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Problem Set No. 1

(Due Date: 15/11/2023)

Problem 1 (*Plane Waves in Anisotropic Media*) [30%]

The plane-wave (algebraic) form of Maxwell's equations for a linear, homogeneous, non-magnetic, and anisotropic medium are

$$\begin{aligned} \vec{k} \times \vec{H} &= -\omega \vec{D}, \\ \vec{k} \times \vec{E} &= \omega \vec{B}, \\ \vec{k} \cdot \vec{D} &= 0, \\ \vec{k} \cdot \vec{B} &= 0, \end{aligned}$$

where $\vec{B} = \mu_0 \vec{H}$, and $\vec{D} = [\epsilon]\vec{E}$.

(a) Show that the wavevector $\vec{k}$ always points in the direction of $\vec{D} \times \vec{B}$.

(b) Show that the magnitude of the wavevector is given by $\vec{k} \cdot \vec{k} = \omega^2 \mu_0 [(\vec{D} \cdot \vec{D})/(\vec{D} \cdot \vec{E})]$.

(c) Show that the complex Poynting vector, $\vec{S} = (1/2)\vec{E} \times \vec{H}^*$, can point in a direction other than that of the wavevector $\vec{k}$.

Problem 2 (*Quarter-Wave Plate*) [30%]

A linearly polarized electromagnetic wave is incident normally at $z = 0$ on the xy-face of a crystal so that it propagates along its z-axis. The xyz coordinate system is the principal axes system of the crystal. The corresponding permittivity tensor is diagonal with elements ϵ_{xx}, ϵ_{yy}, and ϵ_{zz}. If the wave is initially polarized so that it has equal components along the x and y axes, what is the state of its polarization at the plane $z = z_0$ where

$$k_0(n_{xx} - n_{yy})z_0 = \frac{\pi}{2},$$

and $\epsilon_{ww} = \epsilon_0 n_{ww}^2$ ($w = x, y, z$). Assume that $n_{xx} > n_{yy}$ and $n_{xx} = n_s$ (slow axis along x direction), $n_{yy} = n_f$ (fast axis along the y direction).

Problem 3 (*Polarization Transformation by a Wave Plate*) [40%]

A wave plate is characterized by its phase retardation Γ and the azimuth angle ψ.

((a) Find the polarization state of the emerging beam, assuming that the incident beam is polarized along the x direction. Use a complex number to represent the resulting polarization state obtained.(I.e. define $p = E_y/E_x$).

(b) The polarization state of the incident x-polarized beam is represented by a point at the origin of the complex plane. Show that the transformed polarization state can be anywhere on the complex plane, provided Γ can be varied from 0 to 2π and ψ can be varied from 0 to $\pi/2$. Physically, this means that any polarization state can be produced from linearly polarized light, provided a proper wave plate is available. To show this find the wave-plate parameters Γ and ψ for any given r and φ. For a numerical example find the Γ and ψ for the (r, φ) pairs $(1, \pi/2)$, $(1, -\pi/2)$, $(\infty, 0)$, and $(2, -\pi/2)$. Make also plots of r and φ as a function of Γ and ψ.

(c) Show that the Jones matrix W of a wave plate is unitary, that is $W^\dagger W = 1$ where $W^\dagger = (W^T)^*$ (T denotes transposed and $*$ denotes conjugate).

(d) Let $\vec{V}_1'$ and $\vec{V}_2'$ be the transformed Jones vectors of $\vec{V}_1$ and $\vec{V}_2$, respectively. Show that if $\vec{V}_1$ and $\vec{V}_2$ are orthogonal, so are $\vec{V}_1'$ and $\vec{V}_2'$. ($\vec{A}$ and $\vec{B}$ are orthogonal if $\vec{A} \cdot \vec{B}^* = 0$).