05/12/2023

Laser Fundamentals

Electro-Optics & Applications

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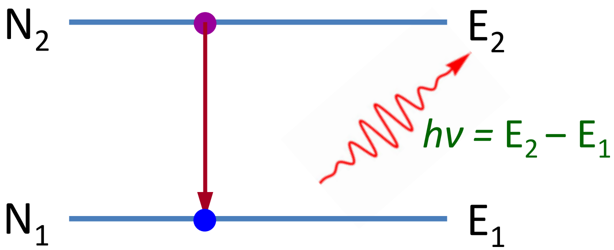
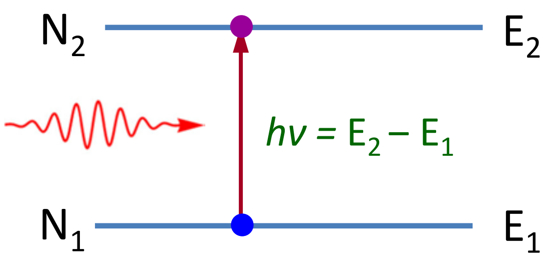
Radiative Processes

Spontaneous Emission



$$\left.\frac{dN_2}{dt}\right|_{sp.em.} = -A_{21}N_2 = -\left.\frac{dN_1}{dt}\right|_{sp.em.}$$

Absorption

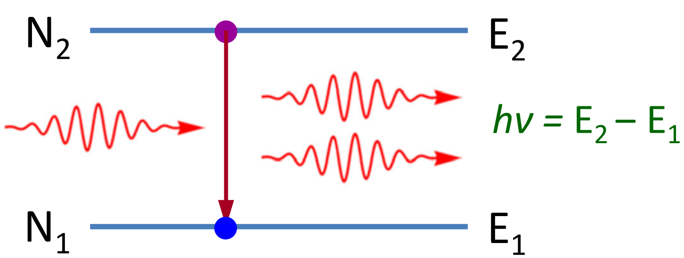


$$\left.\frac{dN_1}{dt}\right|_{abs.} = -B_{12}\rho(\nu)N_1 = -\left.\frac{dN_2}{dt}\right|_{abs.}$$

For Blackbody Radiation:

$$\rho(\nu) = \frac{8\pi n^3\nu^2}{c^3}\frac{h\nu}{e^{h\nu/k_BT}-1}$$

Stimulated Emission



$$\left.\frac{dN_2}{dt}\right|_{st.em.} = -B_{21}\rho(\nu)N_2 = -\left.\frac{dN_1}{dt}\right|_{st.em.}$$

Radiative Processes of Stimulated Emission

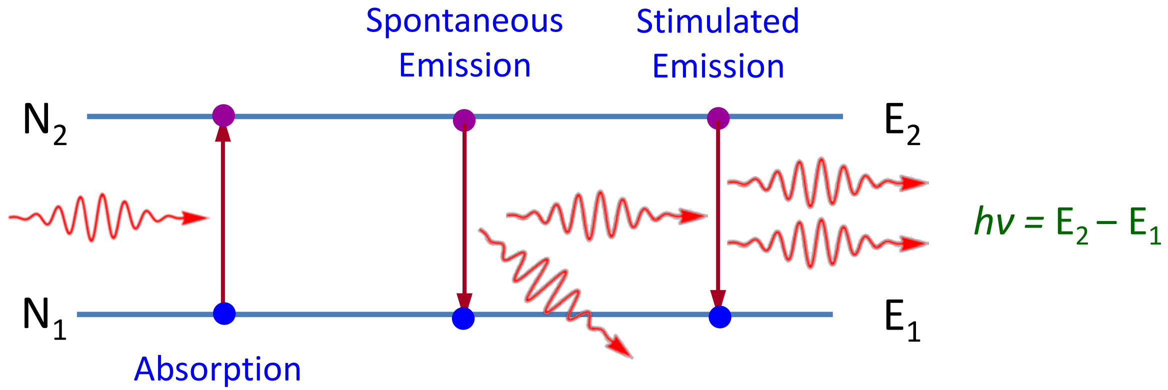
Basic Principle of laser Operation

http://www.laserfest.org/lasers/images/nero1.jpg

L ight
A mplification
S timulated
E mission
R adiation

Radiative Processes

Relations Between Einstein's Coefficients



$$\frac{dN_2}{dt} = +B_{12}\rho(\nu)N_1 - A_{21}N_2 - B_{21}\rho(\nu)N_2 = -\frac{dN_1}{dt}$$

$$\rho(\nu) = \frac{8\pi n^3\nu^2}{c^3}\frac{h\nu}{e^{h\nu/k_BT}-1}$$

Thermodynamic Equilibrium

$$\frac{N_2}{N_1} = \frac{B_{12}\rho(\nu)}{A_{21}+B_{21}\rho(\nu)}$$

Boltzmann Statistics

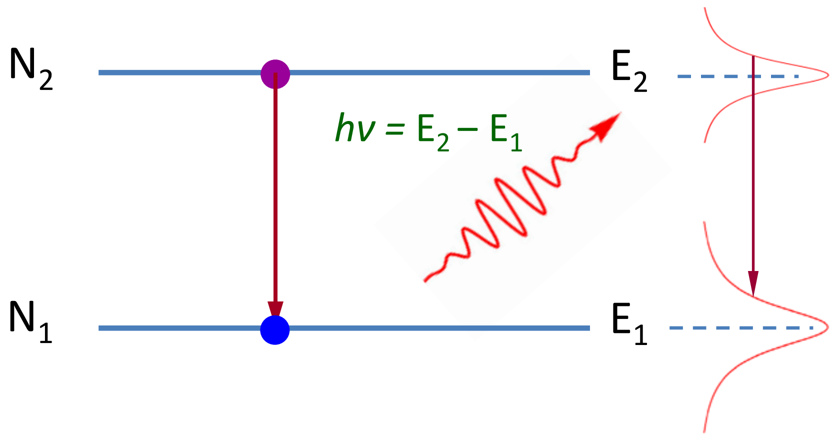
$$\frac{N_2}{N_1} = \frac{g_2}{g_1}e^{-h\nu/k_BT}$$

Einstein's Coefficients

$$g_2 B_{21} = g_1 B_{12}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h\nu^3}{c^3}$$

Lineshape Function



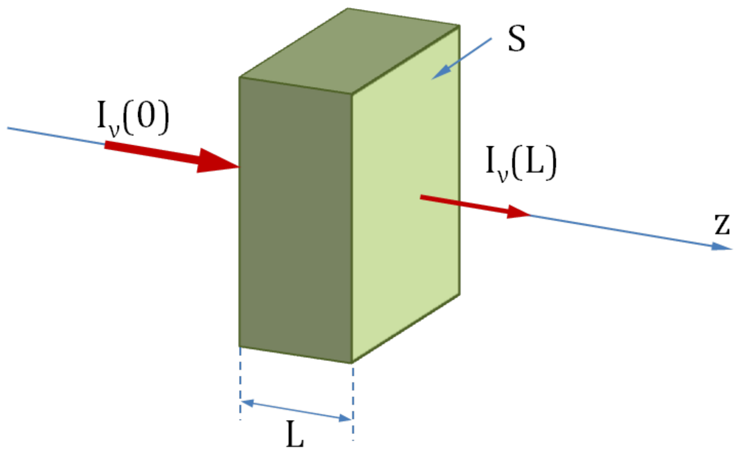
Lineshape Function, *g(v)* = Probability *g(v)dv* for a photon to be:

- Spontaneously Emitted between *v* and *v+dv*
- Absorbed between *v* and *v+dv*
- Stimulating Emitted between *v* and *v+dv*

$$\int_{-\infty}^{+\infty} g(\nu)d\nu = 1$$

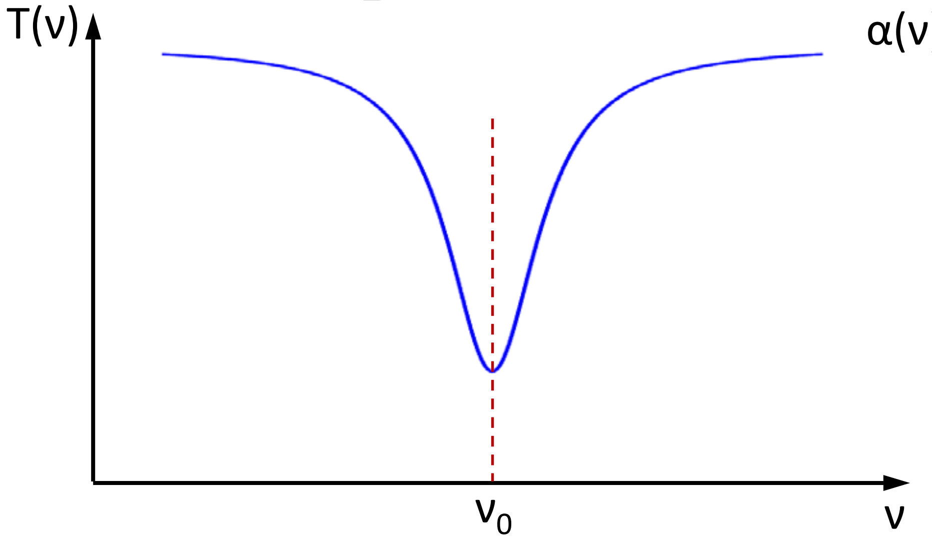
Lineshape Function Measurement

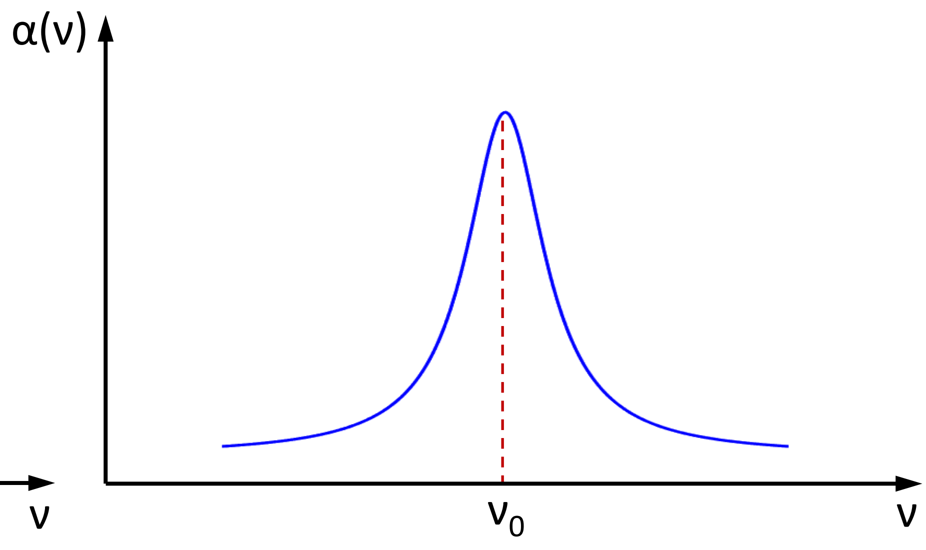
Slab of material of width L



$$T(\nu) = \frac{I_\nu(L)}{I_\nu(0)} = e^{-\alpha(\nu)L}$$

$$\alpha(\nu) = \frac{1}{L}\ln\left(\frac{1}{T(\nu)}\right)$$

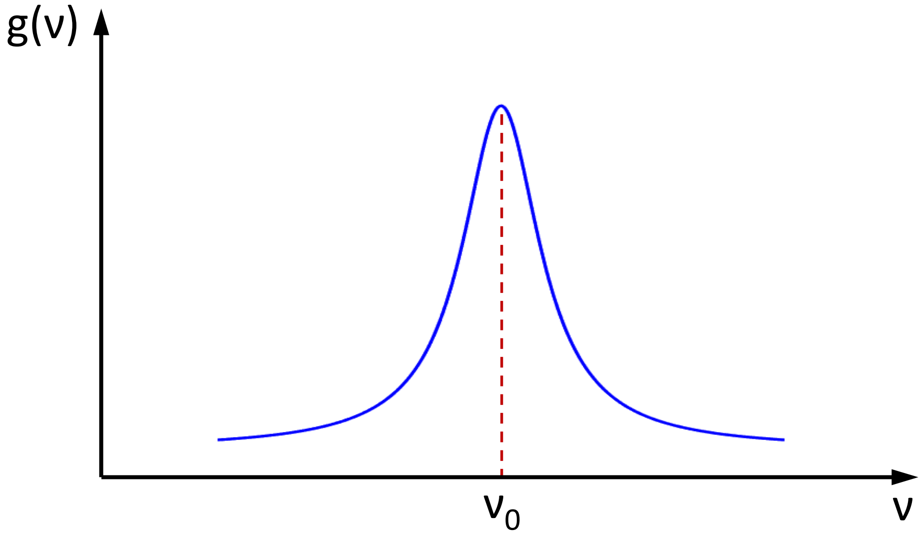




Lineshape Function Measurement

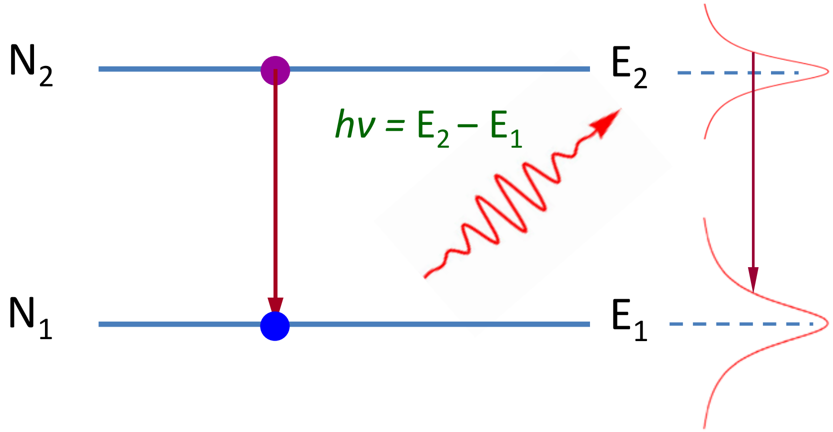
Normalization of *Absorption Coefficient α* gives the Lineshape function

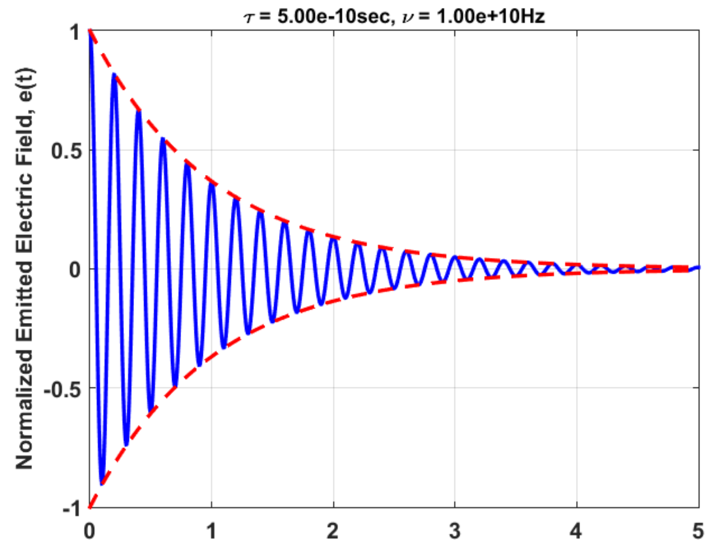
$$g(\nu) = \frac{\alpha(\nu)}{\int \alpha(\nu) d\nu}$$

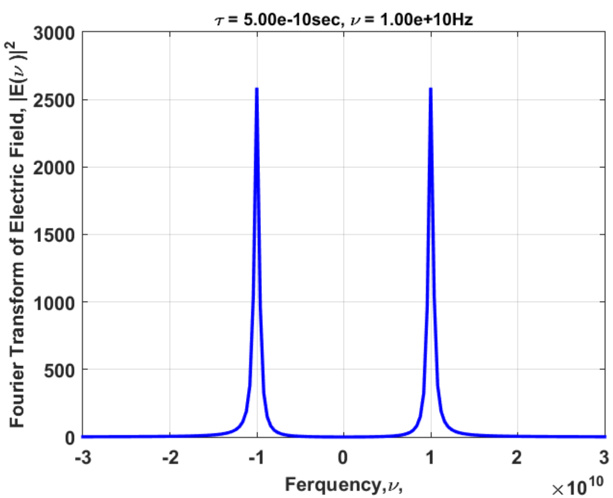


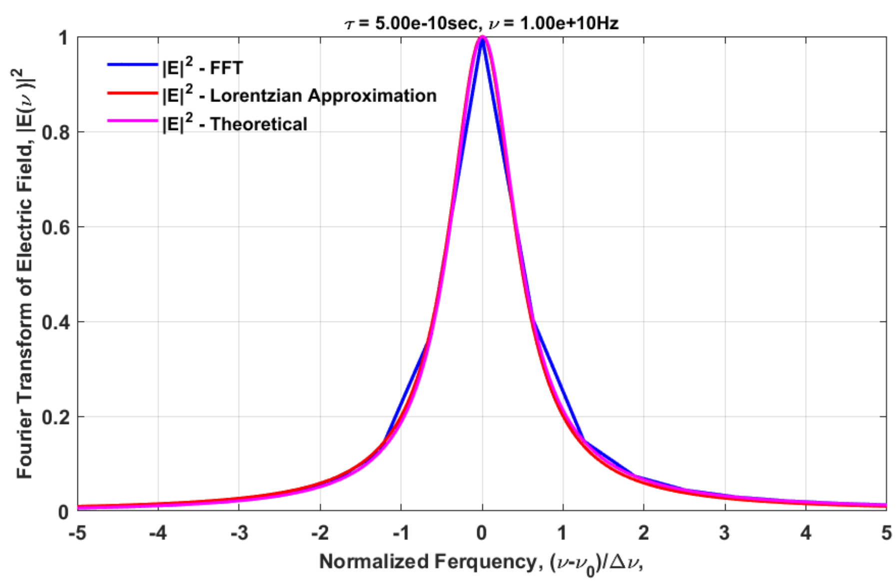
Damped Oscillation

$$\mathscr{E}(t) = E_0 e^{-t/\tau} \cos(2\pi\nu_0 t)$$







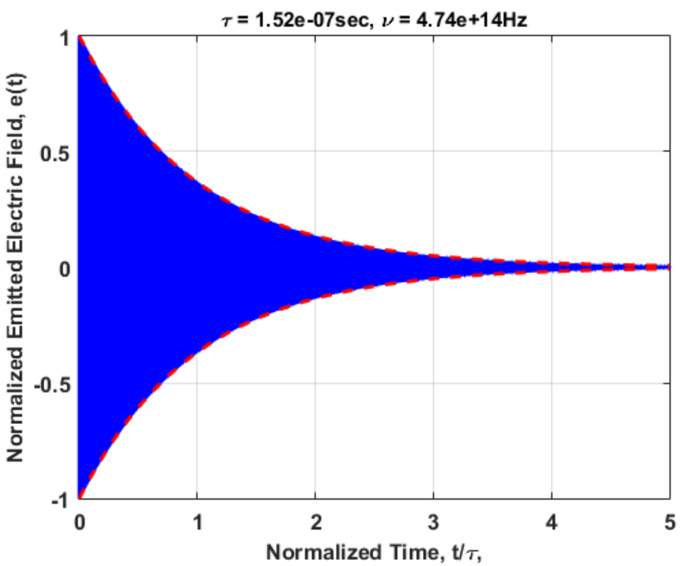


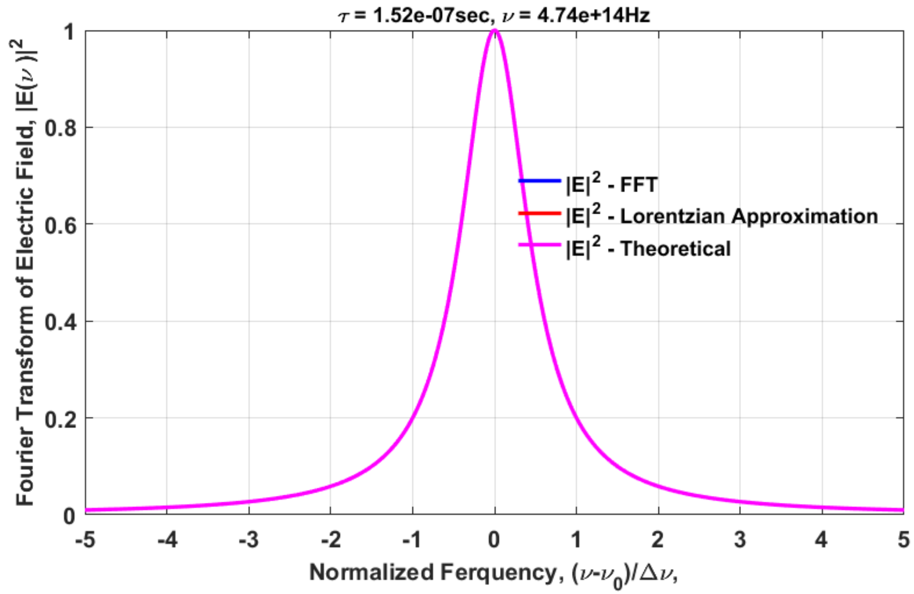
Damped Oscillation

(He-Ne laser transition $\lambda_0 = 0.6328\mu m$)

$$\mathcal{E}(t) = E_0 e^{-t/\tau}\cos(\omega_0 t) \xrightarrow{\mathcal{F}} |E(\omega)|^2 = \left| \frac{\frac{1}{\tau} + j\omega}{\omega_0^2 + (\frac{1}{\tau} + j\omega)^2} \right|^2 \simeq \frac{A}{(\omega - \omega_0)^2 + \left(\frac{1}{\tau}\right)^2}$$

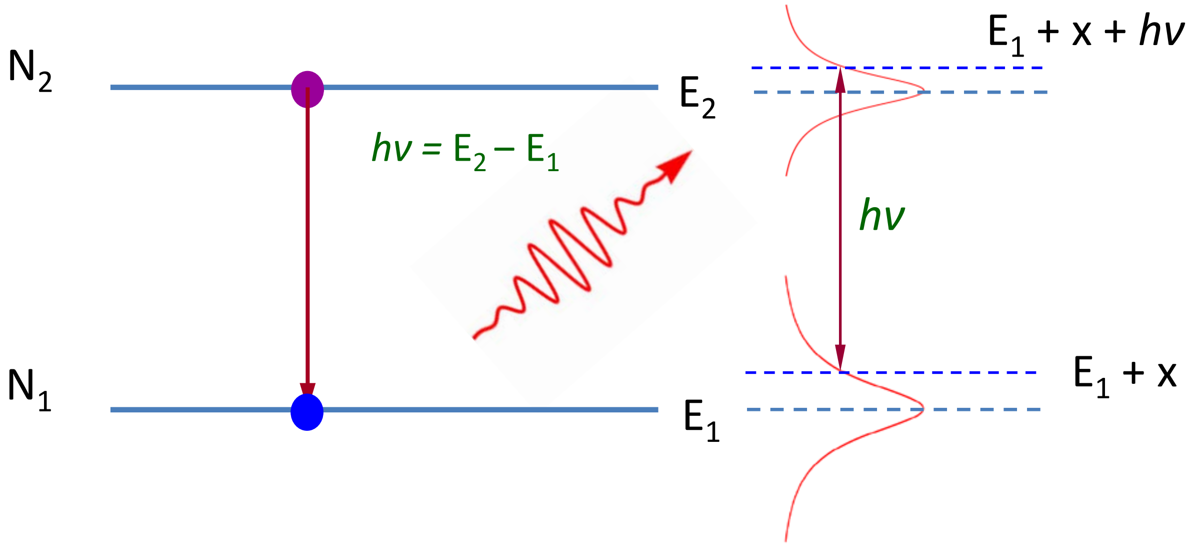
$$\boxed{\Delta\nu = \frac{1}{\pi\tau}}$$





Lineshape Function

Spontaneous Emission Lifetime



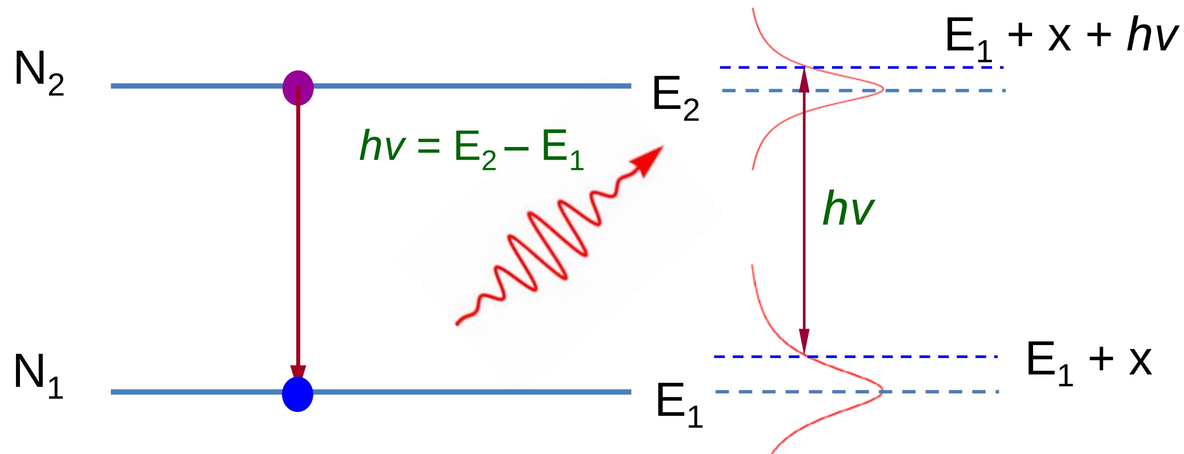
Heisenberg Uncertainty Principle $\Delta E \; \Delta \tau \simeq \hbar$

$$E_1 \longrightarrow \mathcal{L}_1(E) = \frac{\Delta E_1}{2\pi} \frac{1}{(E - E_1)^2 + (\Delta E_1/2)^2}$$

$$E_2 \longrightarrow \mathcal{L}_2(E) = \frac{\Delta E_2}{2\pi} \frac{1}{(E - E_2)^2 + (\Delta E_2/2)^2}$$

Lineshape Function

Spontaneous Emission Lifetime



$$
\begin{aligned}
g(h\nu) &= \int_{-\infty}^{+\infty} \frac{\Delta E_1}{2\pi} \frac{1}{(E_1 + x - E_1)^2 + (\Delta E_1/2)^2} \frac{\Delta E_2}{2\pi} \frac{1}{(E_1 + h\nu + x - E_2)^2 + (\Delta E_2/2)^2} dx \\
&= \int_{-\infty}^{+\infty} \left[\frac{\Delta E_1}{2\pi} \frac{1}{x^2 + (\Delta E_1/2)^2}\right] \left[\frac{\Delta E_2}{2\pi} \frac{1}{(\xi - x)^2 + (\Delta E_2/2)^2}\right] dx \\
&= \mathcal{L}_1 * \mathcal{L}_2 = \mathcal{L}(\xi) = \mathcal{L}(-\xi) = \\
&= \frac{\Delta E_1 + \Delta E_2}{2\pi} \frac{1}{\left[h\nu - (E_2 - E_1)\right]^2 + \left[(\Delta E_1 + \Delta E_2)/2\right]^2}
\end{aligned}
$$

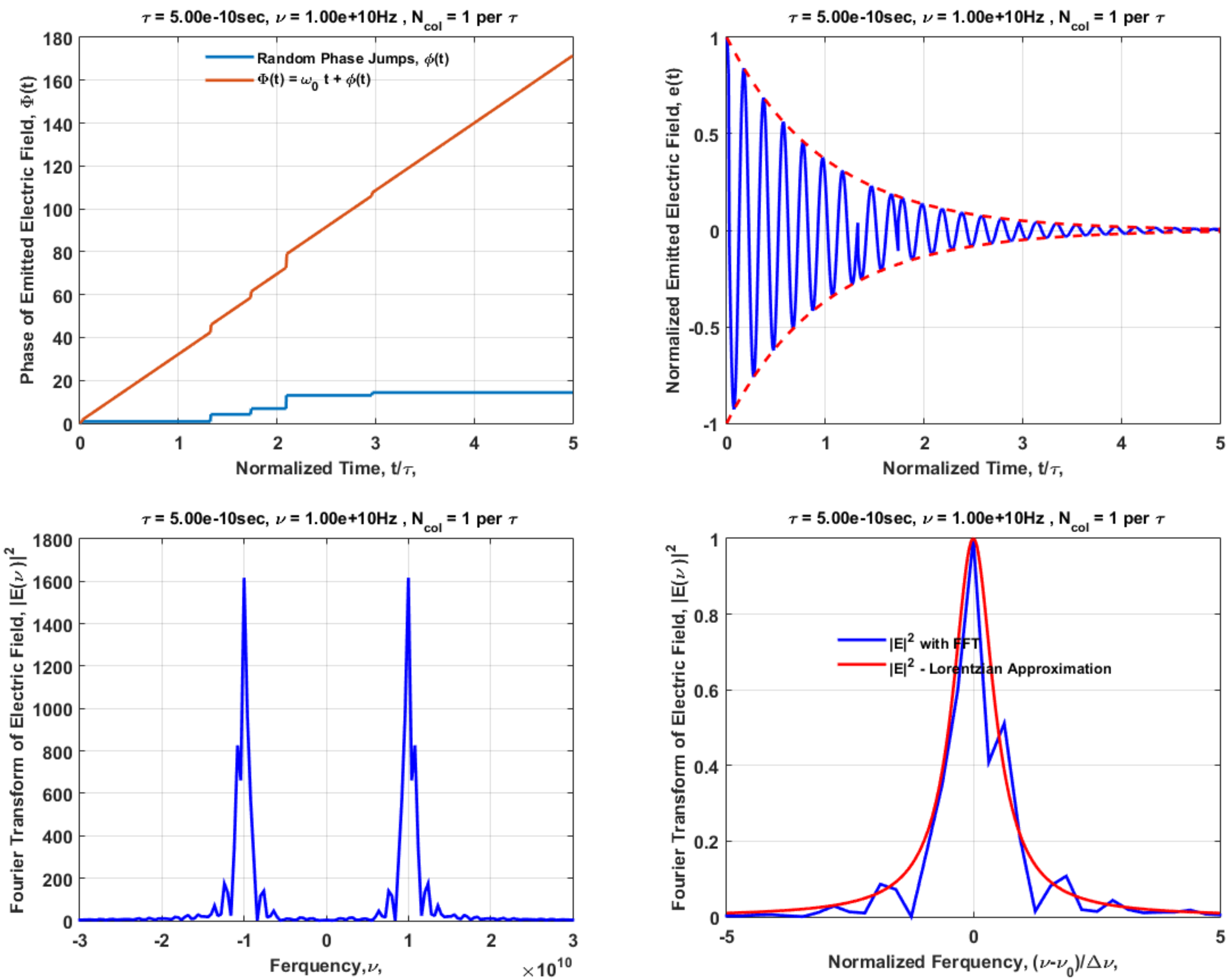
where $\xi = E_2 - E_1 - h\nu,$

$$
\begin{aligned}
\mathcal{F}\{\mathcal{L}_1(E)\} &= e^{-\pi(\Delta E_1/2)|u|}, \\
\mathcal{F}\{\mathcal{L}_2(E)\} &= e^{-\pi(\Delta E_2/2)|u|}, \\
\mathcal{F}\{\mathcal{L}(E)\} &= e^{-\pi[(\Delta E_1/2)+(\Delta E_2/2)]|u|}
\end{aligned}
$$

Damped Oscillation with Elastic Collisions

$$\mathcal{E}(t) = E_0 e^{-t/\tau} \cos\left(\omega_0 t + \phi(t)\right)$$

$$\Delta\nu = \frac{1}{\pi\tau}\left(1 + N_{col}\right)$$



Damped Oscillation with Elastic Collisions

$$\mathcal{E}(t) = E_0 e^{-t/\tau} \cos\left(\omega_0 t + \phi(t)\right)$$

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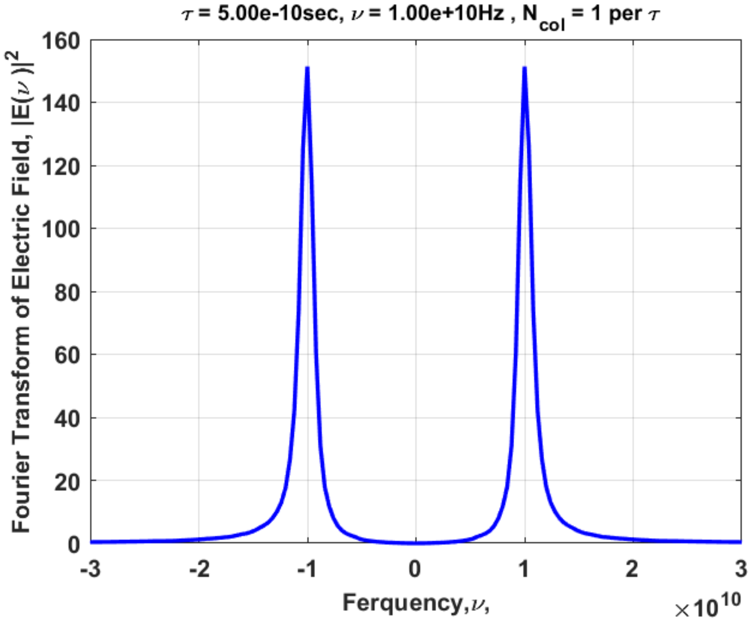
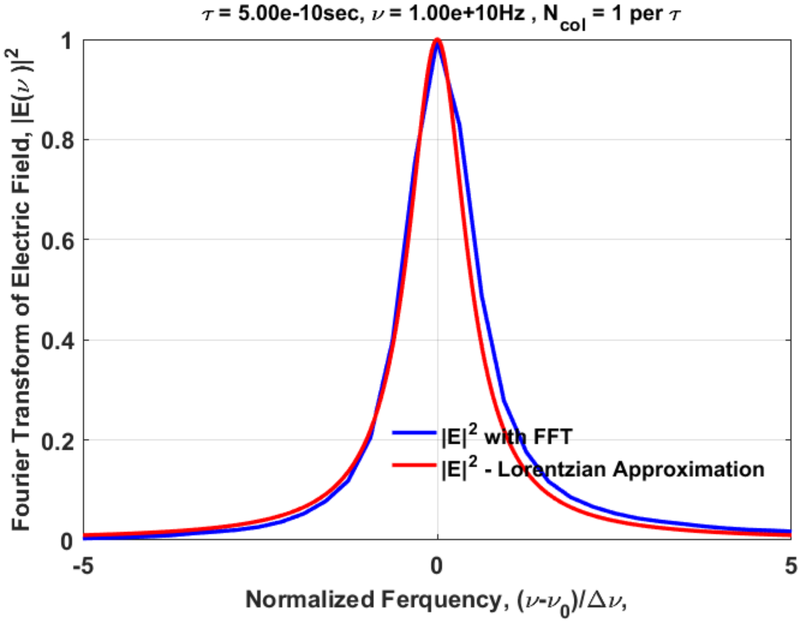
Average over **1000** random events





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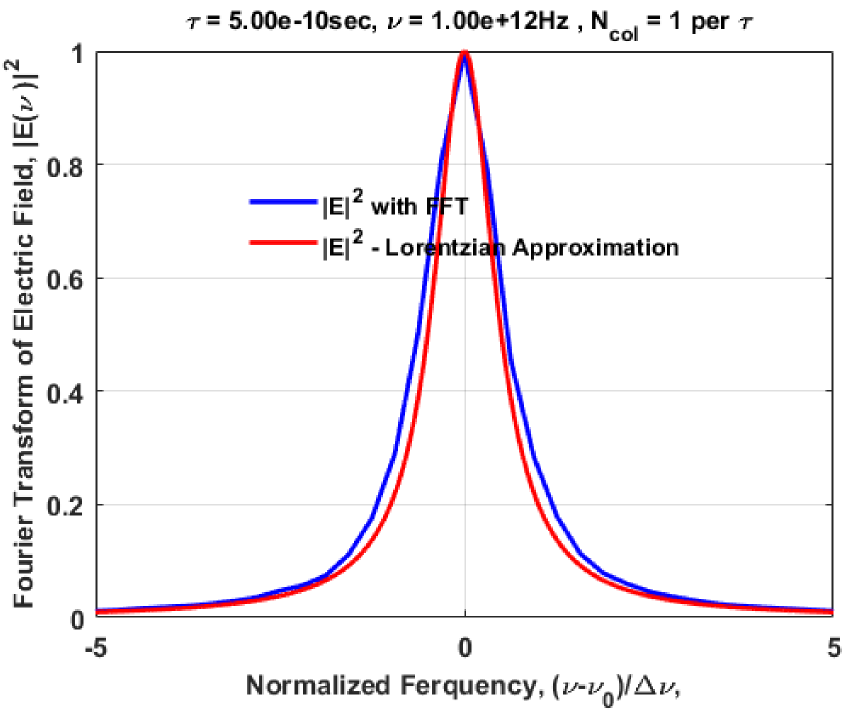

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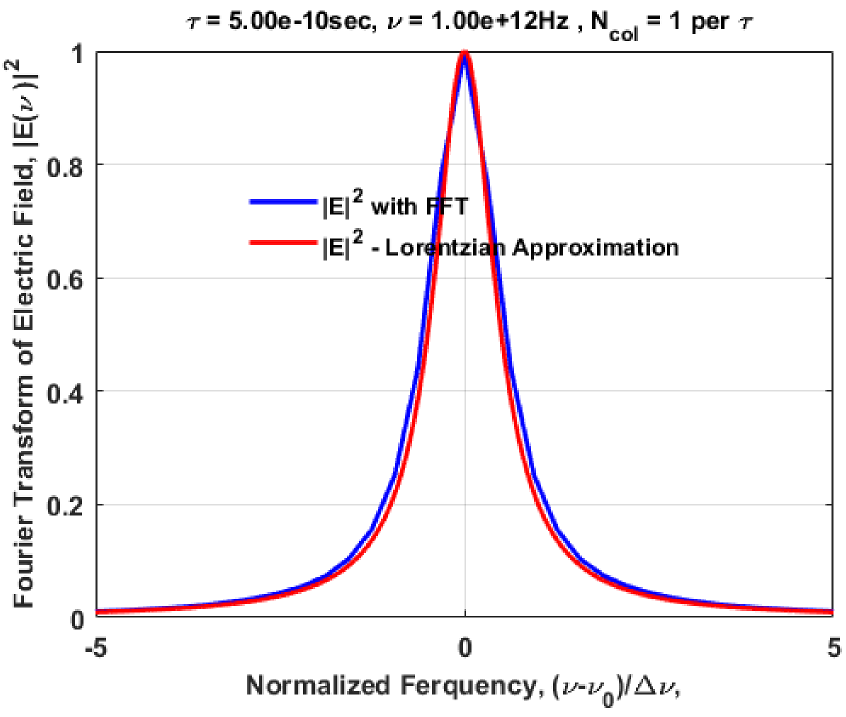

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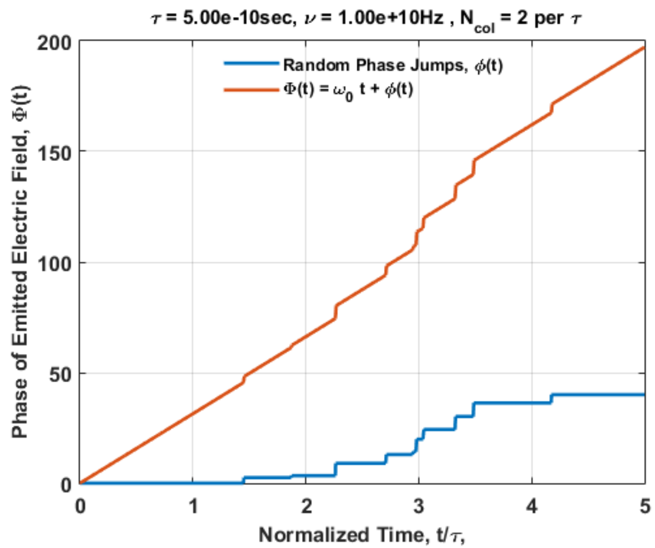

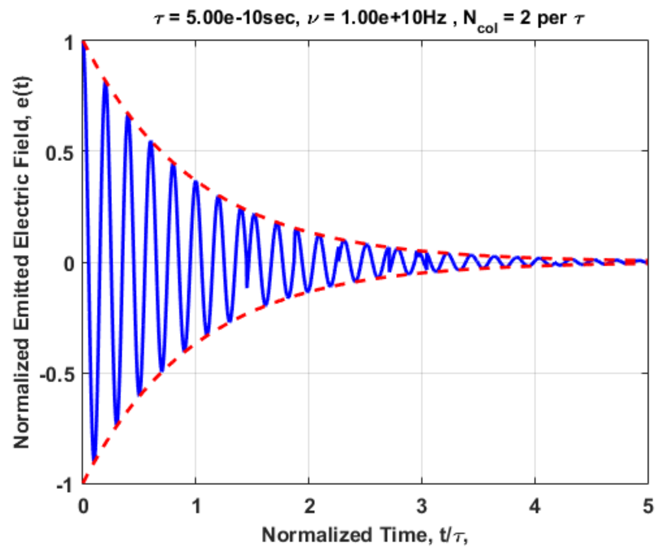

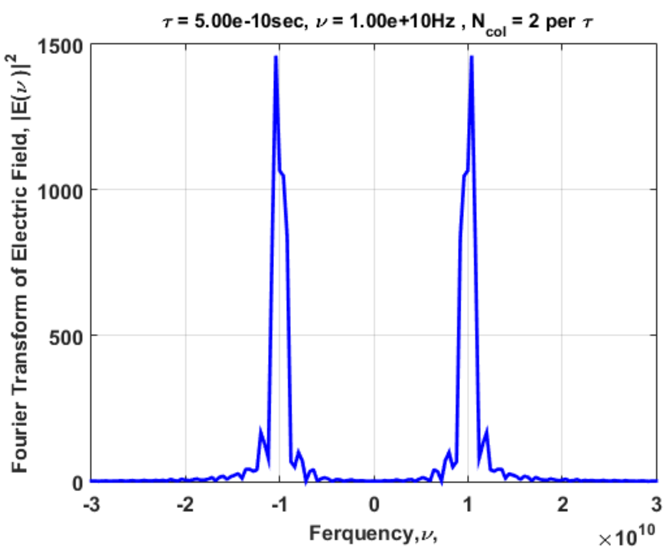

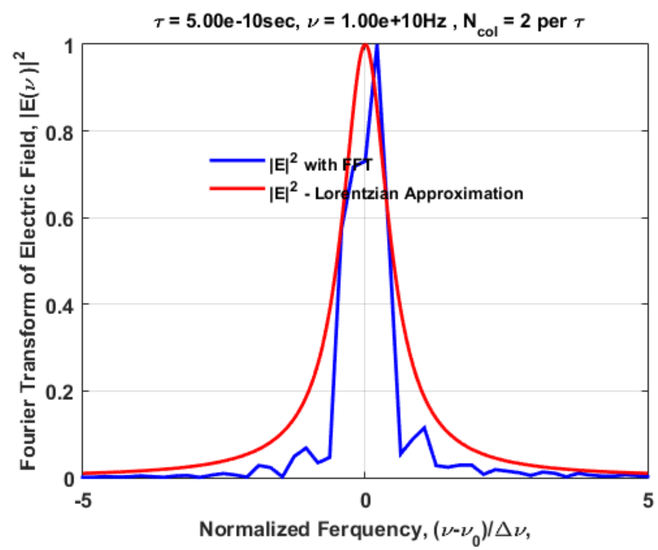

Damped Oscillation with Elastic Collisions

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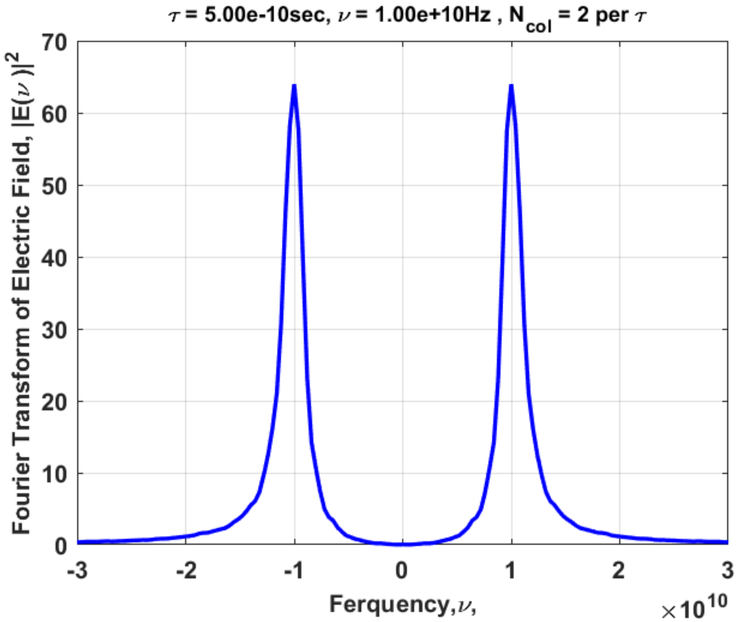
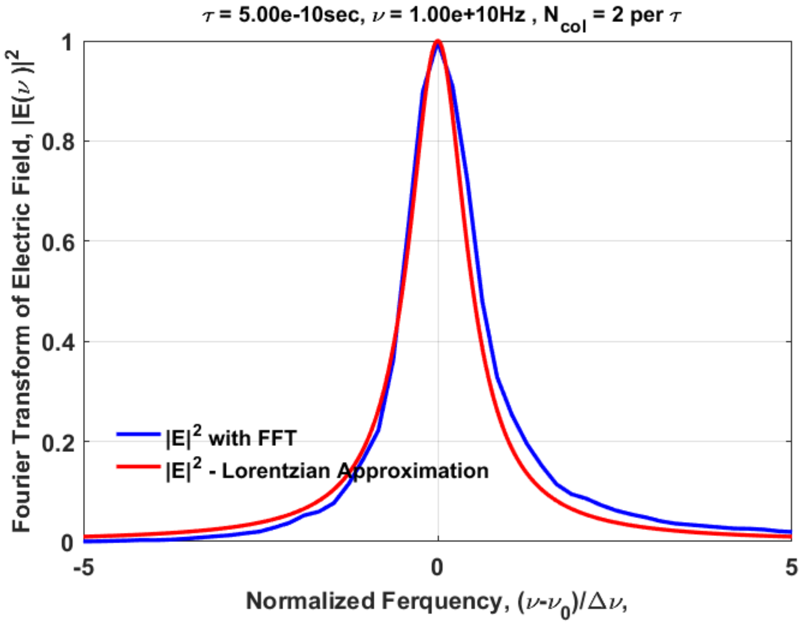




Homogeneous Spectrum Broadening

Characteristics of Homogeneous Broadening

- Each atom in the system has a common emitting spectrum width *Δν.*
- *The lineshape g(ν)* describes the response of any of the atoms (indistinguishable)
- Finite interaction lifetime of the absorbing and emitting atoms

Mechanisms of Homogeneous Broadening

- The spontaneous lifetime of the excited state
- Elastic collisions of an atom with other atoms or with the crystal (phonons)
- Pressure broadening of atoms in a gas

Homogeneous Broadening can be described with a ***Lorentzian Lineshape***

$$g(\nu) = \frac{\Delta\nu}{2\pi} \frac{1}{(\nu - \nu_0)^2 + (\Delta\nu/2)^2}$$

$$\Delta\nu = \frac{\sigma}{2\pi} = \frac{1}{\pi\tau}$$

$$\Delta\nu = \frac{1}{\pi}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_{1c}} + \frac{1}{\tau_{2c}}\right)$$

Inhomogeneous Spectrum Broadening

Features of Inhomogeneous Broadening

- Individual atoms are distinguishable, each having a slightly different frequency due to "seeing" slightly different environment
- The observed spectrum of spontaneous emission reflects the spread in the individual transition frequencies (not only the broadening due to the finite lifetime of the excited state)

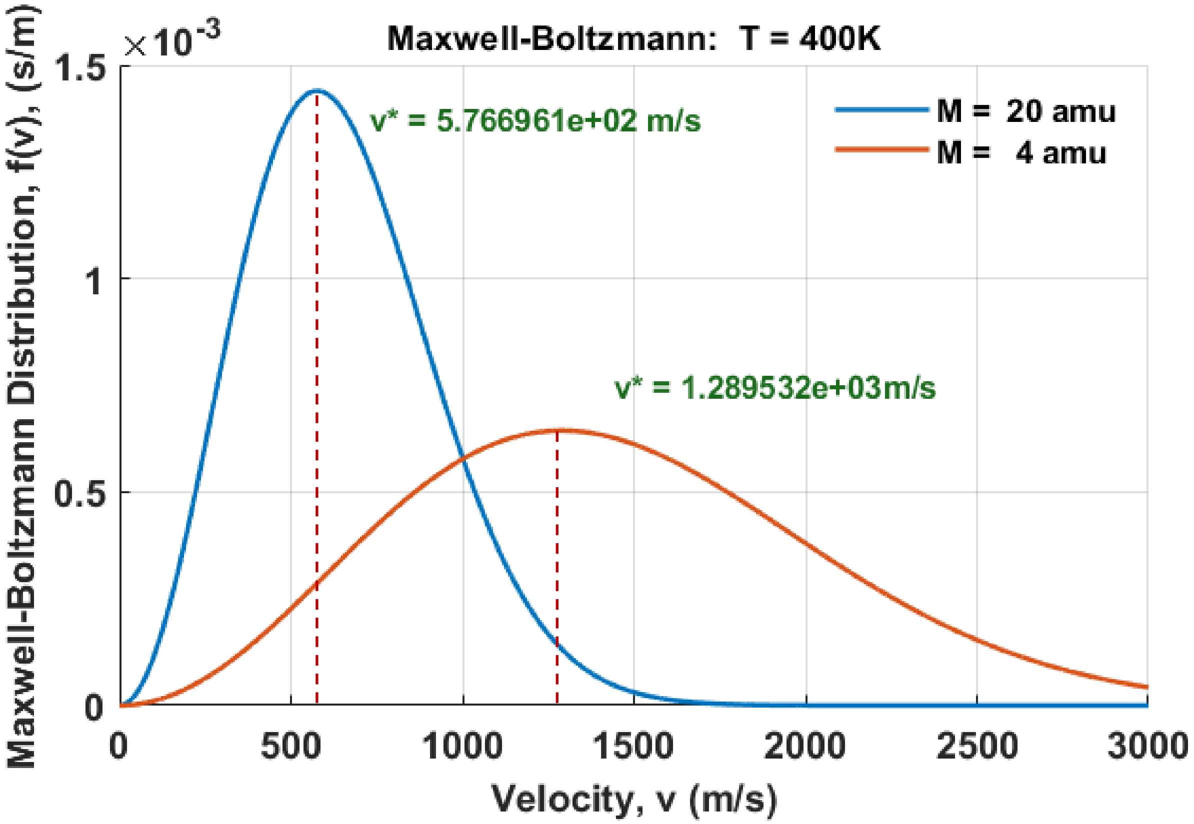
Example Mechanisms of Inhomogeneous Broadening

- The energy levels of impurity in a host crystal
- Random strain
- Crystal imperfections
- Doppler effect in gases

Maxwell-Boltzmann Velocity Distribution

$$f(v)d^3v = \left(\frac{M}{2\pi k_B T}\right)^{3/2} e^{-M|v|^2/2k_B T} d^3v = \left(\frac{M}{2\pi k_B T}\right)^{3/2} 4\pi v^2 e^{-M|v|^2/2k_B T} dv$$

$$|v|^2 = v_x^2 + v_y^2 + v_z^2, \quad \text{and} \quad \int_{v_x=-\infty}^{\infty}\int_{v_y=-\infty}^{\infty}\int_{v_z=-\infty}^{\infty} f(v_x, v_y, v_z)dv_x dv_y dv_z = 1$$



Maxwell-Boltzmann Velocity Distribution

Velocity Distributions along x-axis (laser light propagation)

$$
\begin{aligned}
f(v_x)dv_x &= \left(\frac{M}{2\pi k_B T}\right)^{3/2} \exp\left[-\frac{Mv_x^2}{2k_BT}\right] dv_x \int_{v_y=-\infty}^{\infty}\int_{v_z=-\infty}^{\infty} \exp\left[-\frac{M(v_y^2+v_z^2)}{2k_BT}\right] dv_y dv_z = \\
&= \left(\frac{M}{2\pi k_B T}\right)^{3/2} \exp\left[-\frac{Mv_x^2}{2k_BT}\right] dv_x \, \pi \frac{2k_BT}{M} = \\
&= \left(\frac{M}{2\pi k_B T}\right)^{1/2} \exp\left[-\frac{Mv_x^2}{2k_BT}\right] dv_x
\end{aligned}
$$

Doppler Effect

$$
\nu = \nu_0 \frac{1 \pm \dfrac{v_x}{c}}{\sqrt{1-\left(\dfrac{v_x}{c}\right)^2}} \simeq \nu_0 \left(1 \pm \frac{v_x}{c}\right)
$$

$$
\nu = \nu_0\left(1+\frac{v_x}{c}\right) \Longrightarrow v_x = \frac{c}{\nu_0}(\nu-\nu_0) \quad \text{and} \quad dv_x = \frac{c}{\nu_0} d\nu
$$

Inhomogeneous Spectrum Broadening

Velocity Distributions along x-axis (laser light propagation)

$$
\begin{aligned}
f(v_x)dv_x &= \left(\frac{M}{2\pi k_B T}\right)^{1/2} \exp\left[-\frac{M v_x^2}{2k_B T}\right] dv_x \quad \Longrightarrow \\
f(\nu)d\nu &= \left(\frac{M}{2\pi k_B T}\right)^{1/2} \exp\left[-\frac{M}{2k_B T}\left(\frac{c}{\nu_0}\right)^2 (\nu-\nu_0)^2\right] \frac{c}{\nu_0} d\nu
\end{aligned}
$$

Lorentzian Lineshape for each velocity group

$$
g_H(\nu) = \frac{(\Delta\nu_H/2\pi)}{(\nu-\nu')^2 + (\Delta\nu_H/2)^2}, \qquad \nu' = \nu_0\left(1 + \frac{v_x}{c}\right)
$$

Maxwell-Boltzmann velocity distribution

$$
\frac{dN}{N} = \left(\frac{M}{2\pi k_B T}\right)^{1/2} \exp\left(-\frac{M v_x^2}{2k_B T}\right) dv_x
$$

Inhomogeneous Spectrum Broadening

Average Lineshape Function

$$\Delta\nu_G = 2\nu_0\left(\frac{2k_BT}{Mc^2}\ln(2)\right)^{1/2}$$

$$\begin{aligned}
g_D(\nu) &= \int_{-\infty}^{+\infty} g_H(\nu)\frac{dN}{N} \\
&= \int_{-\infty}^{+\infty}\frac{(\Delta\nu_H/2\pi)}{(\nu-\nu')^2+(\Delta\nu_H/2)^2}\left(\frac{M}{2\pi k_BT}\right)^{1/2}\exp\left(-\frac{Mc^2}{2k_BT\nu_0^2}(\nu'-\nu_0)^2\right)\frac{c}{\nu_0}d\nu' \\
&= \int_{-\infty}^{+\infty}\underbrace{\left\{\frac{(\Delta\nu_H/2\pi)}{(\nu-\nu')^2+(\Delta\nu_H/2)^2}\right\}}_{g_H(\nu')}\underbrace{\left\{\frac{1}{\Delta\nu_G/2}\left(\frac{\ln(2)}{\pi}\right)^{1/2}\exp\left(-\ln(2)\left(\frac{(\nu'-\nu_0)}{\Delta\nu_G/2}\right)^2\right)\right\}}_{g_{MB}(\nu')}d\nu' \\
&= g_H * g_{MB}
\end{aligned}$$

Voigt Lineshape Function

$$\begin{aligned}
g_H * g_{MB}(\nu) &= \underbrace{\frac{\sqrt{(\ln 2)/\pi}}{\Delta\nu_G/2}\mathcal{K}\left[x(\nu),y\right]}_{Voigt\ \ function} = \frac{\sqrt{(\ln 2)/\pi}}{\Delta\nu_G/2}\frac{y}{\pi}\int_{-\infty}^{+\infty}\frac{e^{-t^2}}{[x(\nu)-t]^2+y^2}dt \\
x(\nu) &= \sqrt{\ln 2}\,\frac{\nu-\nu_0}{\Delta\nu_G/2} \\
y &= \sqrt{\ln 2}\,\frac{\Delta\nu_H}{\Delta\nu_G}
\end{aligned}$$

F. Schreir, “Optimized implementations of rational approximations for the Voigt and complex error function”, Journal of Quantitative Spectroscopy & Radiative Transfer 112 (2011) 1010–1025

Inhomogeneous Spectrum Broadening

If the Lorentzian Lineshape is much narrower than the Maxwell-Boltzmann then:

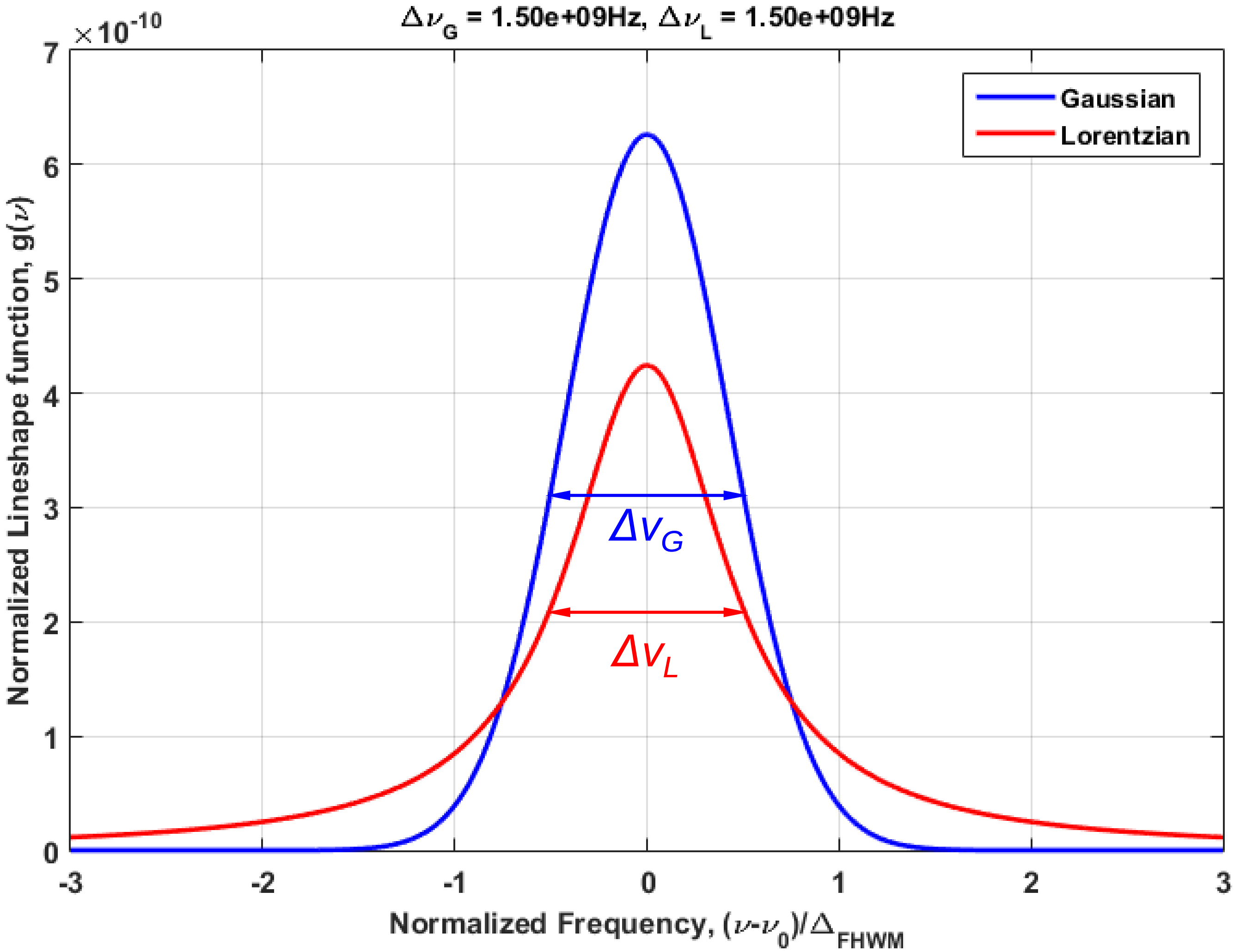
$$g_D(\nu) \simeq \frac{c}{\nu_0}\left(\frac{M}{2\pi k_B T}\right)^{1/2} \exp\left(-\frac{M}{2k_B T}\left(\frac{c}{\nu_0}\right)^2 (\nu-\nu_0)^2\right)$$

$$g_D(\nu) = \frac{2}{\Delta\nu_D}\left(\frac{\ln(2)}{\pi}\right)^{1/2} \exp\left(-\ln(2)\left(\frac{2(\nu-\nu_0)}{\Delta\nu_D}\right)^2\right)$$

$$\Delta\nu_D = 2\nu_0\left(\frac{2k_B T}{Mc^2}\ln(2)\right)^{1/2}$$

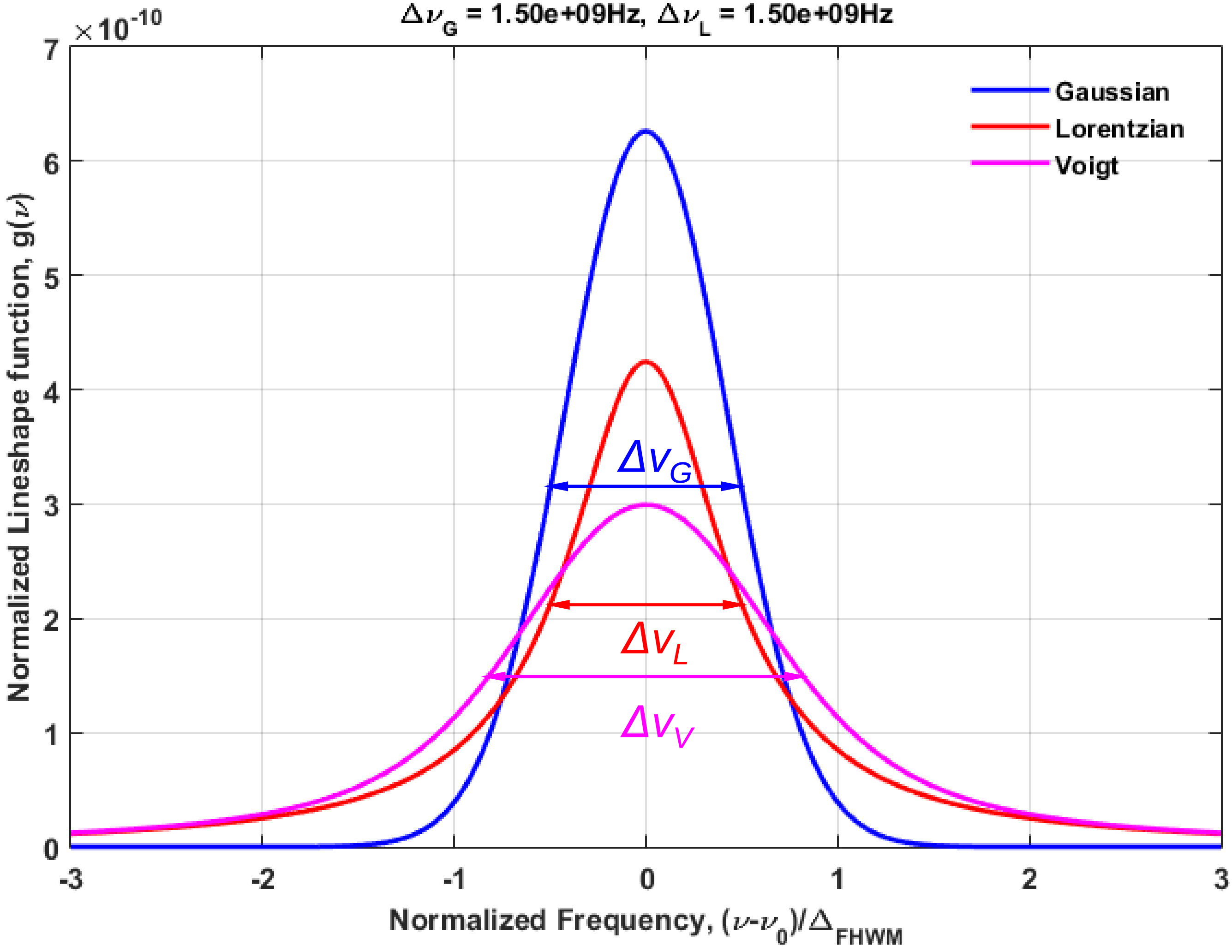
Inhomogeneous Spectrum Broadening

Average Lineshape Function Comparison



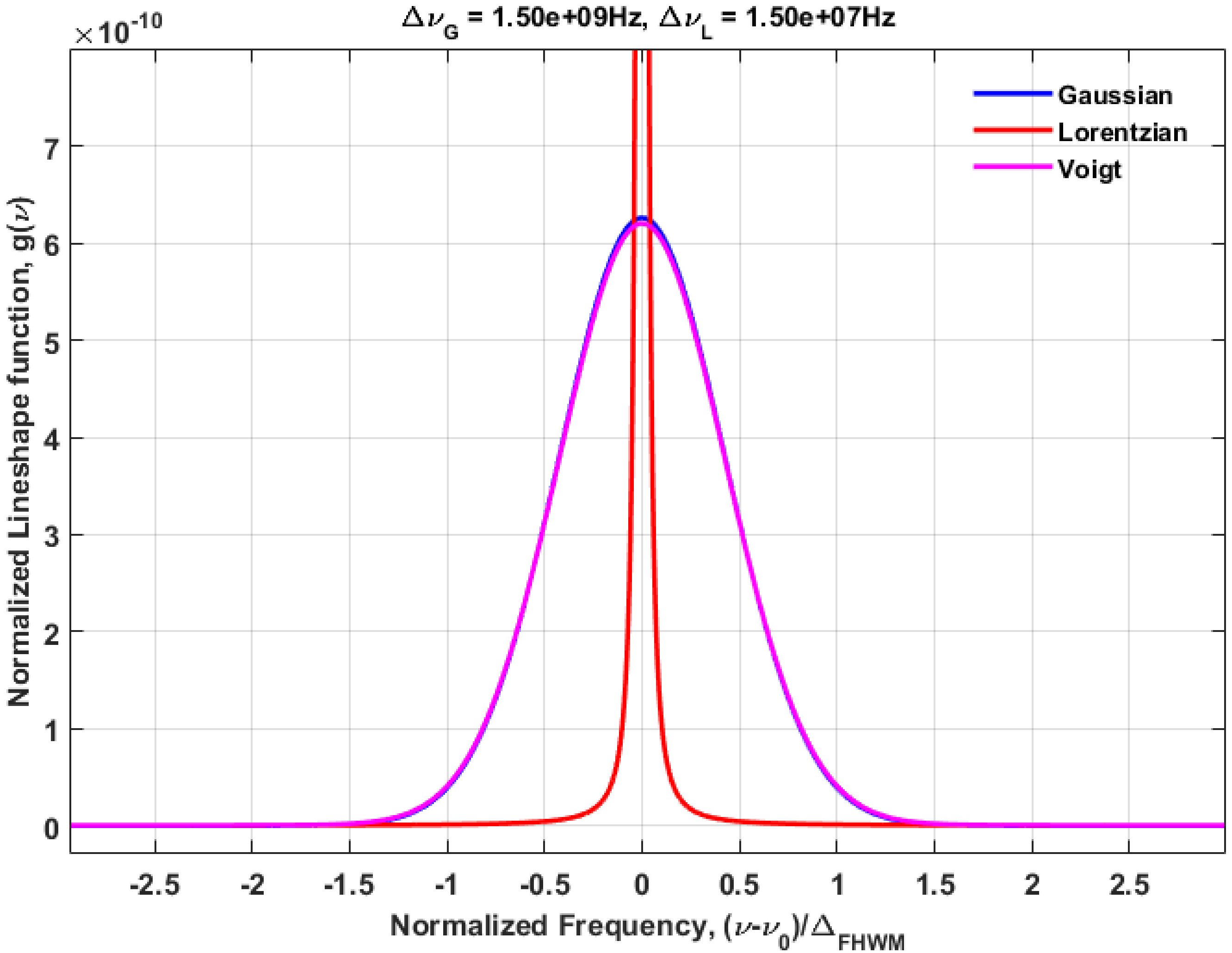
Inhomogeneous Spectrum Broadening

Average Lineshape Function Comparison

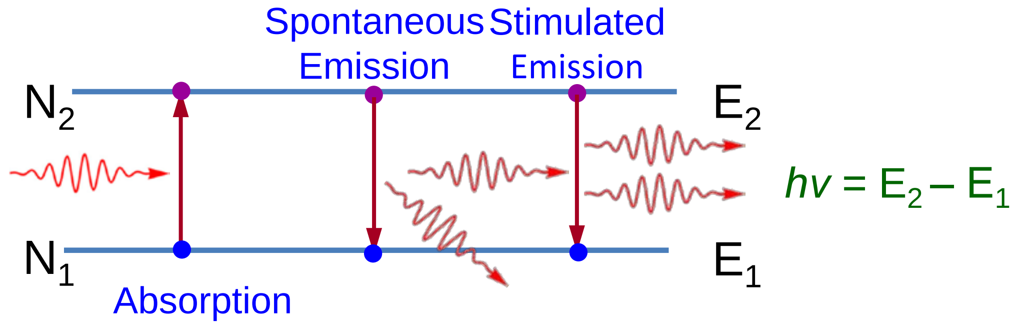


Inhomogeneous Spectrum Broadening

Average Lineshape Function Comparison



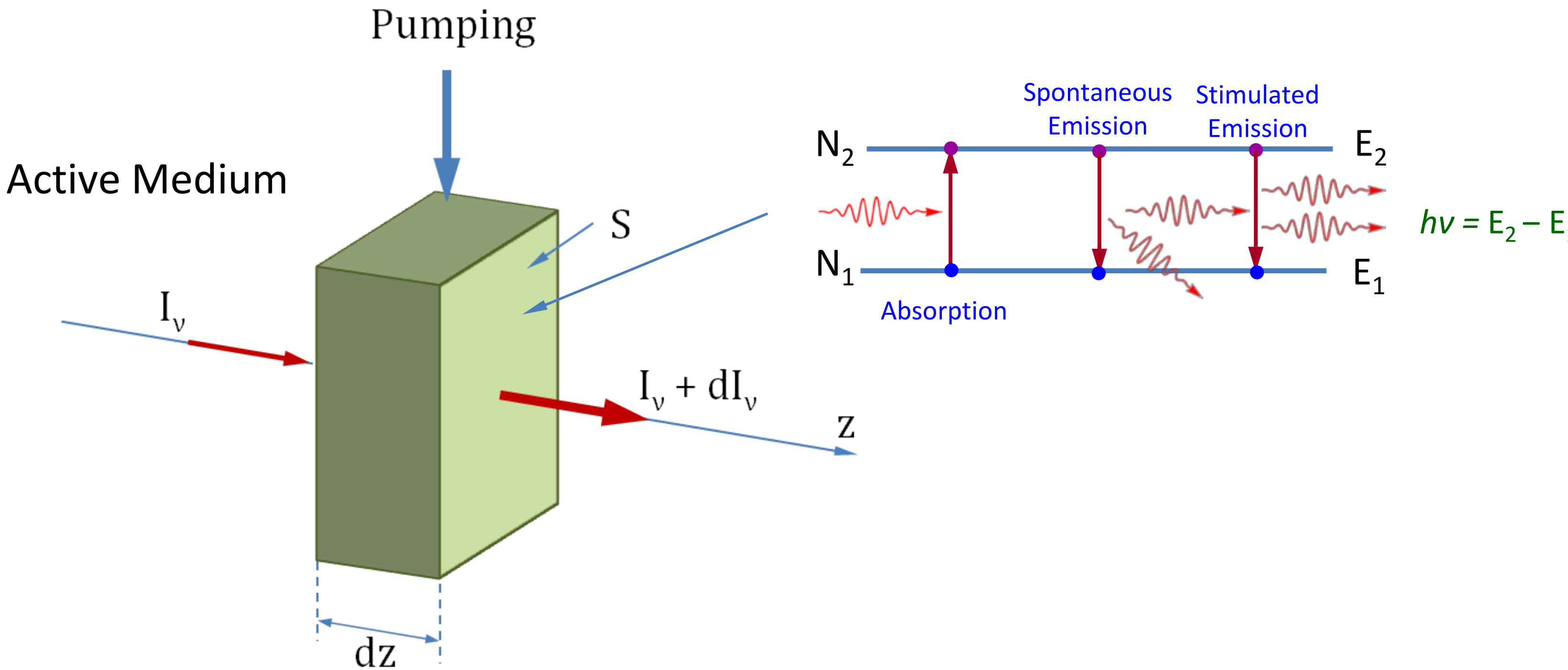
Stimulated Emission Cross-Section and Induced Rate



$$
\begin{aligned}
\frac{dN_2}{dt} &= -A_{21}N_2 - B_{21}\rho(\nu)N_2 + B_{12}\rho(\nu)N_1 \xrightarrow{g(\nu')d\nu'} \\
&= -A_{21}N_2 \underbrace{\int_{-\infty}^{+\infty} g(\nu')d\nu'}_{1} - B_{21}N_2 \int_{-\infty}^{+\infty} \rho(\nu')g(\nu')d\nu' + B_{12}N_1 \int_{-\infty}^{+\infty} \rho(\nu')g(\nu')d\nu' \\
&= -A_{21}N_2 - B_{21}N_2 \int_{-\infty}^{+\infty} \underbrace{\rho_\nu \delta(\nu' - \nu)}_{\substack{\text{Laser} \\ \text{Light Density}}} g(\nu')d\nu' + B_{12}N_1 \int_{-\infty}^{+\infty} \underbrace{\rho_\nu \delta(\nu' - \nu)}_{\substack{\text{Laser} \\ \text{Light Density}}} g(\nu')d\nu' \\
&= -A_{21}N_2 - B_{21}\rho_\nu g(\nu)N_2 + -B_{21}\rho_\nu g(\nu)N_1 \\
&= -A_{21}N_2 - B_{21}\rho_\nu g(\nu)\left[N_2 - \frac{g_2}{g_1}N_1\right] = -A_{21}N_2 - B_{21}I_\nu \frac{n}{c} g(\nu)\left[N_2 - \frac{g_2}{g_1}N_1\right] \\
&= -A_{21}N_2 - A_{21}\frac{c^3}{8\pi n^3 h\nu^3} I_\nu \frac{n}{c} g(\nu)\left[N_2 - \frac{g_2}{g_1}N_1\right] \\
&= -A_{21}N_2 - \underbrace{\left[A_{21}\frac{\lambda_0^2}{8\pi n^2} g(\nu)\right]}_{\substack{\sigma_{SE}(\nu) \\ \text{Stimulated Emission} \\ \text{Cross-section}}} \frac{I_\nu}{h\nu}\left[N_2 - \frac{g_2}{g_1}N_1\right] \\
&= -A_{21}N_2 - \underbrace{\sigma_{SE}(\nu)\frac{I_\nu}{h\nu}}_{\substack{W_i \\ \text{Induced Rate}}}\left[N_2 - \frac{g_2}{g_1}N_1\right] \\
&= \underbrace{-A_{21}N_2}_{\substack{\text{Spontaneous} \\ \text{Emission}}} - \underbrace{W_i\left[N_2 - \frac{g_2}{g_1}N_1\right]}_{\substack{\text{Stimulated Emission} \\ -\text{Absorption}}}
\end{aligned}
$$

$$
\begin{aligned}
\sigma_{SE}(\nu) &= A_{21}\frac{\lambda_0^2}{8\pi n^2} g(\nu) \\
W_i &= \sigma_{SE}(\nu)\frac{I_\nu}{h\nu}
\end{aligned}
$$

Gain Definition

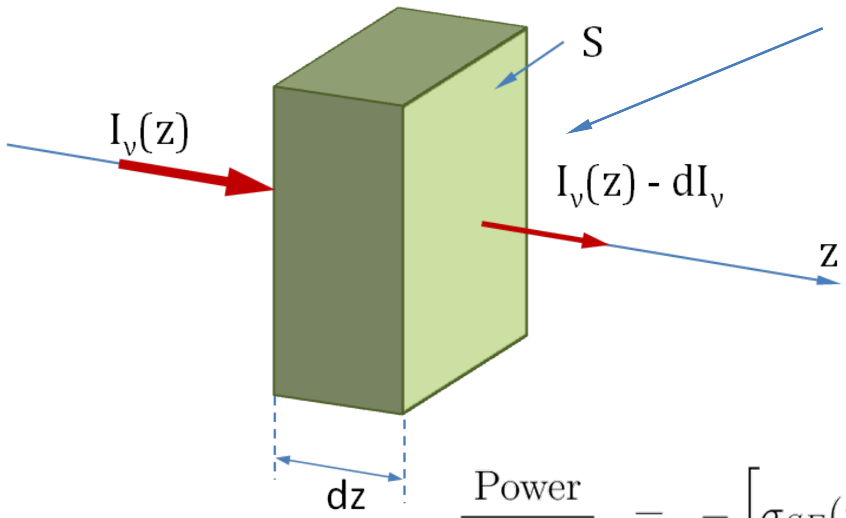


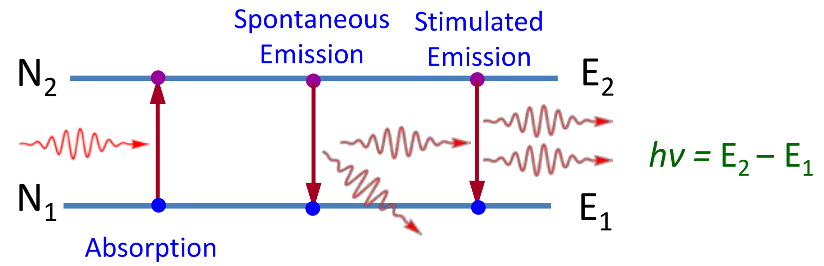
$$\frac{\text{Power}}{\text{Volume}} = \underbrace{\left[\sigma_{SE}(\nu)\frac{I_\nu}{h\nu}\right]\left[N_2 - \frac{g_2}{g_1}N_1\right]}_{\text{Rate of Produced Photons}} \underbrace{h\nu}_{\substack{\text{Energy per}\\ \text{Photon}}} \Longrightarrow$$

$$\frac{dP}{dV} = \underbrace{\sigma_{SE}(\nu)\left[N_2 - \frac{g_2}{g_1}N_1\right]}_{\text{Gain, } \gamma(\nu)} I_\nu$$

$$\frac{dP}{Sdz} = \boxed{\frac{dI_\nu}{dz} = \gamma(\nu) I_\nu}$$

Absorption Definition





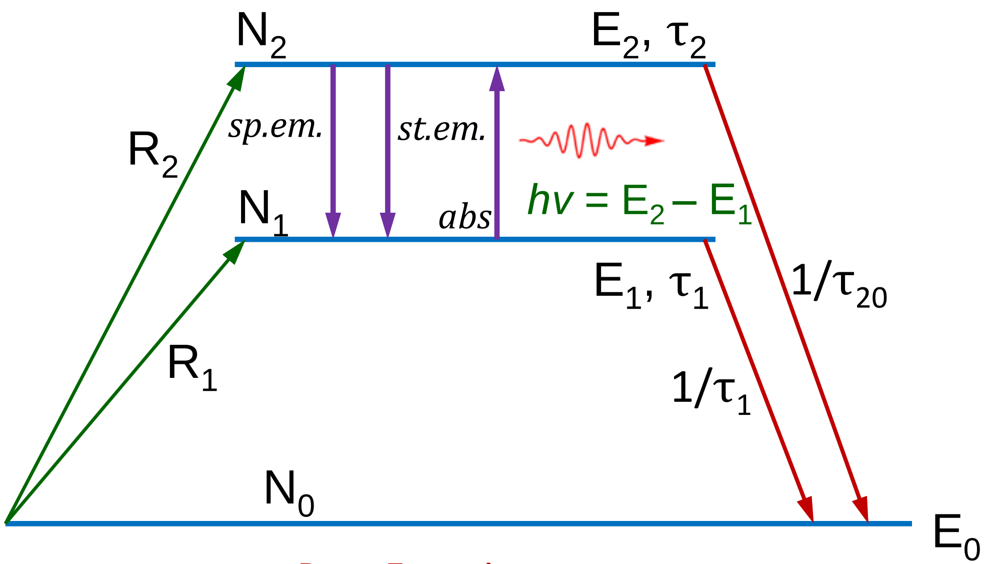
$$\frac{\text{Power}}{\text{Volume}} = -\underbrace{\left[\sigma_{SE}(\nu)\frac{g_2}{g_1}\frac{I_\nu}{h\nu}\right]\left[N_1 - \frac{g_1}{g_2}N_2\right]}_{\text{Rate of Absorbed Photons}} \underbrace{h\nu}_{\substack{\text{Energy per}\\ \text{Photon}}} \Longrightarrow$$

$$\sigma_{abs}(\nu) = \sigma_{SE}(\nu)\frac{g_2}{g_1},$$

$$\frac{dP}{dV} = \underbrace{\sigma_{abs}(\nu)\left[N_1 - \frac{g_1}{g_2}N_2\right]}_{\text{Absorption, }\alpha(\nu)} I_\nu$$

$$\frac{dP}{Sdz} = \boxed{\frac{dI_\nu}{dz} = -\alpha(\nu)I_\nu}$$

Gain Saturation



Rate Equations

$$\frac{dN_2}{dt} = R_2 - A_{21}N_2 - W_i(N_2 - N_1) - \frac{N_2}{\tau_{20}} \qquad \left(W_i = \sigma(\nu)\frac{I_n}{h\nu}\right)$$

$$= R_2 - \frac{N_2}{\tau_2} - W_i(N_2 - N_1) \qquad \left(\frac{1}{\tau_2} = \frac{1}{\tau_{20}} + A_{21}\right)$$

$$\frac{dN_1}{dt} = R_1 + W_i(N_2 - N_1) - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}} \qquad \left(A_{21} = \frac{1}{\tau_{21}}\right)$$

$$\frac{dN_0}{dt} = -R_1 - R_2 + \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{20}}$$

$$\frac{d}{dt}\left[N_2(t) + N_1(t) + N_0(t)\right] = 0 \Longrightarrow N_2(t) + N_1(t) + N_0(t) = constant$$

Mass Conservation

Gain Saturation

Steady-State Solution $\dfrac{dN_1}{dt} = \dfrac{dN_2}{dt} = 0$

$$
\begin{aligned}
N_2 &= \frac{1}{\mathcal{D}}\left[R_2\left(\frac{1}{\tau_1} + \frac{\sigma I_\nu}{h\nu}\right) + R_1\left(\frac{\sigma I_\nu}{h\nu}\right)\right] \\
N_1 &= \frac{1}{\mathcal{D}}\left[R_1\left(\frac{1}{\tau_2} + \frac{\sigma I_\nu}{h\nu}\right) + R_2\left(\frac{1}{\tau_{21}} + \frac{\sigma I_\nu}{h\nu}\right)\right] \\
\mathcal{D} &= \frac{1}{\tau_1\tau_2}\left[1 + \left(\tau_1 + \tau_2 - \frac{\tau_1\tau_2}{\tau_{21}}\right)\frac{\sigma I_\nu}{h\nu}\right]
\end{aligned}
$$

$$
\begin{aligned}
N_2 - N_1 &= \frac{1}{\mathcal{D}}\left[\frac{R_2}{\tau_1} - \frac{R_1}{\tau_2} - \frac{R_2}{\tau_{21}}\right] \\
&= \frac{R_2\left(\tau_2 - \dfrac{\tau_1\tau_2}{\tau_{21}} - \dfrac{R_1}{R_2}\tau_1\right)}{1 + \dfrac{I_\nu}{I_s(\nu)}}
\end{aligned}
$$

$$
I_s(\nu) = \frac{h\nu}{\sigma\left(\tau_1 + \tau_2 - \dfrac{\tau_1\tau_2}{\tau_{21}}\right)}
$$

Gain Saturation

Below Threshold, $I_\nu \approx 0$, $R_1 = 0$

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2}$$

$$\frac{dN_1}{dt} = \frac{N_2}{\tau_{21}} - \frac{N_1}{\tau_1}$$

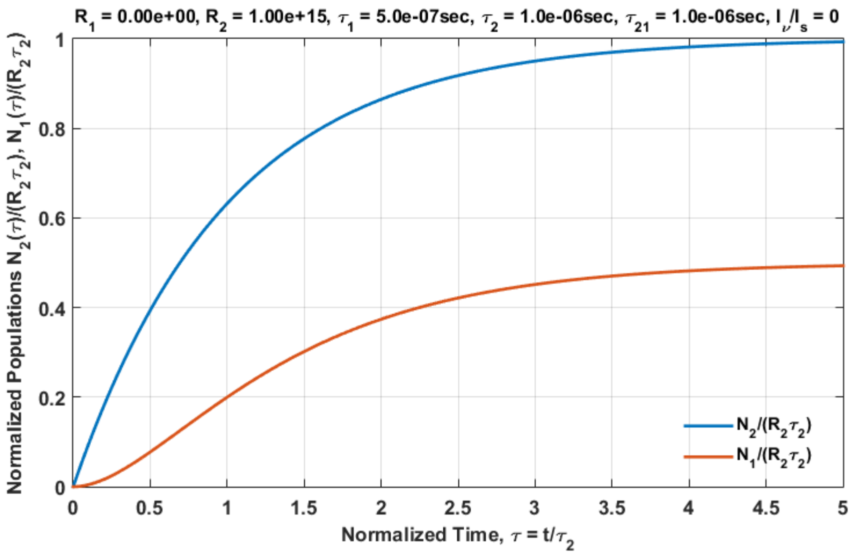
$$R_2(t) = R_{20}u(t)$$

$$N_2(t) = R_{20}\tau_2\left(1 - e^{-t/\tau_2}\right)u(t)$$

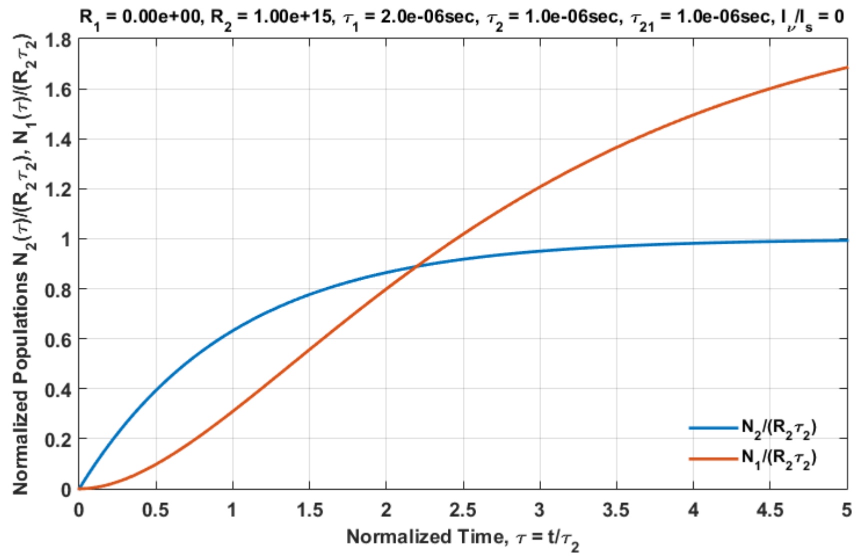
$$N_1(t) = \phi_{21}R_{20}\tau_1\left[1 + \frac{\tau_1/\tau_2}{1-\tau_1/\tau_2}e^{-t/\tau_1} - \frac{1}{1-\tau_1/\tau_2}e^{-t/\tau_2}\right]u(t)$$

$$\phi_{21} = \frac{1/\tau_{21}}{1/\tau_2} = \frac{\tau_2}{\tau_{21}} \qquad \text{(Branching Ratio)}$$

$\tau_2/\tau_1 = 2$



$\tau_2/\tau_1 = 0.5$



Gain Saturation

Above Threshold, $I_\nu \neq 0$, $R_1 = 0$, $\tau_1 = 0$

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2}\left[1 + \frac{\sigma\tau_2}{h\nu} I_\nu\right]$$

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_2}\left[1 + \frac{I_\nu}{I_s}\right] \quad \text{where} \quad \underbrace{I_s = \frac{h\nu}{\sigma\tau_2}}_{\text{Saturation Intensity}}$$

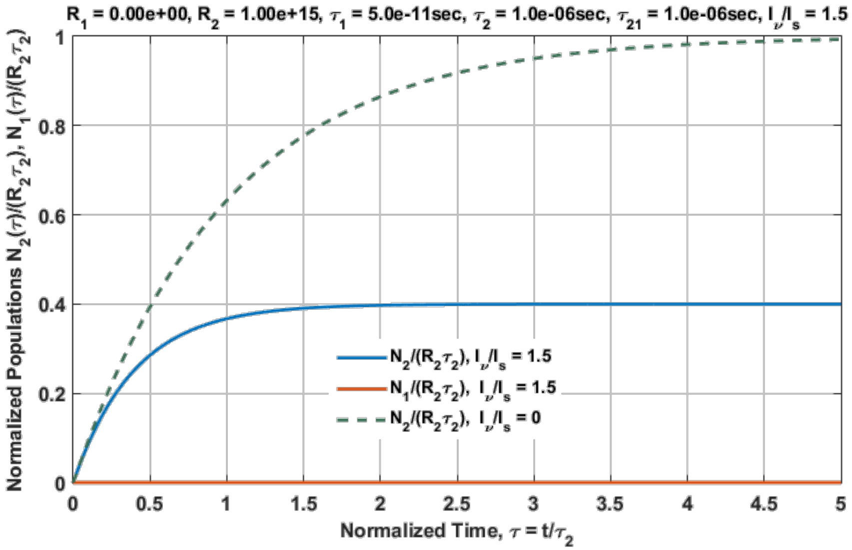
$$\frac{dN_1}{dt} = \frac{N_2}{\tau_{21}} - \frac{N_1}{\tau_1 = 0} \Longrightarrow N_1(t) = 0$$

$$R_2(t) = R_{20}u(t)$$

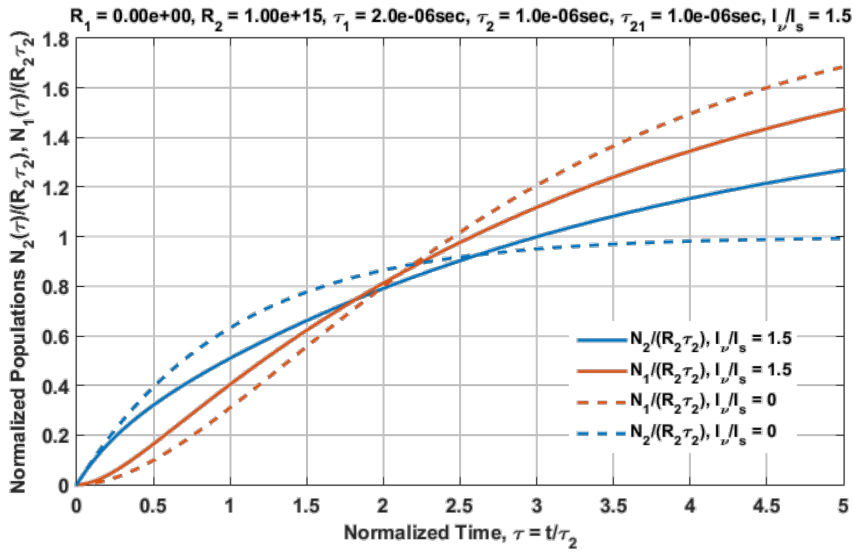
$$N_2(t) = \frac{R_{20}\tau_2}{1 + \frac{I_\nu}{I_s}}\left\{1 - \exp\left[-\frac{t}{\tau_2}\left(1 + \frac{I_\nu}{I_s}\right)\right]\right\} u(t)$$

$$N_1(t) = 0$$

$\tau_2/\tau_1 = 2\cdot10^{+6}$



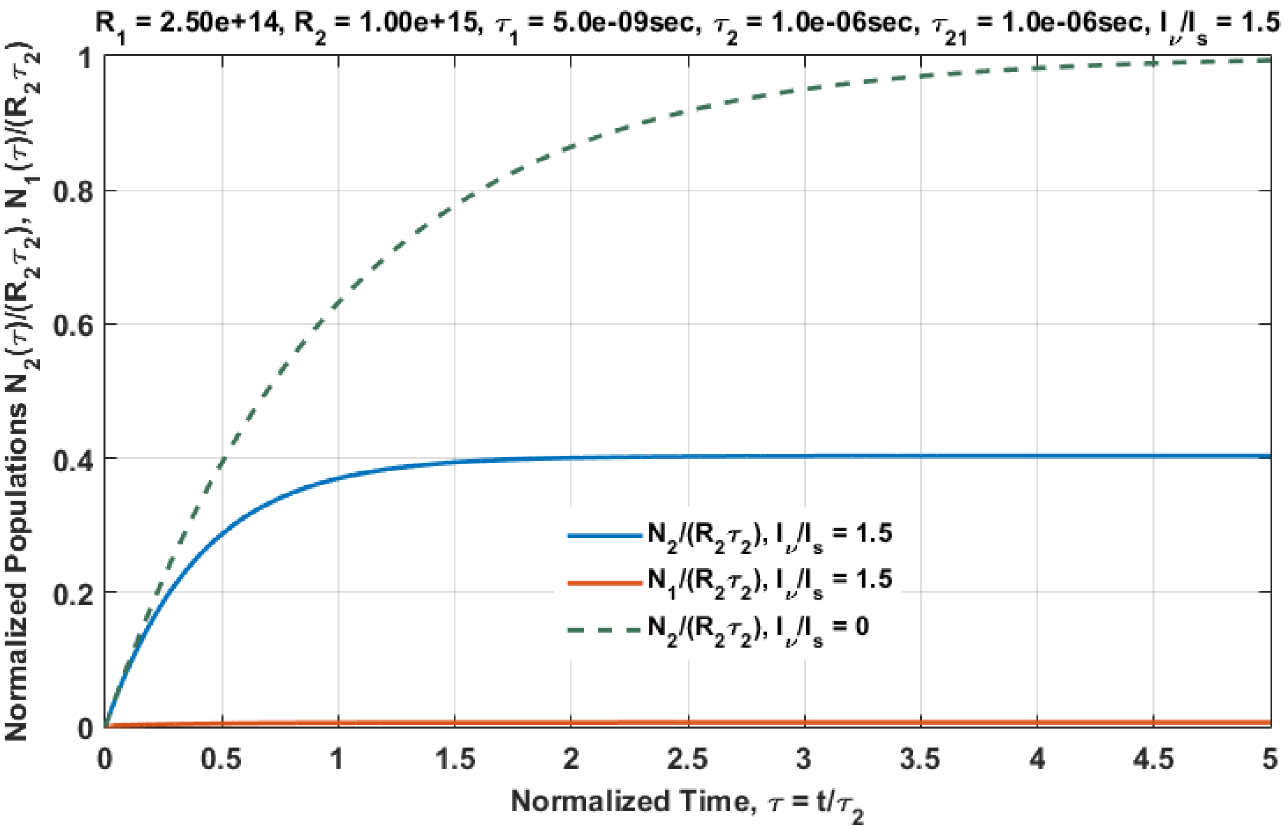
$\tau_2/\tau_1 = 0.5$



Gain Saturation

Above Threshold, $I_\nu \neq 0$

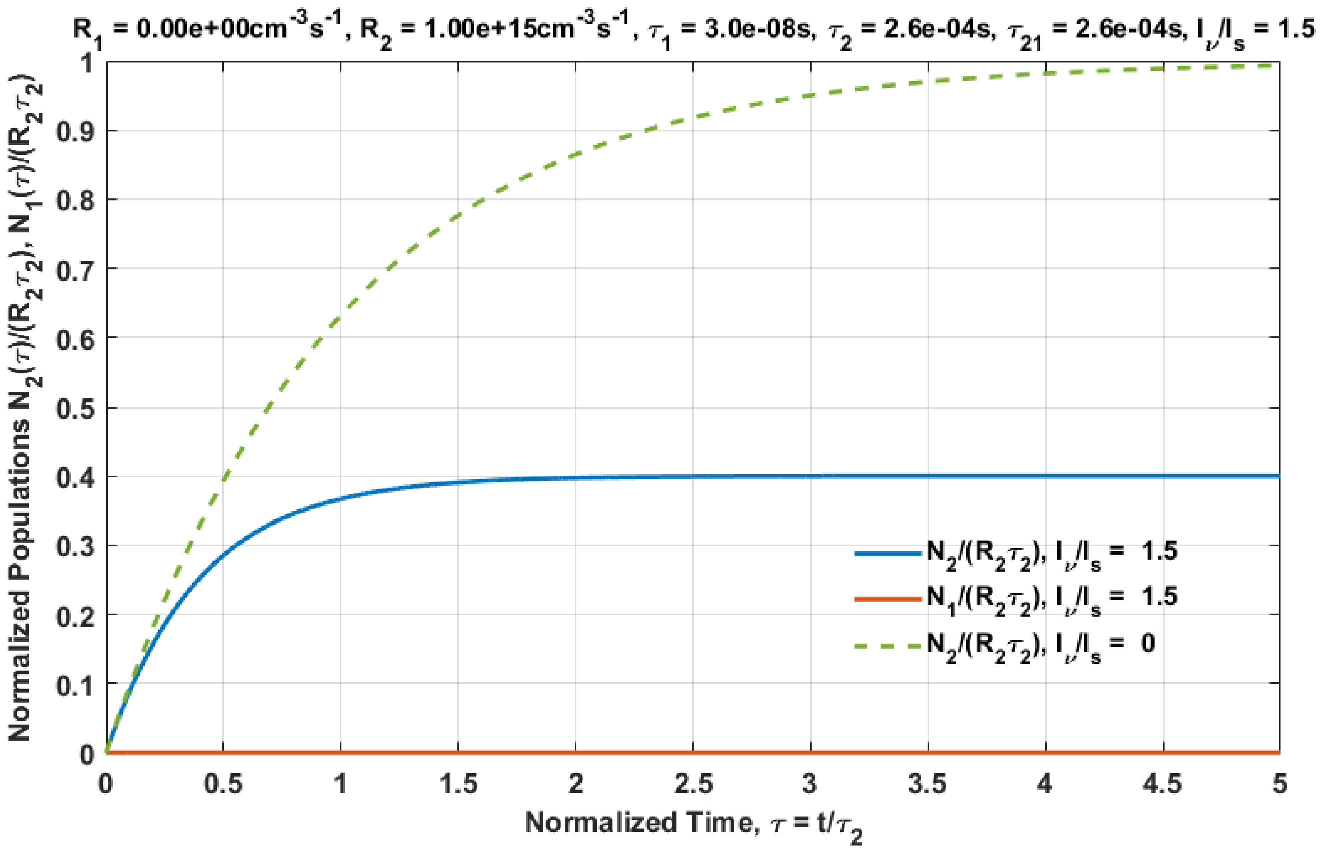
$\tau_2/\tau_1 = 200$



Gain Saturation

Above Threshold, $I_\nu \neq 0$

Nd-YAG Laser: $\tau_1 = 30\text{ns}$, $\tau_2 = 255\mu\text{s}$, $\tau_2/\tau_1 = 8500$



Gain Saturation

Homogeneous Broadening

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1 + \dfrac{I_\nu}{I_s(\nu)}} = \frac{\gamma_0(\nu)}{1 + \bar{g}_H(\nu)\dfrac{I_\nu}{I_s(\nu_0)}}$$

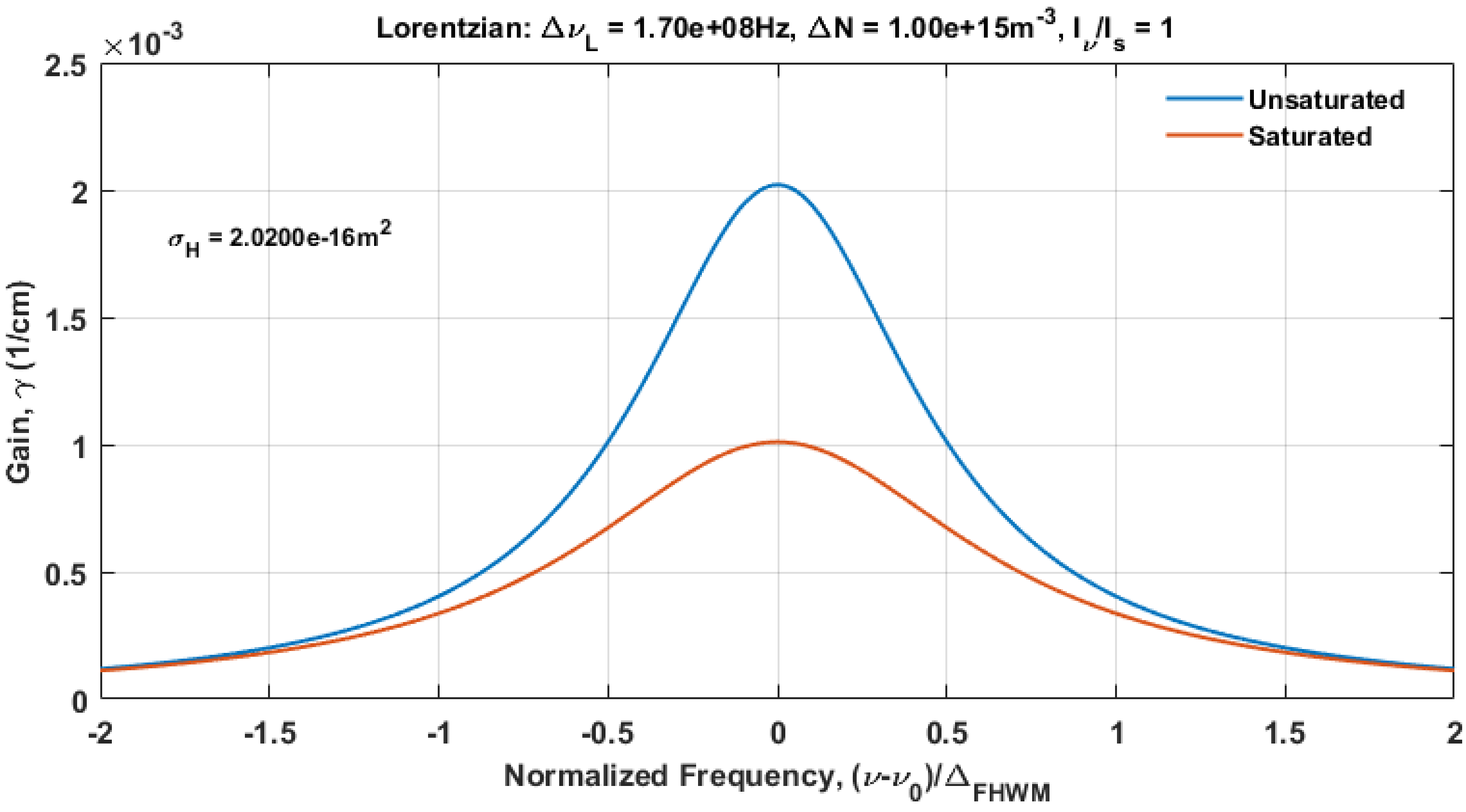
$$\bar{g}_H(\nu) = \frac{g_H(\nu)}{g_H(\nu_0)} = \frac{(\Delta\nu_H/2)^2}{(\nu-\nu_0)^2 + (\Delta\nu_H/2)^2} \quad \text{and} \quad g_H(\nu) = \frac{\Delta\nu_H/2\pi}{(\nu-\nu_0)^2 + (\Delta\nu_H/2)^2}$$

$$\begin{aligned}
\gamma(\nu) &= \left\{A_{21}\frac{\lambda_0^2}{8\pi n^2}\left(N_2 - \frac{g_2}{g_1}N_1\right)_0\right\} \frac{g_H(\nu)}{1 + \bar{g}_H(\nu)\dfrac{I_\nu}{I_s(\nu_0)}} = \\
&= \left\{A_{21}\frac{\lambda_0^2}{8\pi n^2}\left(N_2 - \frac{g_2}{g_1}N_1\right)_0\right\} \frac{\Delta\nu_H/2\pi}{(\nu-\nu_0)^2 + \left(\dfrac{\Delta\nu_H}{2}\right)^2\left[1 + \dfrac{I_\nu}{I_s(\nu_0)}\right]} \\
&= \left\{A_{21}\frac{\lambda_0^2}{8\pi n^2}\left(N_2 - \frac{g_2}{g_1}N_1\right)_0\right\} \frac{1}{\sqrt{1 + \dfrac{I_\nu}{I_s(\nu_0)}}} \frac{\Delta\nu_H\sqrt{1 + I_\nu/I_s(\nu_0)}/2\pi}{(\nu-\nu_0)^2 + \left(\Delta\nu_H\sqrt{1 + I_\nu/I_s(\nu_0)}/2\right)^2}
\end{aligned}$$

Gain Saturation

Homogeneous Broadening

Example: $A_{21} = 3.39 \times 10^6$ 1/s, $\Delta N = 10^{15}$ cm^{-3}, **$\Delta\nu_H$ = 170MHz**, $\lambda_0 = 0.6328\mu m$



Gain Saturation

Inhomogeneous Broadening

Doppler Broadening

$$\int_{-\infty}^{+\infty} g_G(\nu)d\nu = 1 \quad \text{where} \quad g_G(\nu) = \frac{2}{\Delta\nu_D}\left(\frac{\ln 2}{\pi}\right)^{1/2} \exp\left[-\ln 2\left(\frac{2(\nu-\nu_0)}{\Delta\nu_D}\right)^2\right]$$

$$\gamma(\nu) = \left\{A_{21}\frac{\lambda_0^2}{8\pi n^2}\left(N_2 - \frac{g_2}{g_1}N_1\right)_0\right\} g(\nu) \qquad \text{where}$$

$$g(\nu) = \int_{-\infty}^{+\infty} g_G(\nu')\frac{g_H(\nu')}{1+\bar{g}_H(\nu')\frac{I_{\nu'}}{I_s(\nu)}}d\nu' =$$

$$= \int_{-\infty}^{+\infty} g_G(\nu')\frac{1}{\sqrt{1+\frac{I_{\nu'}}{I_s(\nu)}}} \frac{\Delta\nu_H\sqrt{1+I_{\nu'}/I_s(\nu)}/2\pi}{(\nu'-\nu)^2+\left(\Delta\nu_H\sqrt{1+I_{\nu'}/I_s(\nu)}/2\right)^2}d\nu' \simeq$$

$$= \frac{1}{\sqrt{1+\frac{I_\nu}{I_s(\nu)}}}\int_{-\infty}^{+\infty} g_G(\nu') \frac{\Delta\nu_H\sqrt{1+I_\nu/I_s(\nu)}/2\pi}{(\nu'-\nu)^2+\left(\Delta\nu_H\sqrt{1+I_\nu/I_s(\nu)}/2\right)^2}d\nu' =$$

$$= \frac{1}{\sqrt{1+\frac{I_\nu}{I_s(\nu)}}}\underbrace{g_G(\nu) * g_H\left(\nu, \Delta\nu_H\sqrt{1+I_\nu/I_s(\nu)}\right)}_{\text{Voigt}} = \frac{1}{\sqrt{1+\frac{I_\nu}{I_s(\nu)}}}g_V(\nu)$$

Gain Saturation

Inhomogeneous Broadening

$$\gamma(\nu) = \left\{ A_{21}\frac{\lambda_0^2}{8\pi n^2}\left(N_2 - \frac{g_2}{g_1}N_1\right)_0 \right\} \frac{g_V(\nu)}{\sqrt{1+\dfrac{I_\nu}{I_s(\nu)}}} = \frac{\gamma_0(\nu)}{\sqrt{1+\bar{g}_V(\nu)\dfrac{I_\nu}{I_s(\nu_0)}}}$$

Limiting Cases

$$\Delta\nu_G \gg \Delta\nu_H$$

$$\gamma(\nu) = \left\{ A_{21}\frac{\lambda_0^2}{8\pi n^2}\left(N_2 - \frac{g_2}{g_1}N_1\right)_0 \right\} \frac{g_G(\nu)}{\sqrt{1+\dfrac{I_\nu}{I_s(\nu)}}} = \frac{\gamma_0(\nu)}{\sqrt{1+\bar{g}_G(\nu)\dfrac{I_\nu}{I_s(\nu_0)}}}$$

$$\Delta\nu_G \ll \Delta\nu_H$$

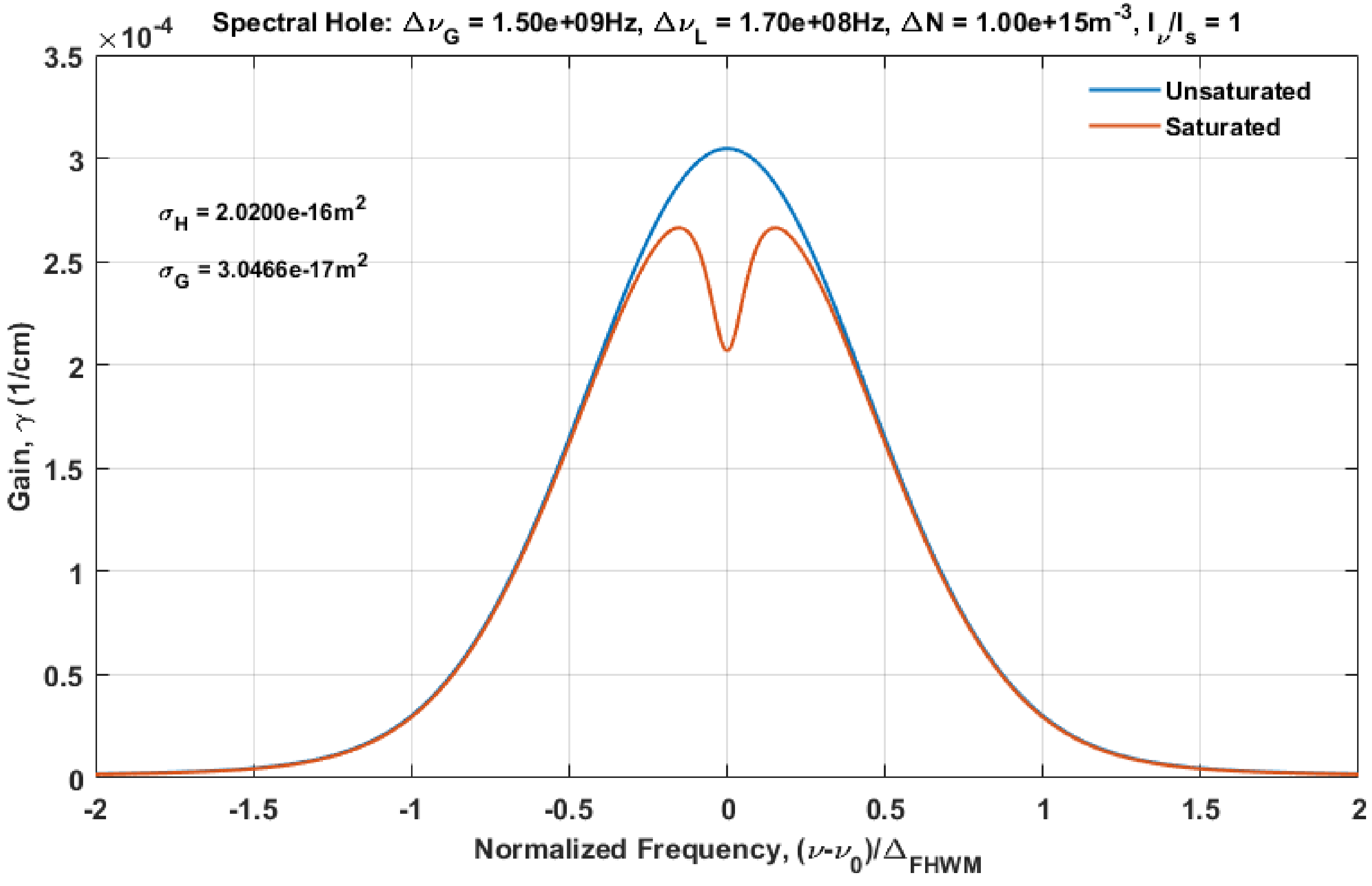
$$\gamma(\nu) = \left\{ A_{21}\frac{\lambda_0^2}{8\pi n^2}\left(N_2 - \frac{g_2}{g_1}N_1\right)_0 \right\} \frac{g_H(\nu)}{\sqrt{1+\dfrac{I_\nu}{I_s(\nu)}}} = \frac{\gamma_0(\nu)}{1+\bar{g}_H(\nu)\dfrac{I_\nu}{I_s(\nu_0)}}$$

Gain Saturation

Inhomogeneous Broadening

Example: $A_{21} = 3.39 \times 10^6$ 1/s, $\Delta N = 10^{15}$ cm^{-3}, **$\Delta\nu_H$ = 170MHz**, $\Delta\nu_G$ = 1.5GHz λ_0 = 0.6328μm

He-Ne Laser

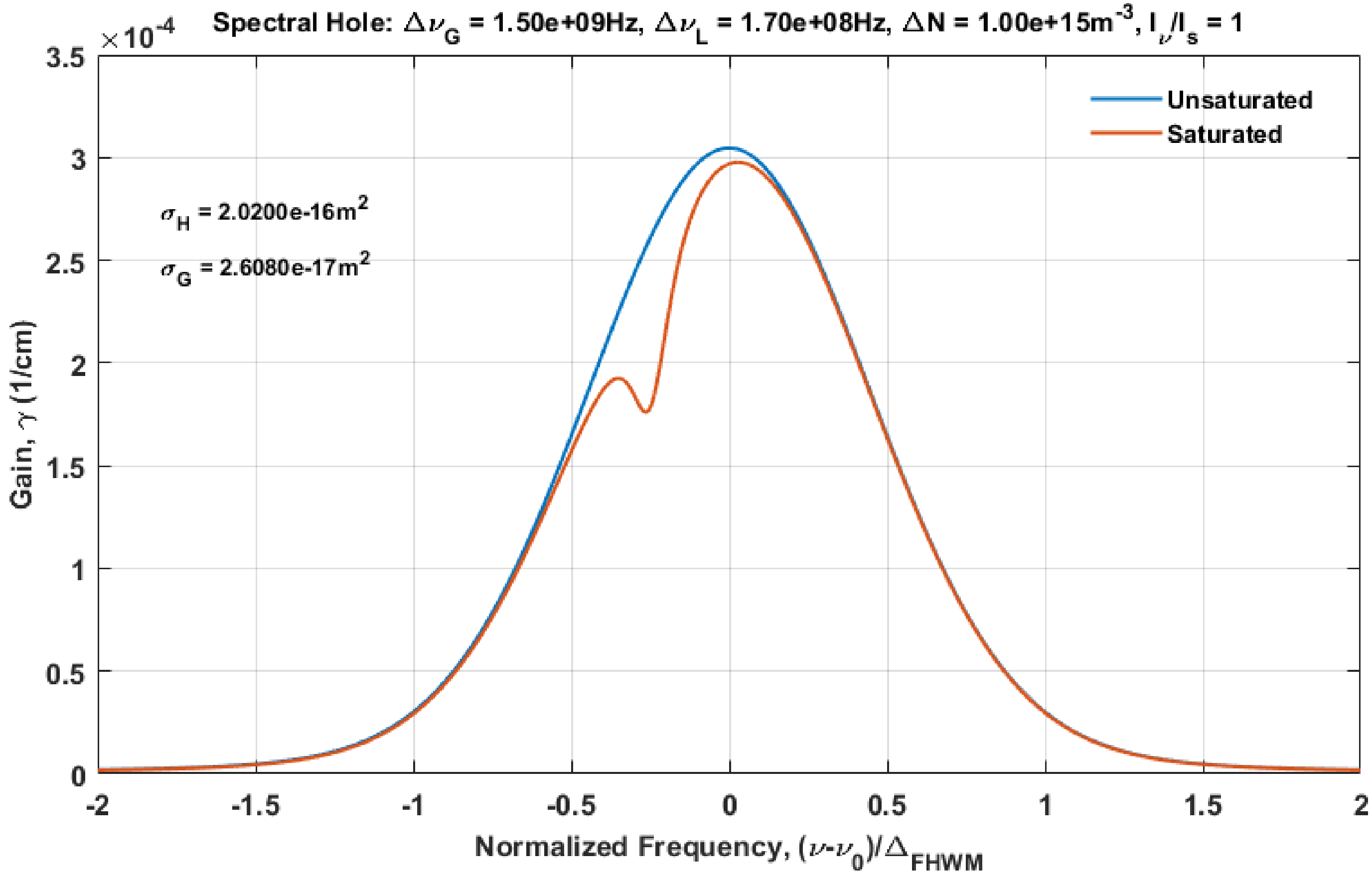


Gain Saturation

Inhomogeneous Broadening

Example: $A_{21} = 3.39 \times 10^6$ 1/s, $\Delta N = 10^{15}$ cm^{-3}, **$\Delta\nu_H$ = 170MHz**, $\Delta\nu_G$ = 1.5GHz λ_0 = 0.6328μm

He-Ne Laser

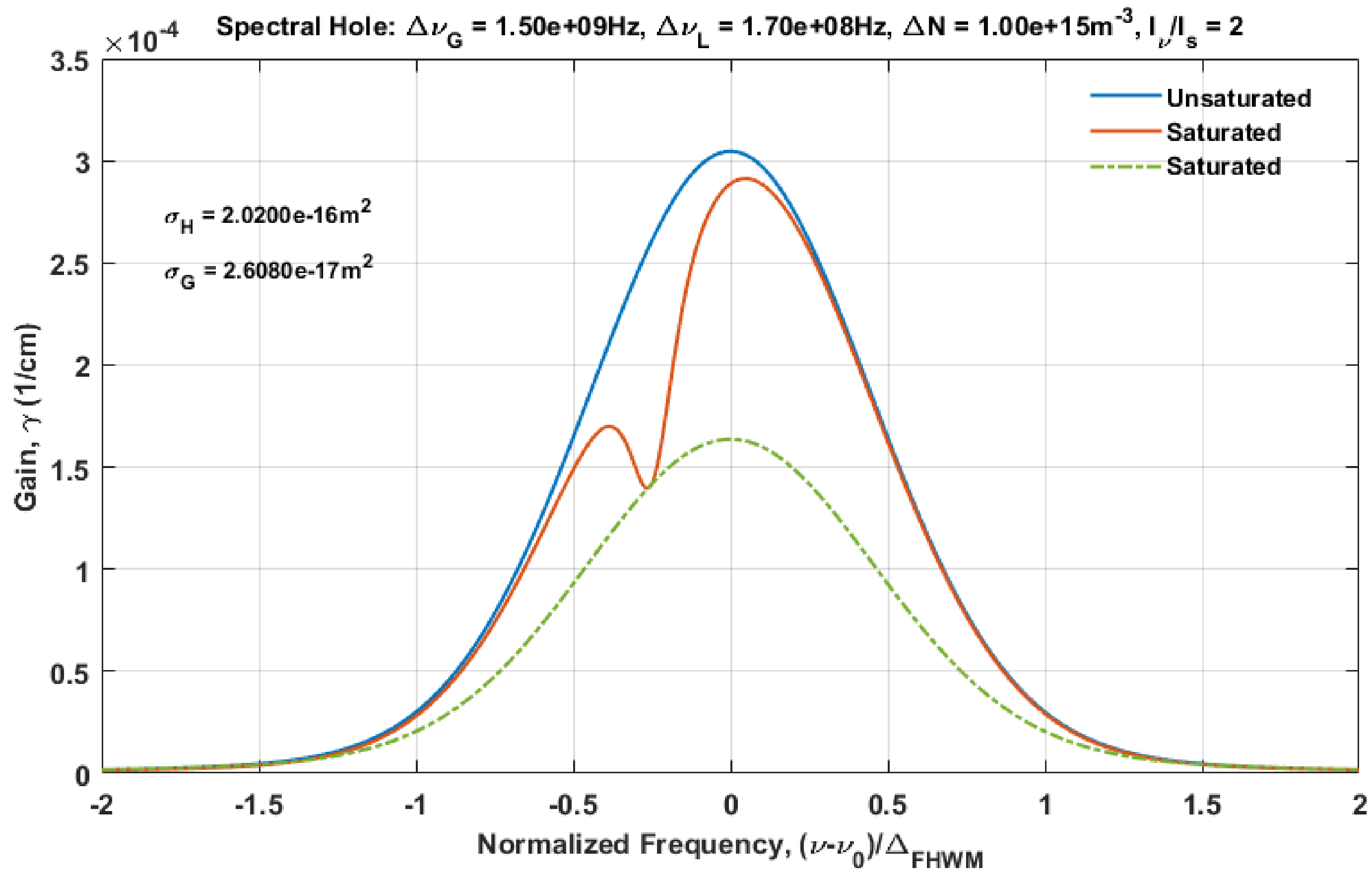


Gain Saturation

Inhomogeneous Broadening

Example: $A_{21} = 3.39 \times 10^6$ 1/s, $\Delta N = 10^{15}$ cm^{-3}, **$\Delta\nu_H$ = 170MHz**, $\Delta\nu_G$ = 1.5GHz λ_0 = 0.6328μm

He-Ne Laser

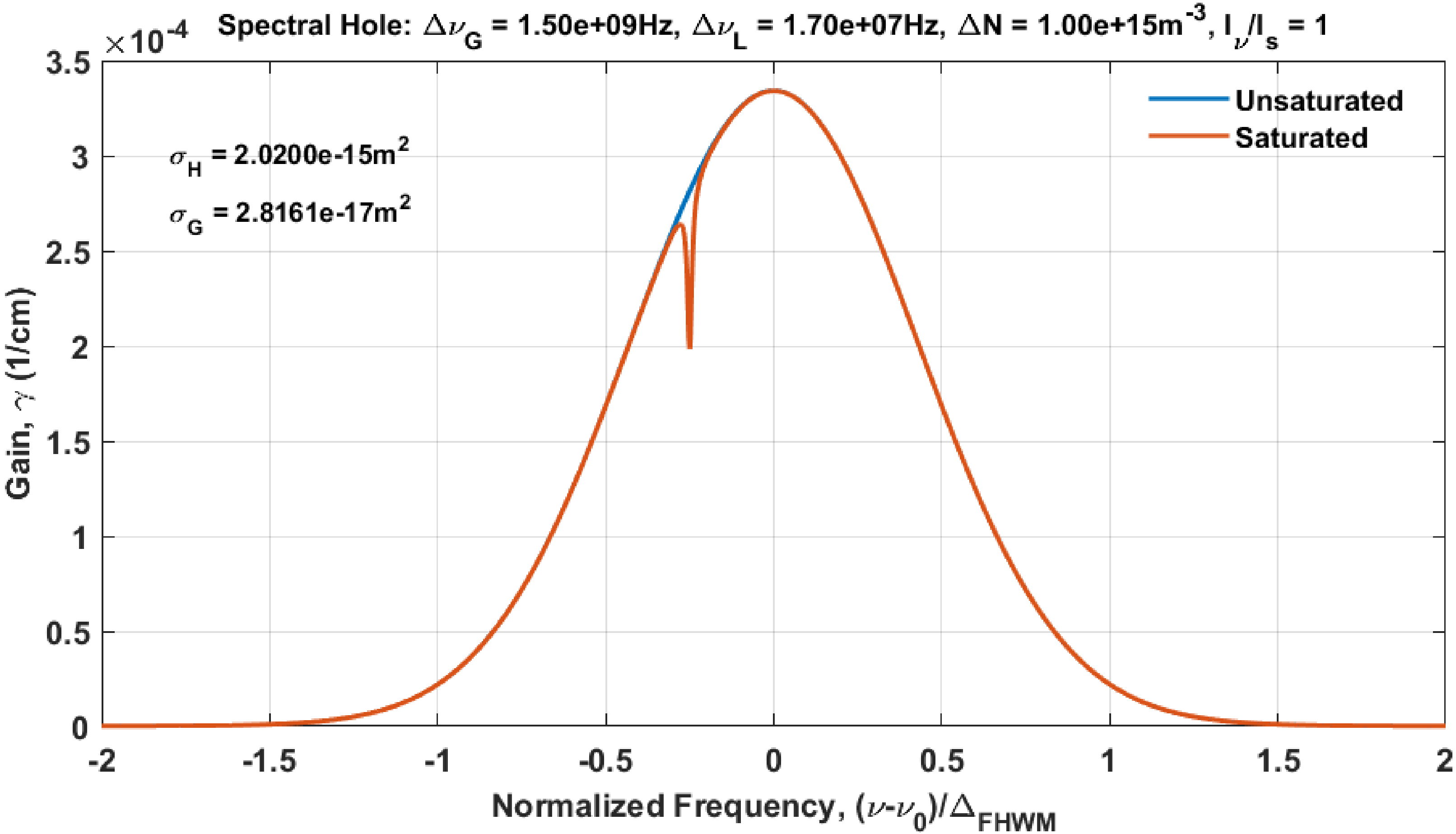


Gain Saturation

Inhomogeneous Broadening

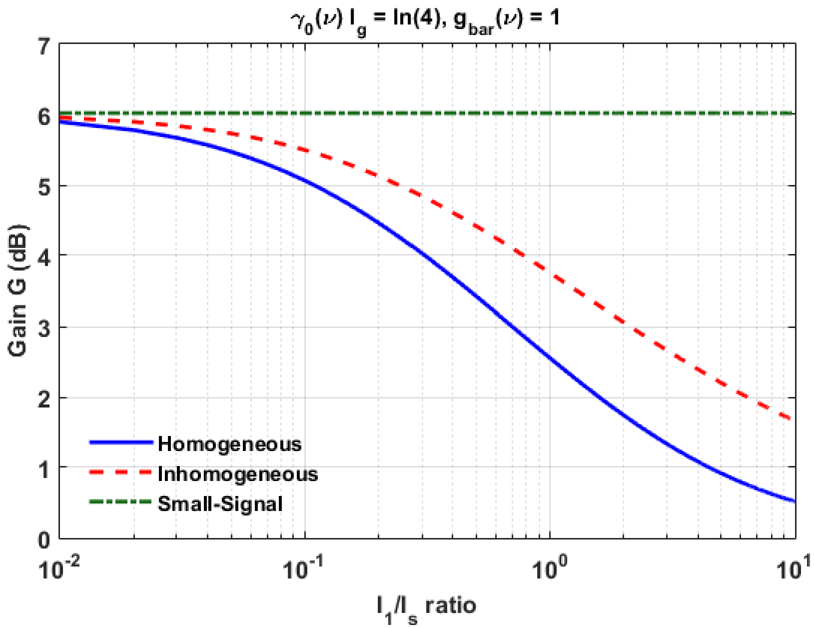
Example: $A_{21} = 3.39 \times 10^6$ 1/s, $\Delta N = 10^{15}$ cm^{-3}, **$\Delta\nu_H$ = 17MHz**, $\Delta\nu_G$ = 1.5GHz λ_0 = 0.6328μm

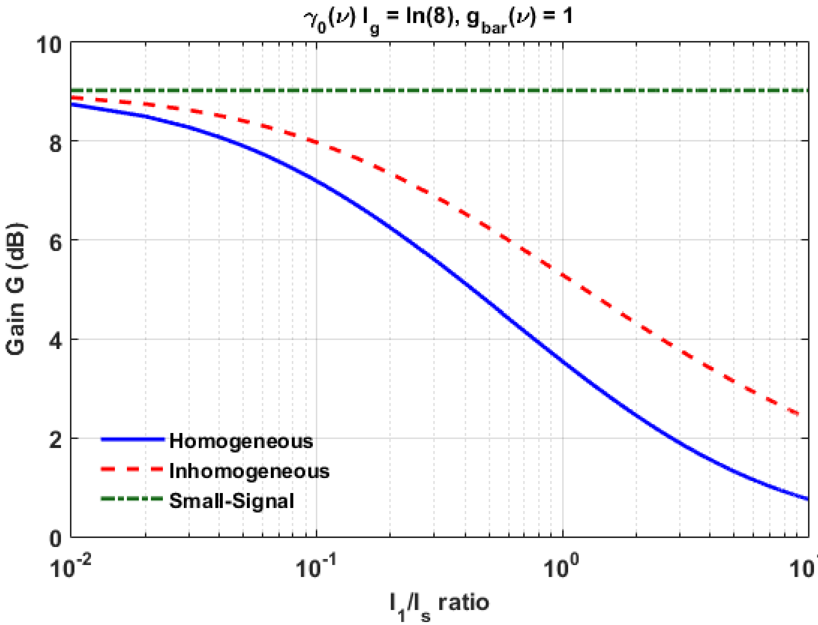
He-Ne Laser



Gain Saturation

Homogeneous & and Inhomogeneous Broadening





Homogeneous Broadening

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{1 + \bar{g}(\nu)\dfrac{I_\nu}{I_s}}$$

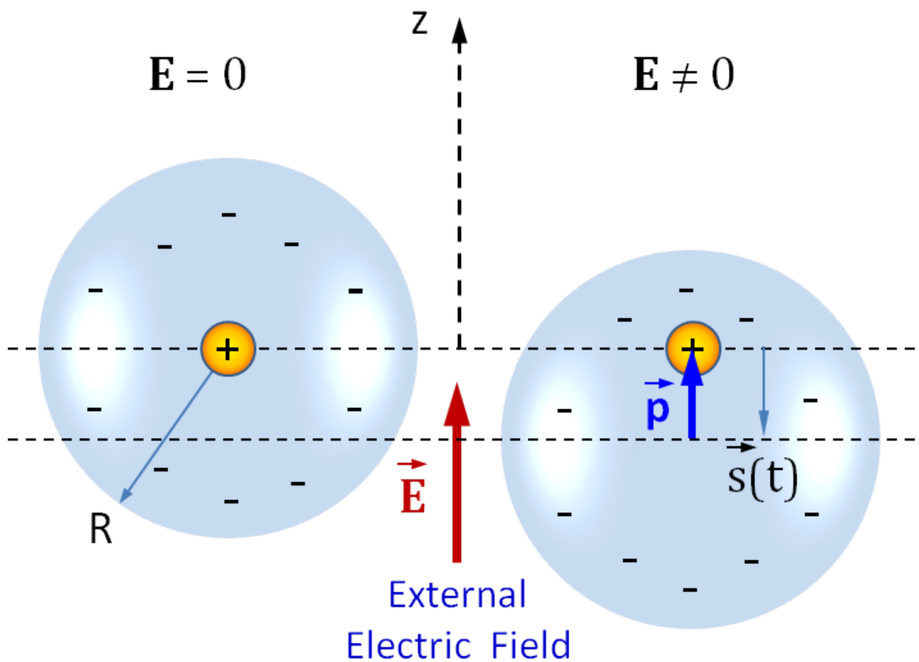
Inhomogeneous Broadening

$$\gamma(\nu) = \frac{\gamma_0(\nu)}{\sqrt{1 + \bar{g}(\nu)\dfrac{I_\nu}{I_s}}}$$

From J. T. Verdeyen, "Laser Electronics" 3rd Ed. Prentice Hall, 1995

Classical Electron Oscillator Model

Simple Atomic Model



Electron Motion Equation

$$m_e \frac{d^2\vec{s}(t)}{dt^2} = \underbrace{-e\vec{E}(t)}_{\text{Lorentz Force}} - \underbrace{K\vec{s}(t)}_{\text{Restoring Force}} - \underbrace{\beta\frac{d\vec{s}(t)}{dt}}_{\text{Damping}}$$

$$\begin{aligned} \frac{d^2\vec{s}(t)}{dt^2} + \sigma\frac{d\vec{s}(t)}{dt} + \omega_0^2\vec{s}(t) &= -\frac{e}{m_e}\vec{E}(t) \\ \sigma &= \beta/m_e \\ \omega_0 &= \sqrt{K/m_e} \end{aligned}$$

Classical Electron Oscillator Model

Electron Motion Equation

$$\frac{d^2\vec{s}(t)}{dt^2} + \sigma\frac{d\vec{s}(t)}{dt} + \omega_0^2\vec{s}(t) = -\frac{e}{m_e}\vec{E}(t)$$

Fourier Transform Pairs

$$\vec{s}(t) \xrightarrow{\mathscr{F}} \vec{S}(\omega)$$

$$\vec{E}(t) \xrightarrow{\mathscr{F}} \vec{E}(\omega)$$

$$\vec{S}(\omega) = \frac{-e/m_e}{(\omega_0^2-\omega^2)+j\omega\sigma}\vec{E}(\omega)$$

$$\text{for} \quad \omega \simeq \omega_0 \Longrightarrow \vec{S}(\omega) \simeq \frac{-e/m_e}{2\omega_0(\omega_0-\omega)+j\omega_0\sigma}\vec{E}(\omega)$$

Electric Dipole Moment

$$\vec{p}(t) = e\,[-\vec{s}(t)] \xrightarrow{\mathscr{F}} \vec{p}(\omega) = -e\vec{S}(\omega) = \frac{e^2/m_e}{2\omega_0(\omega_0-\omega)+j\omega_0\sigma}\vec{E}(\omega)$$

Macroscopic Polarization

$$\vec{P}(t) = \vec{p}(t)N \xrightarrow{\mathcal{F}} \vec{P}(\omega) = \vec{p}(\omega)N = \frac{-j(Ne^2/\omega_0\sigma m_e)}{1+j\dfrac{2(\omega-\omega_0)}{\sigma}}\vec{E}(\omega)$$

Classical Electron Oscillator Model

Macroscopic Polarization

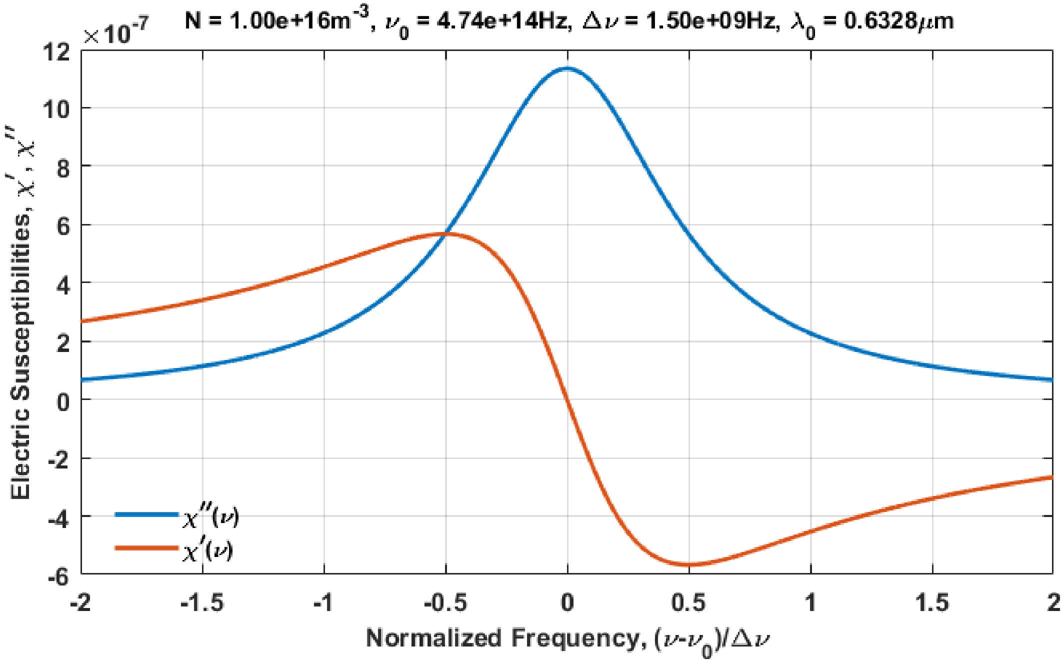
$$\vec{P}(\omega) = \epsilon_0 \left[\chi'(\omega) - j\chi''(\omega)\right] \vec{E}(\omega) = \frac{-j(Ne^2/\omega_0\sigma m_e)}{1 + j\dfrac{2(\omega-\omega_0)}{\sigma}} \vec{E}(\omega)$$

$$\chi''(\nu) = \frac{Ne^2}{16\pi^2 m_e \nu_0 \epsilon_0} \frac{\Delta\nu}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}$$

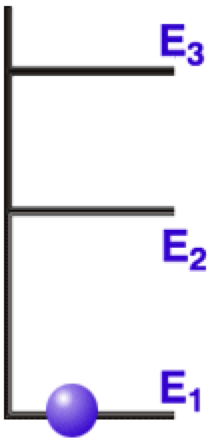
$$\chi'(\nu) = \frac{2(\nu_0-\nu)}{\Delta\nu}\chi''(\nu) = \frac{Ne^2}{8\pi^2 m_e \nu_0 \epsilon_0} \frac{\nu_0-\nu}{(\nu-\nu_0)^2 + (\Delta\nu/2)^2}$$

$$\nu = \frac{\omega}{2\pi} \qquad \Delta\nu = \frac{\sigma}{2\pi}$$

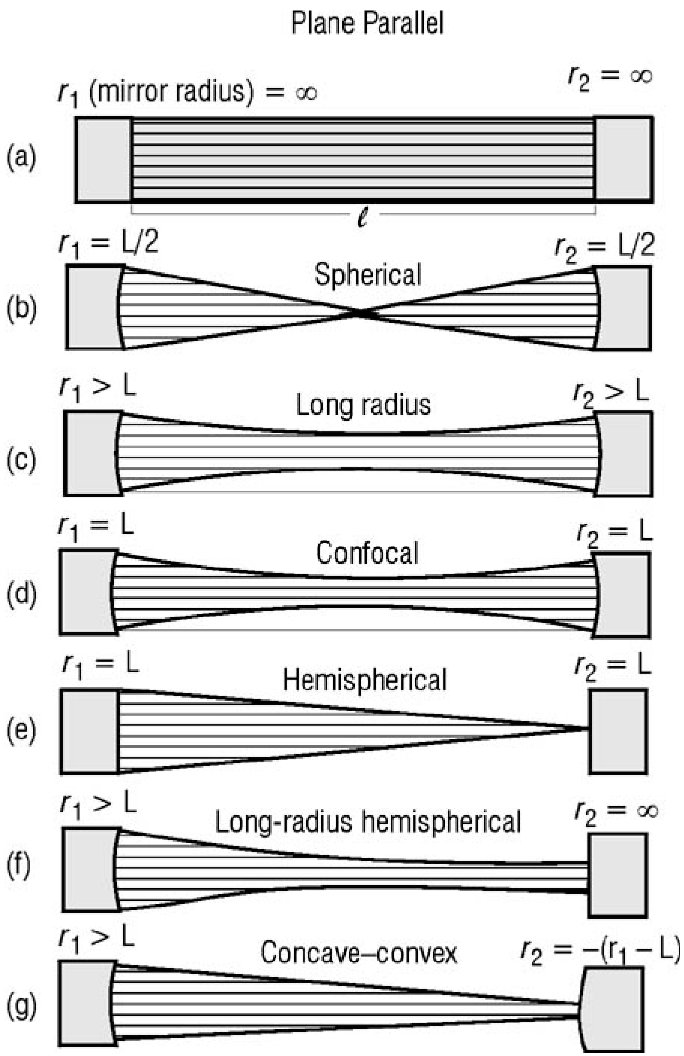
He-Ne laser Example (inversion of $N = 10^{10}$ cm^{-3})



Stimulated Emission



Common Resonators Types



(a) Parallel plane cavity: Highest mode volume and highest diffraction loss. Difficult to align.

(b) The spherical cavity : Represents the functional "opposite" of the plane parallel cavity (a). It is easiest to align, has the lowest diffraction loss, and has the smallest mode volume. CW dye lasers are equipped with this type of cavity because a focused beam is necessary to cause efficient stimulated emission of these lasers. The spherical cavity is not commonly used with any other type of laser.

(c) The long radius cavity: Improves on the mode volume, but does so at the expense of a more difficult alignment and a slightly greater diffraction loss than that of the confocal cavity. This type of cavity is suitable for any CW laser application, but few commercial units incorporate the long radius cavity.

(d) The confocal cavity: A compromise between the plane parallel and the spherical cavities. The confocal cavity combines the ease of alignment and low diffraction loss of the spherical cavity with the increased mode volume of the plane parallel. Confocal cavities can be utilized with almost any CW laser, but are not in common use.

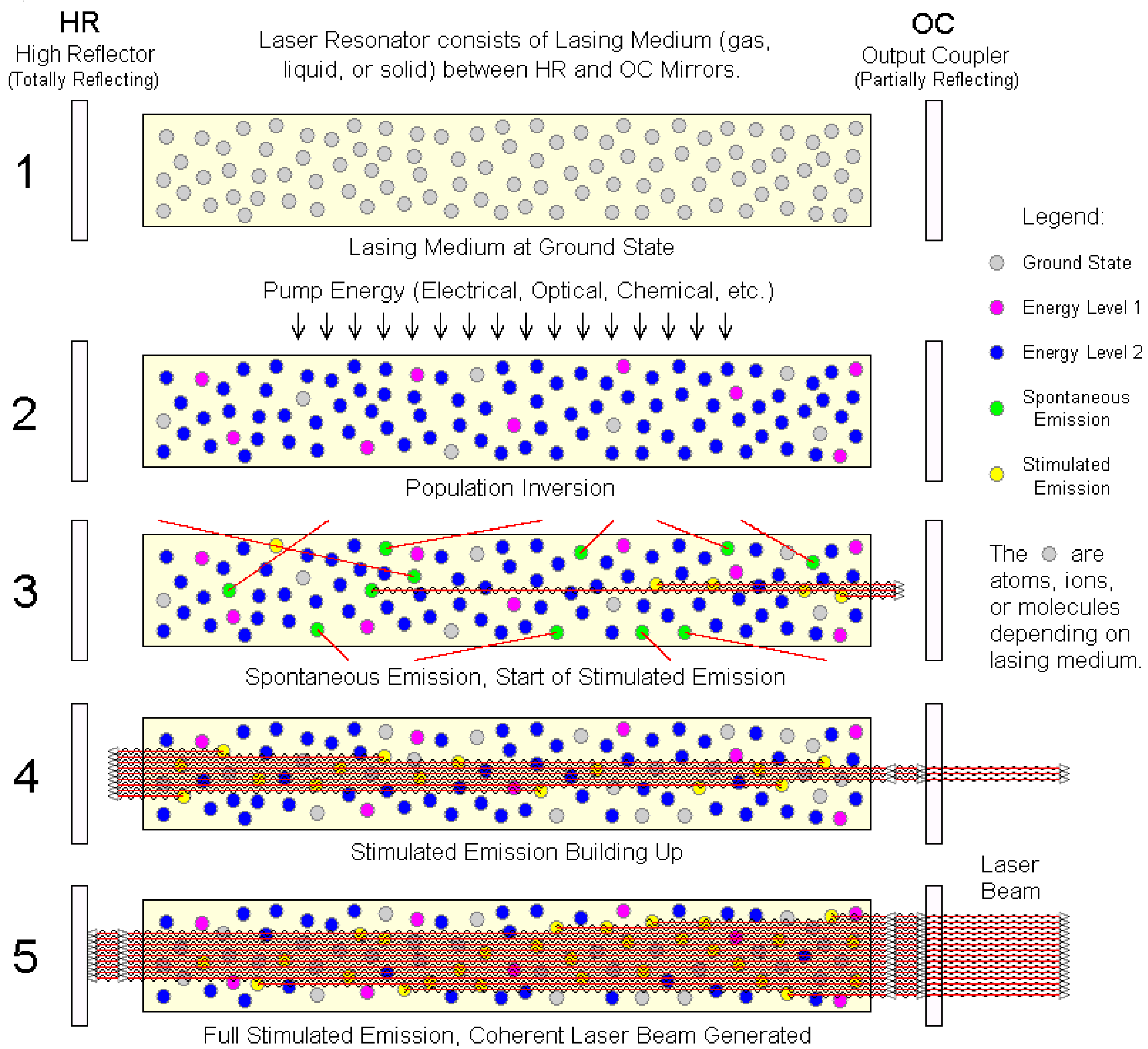
(e) The hemispherical cavity : Actually is one half of the spherical cavity, and the characteristics of the two are similar. The advantage of this type of cavity over the spherical cavity is the cost of the mirrors. The hemispherical cavity is used with most low power He-Ne lasers because of low diffraction loss, ease of alignment, and reduced cost.

(f) The long-radius-hemispherical cavity : Combines the cost advantage of the hemispherical cavity with the improved mode volume of the long-radius cavity. Most CW lasers (except low-power He-Ne lasers) employ this type of cavity. In most cases, $r_1 \geq 2L$.

(g) The concave-convex cavity : Normally is used only with high power CW CO_2 lasers. In practice, the diameter of the convex mirror is smaller than that of the beam. The output beam is formed by the part of the beam that passes around the mirror and, consequently, has a "doughnut" configuration. The beam must pass around the mirror because mirrors that will transmit the intense beams of these high-power lasers cannot be constructed.

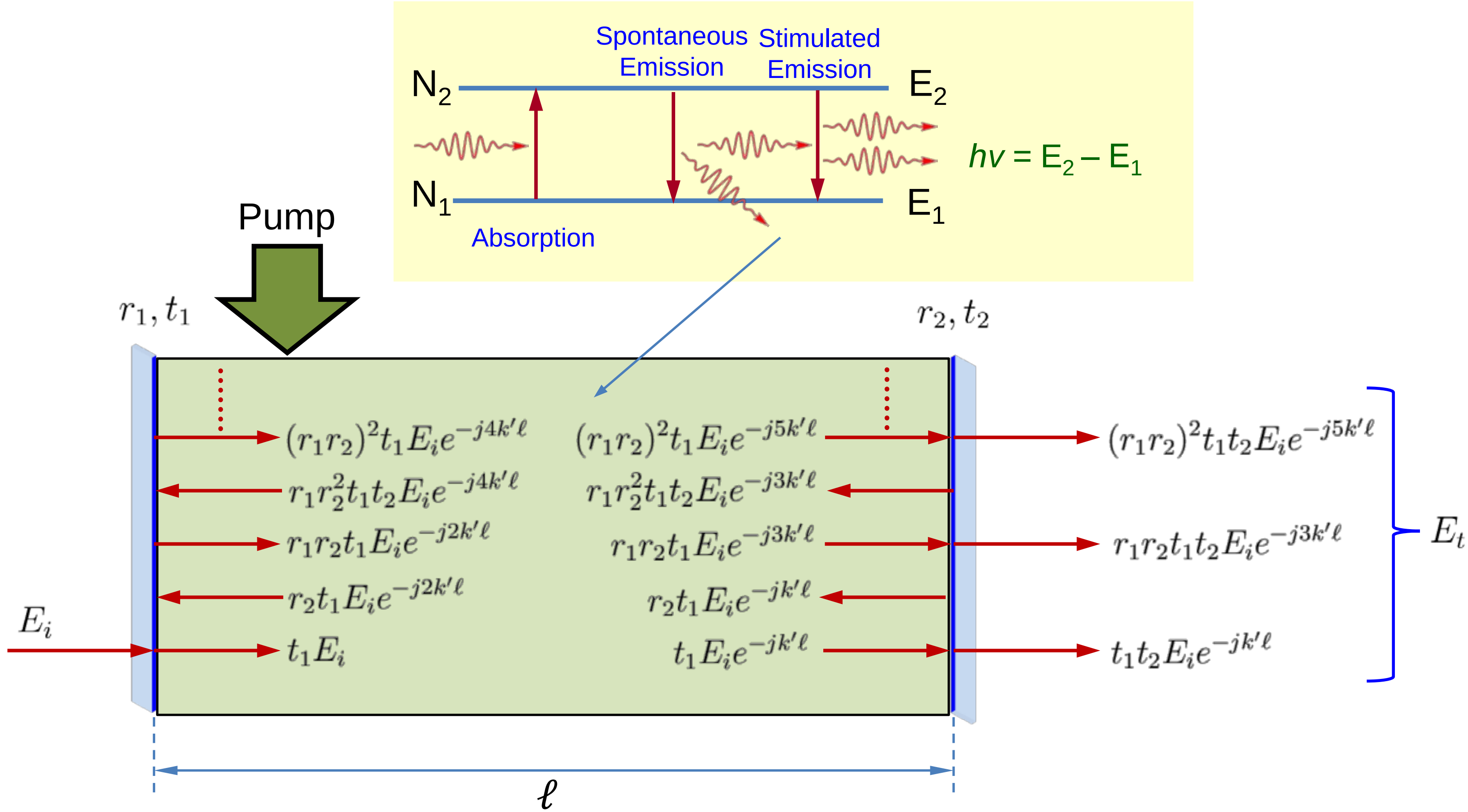
http://www.repairfaq.org/sam/laserioi.htm#ioiresc

Laser Operation Basics



http://ehs.oregonstate.edu/laser/training/how-laser-works

Fabry-Perot Laser



$$E_t = E_i \, \frac{t_1 t_2 e^{-jk'\ell}}{1 - r_1 r_2 e^{-j2k'\ell}}$$

Fabry-Perot Laser

Resonance Conditions

$$r_1 r_2 e^{(\gamma-\alpha)\ell} = 1 \qquad \text{Amplitude Condition}$$

$$2k\ell\left(1+\frac{\chi_e'}{2n^2}\right) = 2\pi m \qquad \text{Phase Condition}$$

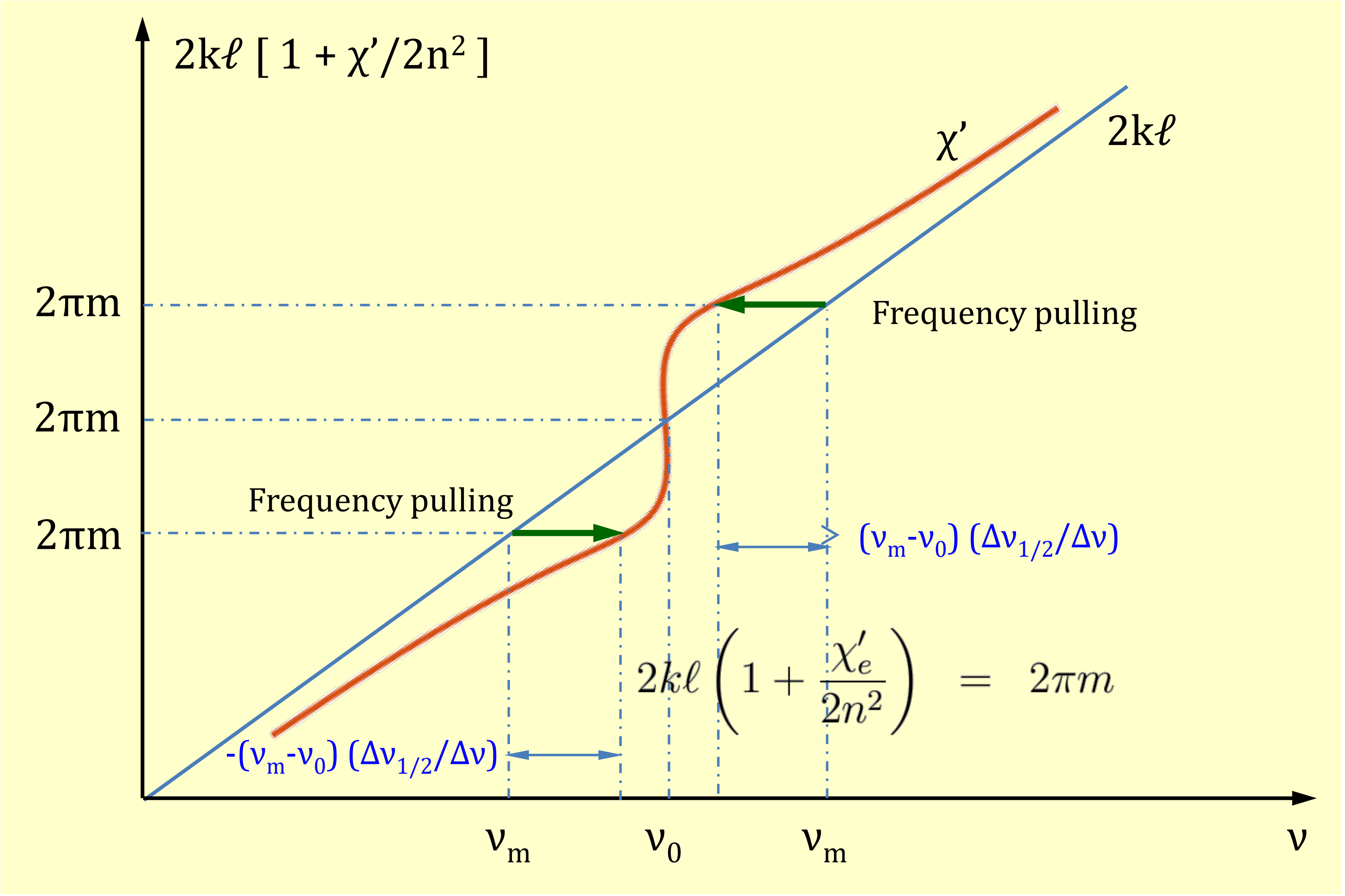
$$\gamma = -\frac{k\chi_e''}{n^2}$$

Threshold Gain

$$\gamma_{th} = \alpha - \frac{1}{\ell}\ln(r_1 r_2) = \alpha - \frac{1}{2\ell}\ln(R_1 R_2)$$

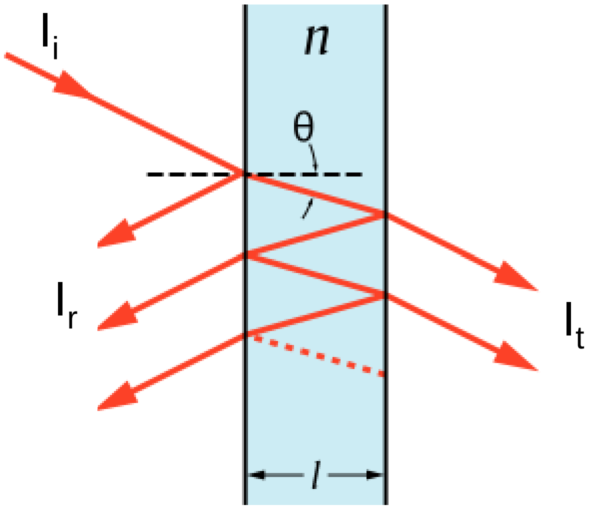
Fabry-Perot Laser

Frequency Pulling



$$\nu \simeq \nu_m - (\nu_m - \nu_0)\frac{\Delta\nu_{1/2}}{\Delta\nu}$$

Fabry-Perot with Gain



https://en.wikipedia.org/wiki/File:Etalon-2.svg

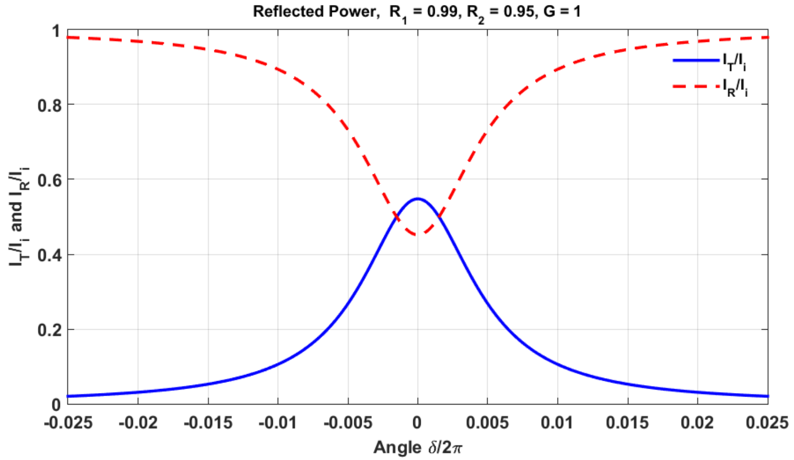
$$\delta = 2k_0 n\ell\cos\theta$$

$$\frac{I_t}{I_i} = \frac{G(1-R_1)(1-R_2)}{(1-G\sqrt{R_1R_2})^2 + 4G\sqrt{R_1R_2}\sin^2(\delta/2)}$$

$$\frac{I_r}{I_i} = \frac{(\sqrt{R_1}-\sqrt{R_2})^2 + 4G\sqrt{R_1R_2}\sin^2(\delta/2)}{(1-G\sqrt{R_1R_2})^2 + 4G\sqrt{R_1R_2}\sin^2(\delta/2)}$$

$$G_{max} = \frac{1}{\sqrt{R_1R_2}}$$

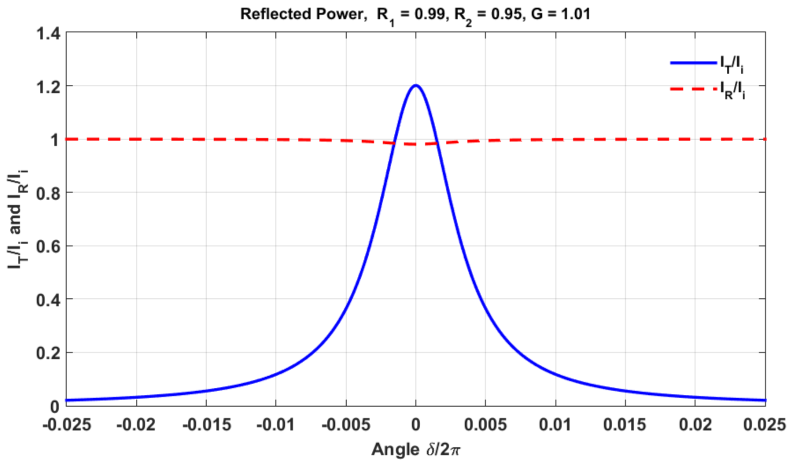
G = 1 (no gain)

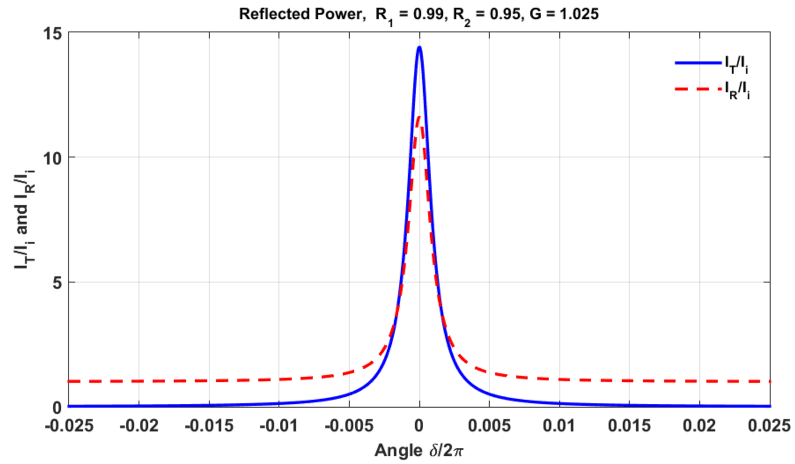


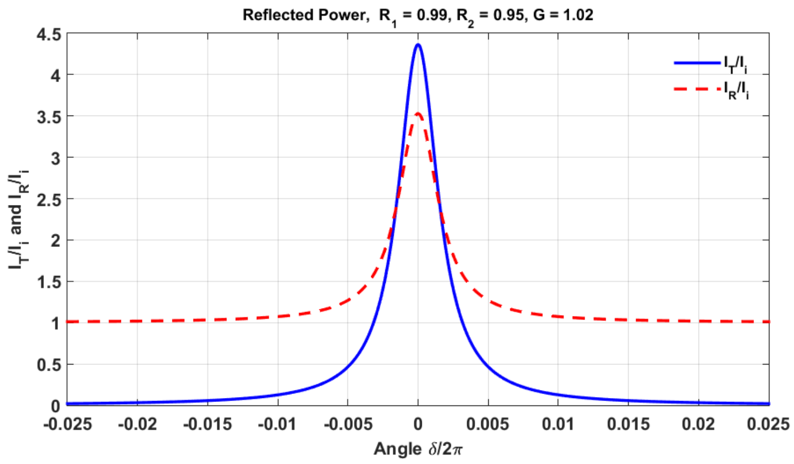
Fabry-Perot with Gain

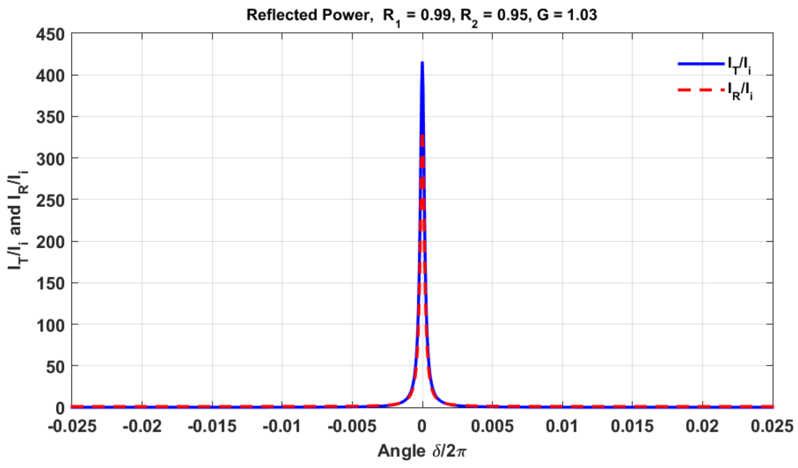
$$G_{max} = \frac{1}{\sqrt{R_1 R_2}}$$

$G_{max} = 1.031147$



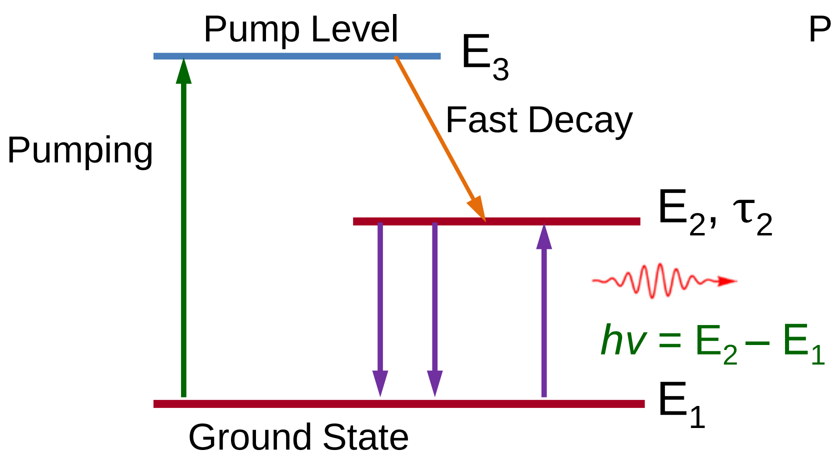




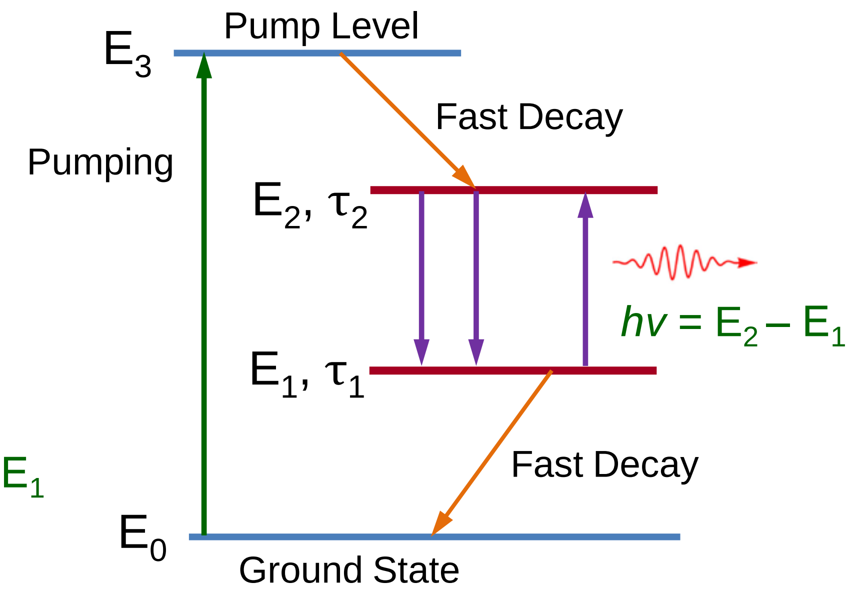


3-Level and 4-Level Lasers

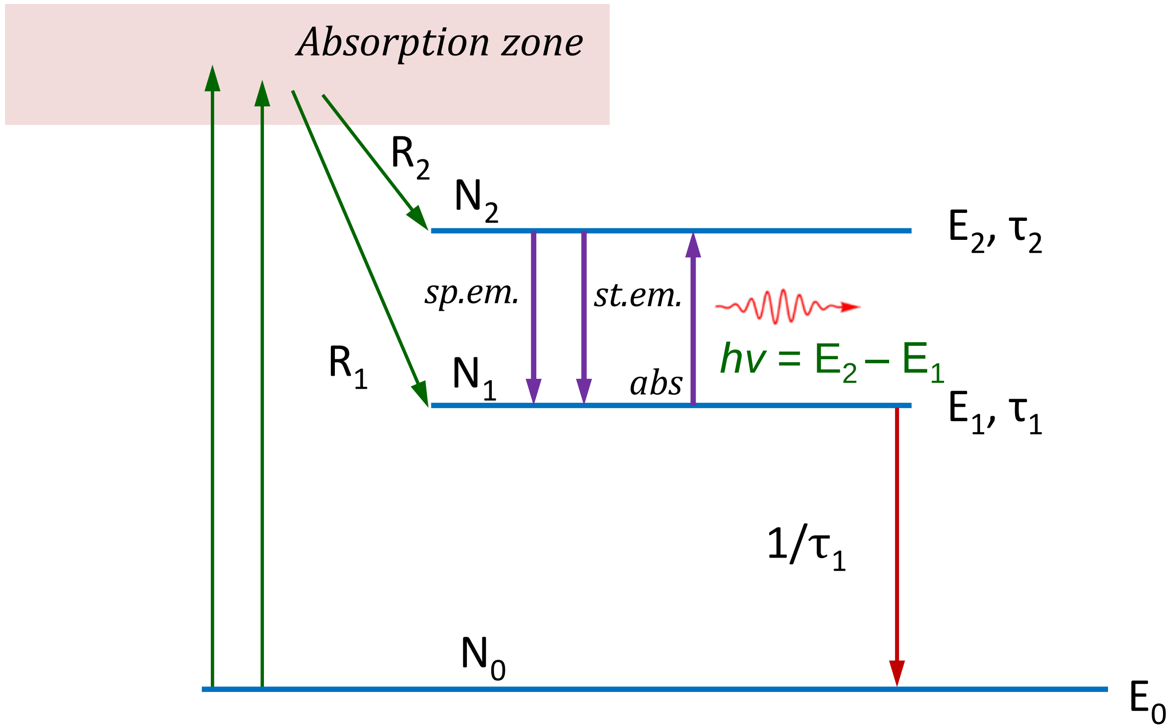
3-Level System



4-Level System



Laser Power Considerations



$$\frac{dN_2}{dt} = R_2 - A_{21}N_2 - W_i(N_2 - N_1) \qquad \left(W_i = \sigma(\nu)\frac{I_\nu}{h\nu}\right)$$

$$= R_2 - \frac{N_2}{\tau_2} - W_i(N_2 - N_1) \qquad \left(\frac{1}{\tau_2} = A_{21}\right)$$

$$\frac{dN_1}{dt} = R_1 + W_i(N_2 - N_1) - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_2}$$

Laser Power Considerations

Steady-State

$$N_2 - N_1 = \frac{R_2\left[1 - \frac{\tau_1}{\tau_2}\left(1 + \frac{R_1}{R_2}\right)\right]}{W_i + \frac{1}{\tau_2}} = \frac{R_{eff}}{W_i + \frac{1}{\tau_2}}$$

Below Threshold $I_\nu \simeq 0 \rightarrow W_i \simeq 0$

$$N_2 - N_1 = \frac{R_{eff}}{1/\tau_2} \qquad \text{for} \qquad R_{eff} < \frac{N_{th}}{\tau_2} = R_{th}$$

Above Threshold $I_\nu \neq 0 \rightarrow W_i \neq 0$

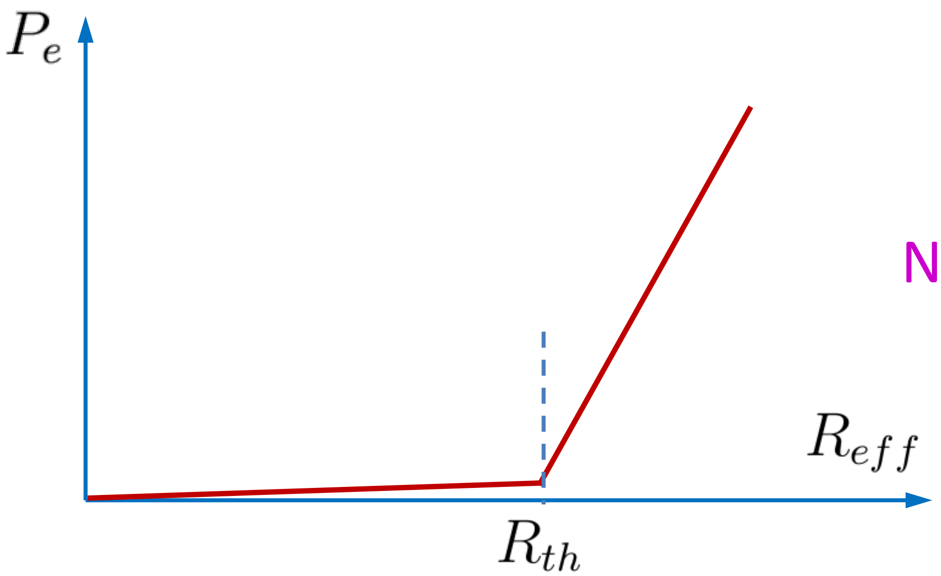
$$N_2 - N_1 = N_{th} = \frac{R_{eff}}{W_i + \frac{1}{\tau_2}} \qquad \text{for} \qquad R_{eff} > \frac{N_{th}}{\tau_2} = R_{th}$$

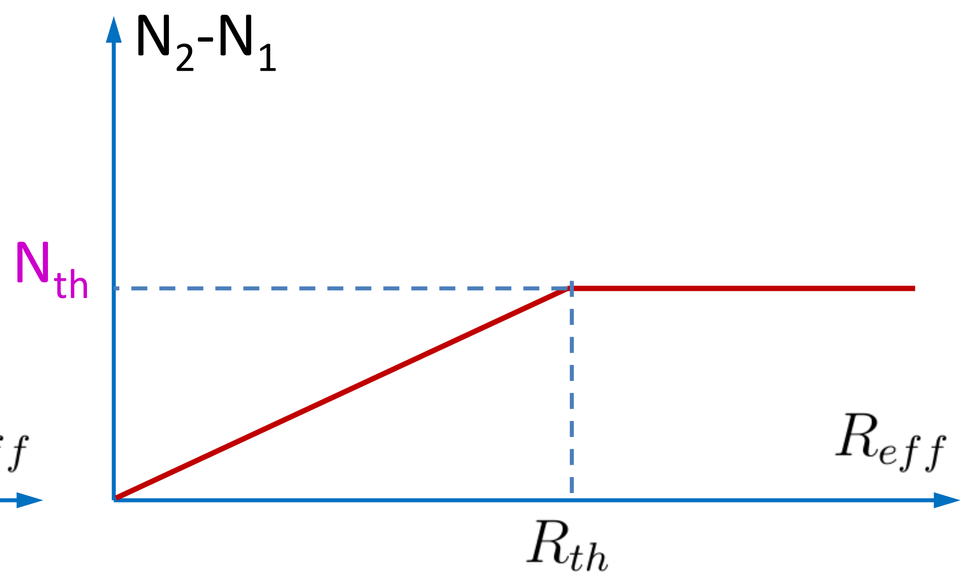
Laser Power Considerations

$$W_i = \frac{R_{eff}}{N_{th}} - \frac{1}{\tau_2}$$

Power due to Stimulated Emission $\quad P_e = W_i\,(N_{th}V)\,h\nu$

$$P_e = P_s\left[\frac{R_{eff}}{N_{th}/\tau_2} - 1\right] \qquad \text{where} \quad P_s = N_{th}Vh\nu\frac{1}{\tau_2} \quad \text{and} \quad R_{eff} > R_{th}$$





Optimum Outcoupling

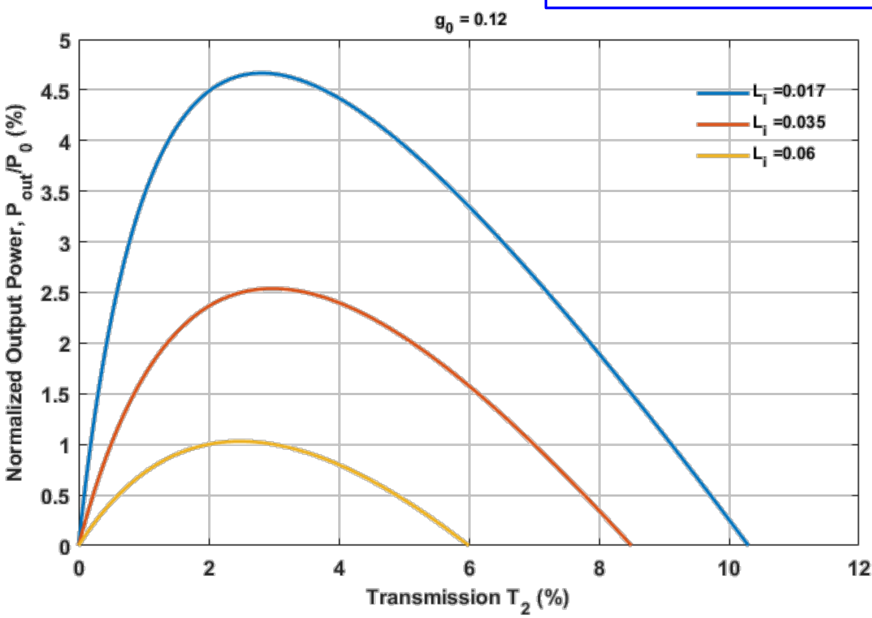
(small losses, small gain, high Q)

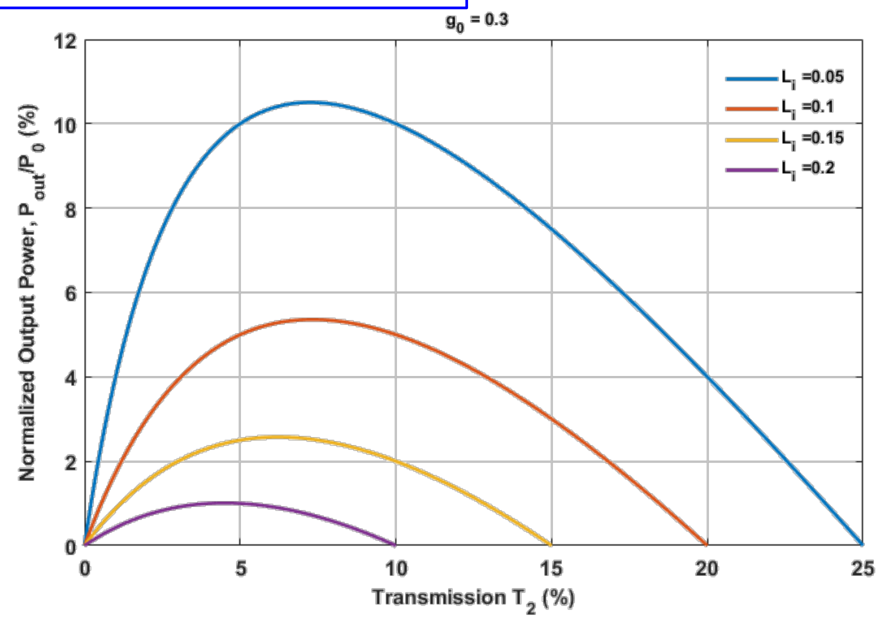
$$P_{out} = P_0 T_2 \left[\frac{g_0}{L_i + T_2} - 1\right], \qquad g_0 = \gamma_0 \ell$$

$$P_0 = \frac{h\nu}{\sigma_{SE}(\nu)\tau_2} S, \qquad S = V/\ell$$

$$T_{2,opt} = \sqrt{g_0 L_i} - L_i$$

$$P_{out,opt} = P_0 \left(\sqrt{g_0} - \sqrt{L_i}\right)^2$$





A. Yariv and P. Yeh, Photonics, 6th Ed., Oxford University Press (2007)

Laser Oscillation for Homogeneous Broadening

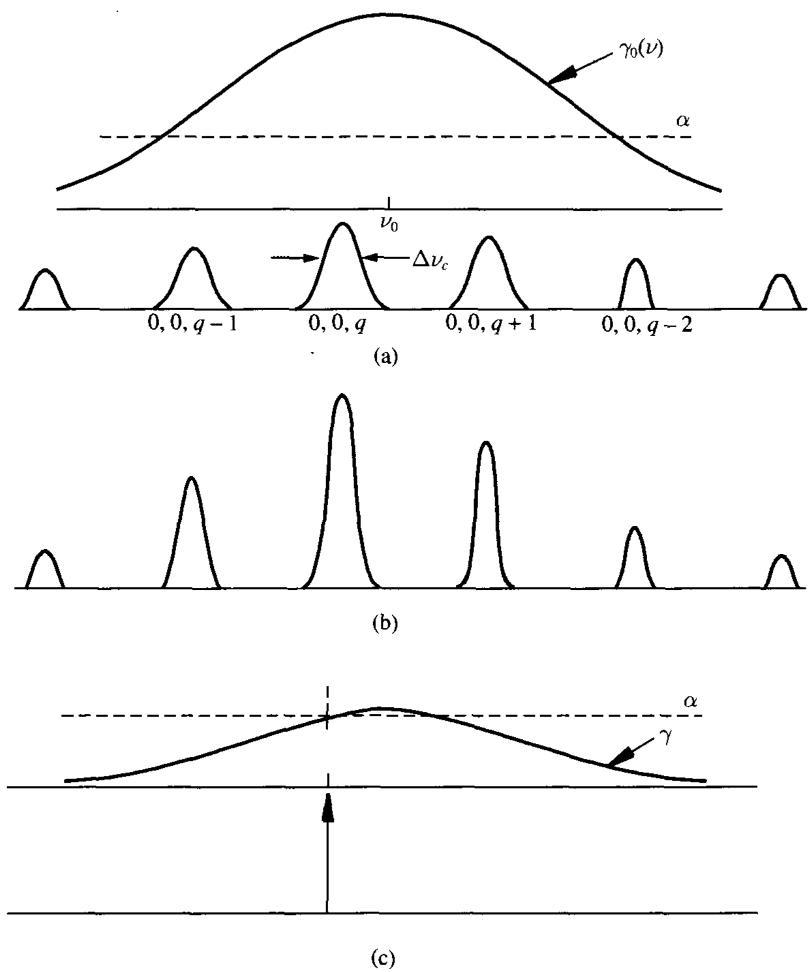
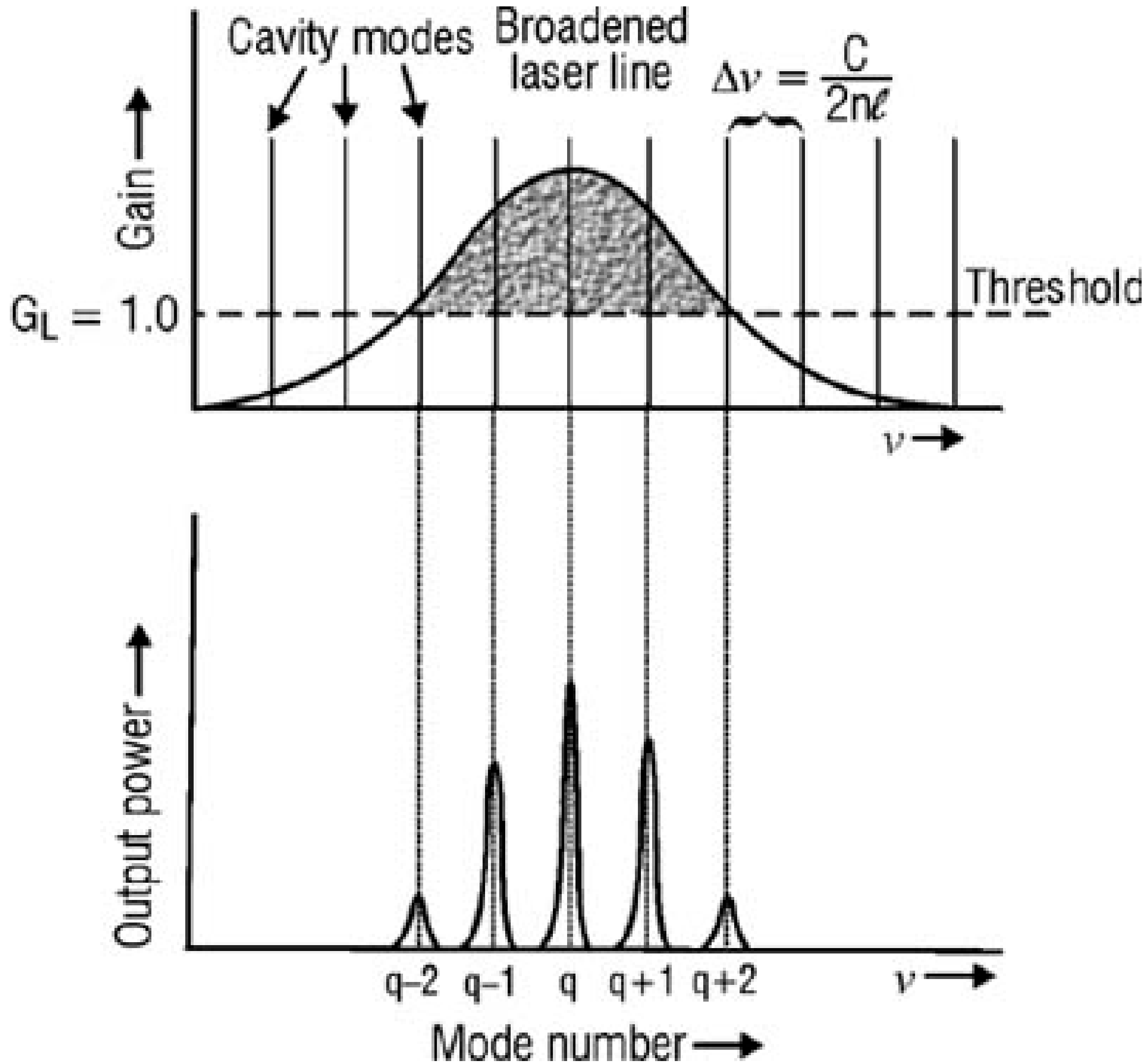


FIGURE 8.3. Evolution of laser oscillation from spontaneous emission: (a) initial; (b) intermediate; and (c) final.

From J. T. Verdeyen, "Laser Electronics" 3rd Ed. Prentice Hall, 1995

Laser Oscillation for Inhomogeneous Broadening



Laser Oscillation for Inhomogeneous Broadening

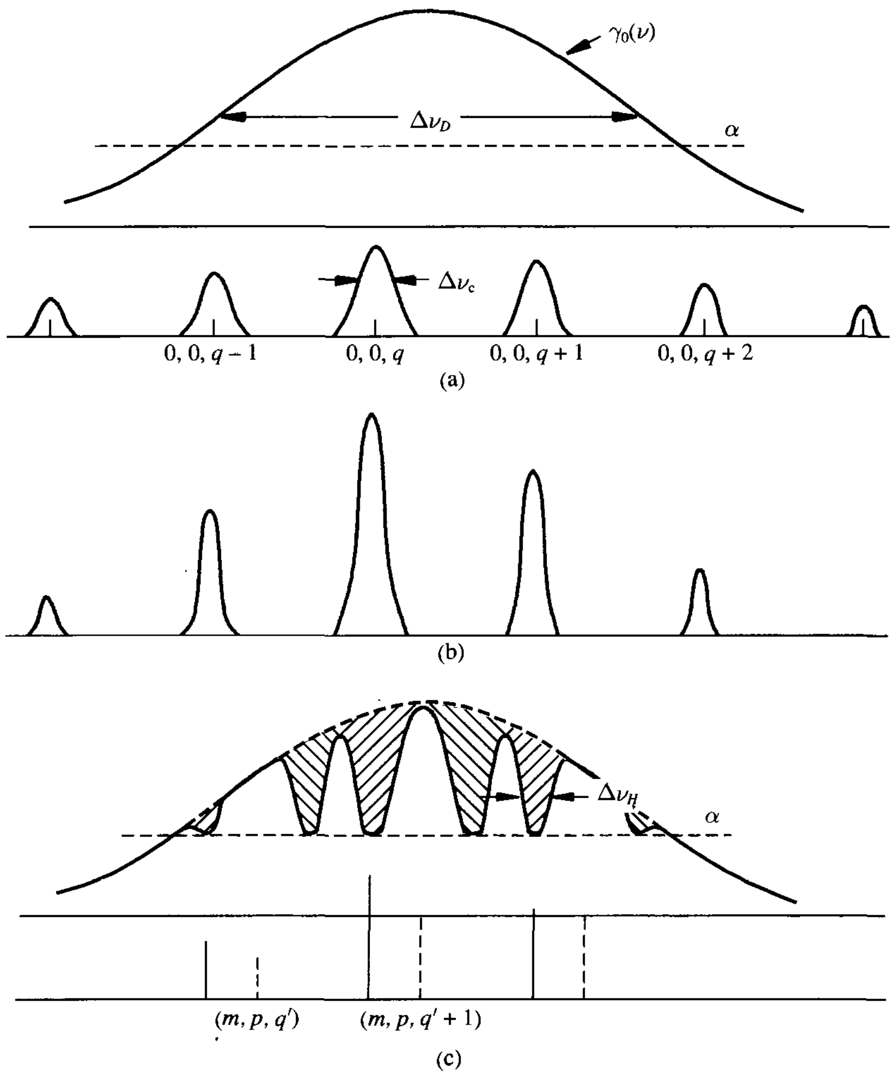
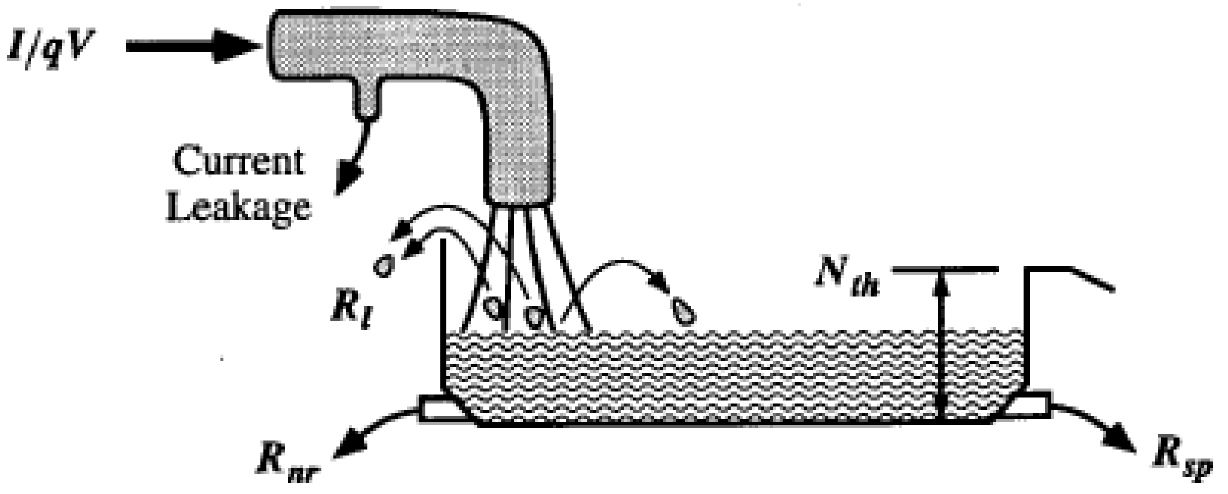


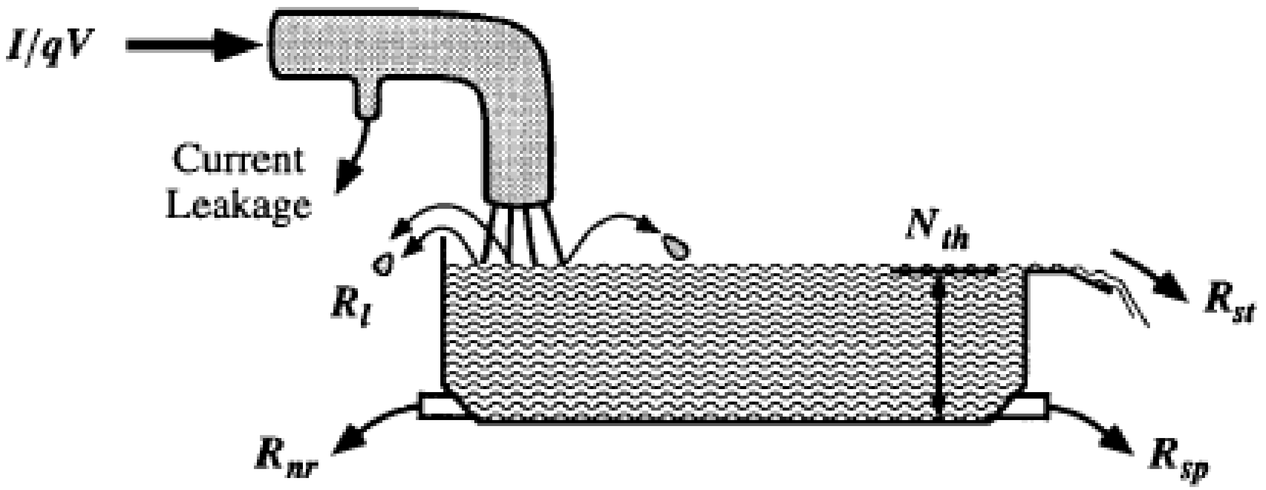
FIGURE 8.12. Evolution of oscillation in a Doppler-broadened transition.

From J. T. Verdeyen, "Laser Electronics" 3rd Ed. Prentice Hall, 1995

Below Threshold



Above Threshold



Coldren and Corzine, Diode Lasers & Photonic Integrated Circuits, J. Wiley (1995)

Mode Selection - Tuning

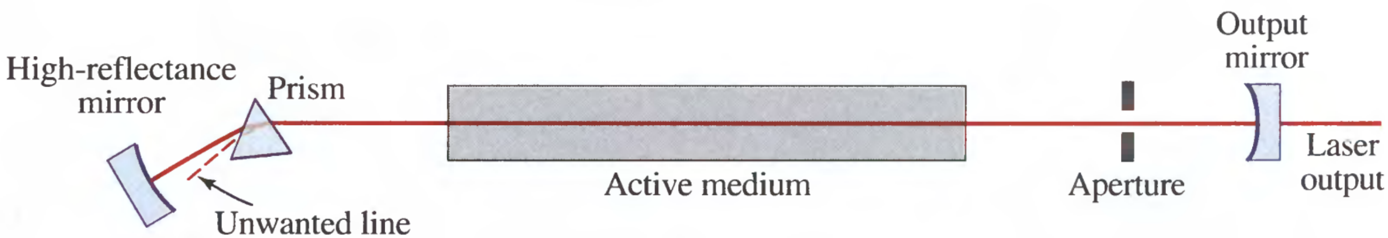


Figure 15.2-12 A particular atomic line may be selected by the use of a prism placed inside the resonator. A transverse mode may be selected by means of a spatial aperture of carefully chosen shape and size.

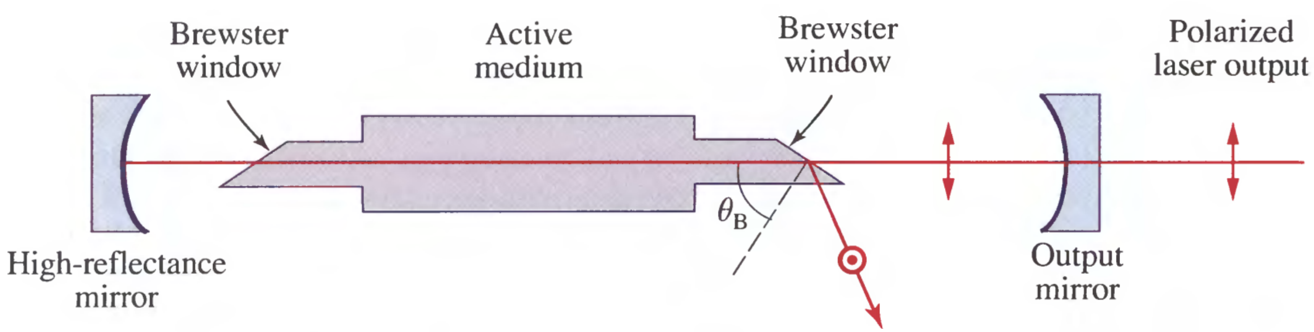


Figure 15.2-13 The use of Brewster windows in a gas laser provides a linearly polarized laser beam. Light polarized in the plane of incidence (the TM wave) is transmitted without reflection loss through a window placed at the Brewster angle. The orthogonally polarized (TE) mode suffers reflection loss and therefore does not oscillate.

From B.E.A. Saleh & M. C. Teich, "Fundamentals of Photonics" 2nd Ed. J. Wiley & Sons,2007.

Mode Selection - Tuning

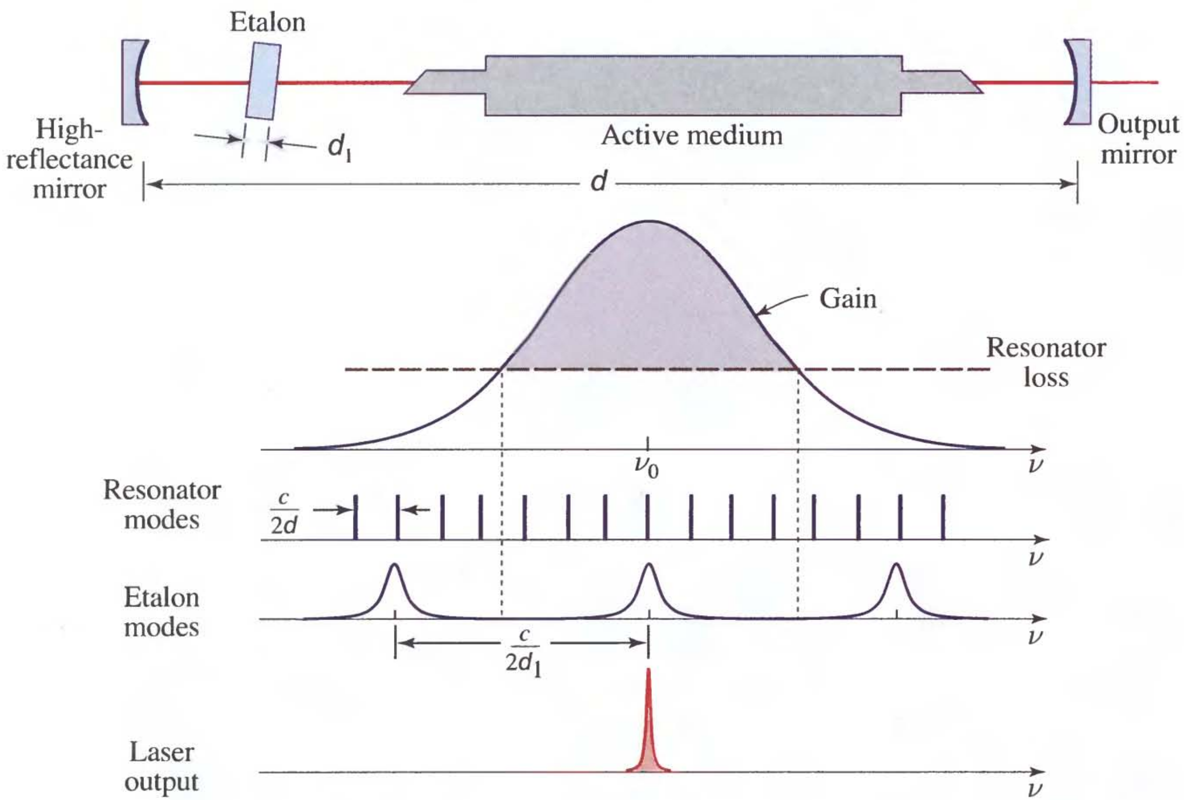
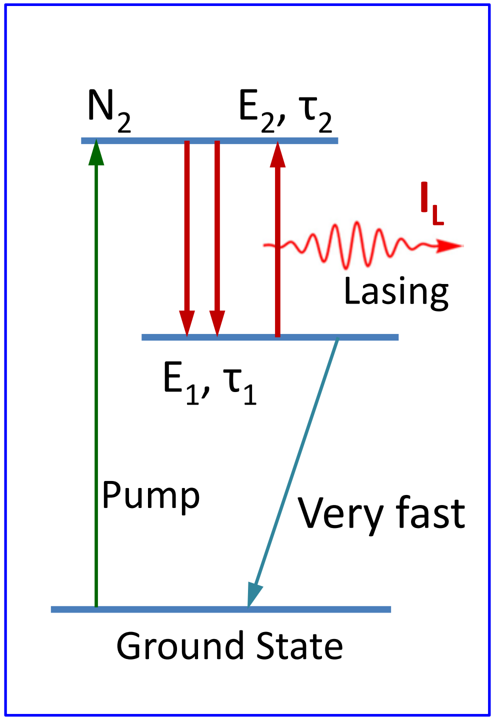
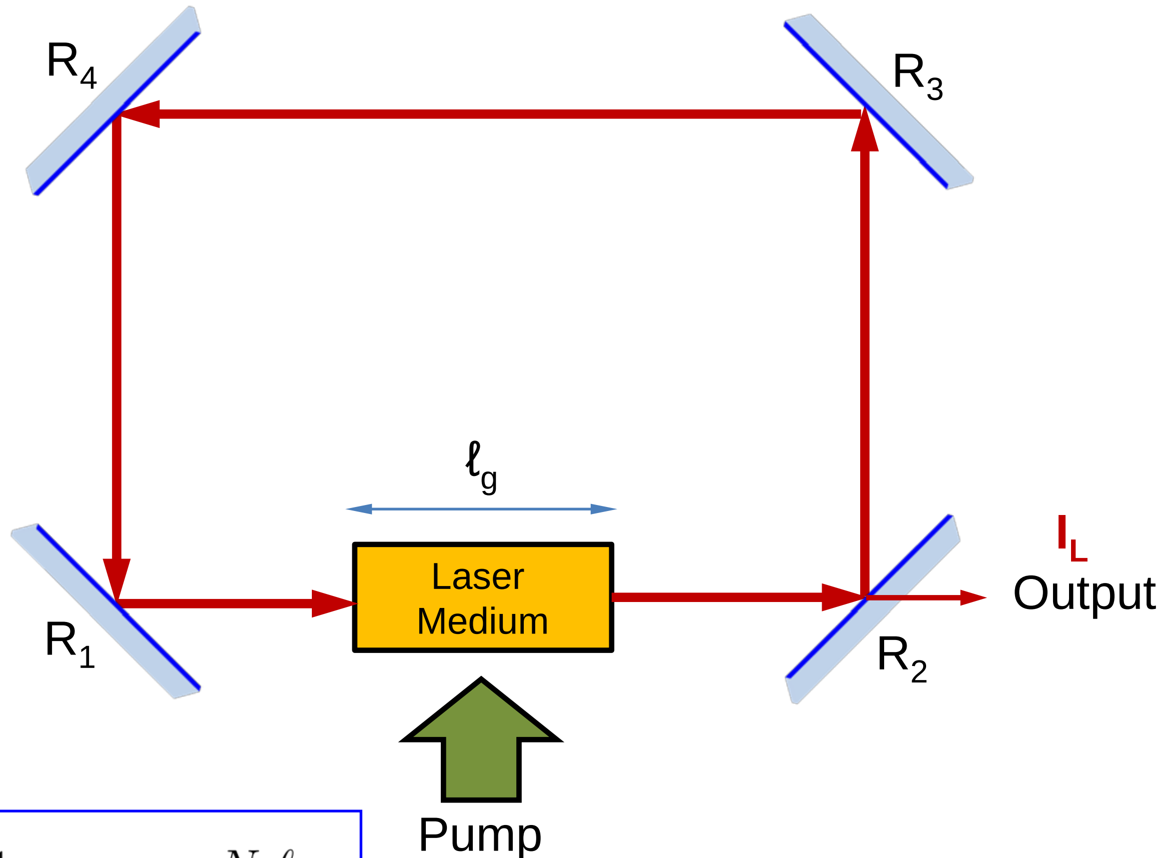


Figure 15.2-14 Longitudinal mode selection by the use of an intracavity etalon. Oscillation occurs at frequencies where a mode of the resonator coincides with an etalon mode; both must, of course, lie within the spectral window where the gain of the medium exceeds the loss.

From B.E.A. Saleh & M. C. Teich, "Fundamentals of Photonics" 2nd Ed. J. Wiley & Sons, 2007

Laser Dynamics – Simplified Two-Level System





$$\frac{dw}{dt} = \frac{S\exp(+N_2\sigma_L\ell_g)-1}{\tau_{RT}}w + \beta h\nu_L\frac{N_2\ell_g}{\tau_2}$$

$$\frac{dN_2}{dt} = \frac{\sigma_P I_P}{h\nu_P}N_0 - \frac{N_2}{\tau_2} - \frac{\sigma_L I_L}{h\nu_L}N_2$$

$$w = I_L\tau_{RT} = N_p h\nu_L/A$$

Laser Dynamics – Simplified Two-Level System

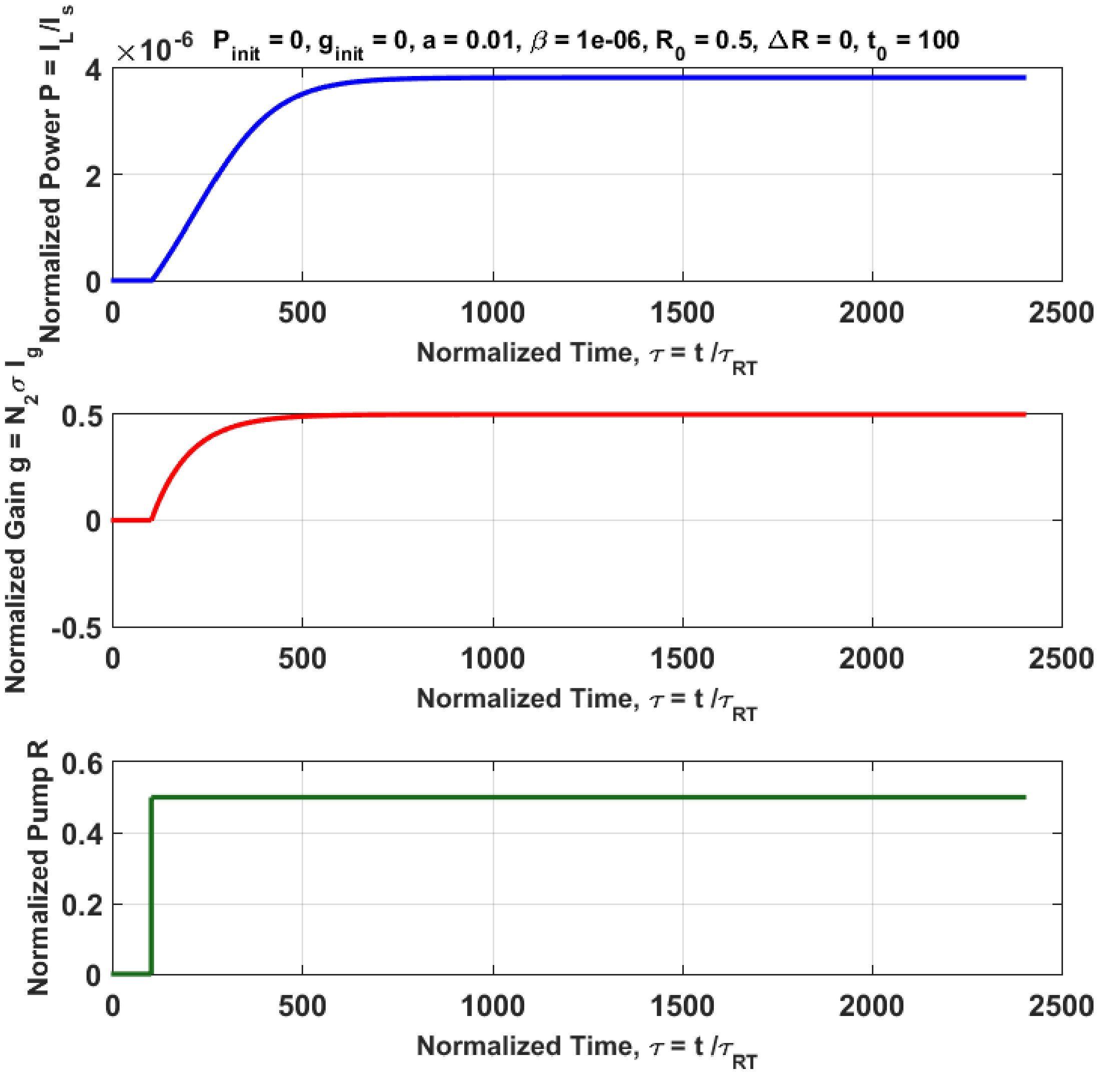
Normalized Rate Equations

$$\frac{dP}{d\tau} = \left(Se^{g} - 1\right)P + \beta g$$
$$\frac{dg}{d\tau} = a\left[R - g(1 + P)\right]$$

$$\tau = \frac{t}{\tau_{RT}} \quad \text{(Normalized time)}$$
$$g = N_2 \sigma_L \ell_g \quad \text{(Line integrated gain)}$$
$$P = \frac{w}{w_s} = \frac{I_L}{I_s} \quad \text{(Normalized energy)}$$
$$R = N_0 \sigma_p \ell_g \frac{\lambda_p}{\lambda_L}\frac{I_p}{I_s} \quad \text{(Normalized pump)}$$
$$a = \frac{\tau_{RT}}{\tau_2}$$
$$S = 1 - L \quad \text{(Survival factor)}$$
$$I_s = \frac{h\nu_L}{\tau_2 \sigma_L} \quad \text{(Saturation Intensity)}$$

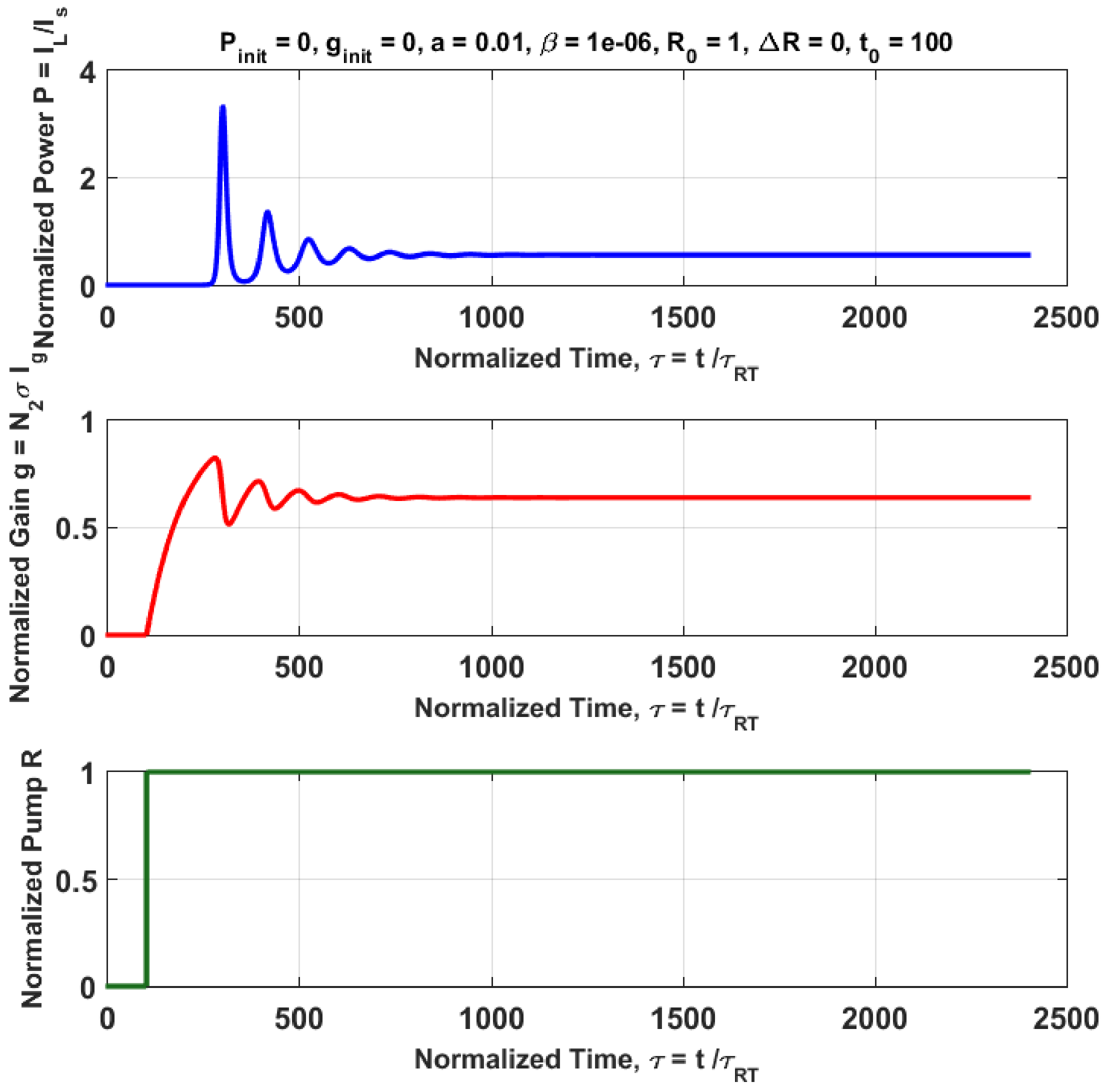
Laser Dynamics – Simplified Two-Level System

Sub-threshold System – g_{th} = 0.64 (S = 0.527292)



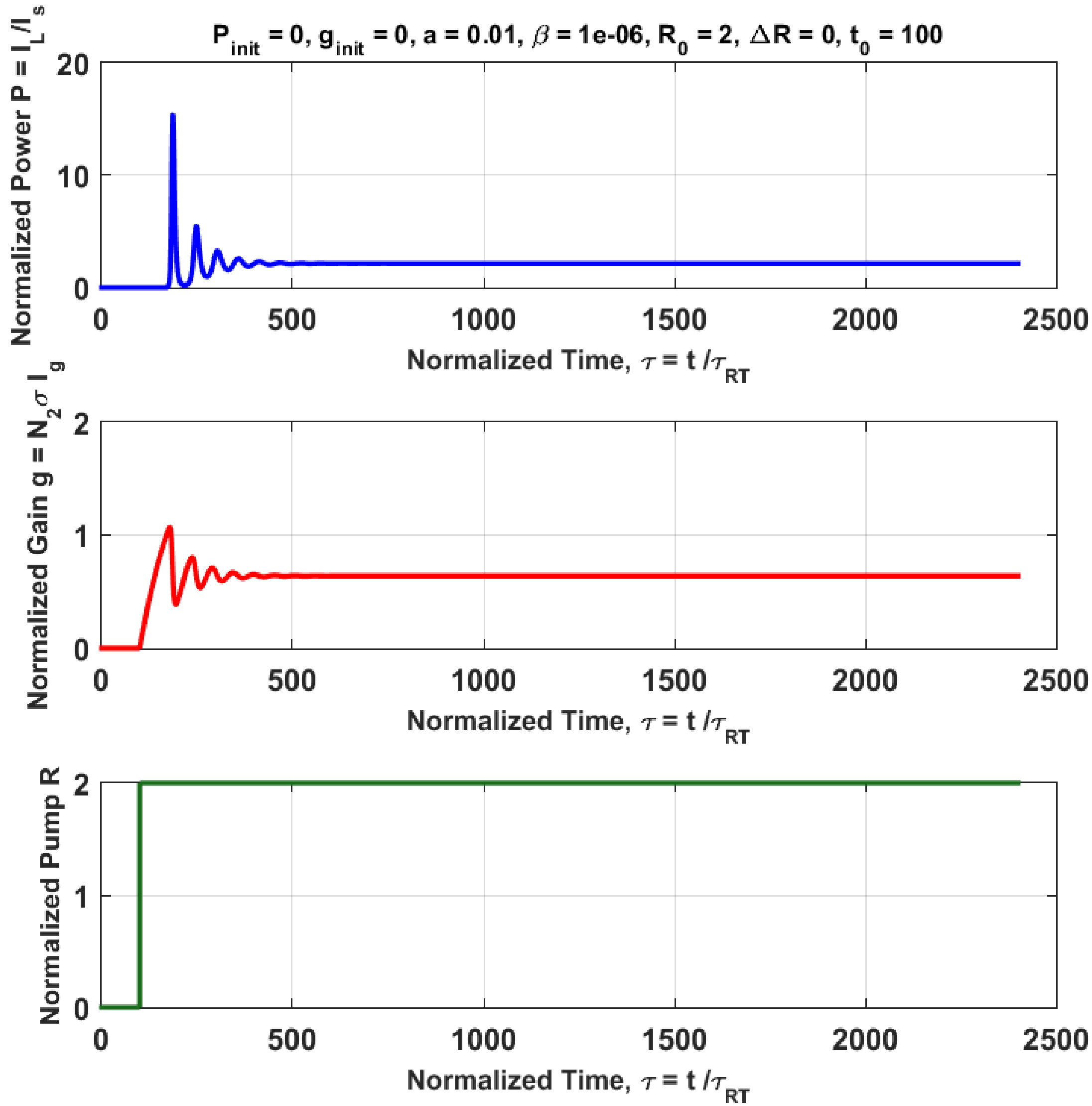
Laser Dynamics – Simplified Two-Level System

Above-threshold System – g_{th} = 0.64 (S = 0.527292)



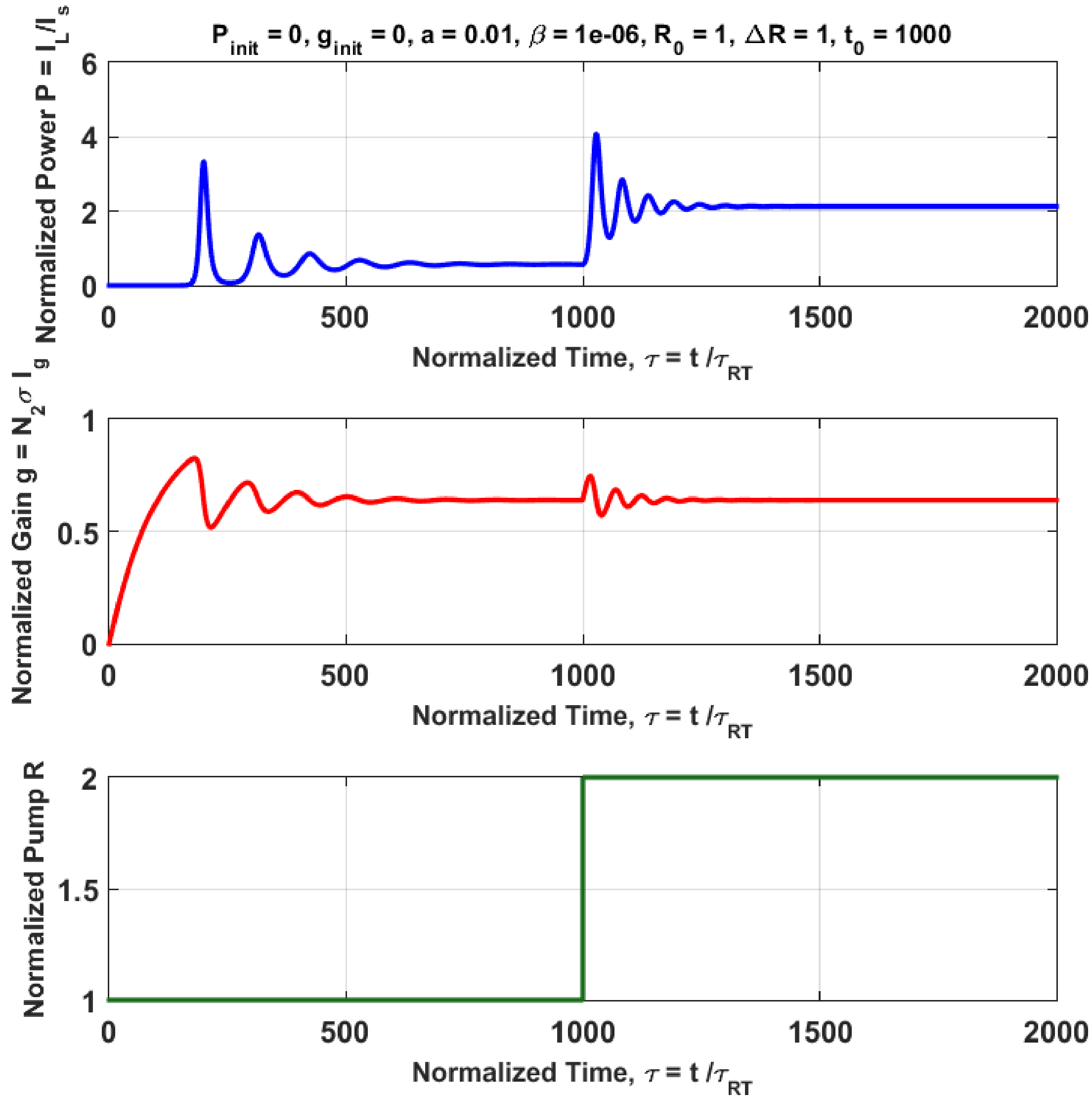
Laser Dynamics – Simplified Two-Level System

Above-threshold System – g_{th} = 0.64 (S = 0.527292)



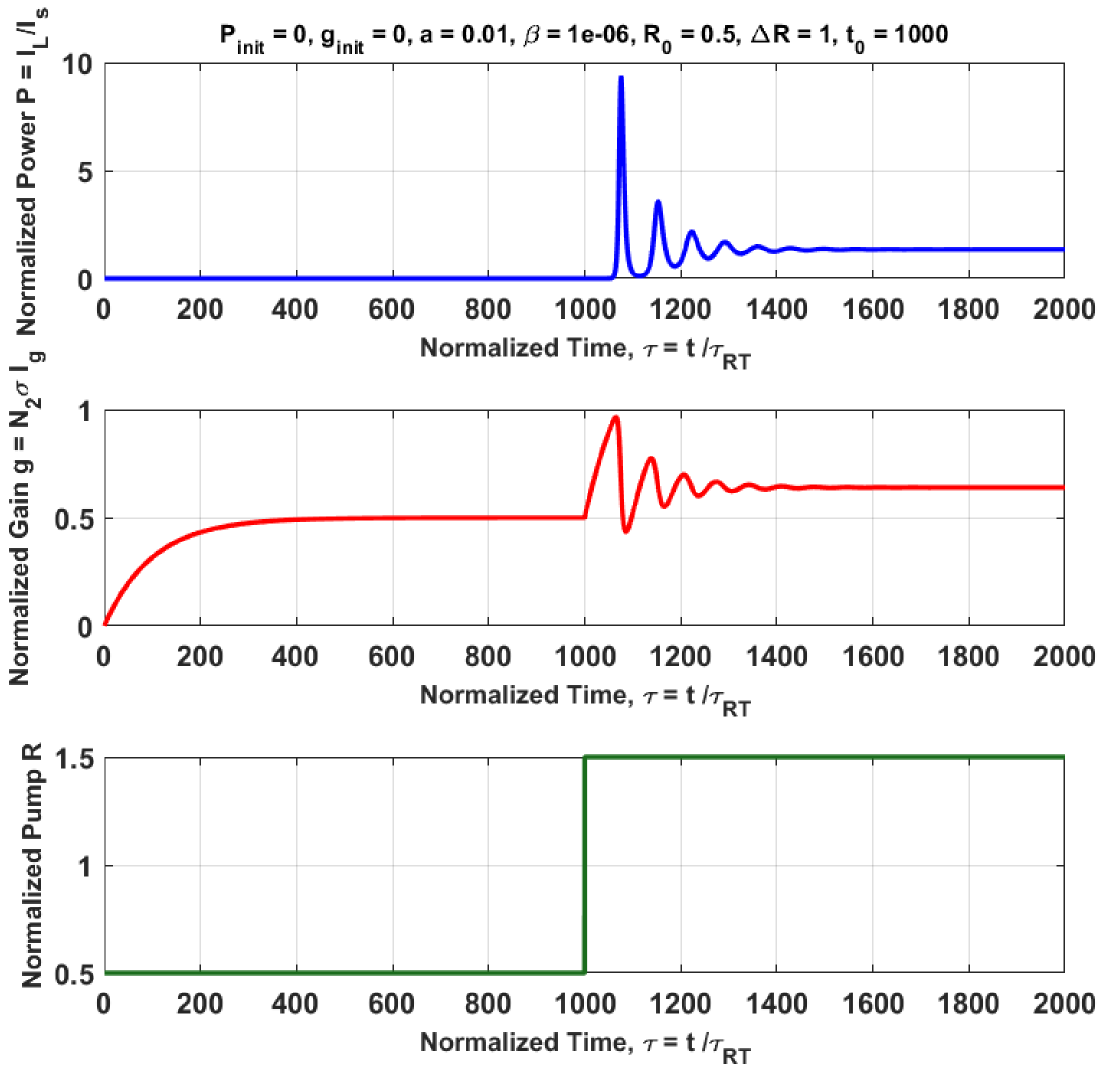
Laser Dynamics – Simplified Two-Level System

Above-threshold System – g_{th} = 0.64 (S = 0.527292) – Step Change



Laser Dynamics – Simplified Two-Level System

Above-threshold System – g_{th} = 0.64 (S = 0.527292) – Step Change



Laser Dynamics – Step Change in Excitation

$$\mathcal{L}\left\{\frac{d\Delta p}{dt} = \Delta g(P_d + \beta)\right\} \Rightarrow s\Delta p(s) = \Delta g(s)(P_d + \beta)$$

$$\mathcal{L}\left\{\frac{d\Delta g}{dt} = a\left[\Delta r(t) - \Delta g(1 + P_d) - g_{th}\Delta p\right]\right\}$$

$$\begin{bmatrix} a g_{th} & s + a(1 + P_d) \\ s & -(\beta + P_d) \end{bmatrix} \begin{bmatrix} \Delta p(s) \\ \Delta g(s) \end{bmatrix} = \begin{bmatrix} \dfrac{a\Delta r}{s} \\ 0 \end{bmatrix}$$

$$\Delta p(s) = \frac{a(\beta + P_d)\Delta r}{s\left[s^2 + a(1 + P_d)s + a g_{th}(\beta + P_d)\right]}$$

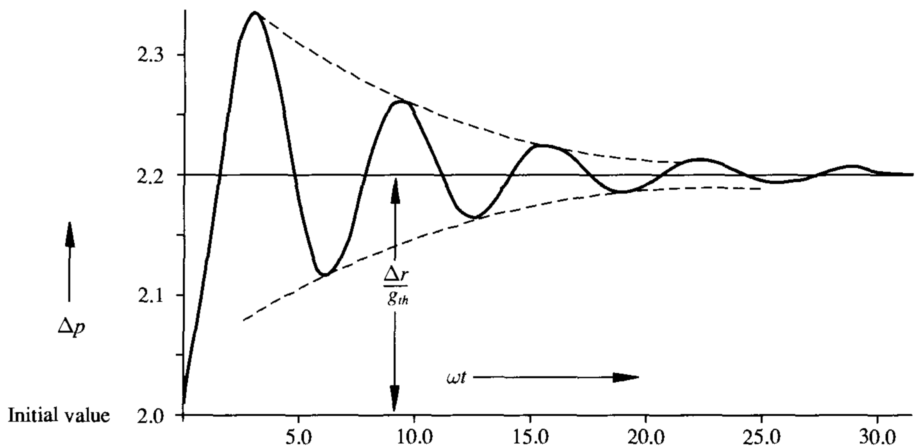


FIGURE 9.8. Damped oscillation while a laser approaches a new steady state. ($\Delta r = 0.2 \times 0.64 = 0.128$)

From J. T. Verdeyen, "Laser Electronics" 3rd Ed. Prentice Hall, 1995

Laser Dynamics – Simplified Two-Level System

Sinusoidal Response – g_{th} = 0.64 (S = 0.527292)

$$R(t) = R_c + \Delta r(t)$$

$$P_c = \frac{R_c}{g_{th}} - 1$$

$$g(t) = g_{th} + \Delta g(t)$$

$$\frac{d^2 \Delta p}{dt^2} + a(1 + P_c)\frac{d\Delta p}{dt} + g_{th} a(P_c + \beta)\Delta p = a(P_c + \beta)\Delta r$$

$$\Delta p(t) = \{p_m \exp[j\omega_m t]\}$$

$$\frac{g_{th} p_m}{r_m} = \frac{g_{th} a(\beta + P_c)}{\left\{\left[g_{th} a(\beta + P_c) - \omega_m^2\right] + j\omega_m\left[a(1 + P_c)\right]\right\}}$$

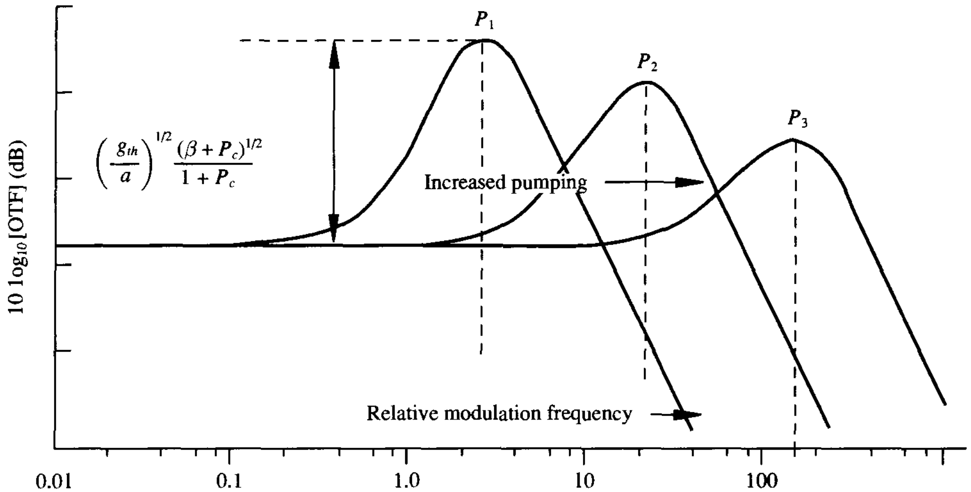
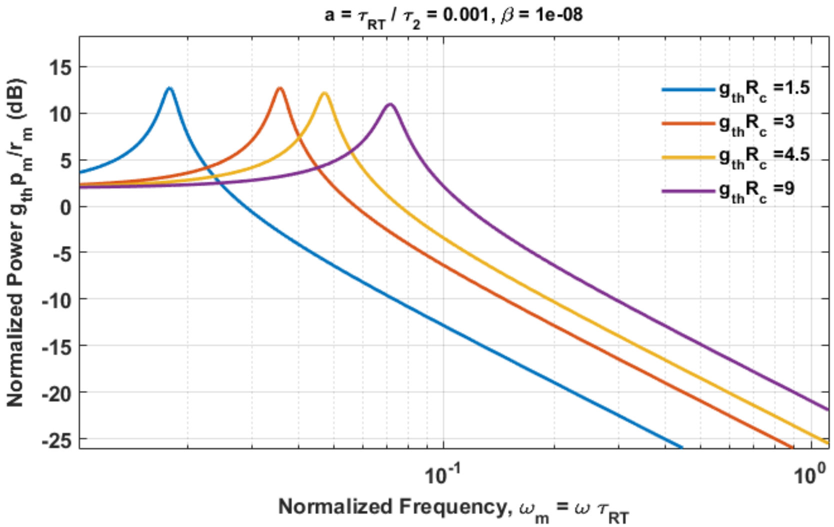


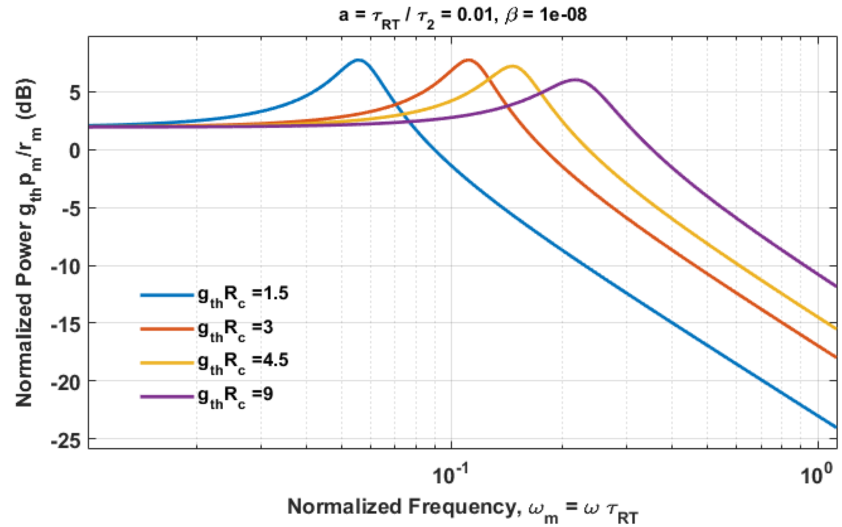
FIGURE 9.7. The relative response of a laser to a sinusoidal modulation of the pump. As the pumping is increased, the power, the resonant frequency, and the resonant width all increase. The "standard" method of plotting the modulation "gain" (in dB relative to that at a low frequency) as a function of the modulation frequency on a logarithmic frequency scale was used.

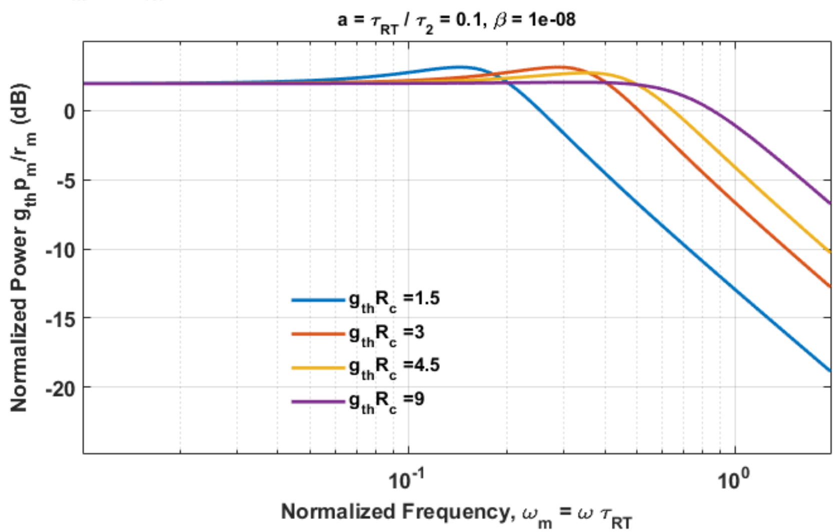
From J. T. Verdeyen, "Laser Electronics" 3rd Ed. Prentice Hall, 1995

Laser Dynamics – Simplified Two-Level System

Sinusoidal Response – g_{th} = 0.64 (S = 0.527292)

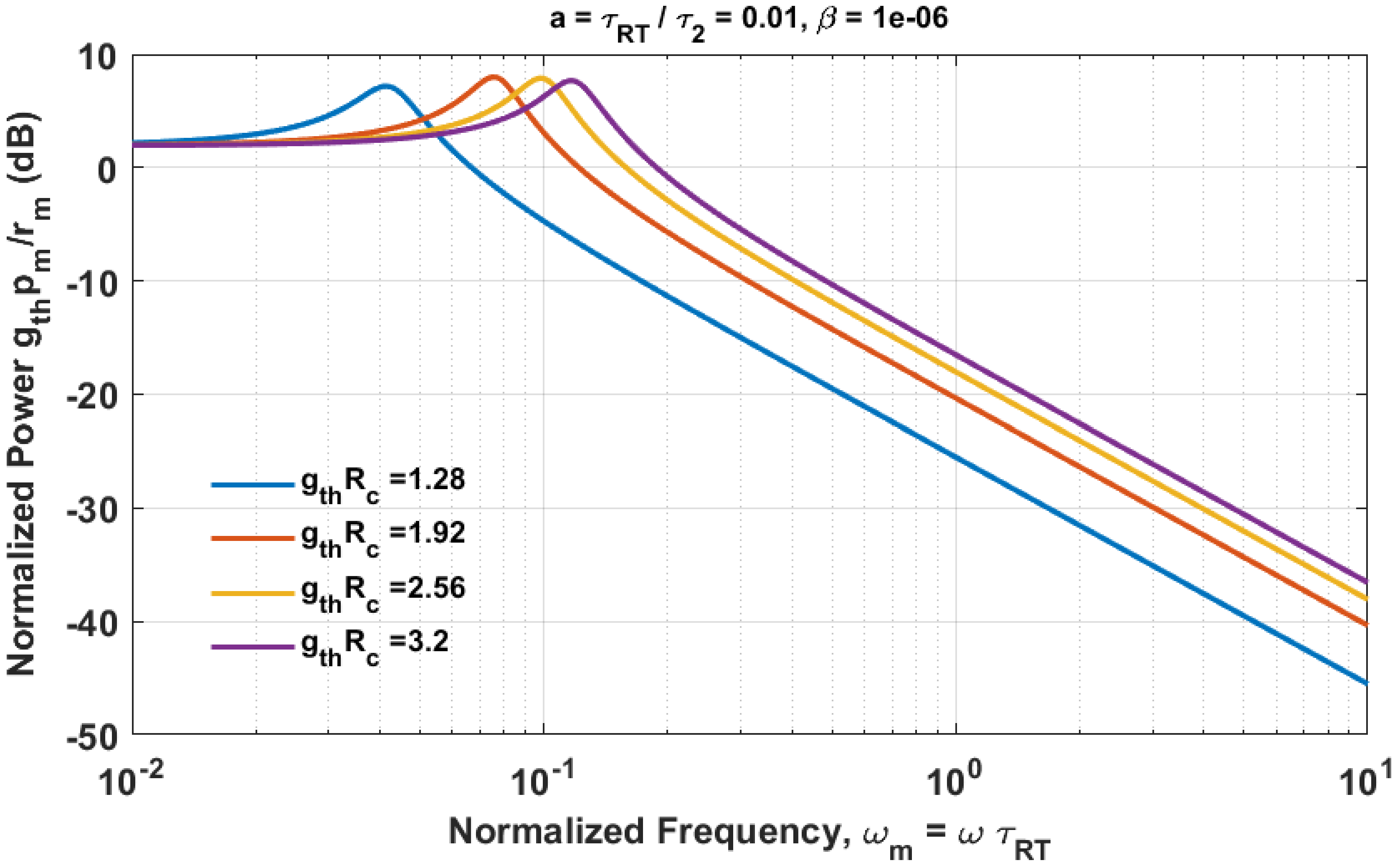






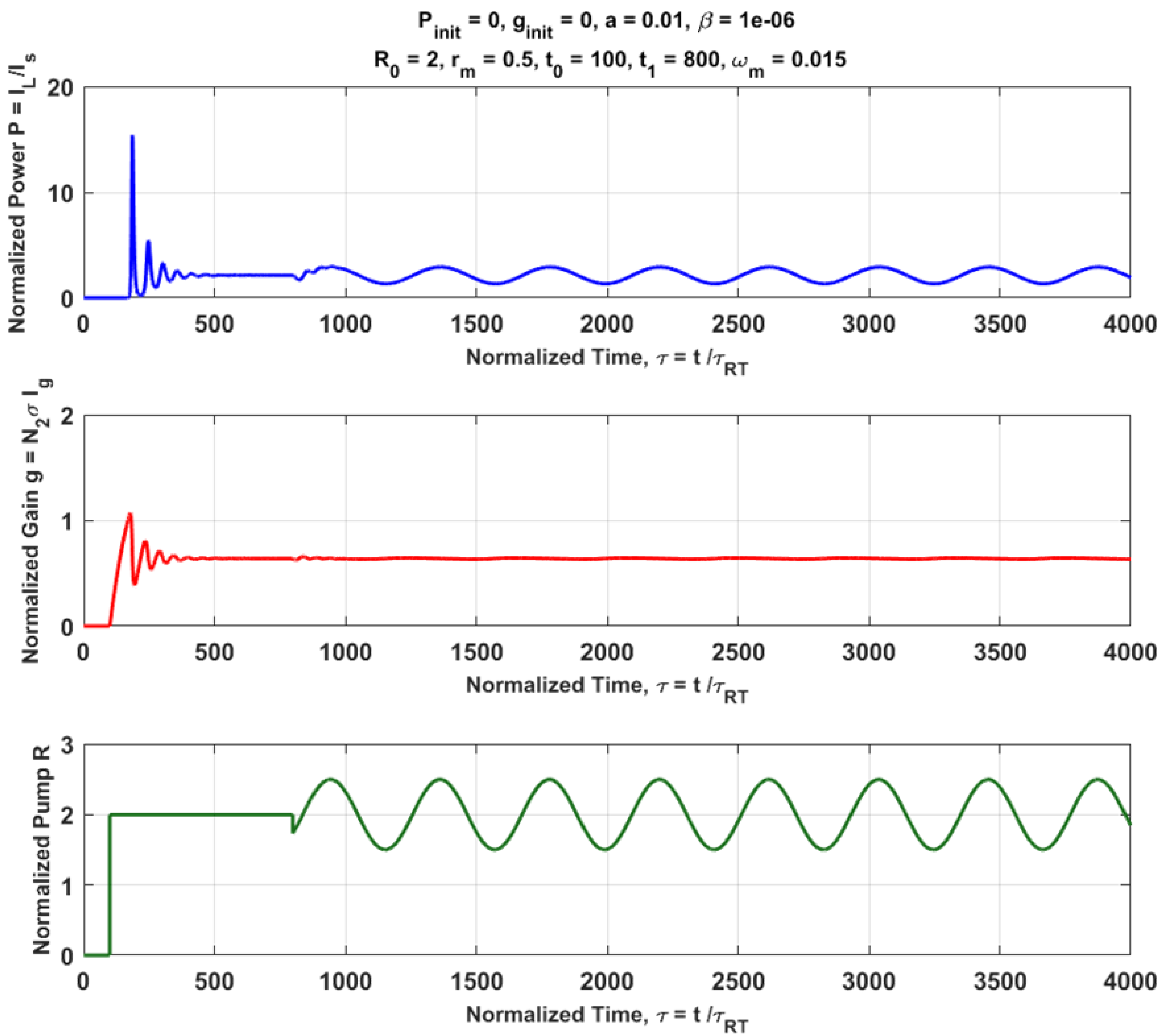
Laser Dynamics – Simplified Two-Level System

Sinusoidal Response – g_{th} = 0.64 (S = 0.527292)



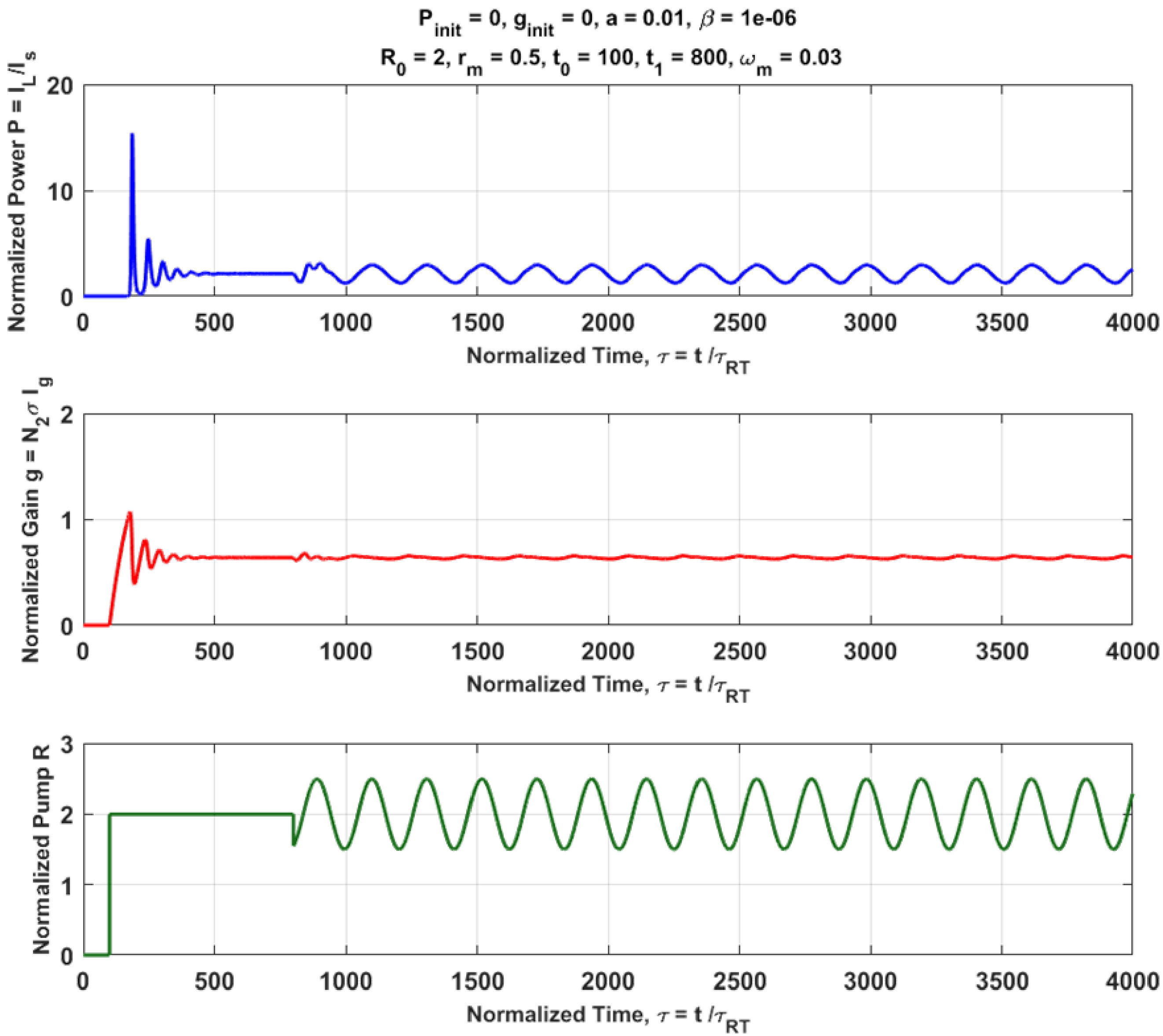
Laser Dynamics – Simplified Two-Level System

Sinusoidal Response – g_{th} = 0.64 (S = 0.527292)



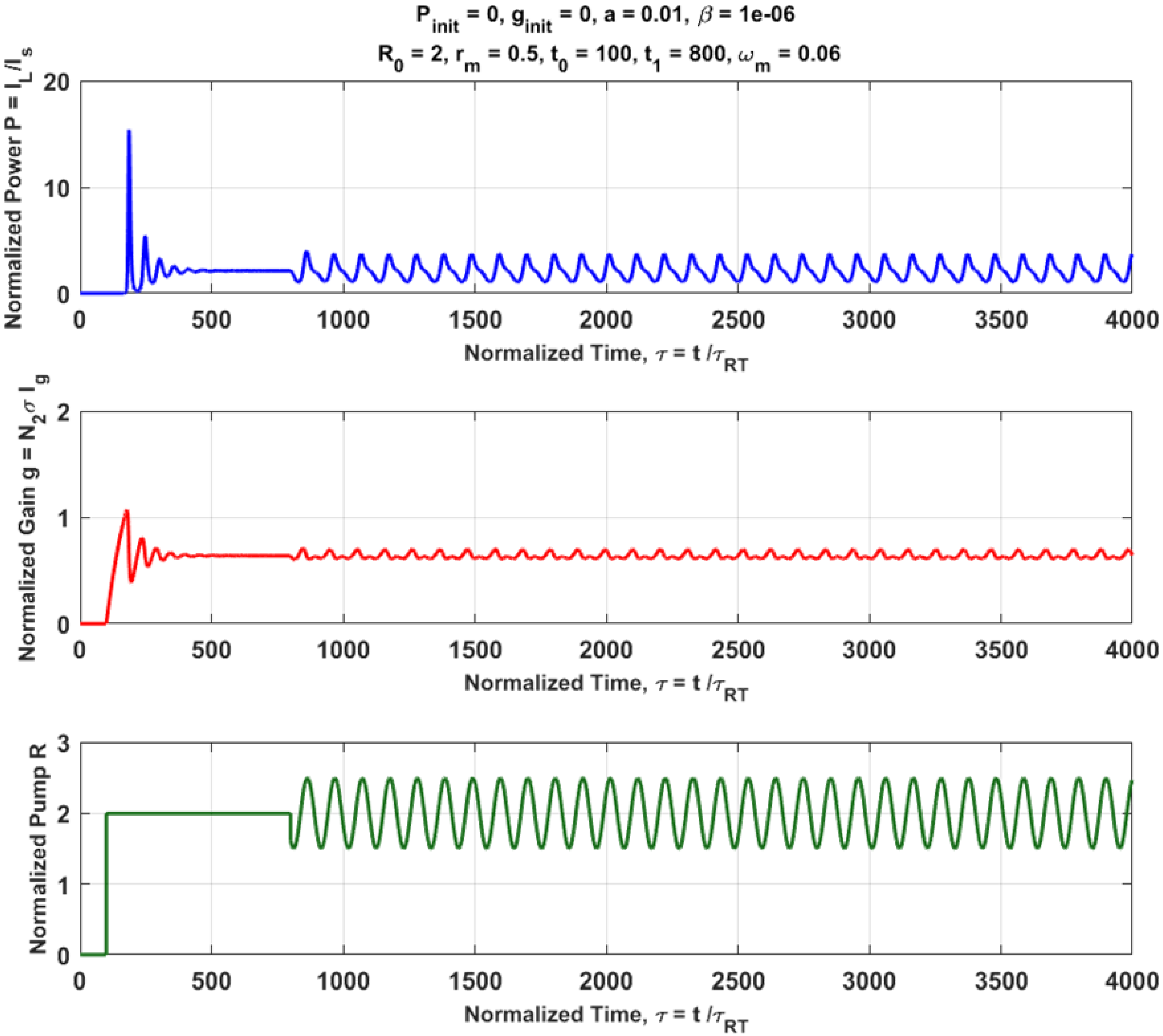
Laser Dynamics – Simplified Two-Level System

Sinusoidal Response – g_{th} = 0.64 (S = 0.527292)



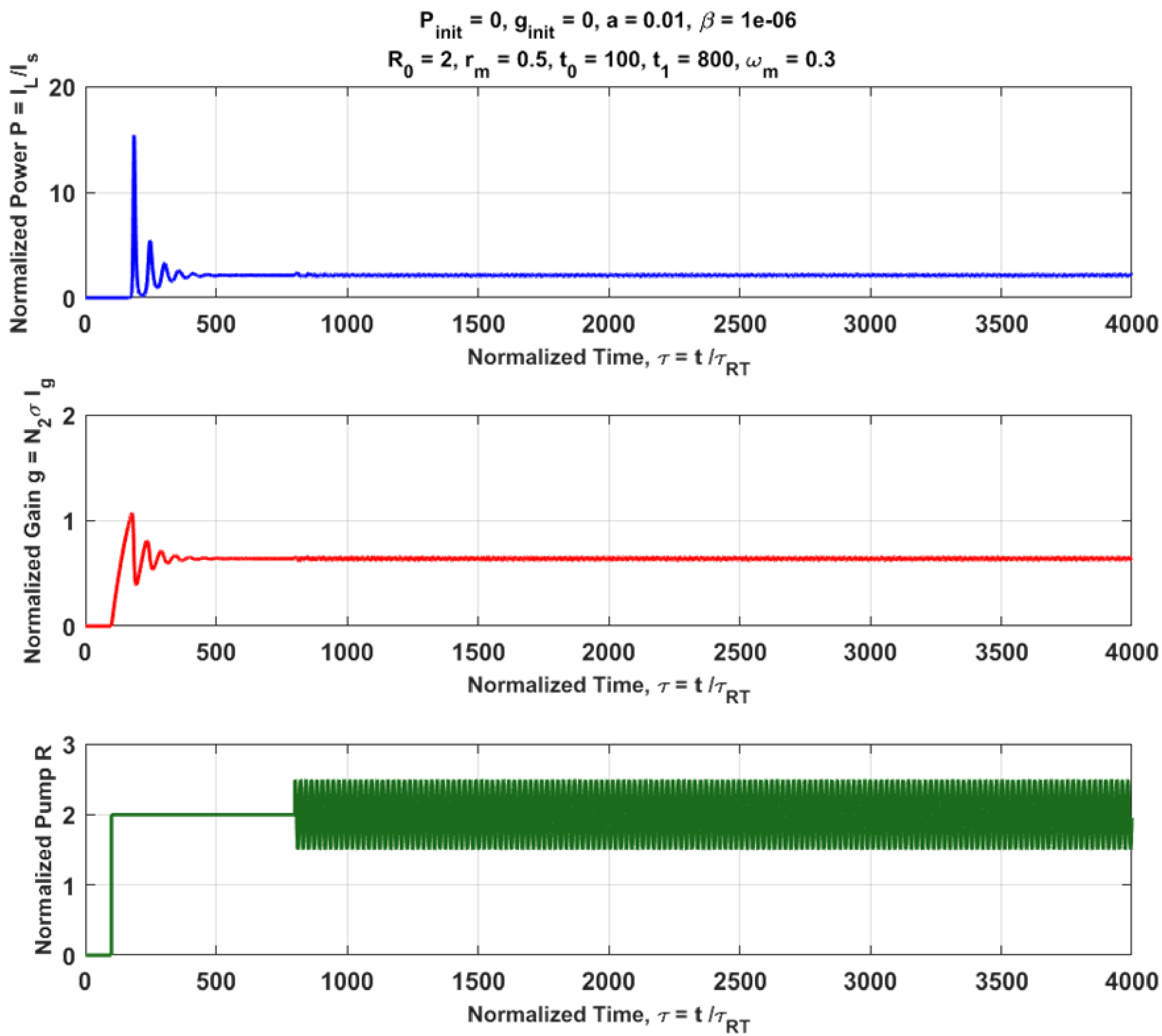
Laser Dynamics – Simplified Two-Level System

Sinusoidal Response – g_{th} = 0.64 (S = 0.527292)



Laser Dynamics – Simplified Two-Level System

Sinusoidal Response – g_{th} = 0.64 (S = 0.527292)



Mode Locking – Time Domain Consideration

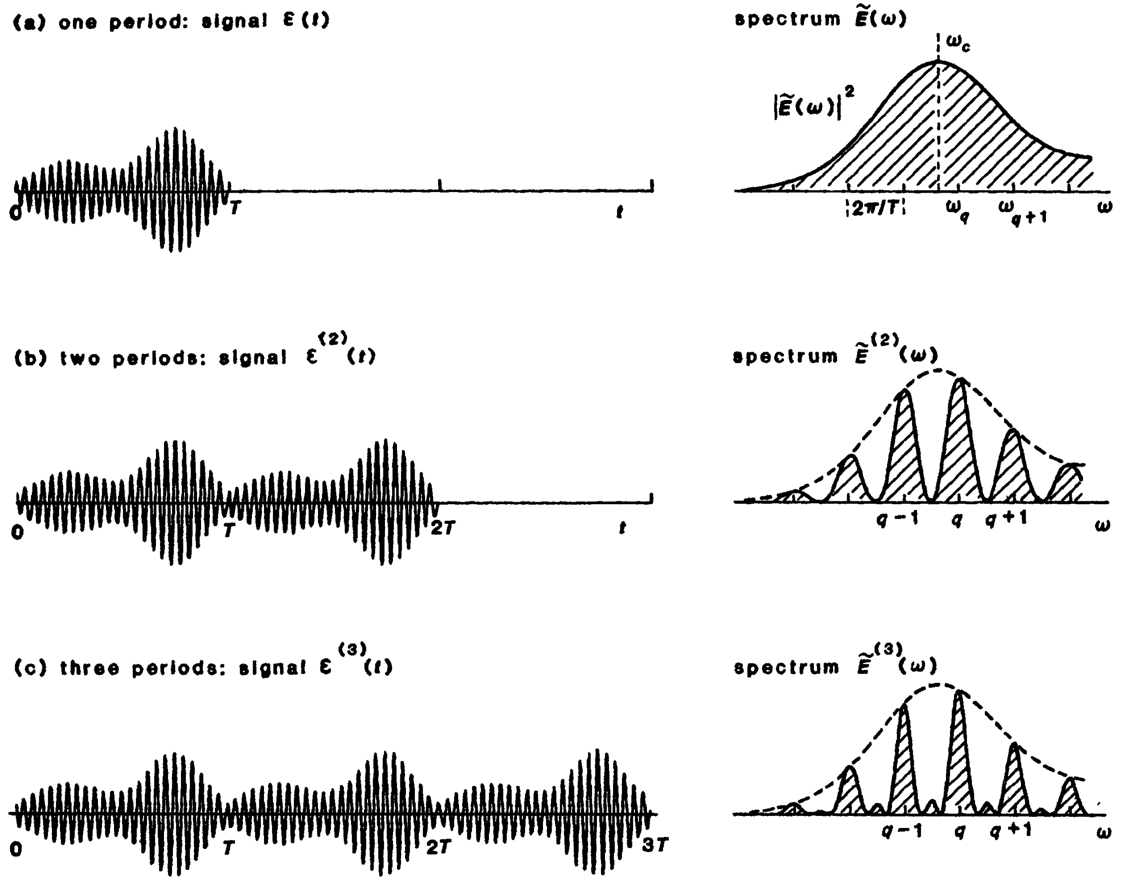


FIGURE 27.2
Examples of a time signal $E(t)$ and its power spectrum for the same signal repeated one, two, or three times in succession. (Note that the carrier frequency ω_c of the sine-wave signal does not coincide with any of the axial modes ω_q.)

From A. Siegman, "Lasers", Univ. Science Books, 1986

Mode Locking – Time Domain Consideration

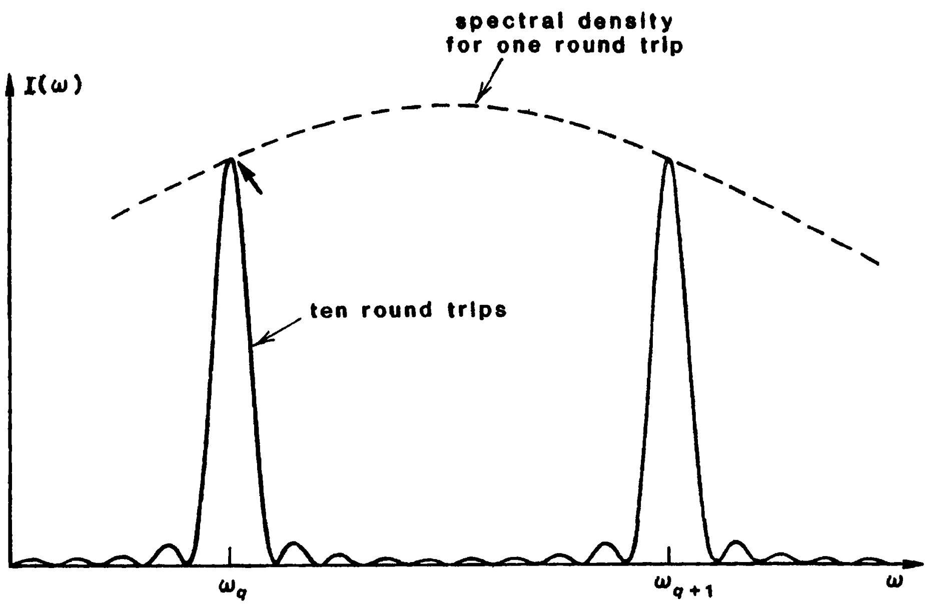


FIGURE 27.3
Power spectral density for the same optical signal repeated 10 times in succession.

From A.E. Siegman, "Lasers", Univ. Science Books, 1986

Mode Locking – Frequency Domain Consideration

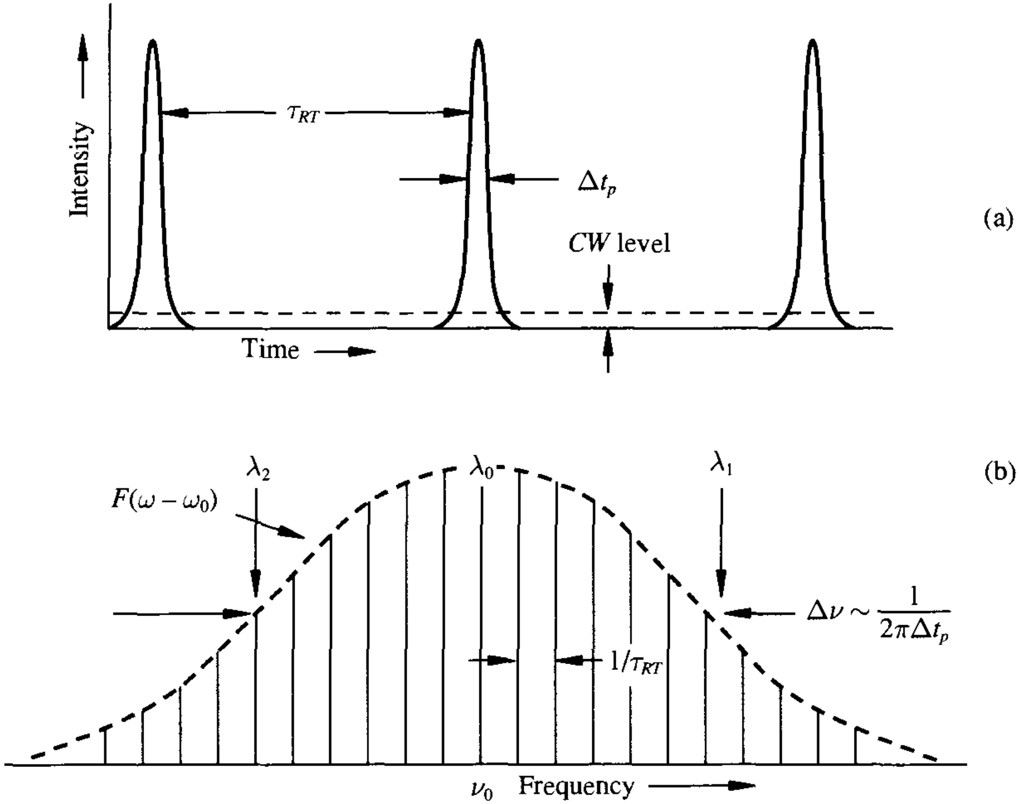


FIGURE 9.15. The (a) time and (b) frequency domain representation of a mode-locked laser.

From J. T. Verdeyen, "Laser Electronics", 3rd Ed., Prentice Hall, 1995

Mode Locking – Frequency Domain Consideration

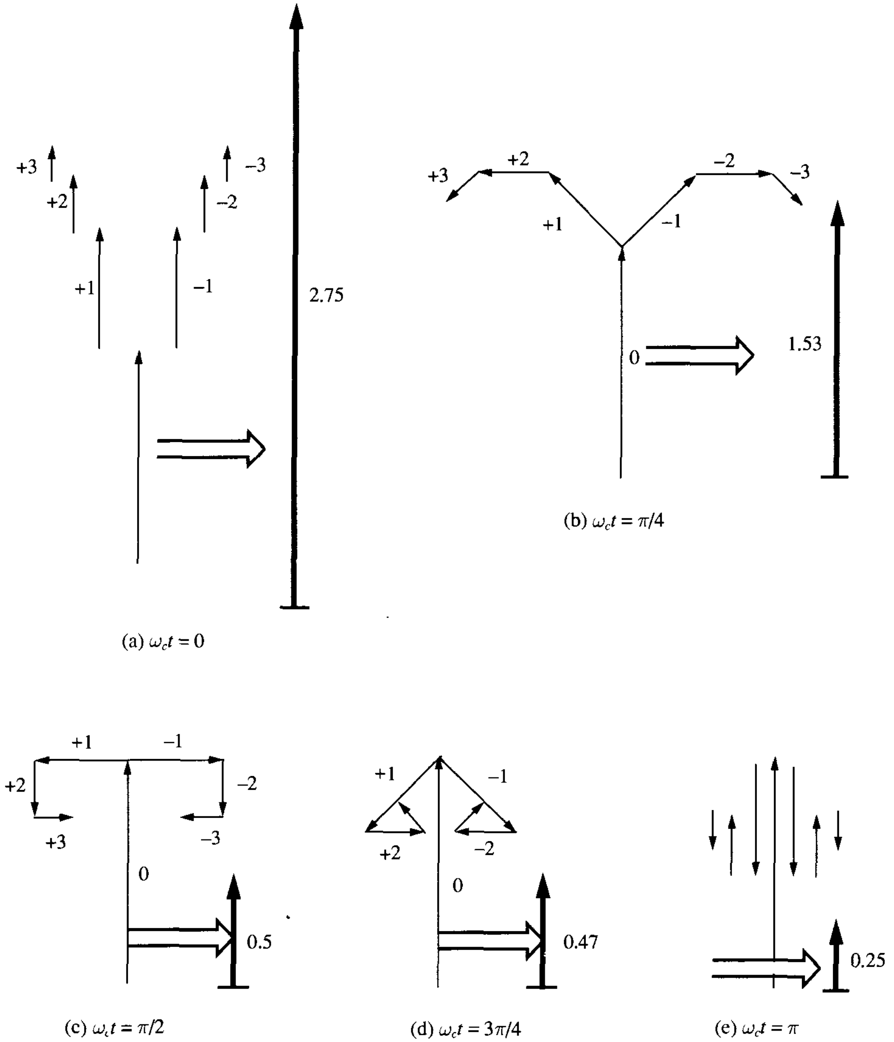


FIGURE 9.16. Phasor addition of the fields of a mode-locked laser. The mode amplitudes were chosen according to a proportional relation: 1 : 0.5 : 0.25 : 0.125.

From J. T. Verdeyen, "Laser Electronics", 3rd Ed., Prentice Hall, 1995

Mode Locking – Frequency Domain Consideration

$$E(t) = \frac{1}{\sqrt{N}} \sum_{m=1}^{N} e^{i(\omega_0 + m\Omega)t} = \frac{1}{\sqrt{N}} e^{i[(\omega_0 + (N+1)\Omega/2]t} \frac{\sin(N\Omega t/2)}{\sin(\Omega t/2)}$$

$$E(t) = \sum_{n=1}^{N} C_n e^{in\Omega t} = \sqrt{\frac{2}{N+1}} \sum_{n=1}^{N} \sin\left(\frac{n\pi}{N+1}\right) e^{in\Omega t}$$

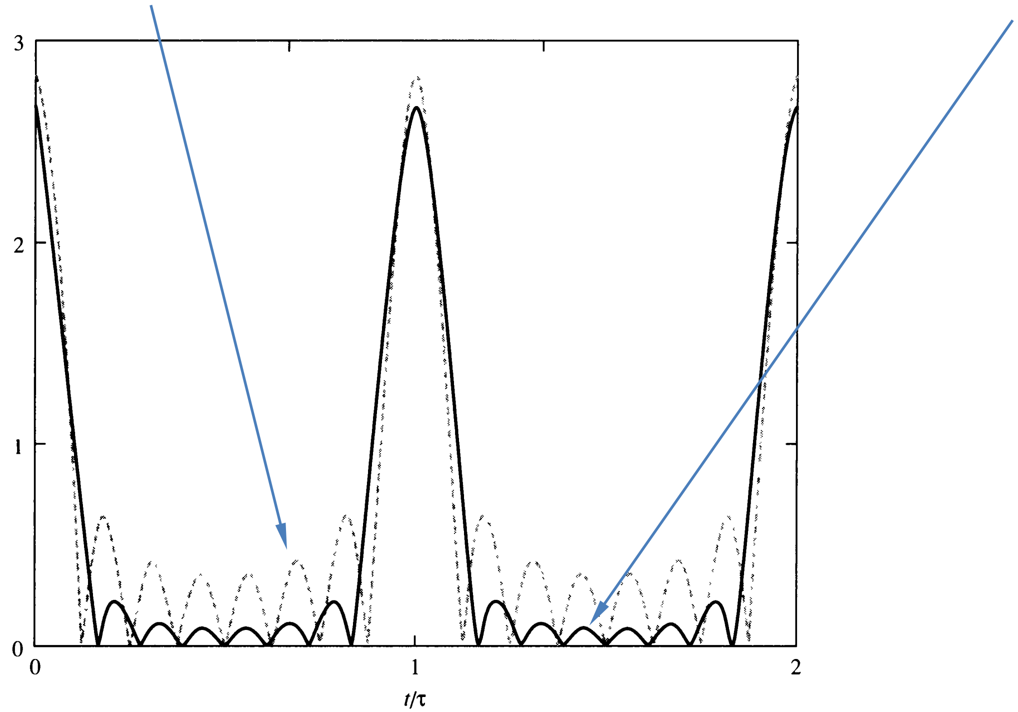


Figure 6.18 Normalized field amplitude (absolute value) profile of mode-locked laser beam with $N = 8$. The solid curve is from Equation (6.6-25), and the dotted curve is from Equation (6.6-7). The field amplitudes are normalized so that $\sum |C_n|^2 = 1$.

From A. Yariv and P. Yeh, "Photonics", 6th Ed., Oxford Univ. Press, 2007

Mode Locking – Frequency Domain Consideration

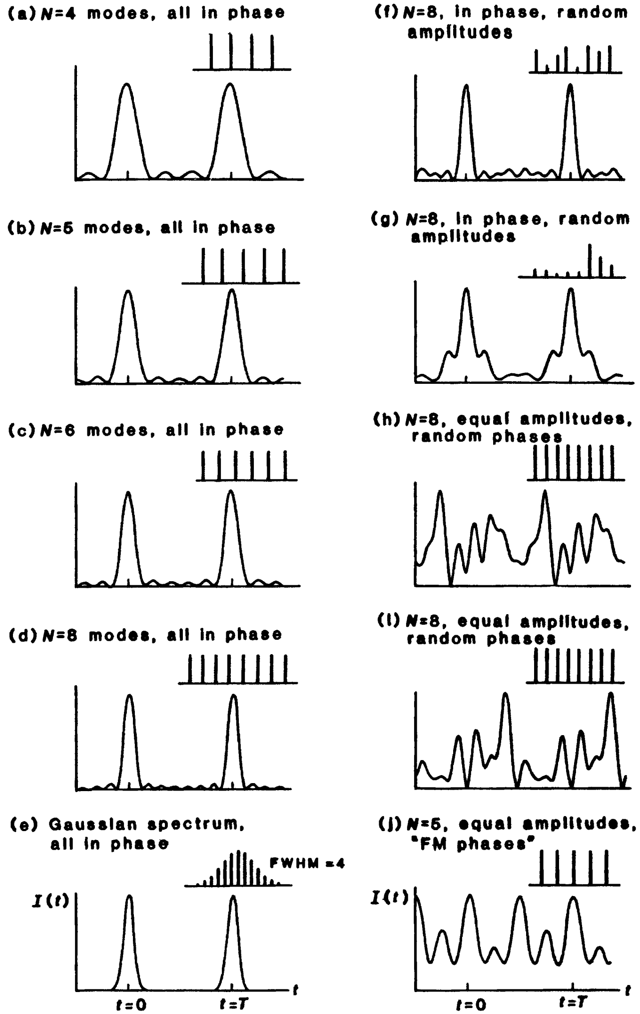
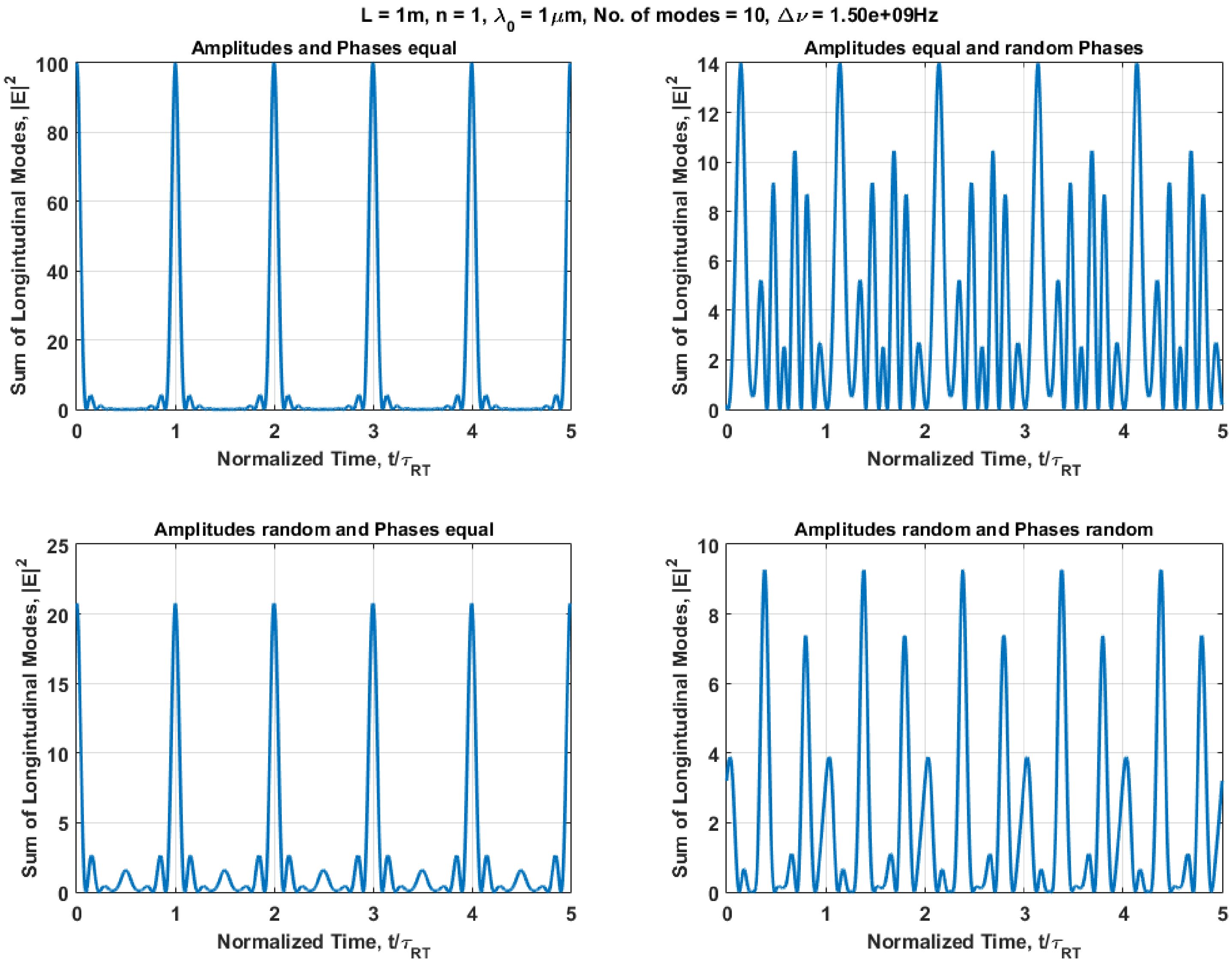


FIGURE 27.7
Examples of the different intensity patterns in time that can be synthesized using N equally spaced frequency components with different relative amplitudes and phase angles.

From A.E. Siegman, "Lasers", Univ. Science Books, 1986

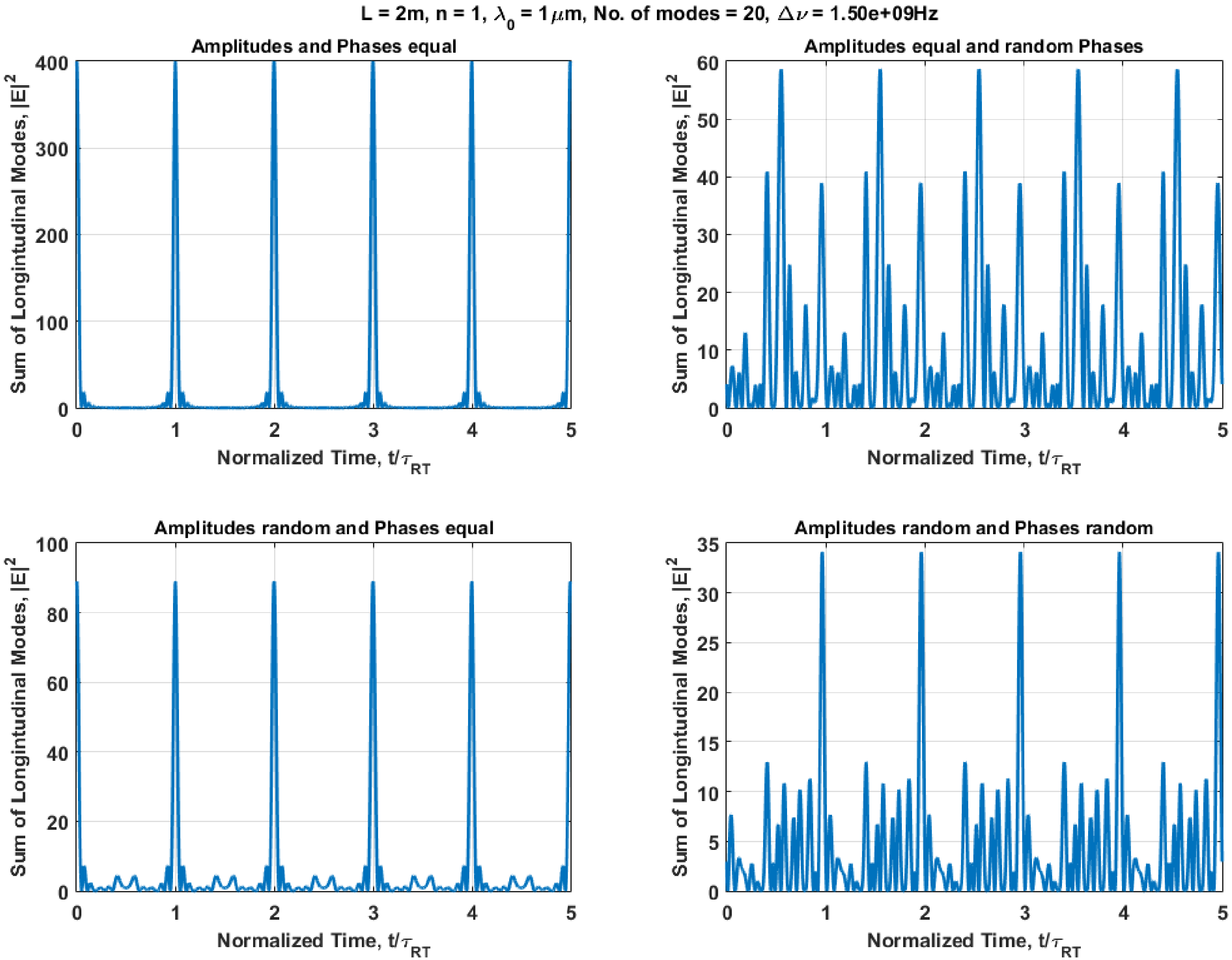
Mode Locking – Frequency Domain Consideration

L = 1m, n = 1, λ_0 = 1μm, Δv = 1.50e+09Hz

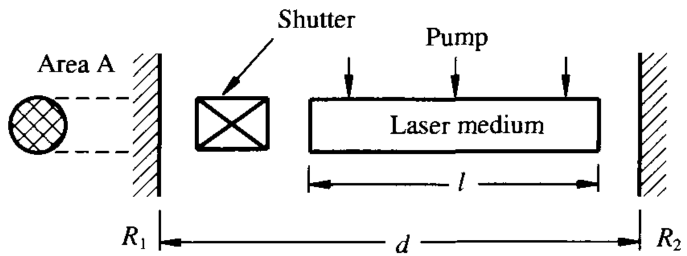


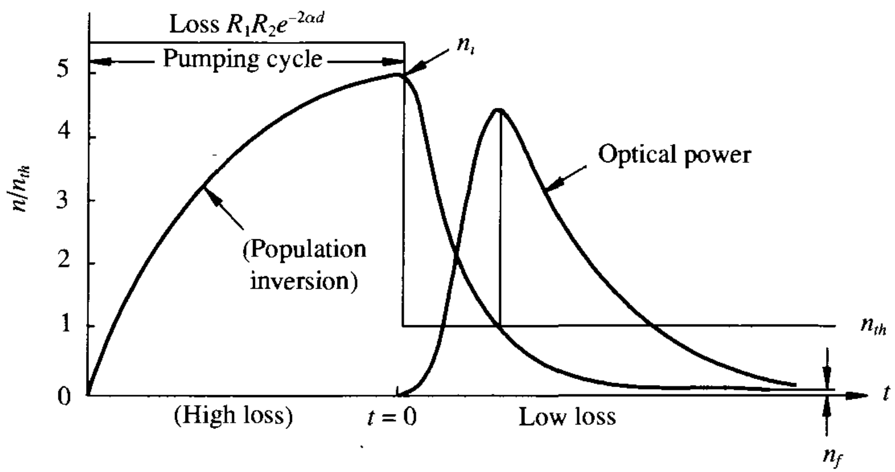
Mode Locking – Frequency Domain Consideration

L = 2m, n = 1, λ_0 = 1μm, Δν = 1.50e+09Hz



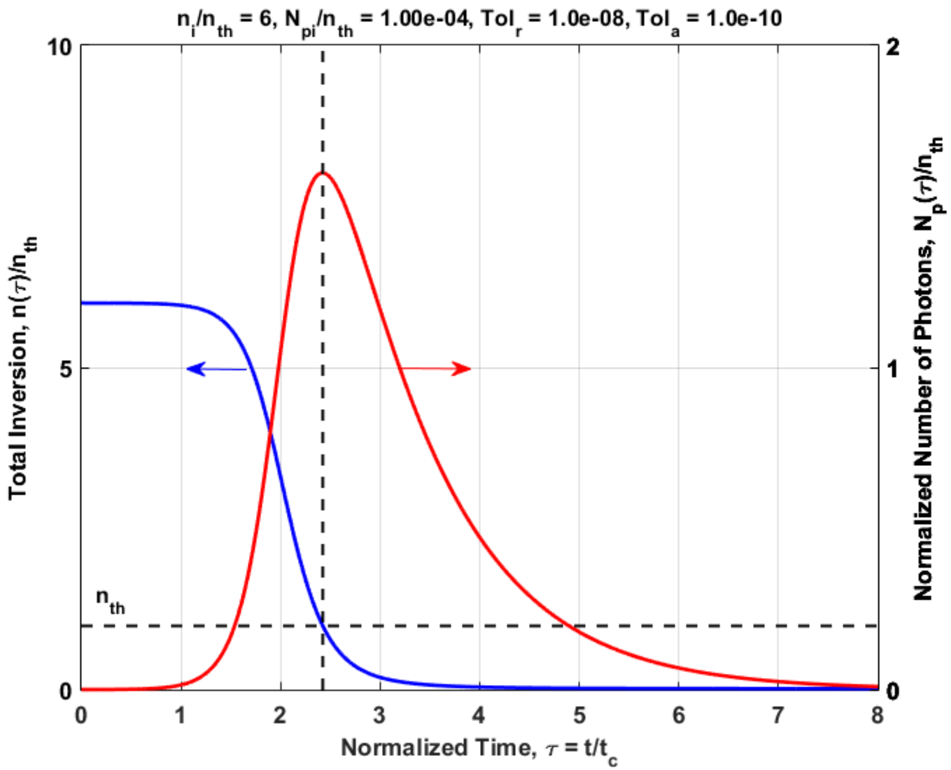
Q-Switching





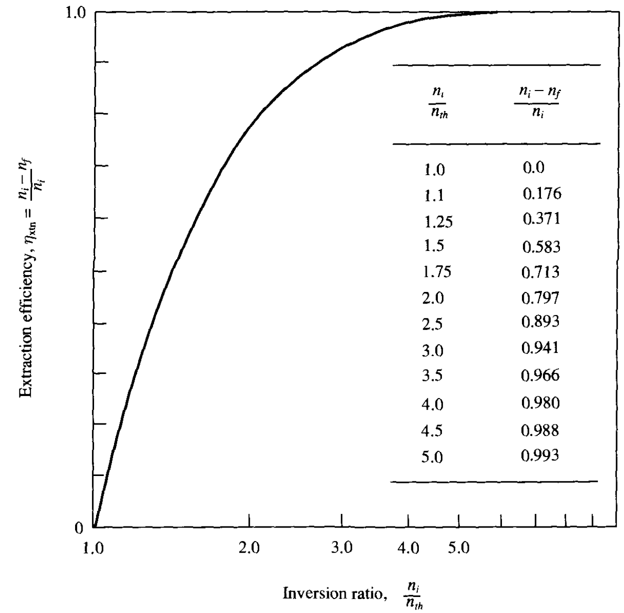
From J. T. Verdeyen, "Laser Electronics", 3rd Ed., Prentice Hall, 1995

Q-Switching Example Outputs

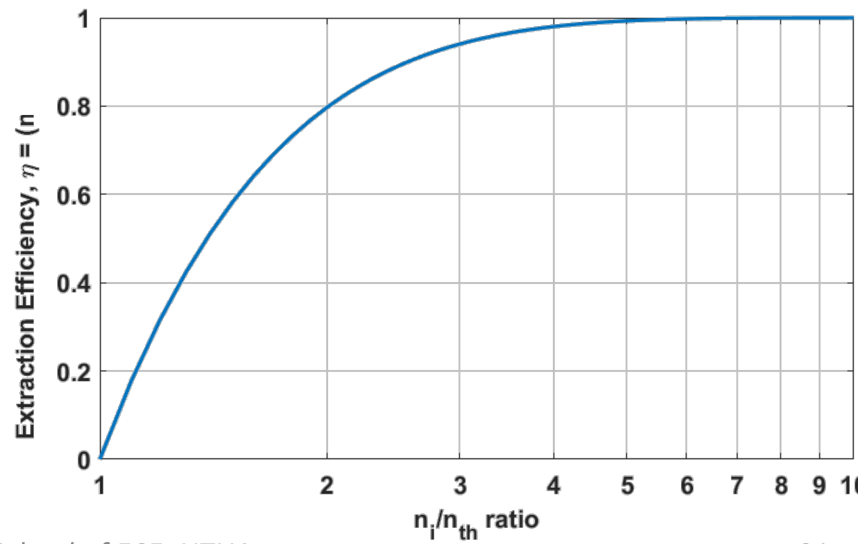


A. Yariv and P. Yeh, Photonics, 6th Ed., Oxford University Press (2007)

Extraction Efficiency

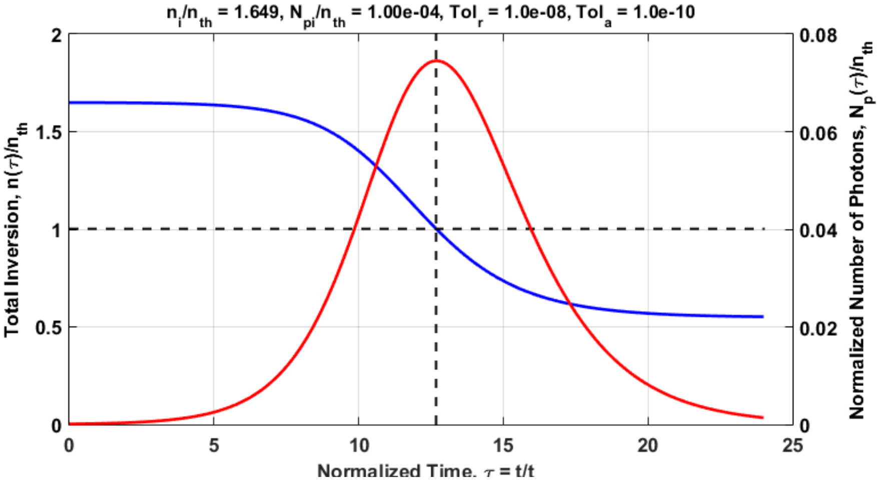


$\frac{n_i}{n_{th}}$	$\frac{n_i - n_f}{n_i}$
1.0	0.0
1.1	0.176
1.25	0.371
1.5	0.583
1.75	0.713
2.0	0.797
2.5	0.893
3.0	0.941
3.5	0.966
4.0	0.980
4.5	0.988
5.0	0.993

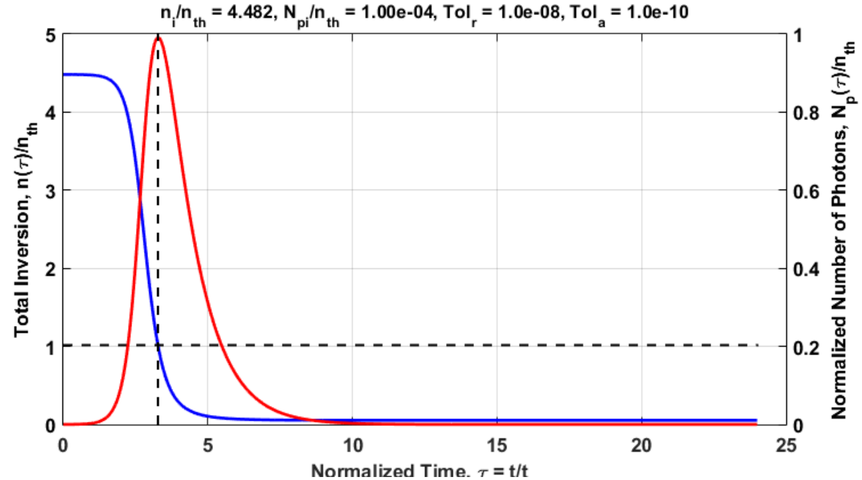


Q-Switching Example Outputs

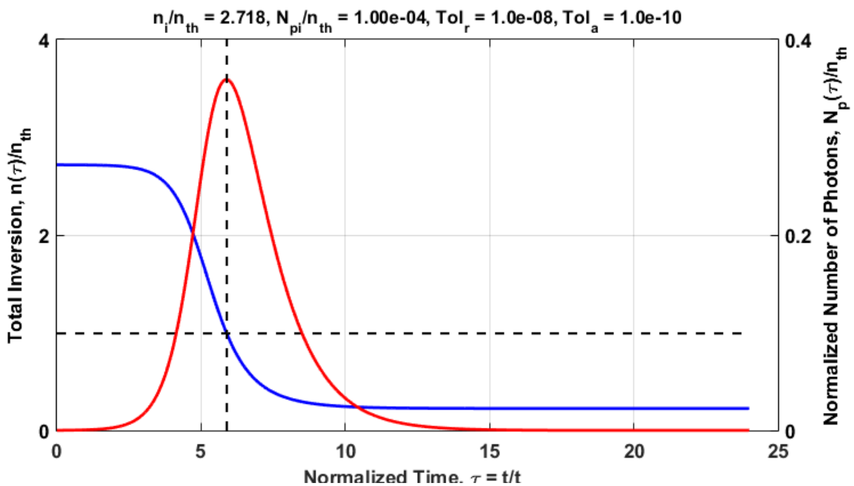
n_i/n_{th}= 1.649



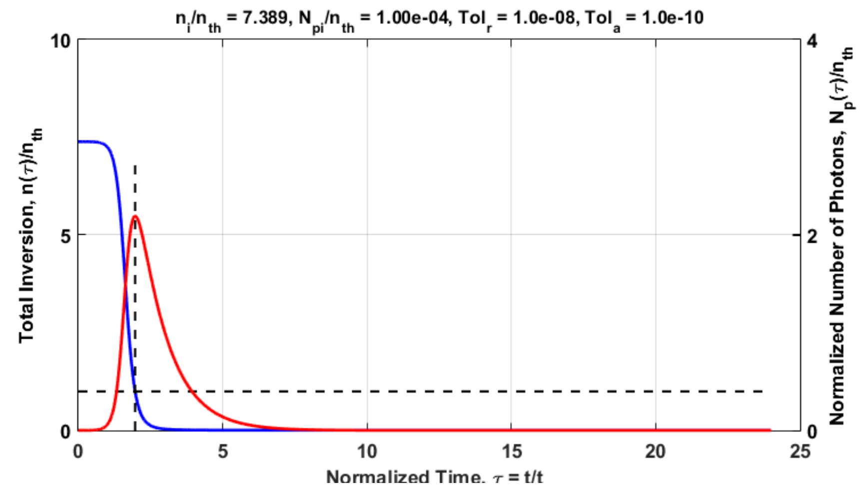
n_i/n_{th}= 4.482



n_i/n_{th}= 2.718



n_i/n_{th}= 7.389



W. G. Wagner and B. A. Lengyel, J. Appl. Phys., vol. 34, Jul. 1963