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Problem Set No. 2

(Due Date: 13/12/2023)

Problem 1 (*Focusing of a lens-like medium*) [20%]

Assume a lenslike medium with a refractive index distribution of the form $n(r) = n_0[1 - (k_2/2k_0)r^2]$ (where $k_0 = 2\pi/\lambda_0$ with λ_0 representing the free space wavelength). The lenslike medium is extended along the z axis from $z = 0$ to $z = \ell$. After $z = \ell$ there is a homogeneous medium of refractive index n_{out}. Use the ABCD matrix for the lens-like medium as it has been derived in class and in the textbook (paraxial approximation).

(a) Show that a family of parallel (to the optical axis) rays entering the lenslike medium at $z = 0$ at different radii will converge upon emerging at $z = \ell$ to a common focus at a distance F from the $z = \ell$ boundary. Specify the inequalities between the problem parameters in order for F to be positive, negative, or zero.

(b) At this point numerical values are assumed. Specifically, $n_0 = 1.50$, $n_{out} = 1.00$, $\lambda_0 = 0.5\,\mu$m. Make a graph of the focal distance F versus the lens-like medium length for $k_2 = 0.01\,\mu\text{m}^{-1}$ and for $k_2 = 0.001\,\mu\text{m}^{-1}$. Since the focal distance is periodic with respect to the length ℓ vary ℓ only within the first period.

(c) Assume again that $n_0 = 1.50$, $n_{out} = 1.00$, $\lambda_0 = 0.5\,\mu$m, $k_2 = 0.01\,\mu\text{m}^{-1}$, and $r_0 = 10\,\mu$m. Make a graph of the ray path through the system for $\ell = 150\,\mu$m, $250\,\mu$m, and $300\,\mu$m.

Problem 2 (*Propagation in a lens-like medium*) [20%]

This problem is the same physical problem with the previous one. The purpose is to show the differences between various ray approximations. The ABCD matrix of the lens-like medium was based on the following differential equation (paraxial approximation with small $k_2/2k_0$):

$$n_0 \frac{d^2 r}{dz^2} \simeq -n_0 \frac{k_2}{k_0} r. \qquad (1)$$

A slighly better paraxial approximation is the following:

$$n_0 \left(1 - \frac{k_2}{2k_0} r^2\right) \frac{d^2 r}{dz^2} \simeq -n_0 \frac{k_2}{k_0} r. \qquad (2)$$

Finally, the exact eikonal equation (without the paraxial approximation) is

$$\frac{d}{ds}\left(n \frac{d\vec{r}}{ds}\right) = \vec{\nabla} n. \qquad (3)$$

Equation (1) can be solved analytically as it was shown in the notes and in the textbook using the ABCD matrix. However, equations (2) and (3) can be solved only numerically using Runge-Kutta techniques.

For example, Matlab's "ode45" function can easily solve nonlinear systems of differential equations (other similar routines in various computational packages can be used). Solve numerically all three equations and plot the ray position inside the lens-like medium as function of the distance z (where z varies from 0 to $\sim 800\,\mu$m). The values of k_2, n_0, and λ_0 are as defined in problem 1.

(a) For initial conditions $r_0 = 10\,\mu$m and $r_0' = 0$ plot the ray paths that correspond to the three solutions in the same diagram.

(b) Repeat (a) for the initial conditions $r_0 = 10\,\mu$m and $r_0' = 1$.

Hint: For the solution of the eikonal equation (3) see the journal paper: A. Sharma *et al.*, "Tracing rays through graded-index media: a new method," *Applied Optics*, vol. 21, No. 6, pp. 984–987, Mar. 15, 1982.

Problem 3 (*Ray tracing in a stable cavity*) [20%]

Assume the cavity given in Fig. 1. The incoming ray is perpendicular to the flat mirror and displaced from the axis by a specified amount $-r_0$. The relation between the parameters is $d/R_2 = 3/4$.

(a) Is the cavity stable?

(b) Find the transmission matrix ABCD of the unit cell 1 which is comprised of the plane just after the flat mirror and includes the propagation distance d, the reflection from the spherical mirror, the backward traveled propagation distance d and the reflection of the flat mirror. Using this matrix solve for r_s, and r_s' where $s = 0$ corresponds to the values at the input of the first cell. Verify the positions of the rays in the diagram.

(c) Repeat (b) for a unit cell which is comprised of the plane just before the spherical mirror, a reflection from the spherical mirror, a backward propagation distance d, a reflection from the flat mirror, and a forward propagation distance d.

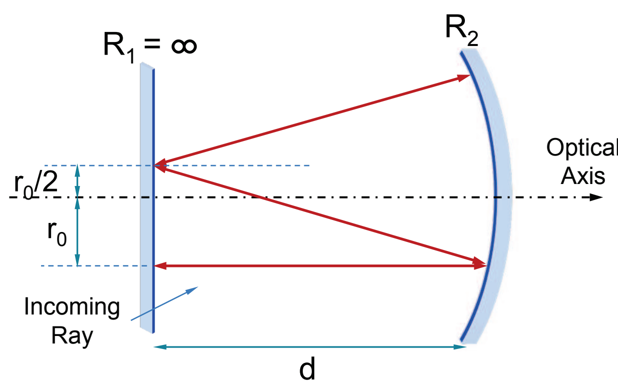

Figure 1: A stable cavity of a planar left mirror $R_1 = \infty$ and a spherical right mirror R_2, at a distance d, where $d/R_2 = 3/4$. The incident beam start normally from the left mirror at a distance r_0 below the optical axis.

Problem 4 (*General focusing of a Gaussian beam*) [20%]

Solve the problem of a Gaussian beam incident on a thin lens of focal length f when the lens is placed in an arbitrary position relative to the input beam (that is, not at its waist). I.e., calculate the location of the waist of the output beam and its minimum spot size. Assume that the minimum waist of the incident beam is to the left of the lens. Evaluate the location and the waist of the output beam for the following parameters: $d = 15\,$mm (where d is the distance of the minimum waist of the incident beam from the lens), $f = 100\,$mm (lens focal length), $\lambda_0 = 1.0\,\mu$m (freespace wavelength), $w_{01} = 1\,$mm (minimum waist of incident beam), and $n = 1$ (refractive index).

Problem 5 (*Focusing of a Gaussian beam on a target through a lens*) [20%]
A Gaussian beam with wavelength λ_0 is incident on a lens placed at distance $z = \ell$ as shown in Fig. 2. Calculate the lens focal length, f, so that the output beam has its minimum waist at the front surface of the sample crystal which is placed at distance L from the lens. Show that given ℓ and L there are two solutions. Show graphs of the two solutions as a function of the input Gaussian beam minimum waist, w_0. For the numerical implementation the following parameters should be assumed: $\lambda_0 = 1\,\mu$m, $n = 1.0$, $\ell = 15\,$mm, $L = 100\,$mm, $w_0 \in [0.02,\ 10]$mm (varying parameter).

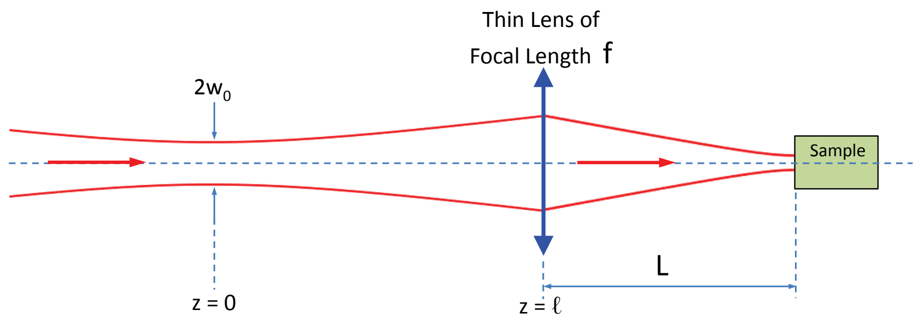

Figure 2: A Gaussian beam of minimum waist w_0 is focused on a sample with the help of a thin lens of focal length f.