression for the separation of the respective circles (of the same m) in the focal plane. What is the smallest wavelength separation $\Delta\lambda_{min} = |\lambda_1 - \lambda_2|_{min}$ that can be resolved by this technique?
(d) [20%] Now assume the following numerical parameters: $\lambda_0 = 0.6328\,\mu$m (red light of He-Ne laser), $\ell = 1\,$cm, $n = 1.50$, $n' = 1.0$ (air), $\mathcal{F} = 10$, and $f = 25\,$cm. Determine the radii of the first 10 rings (starting from the smallest in an increasing order). Determine the normalized intensity along the transverse radial direction in the focal plane of the lens (including the first 10 rings). Then, assume that in the incident beam are now present the wavelengths $\lambda_0 \pm \Delta\lambda$ where $\Delta\lambda = 0.002\,$nm. Plot in the same figure the normalized intensity distributions that correspond to all three wavelengths simultaneously. Can the three families of rings be resolved?

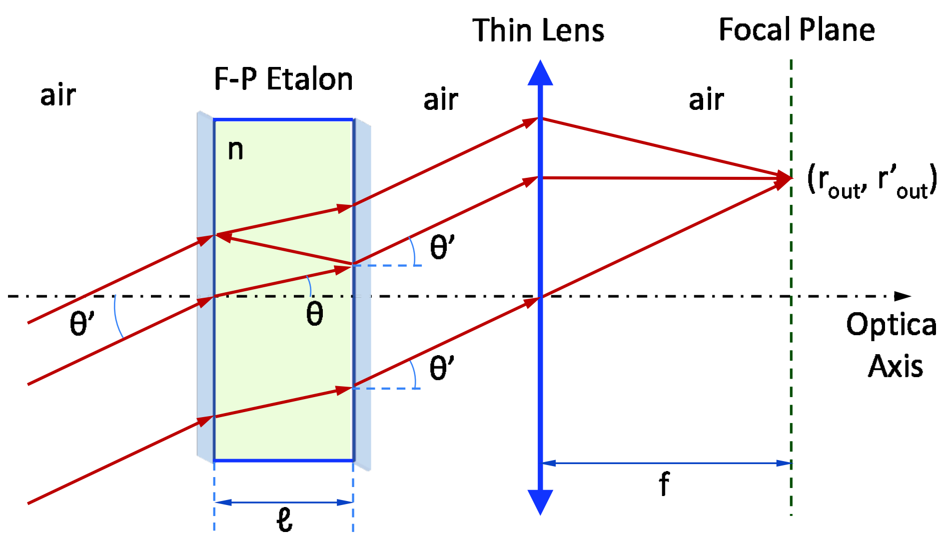

Figure 2: Monochromatic beam in air incident on a Fabry-Perot etalon at an angle θ' passes through and is focused via a thin lens on to its focal plane.

Problem 3 (*Optical Resonator Characteristics through the ABCD Matrix*) [30%]

Consider a resonator with a flat left mirror ($R_1 = \infty$) and a right concave mirror of radius R_2 ($R_2 > 0$ according to the stability criterion convention). The separation of the mirrors is d while the material inside the cavity has a refractive index of n.
(a)[20%] Using the ABCD matrix method find the spot size $w(z_1)$ and the radius of curvature $R(z_1)$ of a stable Gaussian beam, where z_1 is a random point along the axis of the resonator and $0 \leq z_1 \leq d$.

Plot the $1/R(z_1)$ (curvature) as function of the z_1 for $d/R_2 = 0.30, 0.60, 0.90$, $R_2 = 1\,\text{m}$, $n = 1$, and $\lambda_0 = 1\mu\text{m}$.

(b) [10%] Find the analytical expressions (based on the ABCD theory) of the minimum spot size, w_0, and of the Rayleigh distance z_0, for the stable Gaussian beam in the resonator. Furthermore, specify, what is the minimum radius of curvature of the Gaussian beam and for which point z_1 it occurs.

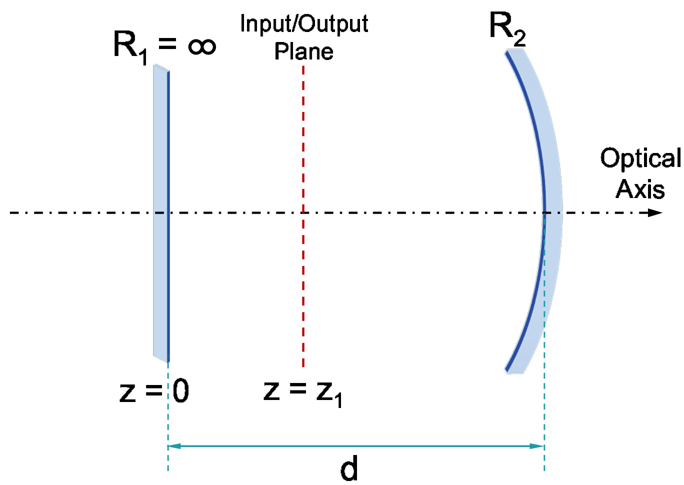

Figure 3: A resonator with flat left mirror ($R_1 = \infty$) and concave right mirror ($R_2 > 0$). The plane at $z = z_1$ is considered as the input and the output plane for a round trip of a Gaussian beam around the resonator cavity.