ΗΛΕΚΤΡΟ-ΟΠΤΙΚΗ ΚΑΙ ΕΦΑΡΜΟΓΕΣ
(ELECTRO-OPTICS)

ΒΑΣΙΚΗ ΘΕΩΡΙΑ ΛΕΙΤΟΥΡΓΙΑΣ ΤΩΝ LASER

(Fundamentals of Laser Operation)

Σημειώσεις

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Fabry-Perot Laser:

For the laser medium the electric displacement vector is:

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} + \vec{P}_{trans} =$$

$$= \epsilon_0 \vec{E} + \epsilon_0 \chi_e \vec{E} + \vec{P}_{trans} = \epsilon \vec{E} + \epsilon_0 \chi(\nu) \vec{E}$$

where $\vec{P}$ is the nonresonant component of the polarization and $\vec{P}_{trans}$ is the polarization due to the atomic transition. (resonance).

Then $\vec{D} = \epsilon \left[1 + \frac{\epsilon_0}{\epsilon} \chi(\nu) \right] \vec{E} = \epsilon' \vec{E}$ where

$$\epsilon' = \epsilon \left[1 + \frac{\epsilon_0}{\epsilon} \chi(\nu) \right]$$

Now consider propagation of electromagnetic plane wave inside the laser medium. The electric field phasor will be of the form

$$\vec{E} = \vec{E}_0 e^{-jk'z} \quad \text{where} \quad k' = \omega\sqrt{\mu\epsilon'} = \omega\sqrt{\mu_0 \epsilon}\left[1 + \frac{\epsilon_0}{\epsilon}\chi(\nu)\right]^{1/2} \simeq$$

$$\simeq \omega\sqrt{\mu_0\epsilon}\left[1 + \frac{\epsilon_0}{2\epsilon}\chi(\nu)\right]$$

$$= k\left[1 + \frac{\epsilon_0}{2\epsilon}\chi(\nu)\right]$$

But $\epsilon = \epsilon_0 \varepsilon_r = \epsilon_0 n^2$ and $\chi(\nu) = \chi'(\nu) - j\chi''(\nu)$. Therefore,

$$k' = k\left[1 + \frac{\chi'(\nu)}{2n^2}\right] - jk\frac{\chi''(\nu)}{2n^2}$$

Then, the electric field phasor is

$$\vec{E} = \vec{E}_0 \, e^{-jk\left(1 + \frac{\chi'(\nu)}{2n^2}\right)z} \quad e^{-\frac{k\chi''(\nu)}{2n^2}z}$$

The term $\exp\left\{-\frac{k\chi''(\nu)}{2n^2}z\right\} = \exp\left\{\frac{\gamma(\nu)}{2}z\right\}$ where

$\gamma(\nu) = -\frac{k\chi''(\nu)}{n^2}$ and we have amplification when $\chi''(\nu) < 0$.

Electron Oscillator Model of an Atomic Transition:

Using the fact that the power per unit volume that is expended by the field on electric dipoles of the medium is $\left(\vec{e} \cdot \frac{\partial \vec{p}}{\partial t}\right)$,

$$\frac{\text{Power}}{\text{volume}} = \frac{\omega \epsilon_0 \chi''(\nu)}{2} |E|^2 = (N_1 - N_2) W_i(\nu) h\nu ,$$

and that $I_\nu = \frac{1}{2} \frac{c}{n} \epsilon |E|^2$ it can be shown that

$$\chi''(\nu) = \frac{(N_1 - N_2)\lambda_0^3}{8\pi^3 n \, t_{spont} \Delta\nu} \cdot \frac{1}{1 + 4(\nu - \nu_0)^2/(\Delta\nu)^2}$$

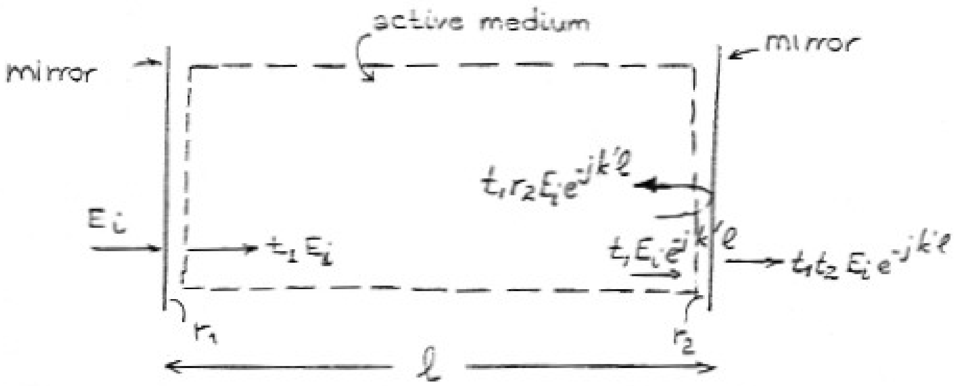
In addition using the classical model of an electronic oscillator we can show that $\chi'(\nu) = \frac{2(\nu_0 - \nu)}{\Delta\nu} \chi''(\nu)$ where $\chi''(\nu)$ has its classically derived value. It is reminded that

$\chi(\nu) = \chi'(\nu) - j\chi''(\nu)$ = electric susceptibility.

In the quantum-mechanical derivation $\chi''(\nu) > 0$ if $N_1 > N_2 \Rightarrow$ absorption prevails or $\chi''(\nu) < 0$ if $N_2 > N_1 \rightsquigarrow$ amplification can occur. (medium with gain).

Fabry-Perot Laser:

Consider the two mirror laser cavity shown in the figure:



The medium propagation constant is

$$k'(\omega) = k - j\underbrace{\frac{\alpha}{2}}_{\text{medium losses}} + \underbrace{k\,\frac{\chi'(\nu)}{2n^2} - j\overbrace{k\,\frac{\chi''(\nu)}{2n^2}}^{=\,-\gamma(\nu)/2}}_{\text{polarization contribution at the transition frequency}}$$

$k = \omega\sqrt{\mu\epsilon}$

It can be shown that the transmitted field is

$$E_t = E_i \frac{t_1 t_2 e^{-jk'\ell}}{1 - r_1 r_2 e^{-j2k'\ell}}$$

In order for an output field to exist without incident field E_i the denominator $1 - r_1 r_2 e^{-j2k'\ell} = 0$ (oscillation condition).

$$1 - r_1 r_2\, e^{-j2\left[k + k\frac{\chi'(\nu)}{2n^2}\right]\ell}\; e^{(\gamma-\alpha)\ell} = 0$$

If $\gamma = \gamma(\nu) = -k\,\frac{\chi''(\nu)}{n^2} = (N_2 - N_1)\,\frac{\lambda_0^2}{8\pi n^2 t_{spont}}\, g(\nu) > \alpha$

then $r_1 r_2 e^{(\gamma-\alpha)\ell}$ can be equal to 1, assuming that $e^{-j(\,)} = 1$

The two conditions are summarized below:

$$\begin{Bmatrix} r_1 r_2 e^{(\gamma-\alpha)\ell} = 1 & \\ 2\left[k + k\,\frac{\chi'(\nu)}{2n^2}\right]\ell = 2m\pi & m = 1, 2, \ldots \end{Bmatrix}$$

Physically, what these two conditions mean is that the wave making a round trip in the cavity returns to its initial position with the same amplitude and phase. The amplitude condition gives the threshold value for the gain γ_t :

$$\gamma_t = \alpha - \frac{1}{\ell} \ln(r_1 r_2)$$

and from that the threshold population inversion can be found,

$$N_t = (N_2 - N_1)_t = \frac{8\pi n^2 t_{spont}}{\lambda_0^2} \frac{1}{g(\nu)} \cdot \left[\alpha - \frac{1}{\ell} \ln(r_1 r_2)\right]$$

The threshold gain can be written in terms of the power reflectivities R_1, R_2 $(R_1 = r_1^2, R_2 = r_2^2)$ as

$$\gamma_t = \alpha - \frac{1}{2\ell} \ln(R_1 R_2)$$

It is reminded that the cavity resonator lifetime t_c is given by $t_c = \dfrac{n/c}{\alpha - \frac{1}{2\ell} \ln(R_1 R_2)}$ and the threshold population can be commonly given as

$$N_t = \frac{8\pi n^3 \nu^2}{c^3 g(\nu)} \left(\frac{t_{spont}}{t_c}\right)$$

Numerical Example:

What is N_t for He-Ne laser at $\lambda_0 = 0.6328\ \mu m$ where $t_{spont} \simeq 10^{-7}$ sec, $\ell = 12$ cm, $\frac{1}{g(\nu_0)} \simeq \Delta\nu = 10^9$ Hz (Doppler broadened width) $R_1 = R_2 = 0.98$, $\alpha \simeq 0$, $n \simeq 1$.

$$t_c \simeq \frac{1/c}{-\frac{1}{2\ell} \ln R_1 R_2} = \frac{1/3 \cdot 10^9}{-\frac{1}{2 \cdot 0.12} \ln(0.98^2)} \text{ sec} = 1.98 \cdot 10^{-8} \text{ sec}$$

$$N_t = \frac{8\pi \cdot n^3}{c \lambda_0^2} \Delta\nu \left(\frac{t_{spont}}{t_c}\right) = 1.05 \cdot 10^9 \ 1/cm^3$$

Oscillation Frequency:

From the phase condition we have

$$k\ell \left[1 + \frac{\chi'(\nu)}{2n^2} \right] = m\pi$$

The resonator longitudinal resonance frequencies are

$$\nu_m = m \frac{c/n}{2\ell}$$

Using the above equations and the relationship between $\chi'(\nu), \chi''(\nu)$ and $\gamma(\nu)$ it can be shown that the lasing frequency ν is

$$\nu \simeq \nu_m - (\nu_m - \nu_o) \frac{\gamma(\nu_m)\, c}{2\pi n\, \Delta\nu} \simeq$$

$$\simeq \nu_m - (\nu_m - \nu_o) \frac{\Delta\nu_{1/2}}{\Delta\nu}$$

where $\Delta\nu_{1/2} = \frac{(\Delta\nu)_m}{F} = \frac{c/n}{2\ell\, F} \simeq \frac{c\,[\alpha - (1/\ell)\ln(R_1R_2)]}{2\pi n}$ is the linewidth of the resonator modes ν_m.

The result of the above equation is that the oscillation frequency is close to ν_m but shifts towards ν_o. This is called frequency pulling.

Numerical Example:

For the previously used He-Ne Laser:

$(\Delta\nu)_m = \frac{c}{2\ell} = 1.25 \cdot 10^9$ Hz $\qquad R_1 = R_2 = R = 0.98$

$F = \frac{\pi\sqrt{R}}{1-R} = 155.5$

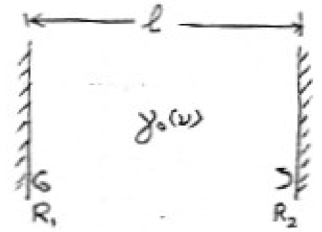
$\Delta\nu_{1/2} = \frac{(\Delta\nu)_m}{F} = 8.04 \cdot 10^6$ Hz $\rightarrow \frac{\Delta\nu_{1/2}}{\Delta\nu} = \frac{8.04 \cdot 10^6}{10^9} = 8.04 \cdot 10^{-3}$

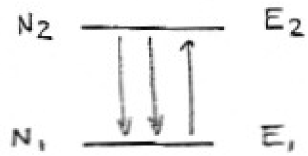
The closest to $\nu_o = \frac{c}{\lambda_o} = 4.7408 \cdot 10^{14}$ Hz ν_m is for $m = 379267$.

($\nu_o \simeq \nu_m = \frac{cm}{2\ell}$). Then $\nu_m - \nu_o = 0.31 \cdot 10^9$ Hz and the oscillation frequency is: $\nu = \nu_m - 0.31 \cdot 10^9 \; 8.04\, 10^{-3} =$

$$= 4.7408 \cdot 10^{14} - 2.5 \cdot 10^6 \text{ Hz}.$$

Laser Oscillation: (another viewpoint)





$E_2 - E_1 = h\nu$

Assume an active medium in a cavity that has a small-signal gain $G = e^{\gamma_0(\nu)\ell}$. The mirror power reflectivities are R_1 and R_2. The laser oscillation builds up from the first photons produced by spontaneous emission.

Consider a high-Q $TEM_{0,0,q}$ modes in the cavity. The separation of these longitudinal modes is $\Delta\nu = (c/n)/2\ell$. The number of active atoms in state 2 (E_2) is N_2. The rate of generation of spontaneous emission photons at the $TEM_{0,0,q}$ longitudinal mode is:

$$\left.\frac{dN_p}{dt}\right|_{sp.em.} = (A_{21} N_2 V)\left[g(\nu)\underbrace{\frac{c/n}{2\ell}}_{\Delta\nu}\right] \frac{1 \text{ mode}}{\underbrace{\left(\frac{8\pi n^3 \nu^2}{c^3}\right)\underbrace{\left(\frac{c/n}{2\ell}\right)}_{\Delta\nu} V}_{\text{total number of EM modes}}}$$

where V is the active volume, and $\Delta\nu = \frac{c/n}{2\ell}$ is the frequency interval around the q-th TEM mode $\nu_q = q\,\frac{c/n}{2\ell}$.

The above equation can be written as:

$$\left.\frac{dN_p}{dt}\right|_{sp.em} = \left[A_{21}\,\frac{\lambda_0^2}{8\pi n^2}\cdot g(\nu)\right]\frac{c}{n}\,N_2 = N_2\,\frac{c}{n}\,\sigma(\nu)$$

where $\sigma(\nu)$ is the stimulated emission cross-section.

Once these photons are in the cavity mode they are being amplified by a factor G per pass in the cavity. Then, the rate of increase of these photons per round-trip time is:

$$\left.\frac{dN_p}{dt}\right|_{st\,em} = \frac{G^2R_1R_2-1}{(2\ell n/c)} N_p$$

Where $\alpha \simeq 0$ (if $\alpha \not\simeq 0$ the $G^2R_1R_2-1$ is replaced by $G^2R_1R_2e^{-2\alpha\ell}-1$).
Then, the rate of generation of N_p due to both spontaneous and stimulated emission is:

$$\frac{dN_p}{dt} = \frac{G^2R_1R_2-1}{2\ell n/c} N_p + N_2 \frac{c}{n}\sigma$$

If $G^2R_1R_2-1<0$ (Losses exceed gain) then we get small amounts of power due to the spontaneous emission term only. Assuming $N_2 = N_2(t) = N_2 u(t)$ [$u(t)$ = step function], then $N_p(t)$ can be expressed by:

$$N_p(t) = \left[\left(N_p(0) + \frac{N_2\frac{c}{n}\sigma}{A}\right)e^{At} - \frac{N_2\frac{c}{n}\sigma}{A}\right]u(t)$$

with $A = (G^2R_1R_2-1)/(2\ell n/c)$. If $G^2R_1R_2-1<0 \sim A<0 \sim$
$N_p(t) \simeq -\frac{N_2\frac{c}{n}\sigma}{A}$.

In order to get an idea about the N_p we can just look at the $N_2\frac{c}{n}\sigma$ which is the rate of spontaneous emission photons into the TEM_{00q} mode. If $\sigma = 3\cdot10^{-12}\,cm^2$ (large stimulated emission cross-section) $c = 3\cdot10^8$ m/s, $n=1$, $N_2 = 10^{12}\,cm^{-3}$ (typical of high-gain lasers), then $N_2\frac{c}{n}\sigma = 9\cdot10^{10}$ 1/sec. The corresponding power in this mode is (by assuming $h\nu \simeq 1eV$):

$$9\cdot10^{10}\;1.60219\cdot10^{-19}\;W = 14.42\cdot10^{-9}W = 14.42\,nW,$$

which is really small.

Interesting effects happen when $G^2R_1R_2-1>0$ (gain exceeds losses). In this case $N_p(t)$ grows exponentially with time if the gain

In a Fabry-Perot etalon with gain the transmitted power around ν_q can be written as

$$P(\nu) = \frac{G_{sat}(1-R_1)(1-R_2)}{[1-G_{sat}(R_1R_2)^{1/2}]^2 + 4G_{sat}(R_1R_2)^{1/2}\sin^2[\underbrace{2\pi(\nu-\nu_q)\ell n/c}_{\delta/2}]}$$

around the longitudinal frequency $\nu_q = q\,\frac{c/n}{2\ell}$.

It is straightforward to show that the FWHM of $P(\nu)$ is given by

$$\Delta\nu_{osc} = \frac{1-G_{sat}(R_1R_2)^{1/2}}{\pi\,[G_{sat}(R_1R_2)^{1/2}]^{1/2}}\left(\frac{c/n}{2\ell}\right)$$

Since $G_{sat}^2(R_1R_2) \simeq 1 \rightarrow 1-G_{sat}^2(R_1R_2) \simeq 2\,(1-G_{sat}(R_1R_2)^{1/2})$

and $\Delta\nu_{osc}$ can be written as

$$\Delta\nu_{osc} \simeq \frac{h\nu}{P_{out}}\left[\frac{c/n}{4\pi\ell}(1-R_1R_2)\right]N_2^{(s)}\frac{c}{n}\sigma \simeq$$

$$\simeq \frac{h\nu}{P_{out}}(\Delta\nu_{1/2})\,N_2^{(s)}\frac{c}{n}\sigma$$

(where for the denominator expressions it is assumed that $[G_{sat}(R_1R_2)^{1/2}]^{1/2} \simeq 1$)

Finally, it can be shown that

$$\Delta\nu_{osc} \simeq 2\pi\frac{h\nu}{P_{out}}(\Delta\nu_{1/2})^2\left(1-\frac{g_2}{g_1}\frac{N_1^{(s)}}{N_2^{(s)}}\right)^{-1}$$

where $\Delta\nu_{1/2} = \frac{(c/n)/2\ell}{F}$ $\quad F = \frac{\pi[(R_1R_2)^{1/2}]^{1/2}}{1-(R_1R_2)^{1/2}}$ and the assumption $(R_1R_2)^{1/4} \simeq 1$.

To get an idea about $\Delta\nu_{osc}$ let's assume that $N_1^{(s)} = 0$

$\nu = 2.42 \cdot 10^{14}$ Hz ($\lambda_0 = 1.24\,\mu m$), $h\nu = 1.eV$, $Q = 5.05 \cdot 10^7$,

$\Delta\nu_{1/2} = 4.77$ MHz, $P_{out} = 10$ mW, $\rightarrow \Delta\nu_{osc} = 2.29 \cdot 10^{-3}$ Hz! In practice, just slight perturbations in laser parameters can increase $\Delta\nu_{osc}$.

FABRY-PEROT CAVITY WITH GAIN

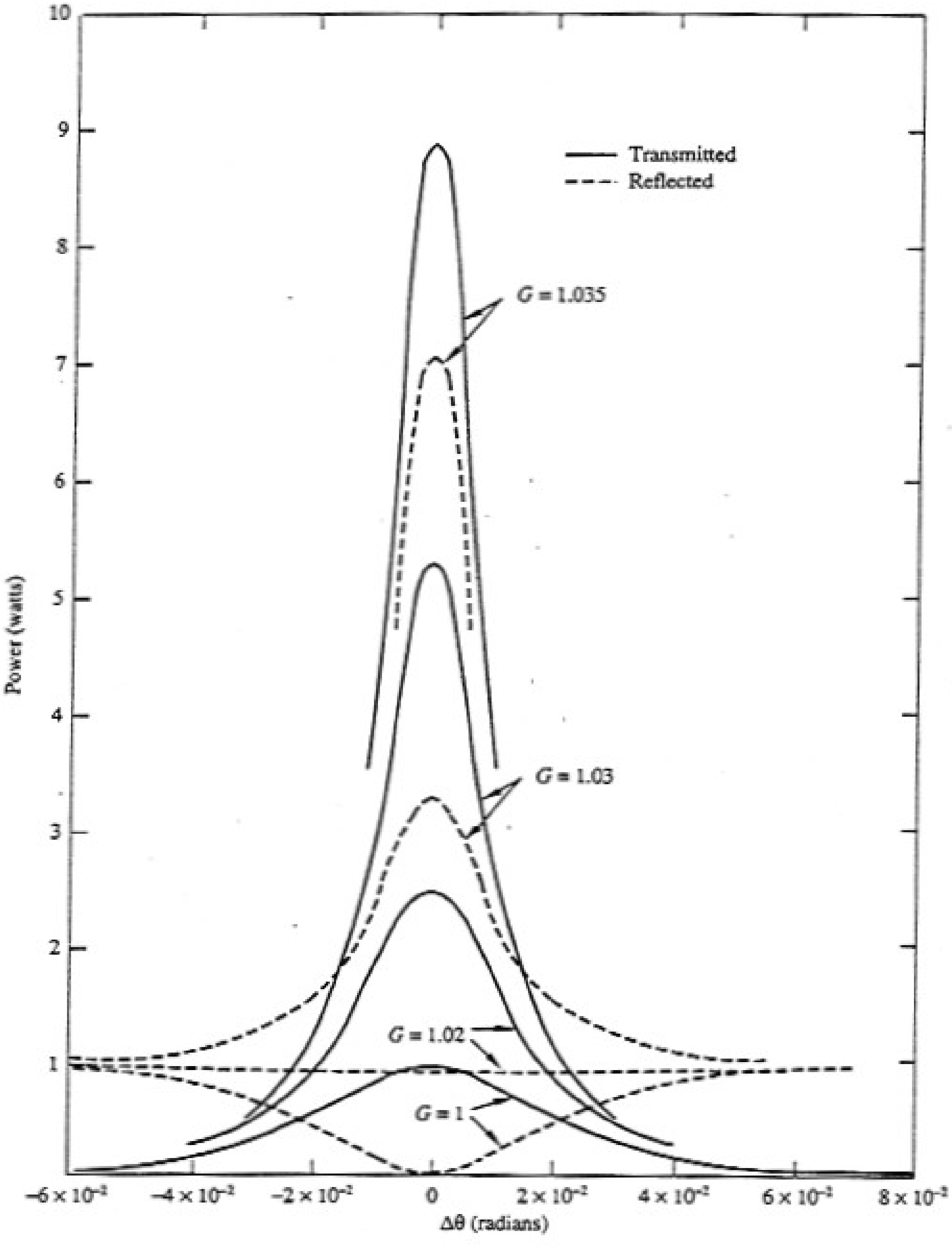
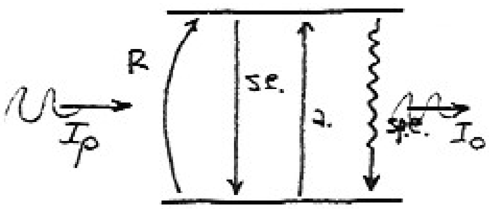


FIGURE 6.8. Response of an active cavity. The parameter G is the single pass power amplification factor.

From J.T. Verdeyen, "Laser Electronics", 3rd ed., Prentice Hall 1995



2-level system:

{ Set $R=0$ only optical pumping is possible. Thus, optical signal $h\nu_p = E_2 - E_1$ }

$$\frac{dN_2}{dt} = -\frac{N_2}{\tau_2} + R - W_i(N_2 - N_1)$$

$$\frac{dN_1}{dt} = \frac{N_2}{\tau_2} + W_i(N_2 - N_1) - R \qquad N_1 + N_2 = N_0$$

Steady-state: $\dfrac{dN_1}{dt} = \dfrac{dN_2}{dt} = 0$

$$\frac{dN_1}{dt} + \frac{dN_2}{dt} = R \Rightarrow \frac{d}{dt}(N_1 + N_2) = R \Rightarrow N_0 = Rt + C$$

so population increases with t
not stable

Thus $\dfrac{dN_1}{dt} = -R + \dfrac{N_2}{\tau_2} + W_i(N_2 - N_1)$ and then

$$\frac{dN_1}{dt} + \frac{dN_2}{dt} = 0 \Rightarrow N_0 = \text{constant} = N_1 + N_2$$

$$0 = -\frac{N_2}{\tau} + R - W_i(N_2 - N_1) = -\frac{N_2}{\tau} + R - W_i[N_2 - (N_0 - N_2)]$$

$$= -\frac{N_2}{\tau_2} + R - W_i[2N_2 - N_0] \Rightarrow$$

$$-N_2\left[\frac{1}{\tau_2} + 2W_i\right] + W_i N_0 + R = 0 \Rightarrow$$

$$N_2\left[\frac{1}{\tau_2} + 2W_i\right] = R + W_i N_0$$

$$N_2 = \frac{R + W_i N_0}{\left[\frac{1}{\tau_2} + 2W_i\right]} \qquad (R=0) \rightarrow N_2 = \frac{W_i}{\frac{1}{\tau_2} + 2W_i} N_0$$

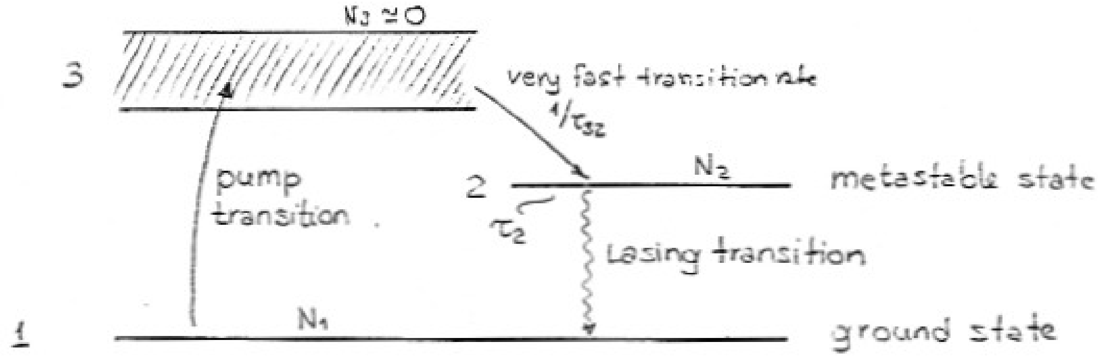
$$N_1 = N_0 - N_2 = \frac{N_0\left[\frac{1}{\tau_2} + 2W_i\right] - R - W_i N_0}{\left[\frac{1}{\tau_2} + 2W_i\right]} =$$

$$N_1 = \frac{N_0\left[\frac{1}{\tau_2} + W_i\right] - R}{1/\tau_2 + 2W_i} \qquad (R=0) \rightarrow N_1 = \frac{\frac{1}{\tau_2} + W_i}{\frac{1}{\tau_2} + 2W_i} N_0$$

$$\Delta N = N_2 - N_1 = \frac{-1/\tau_2}{1/\tau_2 + 2W_i} N_0$$

Three & Four Level Systems:

An idealized 3-level system has the following energy diagram:



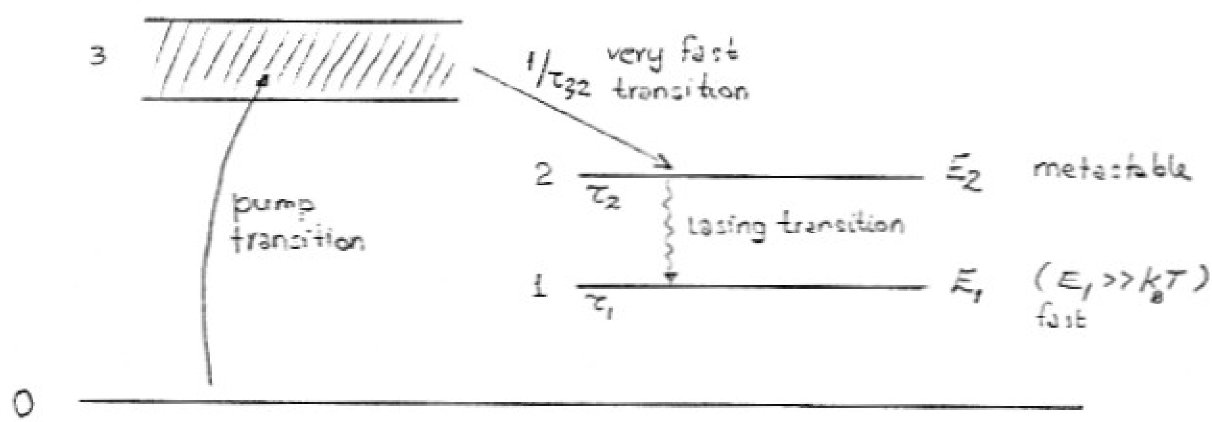
In order to satisfy the oscillation condition the pumping rate should be such that the populations N_1, N_2 satisfy the conditions:

$$\left.\begin{aligned} N_2 &\simeq \frac{N_0}{2} + \frac{N_t}{2} \\ N_1 &\simeq \frac{N_0}{2} - \frac{N_t}{2} \end{aligned}\right\} \Rightarrow N_2 - N_1 = N_t = \text{threshold value}$$

where N_0 is the density of the active atoms.

In most laser systems $N_0 \gg N_t$.

An idealized 4-level system has the following energy diagram:



Where τ_2 = Lifetime of E_2 $\gg$ τ_1 = Lifetime of E_1.

In this case at threshold

$$N_2 \simeq N_t \quad \text{and} \quad N_1 \simeq 0$$

Since the N_2 population depends on the pumping rate we can compare the pumping rates of a 3-level and a 4-level systems:

$$\frac{(N_2)_{3-L}}{(N_2)_{4-L}} \simeq \frac{\frac{N_o}{2} + \frac{N_t}{2}}{N_t} \simeq \frac{N_o}{2N_t} \quad \text{which is usually large } (\sim 100).$$

The minimum power expenditure for a 3-level system to keep $N_2 \simeq N_o/2$ atoms in level 2 is

$$(P_s)_{3-L} \simeq \underbrace{\left(\frac{N_o}{2}V\right)}_{\substack{\text{number}\\ \text{of atoms}\\ \text{that lase}}} \cdot \underbrace{(h\nu)}_{\substack{\text{Lasing}\\ \cancel{\text{power}}\ \text{energy}\\ \text{release/atom}}} \underbrace{\left(\frac{1}{\tau_2}\right)}_{\text{average time}} = \frac{N_o V h\nu}{2\tau_2}$$

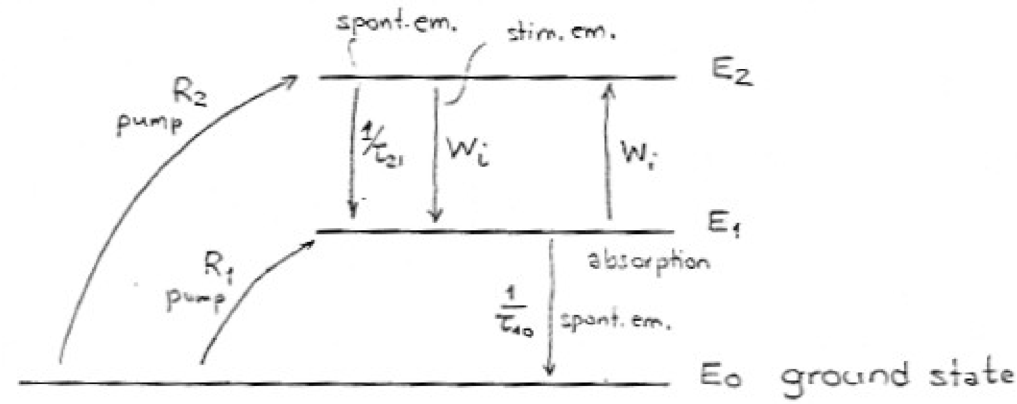
and for a 4-level system is

$$(P_s)_{4-L} \simeq (N_t V)(h\nu)\left(\frac{1}{\tau_2}\right) = \frac{N_t V h\nu}{\tau_2}$$

Usually $\tau_2 \simeq t_{spont}$. In this case $(P_s)_{4-L}$ is also called "critical fluorescence power."

Power in Laser Oscillators:

Assume a 4-level system in which $N_t << N_0$ (ground state population). Then we can assume that the ground state population remains basically unaffected.



Rate Equations:

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{\tau_{21}} - W_i\,(N_2 - N_1)$$

$$\frac{dN_1}{dt} = R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}} + W_i\,(N_2 - N_1)$$

where $W_i = \dfrac{c^2 g(\nu)}{8\pi n^2 h\nu^3 t_{spont}} I_\nu = \sigma(\nu)\dfrac{I_\nu}{h\nu}$ $\quad (\tau_{21} = t_{spont})$.

The steady-state solution of the above rate equations is:

$$N_2 - N_1 = \frac{R_2\left[1 - \frac{\tau_1}{\tau_{21}}\left(1 + \frac{R_1}{R_2}\right)\right]}{W_i + 1/\tau_{21}} = \frac{R_{eff}}{W_i + \frac{1}{\tau_{21}}}$$

Note: This is the same equation with the one on page 59 assuming that $\tau_2 = \tau_{21}$.

A necessary condition for $N_2 - N_1 > 0$ is that $\frac{\tau_1}{\tau_{21}} < 1$ or $\tau_{21} > \tau_1$. In addition, to make pumping more effective $R_1 << R_2$.

Below threshold $I_\nu \simeq 0 \Rightarrow W_i = 0$ and therefore

$$N_2 - N_1 = \frac{R_{eff}}{1/\tau_{21}} \qquad R_{eff} < (N_t/\tau_{21})$$

When $N_2 - N_1 = N_t$ then W_i becomes significant. This is when the gain is enough to make up for the losses and stimulated emission prevails. In the steady-state further increase of $N_2 - N_1$ from N_t is impossible since this would imply that the energy in the resonator would continuously increase with time.

Thus for $R_{eff} \geq N_t (1/\tau_{21})$

$$N_2 - N_1 = N_t = \frac{R_{eff}}{W_i + 1/\tau_{21}} \Rightarrow W_i = \frac{R_{eff}}{N_t} - \frac{1}{\tau_{21}} \qquad [R_{eff} \geq N_t(1/\tau_{21})]$$

The total power generated by stimulated emission is

$$P_e = (N_t V)\, W_i\, (h\nu)$$

where V is the volume of the oscillating mode. Therefore,

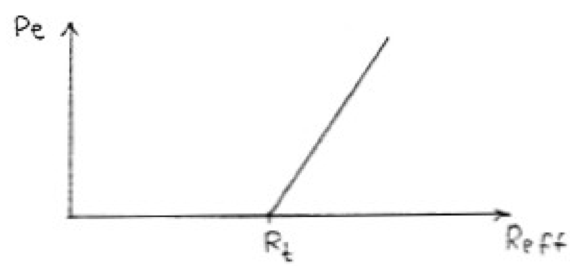
$$\frac{P_e}{V h\nu} = \left(\frac{N_t}{\tau_{21}}\right)\left[\frac{R_{eff}}{N_t/\tau_{21}} - 1\right] \qquad R_{eff} \geq (N_t/\tau_{21})$$

The power generated by spontaneous emission is

$$P_{sp.em} = N_t V\, h\nu\, 1/\tau_{21} \simeq P_s$$

and the lasing power P_e can be re-written as

$$P_e = P_s \left[\frac{R_{eff}}{R_t} - 1\right] \quad \text{for} \quad R_{eff} \geq R_t = (N_t/\tau_{21})$$



Note that for $R_{eff} = 2R_t \sim P_e = P_s$.

Optimum Output Coupling in Laser Oscillators:

The loss of a laser mode inside the resonator can be attributed to two sources: (a) due to absorption, scattering, and (b) due to the coupling of output power through the partially transmitting reflectors. The latter loss corresponds to the output power of the laser. Reducing the output coupling to zero ($R_1 = R_2 = 1$) the threshold gain is also reduced but no useful output can be produced. On the other hand increasing the coupling ($R_1, R_2 \simeq 0$) increases the threshold gain so much that can not be achieved by the available pumping. There is an optimum output coupling for which the laser power can be maximized.

From the population inversion (at steady-state):

$$N_2 - N_1 = \frac{R_{eff}/\omega_{21}}{1 + W_i/\omega_{21}} \qquad \omega_{21} = 1/\tau_{21}$$

But the gain $\gamma(\nu) = \sigma(\nu)(N_2 - N_1) \Rightarrow$

$$\gamma(\nu) = \frac{\sigma(\nu)(R_{eff}/\omega_{21})}{1 + W_i/\omega_{21}} = \frac{\gamma_0}{1 + W_i/\omega_{21}}$$

where γ_0 is the unsaturated gain ($W_i = 0$).

The total emitted power $P_e = (N_t V) h\nu \cdot W_i \Rightarrow W_i = \dfrac{P_e}{N_t V h\nu}$

Then, the gain $\gamma(\nu)$ can be written as:

$$\gamma(\nu) = \frac{\gamma_0}{1 + \frac{P_e}{P_s}} \qquad P_s = (N_t V) . h\nu\, \omega_{21}$$

In one round trip inside the cavity the fraction of the intensity which is lost is $1 - R_1 R_2 e^{-2\alpha \ell} = 1 - r_1^2 r_2^2 e^{-2\alpha\ell} =$

$= 1 - (r_1 r_2 e^{-\alpha \ell})(r_1 r_2 e^{-\alpha\ell})$. Thus, for a single pass the fraction of the intensity loss is $\quad L = 1 - r_1 r_2 e^{-\alpha \ell}$

From the oscillation condition we have:

$$r_1 r_2 e^{(\gamma_t - \alpha)\ell} = 1 \Rightarrow \quad e^{\gamma_t \ell}\left[1 - (1 - r_1 r_2 e^{-\alpha\ell})\right] = 1 \Rightarrow$$

$$e^{\gamma_t \ell}\left[1 - L\right] = 1 \Rightarrow \quad e^{\gamma \ell} = \frac{1}{1-L}$$

For small L $(L \ll 1)$

$$\left.\begin{aligned} e^{\gamma_t \ell} &= \frac{1}{1-L} \simeq 1 + L \\ e^{\gamma_t \ell} &\simeq 1 + \gamma \ell \end{aligned}\right\} \Rightarrow \quad \gamma_t \ell = L$$

After oscillation starts $\gamma = \gamma_t$ and is independent of R_{eff}.

The gain as function of P_e can be written in the form:

$$\gamma_t = \frac{\gamma_0}{1 + \frac{P_e}{P_s}} \Rightarrow \quad \frac{\gamma_0}{\gamma_t} = 1 + \frac{P_e}{P_s} \Rightarrow \quad P_e = P_s\left(\frac{\gamma_0}{\gamma_t} - 1\right) \Rightarrow$$

$$P_e = P_s\left(\frac{\gamma_0 \ell}{L} - 1\right) = P_s\left(\frac{g_0}{L} - 1\right)$$

where $g_0 = \gamma_0 \ell$ = unsaturated gain per pass.

Now let's decompose L into two terms:

$$L = L_i + T$$

where L_i corresponds to unavoidable scattering & absorption losses and T is the transmission of the output mirror.

The fraction of the emitted power P_e that is coupled out of the laser is $\quad P_e \dfrac{T}{L_i + T} = P_o \Rightarrow$

$$P_o = P_s\left(\frac{g_0}{L_i + T} - 1\right)\frac{T}{L_i + T}$$

$$P_s = N_t V h\nu\, w_{21} = \frac{N_t V h\nu}{\tau_{21}} = \frac{8\pi n^2 \nu^2}{c^3 g(\nu)} \; \frac{\cancel{t_{spont}}}{t_c} \; \frac{V h\nu}{\cancel{\tau_{21}}}$$

$$t_c \simeq \frac{n\ell}{(L_i + T)c} \qquad \text{(for small losses)}$$

$$t_c = \frac{n/c}{\alpha - \frac{1}{\ell}\ln(r_1 r_2)} = \frac{\ell(n/c)}{\alpha\ell - \ln(r_1 r_2)} \cong \frac{\ell(n/c)}{r_1 r_2 \alpha\ell - (r_1 r_2 - 1)} \simeq$$

$$= \frac{\ell(n/c)}{1 - r_1 r_2(1-\alpha\ell)} \simeq \frac{\ell(n/c)}{1 - r_1 r_2 e^{-\alpha\ell}} = \frac{\ell(n/c)}{L} = \frac{\ell n}{Lc} =$$

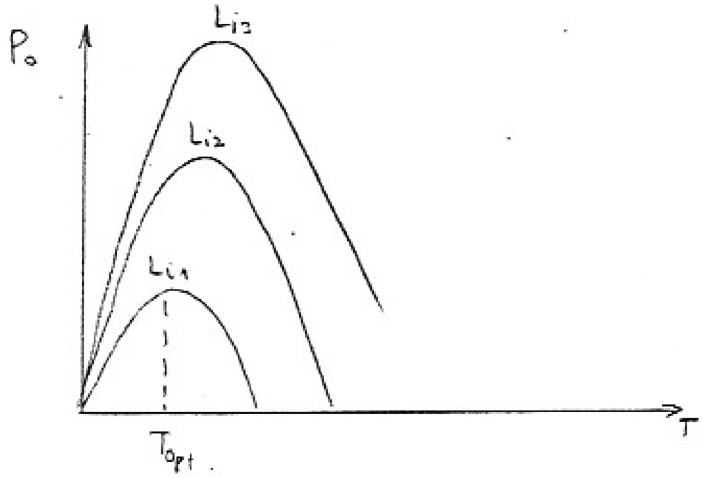
$$= \frac{\ell n}{c(L_i+T)}$$

Therefore,

$$P_o = P_o(T) = \left(\frac{8\pi n^3 \nu^2}{c^3 g(\nu)} V h\nu\right) \frac{c\,\cancel{(L_i+T)}}{\ell n} \cdot \frac{T}{\cancel{L_i+T}} \left(\frac{g_o}{L_i+T} - 1\right) =$$

$$= K\ T \left[\frac{g_o}{L_i+T} - 1\right]$$

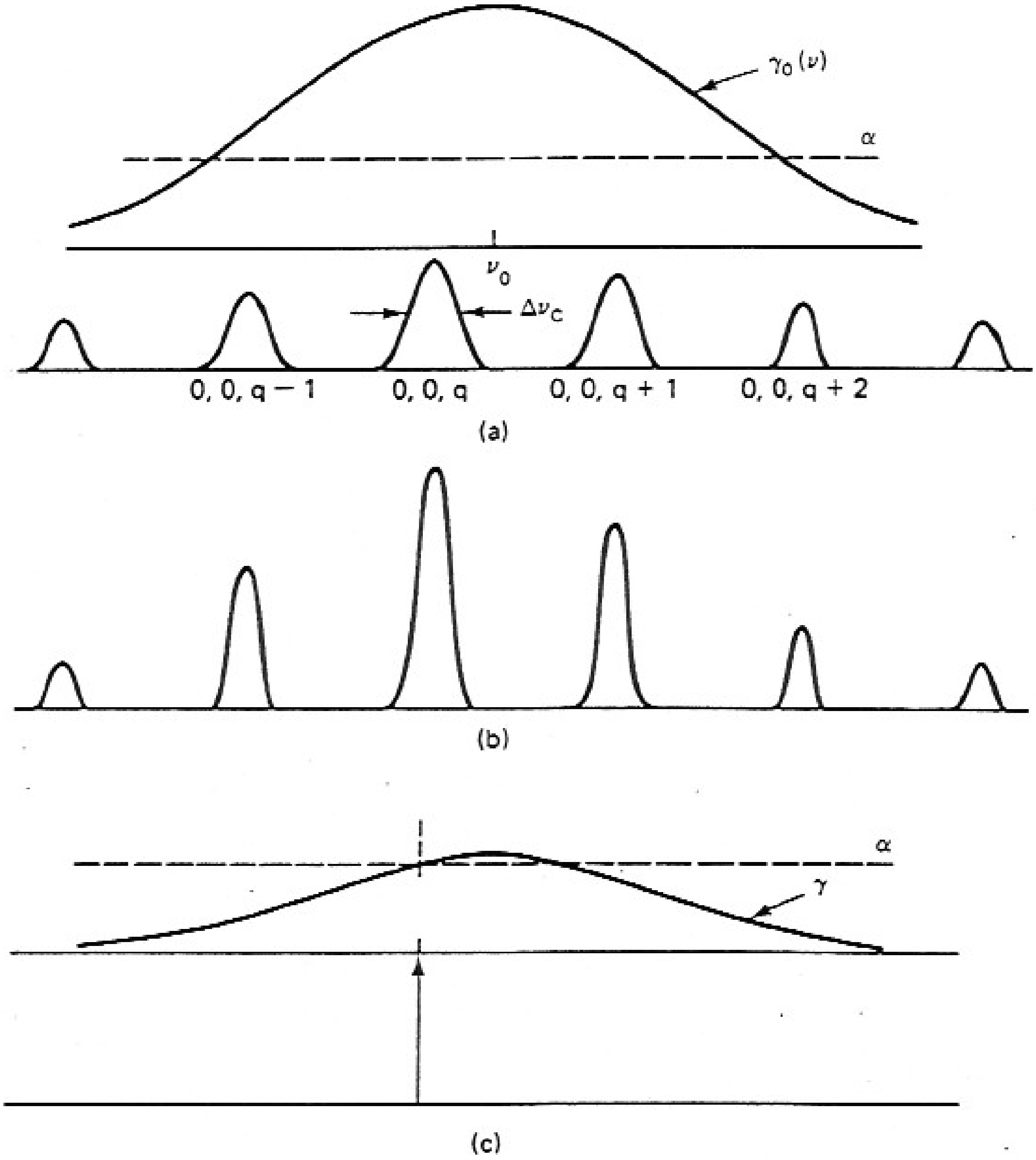
optimum $\frac{\partial P_o}{\partial T} = 0 \implies T_{opt} = -L_i + \sqrt{g_o L_i}$

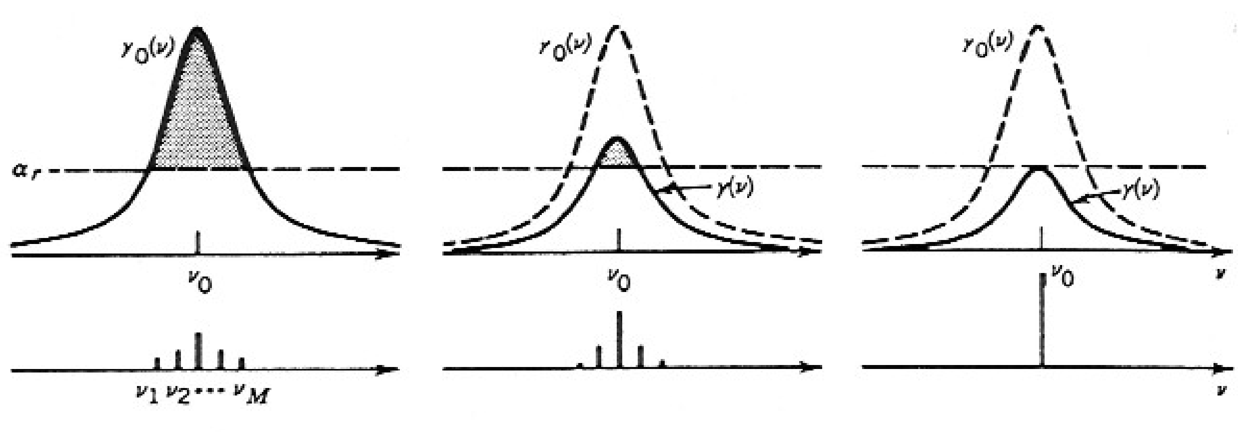


Multimode Laser Oscillation:

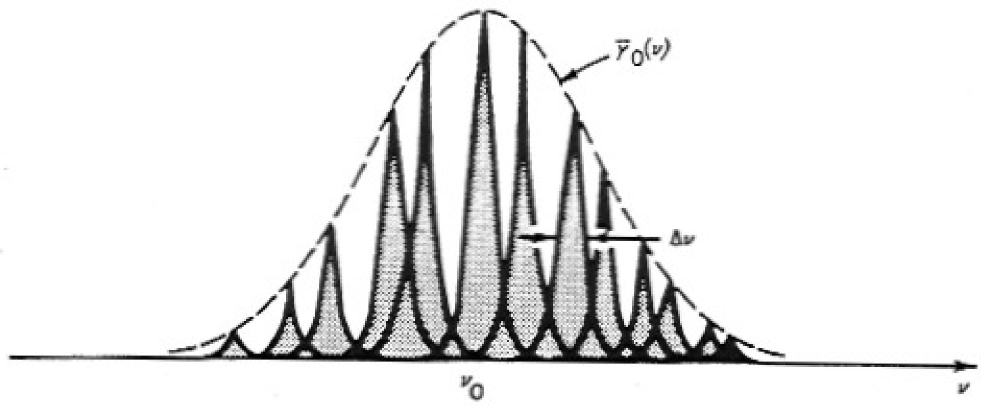
When the gain in the medium exceeds the resonator losses then from all possible electromagnetic modes the ones that are supported by the resonator are going to enhance and increase their power. However, the increase in electromagnetic power causes reduction of the gain (gain saturation) and thus less and less from the resonator modes will be sustained. Actually, in an ideal homogeneously broadened medium only one mode will finally lase (the one closest to the peak of the gain curve). In the steady state then $\gamma_t = \alpha_{losses}$ and this situation is depicted in the next figure. This happens because all atoms are considered identical and only one mode prevails from the competition among all resonator modes that fall within the gain curve.

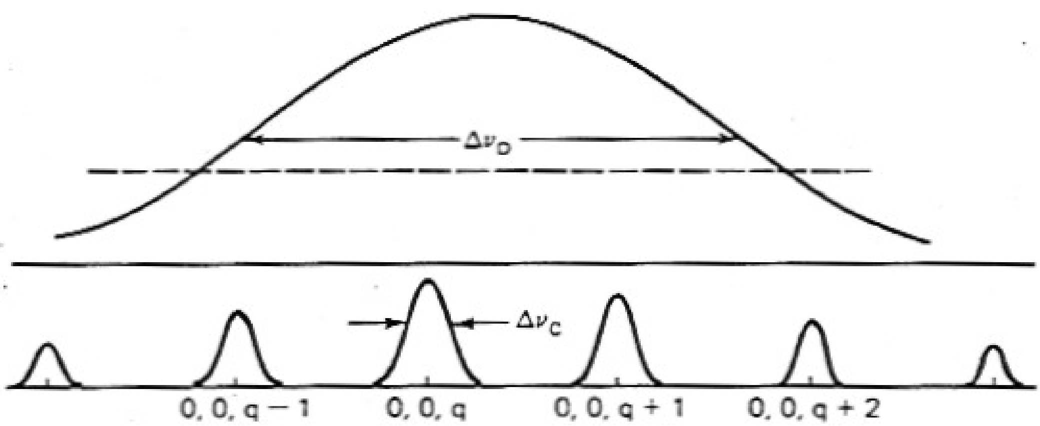
In case that two modes occupy different volume in the resonator, they draw their power from atoms that are in different location in the medium and consequently they do not compete. Thus, it is possible to have more than a single mode. This is called "spatial hole burning."



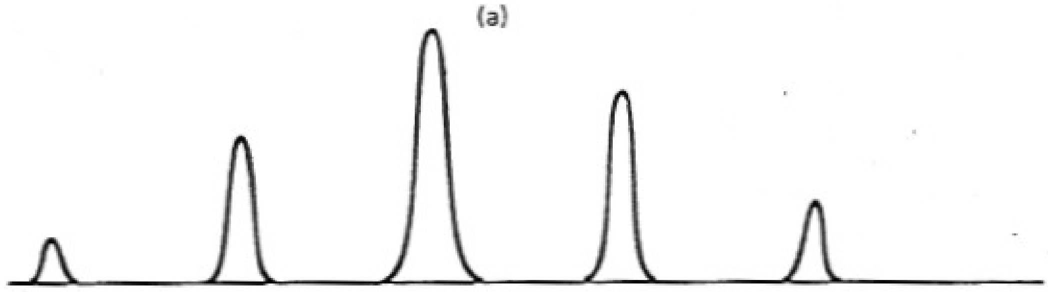


Homogeneously Broadened Medium

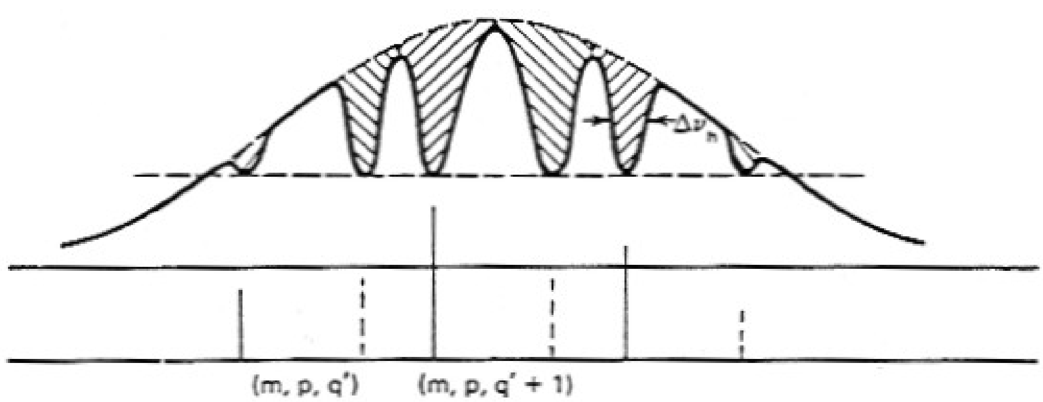




(a)

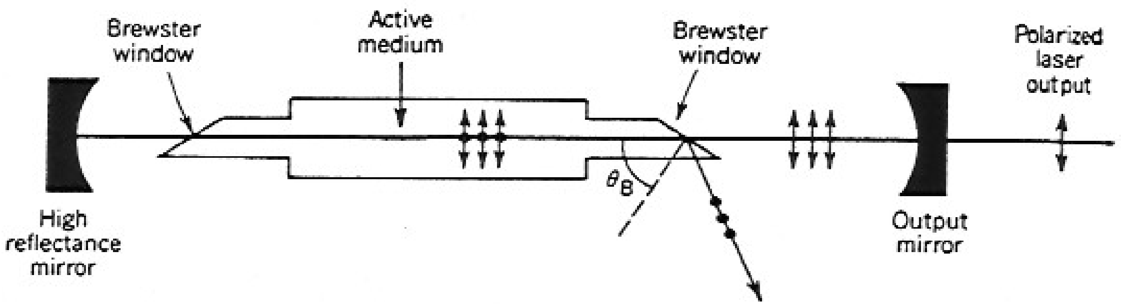


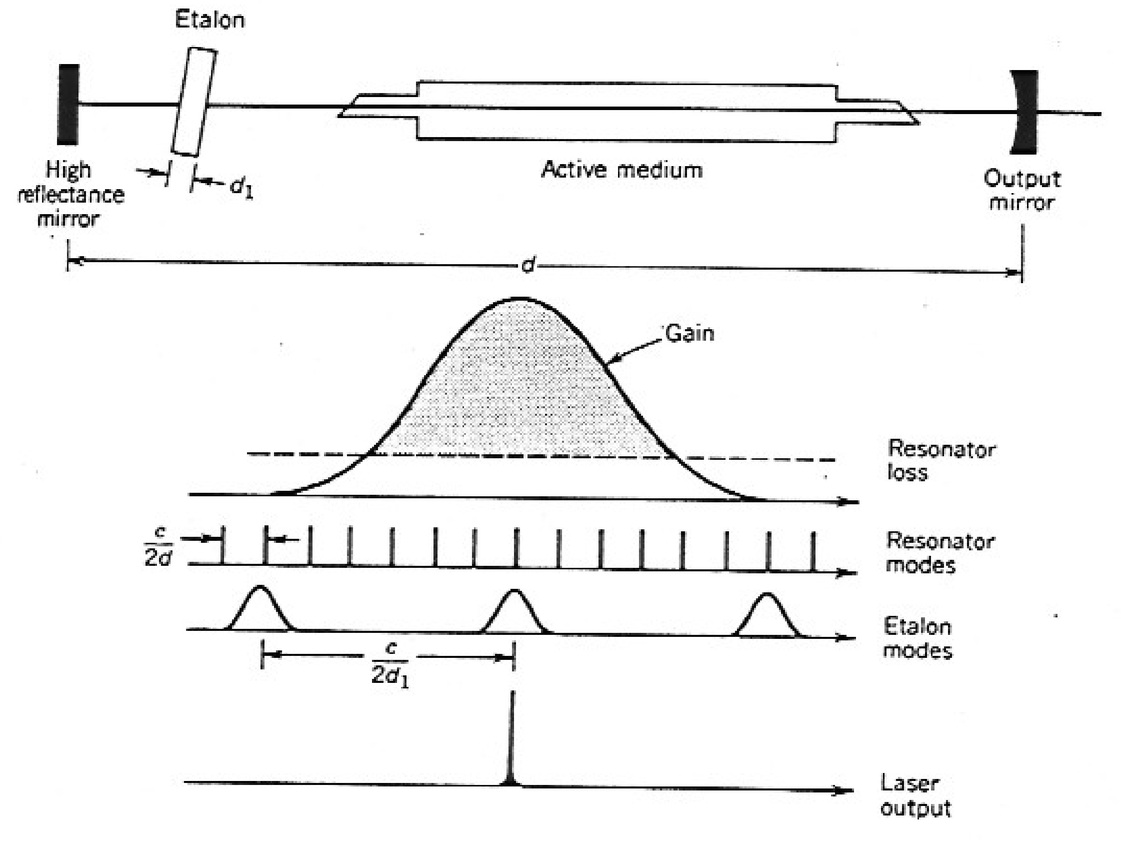
(b)

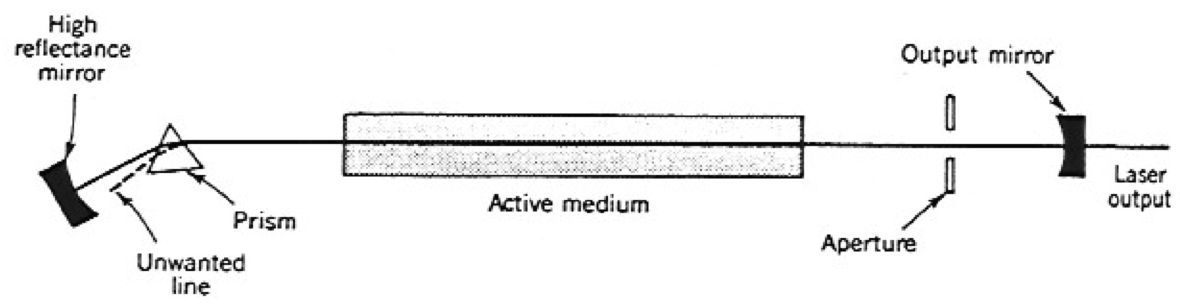


(c)

Inhomogeneously Broadened Medium


Brewster window
Active medium
Brewster window
Polarized laser output
θ_B
High reflectance mirror
Output mirror

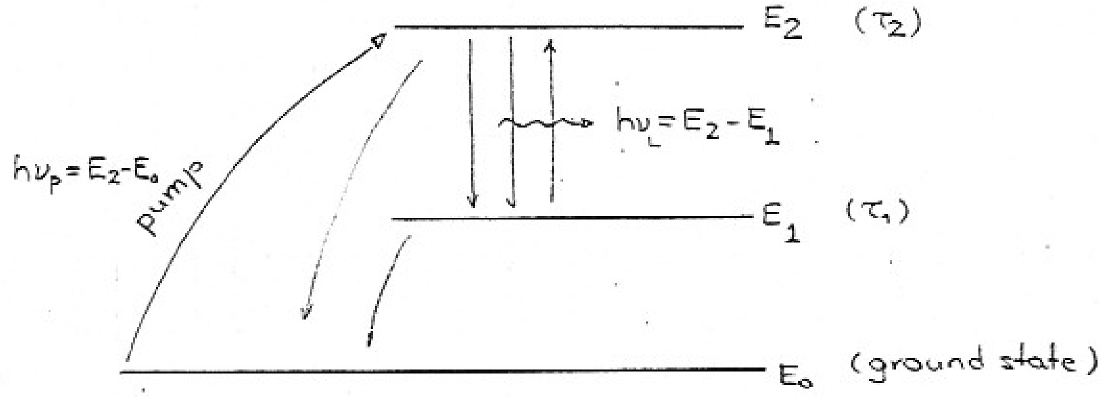

Etalon
High reflectance mirror
d_1
Active medium
Output mirror
d
Gain
Resonator loss
$\frac{c}{2d}$
Resonator modes
Etalon modes
$\frac{c}{2d_1}$
Laser output


High reflectance mirror
Output mirror
Prism
Unwanted line
Active medium
Aperture
Laser output

Laser Dynamics:

Up to this point we have been examining the steady-state response of a laser. However, dynamic phenomena can also be important. For example, a pulse excitation or modulation of the excitation are interesting applications of the dynamic behavior of a laser.

In order to study the dynamic behavior of lasers let's assume that the active medium can be described with the following energy diagram:



The pumping from ground level to E_2 is being done optically with another laser of intensity I_p. For simplicity, $\tau_1 << \tau_2$ and $N_2 >> N_1 \simeq 0$. Then the rate equation for Level E_2 can be written as:

$$\frac{dN_2}{dt} = \underbrace{\underbrace{\frac{\sigma_p I_p}{h\nu_p}}_{W_p} N_0}_{R} - \frac{N_2}{\tau_2} - \frac{\sigma_L I_L}{h\nu_L}(N_2 - \underset{0}{\underset{\simeq}{N_1}}) =$$

$$= \frac{\sigma_p I_p}{h\nu_p} N_0 - \frac{N_2}{\tau_2}\left(1 + \frac{I_L}{I_s}\right)$$

with $I_s = \frac{h\nu_L}{\sigma_L \tau_2}$ (saturation intensity), σ_p is the absorption cross-section of the $E_0 \leftrightarrow E_2$ transition and σ_L is the stimulated emission cross-section of the $E_2 \leftrightarrow E_1$ transition. The last equation describes the dynamics of the inversion $\Delta N = N_2 - N_1 \simeq N_2$. Now we need to describe the dynamics of the photons.

If S is the survival factor of the photons in the cavity (where $S = 1 - L$ and L represents the lost photons fraction), then we can write the following equation for the cavity photons:

$$\frac{dN_P}{dt} = \frac{S e^{\gamma l_g} - 1}{\tau_{RT}} N_P + \beta A_{21} N_2 V \qquad (\beta \text{ is small})$$

where β is the fraction of photons being generated by spontaneous emission into the cavity mode of the laser. In order to transform the above equation in energy in the cavity per unit area we must multiply by $h\nu$ (energy/photon) and divide by the cross-sectional area A. Then, the previous equation can be written as:

$$\frac{d}{dt}(\overbrace{N_P h\nu / A}^{w}) = \frac{S e^{\gamma l_g} - 1}{\tau_{RT}} (N_P h\nu / A) + \beta \frac{1}{\tau_2} N_2 V h\nu \frac{1}{A} \Rightarrow$$

$$\frac{dw}{dt} = \frac{S e^{\gamma l_g} - 1}{\tau_{RT}} w + \beta h\nu \frac{N_2 l_g}{\tau_2}$$

The parameter l_g is the length of the gain medium in a round-trip time. Strictly speaking the above equation is valid only for ring laser configurations where there is only one type of traveling wave.

Now the differential equations for the inversion and photon energy (/area) of the cavity can be written in the following normalized form:

$$\frac{dg}{d\tau} = a\left[R - g(1+P) \right]$$

$$\frac{dP}{d\tau} = (Se^{g} - 1)P + \beta g$$

where :

$\tau = t/\tau_{RT}$ = normalized time

$a = \tau_{RT}/\tau_2$ (usually << 1)

$g = N_2 \sigma_L l_g$ = line integrated gain (γl_g)

$P = \omega/\omega_s = I_L/I_s$ where $\omega = I_L \tau_{RT}$

$R = (N_0 \sigma_p l_g)(\lambda_p/\lambda_L)(I_p/I_s)$ = relative pumping rate

$S = 1 - L$ = survival factor , L = fractional loss per round trip

Case 1: A sub-threshold system

Assume a step function pumping $R = R_a u(t)$ with R_a such that the laser remains below threshold. Then $I_L \simeq 0 \sim P \simeq 0$,

Then

$$\frac{dg}{d\tau} = aR - ag \Rightarrow \frac{dg}{d\tau} + ag = R_a u(\tau) \rightarrow$$

$$g(\tau) = R_a (1 - e^{-a\tau}) u(\tau)$$

As $t \to \infty$ $g \to R_a = g_a < g_{th}$. Then the steady-state solution for photons is

$$P_a = \frac{\beta R_a}{1 - Se^{R_a}} \simeq \frac{\beta R_a}{1 - S} \qquad (\text{for } g_a = R_a << 1)$$

resulting in small P_a (spontaneous emission) due to small β:

(β of the order of 10^{-8} for macrocavities and 10^{-2} for microcavities).

Case 2 : A CW (continuous-wave) laser (threshold conditions)

The pumping rate must be such that $g \geq g_{th}$ initially. Or,

$$S e^{g_{th}} \geq 1 \sim g_{th} \geq \ln\left(\frac{1}{S}\right) = \ln\left(\frac{1}{1-L}\right) \simeq \ln(1+L) \simeq L$$

The threshold pumping must be above R_{th}. The saturated value of g, g_s, at the steady-state is:

$$\frac{dP}{d\tau} = 0 = (S e^{g_s} - 1) P + \beta g_s \Rightarrow$$

$$1 - S e^{g_s} = \beta \frac{g_s}{P} > 0 \rightsquigarrow \quad S e^{g_s} \lessapprox 1$$ as we have also shown previously. Also from $(g_s \simeq g_{th})$

$$\frac{dg}{d\tau} = 0 = a[R - g_{th}(1+P)] \Rightarrow \quad P = \frac{R}{g_{th}} - 1$$

which also has been derived previously.

Case 3 : A sinusoidal modulated pump:

In this case assume that $R > R_{th}$ in order to establish a steady-state for both inversion and relative photon number. Then allow small time-dependent changes in $R(\tau)$, $P(\tau)$, and $g(\tau)$:

$$R(\tau) = R_o + \Delta r(\tau)$$

$$P(\tau) = P_o + \Delta p(\tau)$$

$$g(\tau) = g_{th} + \Delta g(\tau)$$

where $P_o = \frac{R_o}{g_{th}} - 1$. For small $\Delta r, \Delta g, \Delta p$ the photon equation becomes:

$$\frac{d(\Delta p)}{d\tau} = (S e^{g_{th}} e^{\Delta g} - 1)(P_o + \Delta p) + \beta(g_{th} + \Delta g)$$

where $Sg^{th} \simeq 1$, $\Delta g \ll 1$ so that $e^{\Delta g} \simeq 1 + \Delta g$ ~

$$\frac{d(\Delta p)}{d\tau} \simeq \Delta g \, (P_o + \beta)$$

The population inversion equation becomes:

$$\frac{d}{d\tau}(\Delta g) = a\left[\, \Delta r - \Delta g\,(1 + P_o) - g_{th}\,\Delta p \,\right]$$

Eliminating Δg from the set of the last two equations results in:

$$\frac{d^2}{d\tau^2}(\Delta p) + a\left[\,1 + P_o\,\right]\frac{d}{d\tau}(\Delta p) + a g_{th}(P_o + \beta)\,\Delta p = a(P_o + \beta)\,\Delta r$$

Now assume that $\Delta r(\tau) = \mathrm{Re}\{ r_m e^{j\omega_m \tau} \}$ with ω_m a normalized frequency. The response $\Delta p(\tau) = \mathrm{Re}\{ p_m e^{j\omega_m \tau} \}$, $\omega_m / \tau_{RT} = \omega_m' =$ real modulation frequency. Using phasors we get:

$$\frac{p_m}{r_m} = \frac{a(\beta + P_o)}{\left[g_{th} a(\beta + P_o) - \omega_m^2 \right] + j\omega_m a(1 + P_o)}$$

A resonance occurs at $\omega_{mr}^2 = g_{th} a(\beta + P_o)$ and the optical transfer function $\mathrm{OTF}(\omega_m = \omega_{mr}) = \frac{p_m}{r_m}(\omega_{mr}) = -j \frac{1}{g_{th}} \left(\frac{g_{th}}{a}\right)^{1/2} \frac{(\beta + P_o)^{1/2}}{1 + P_o}$

A sample response is shown in the next figure.

SINUSOIDAL PUMP MODULATION

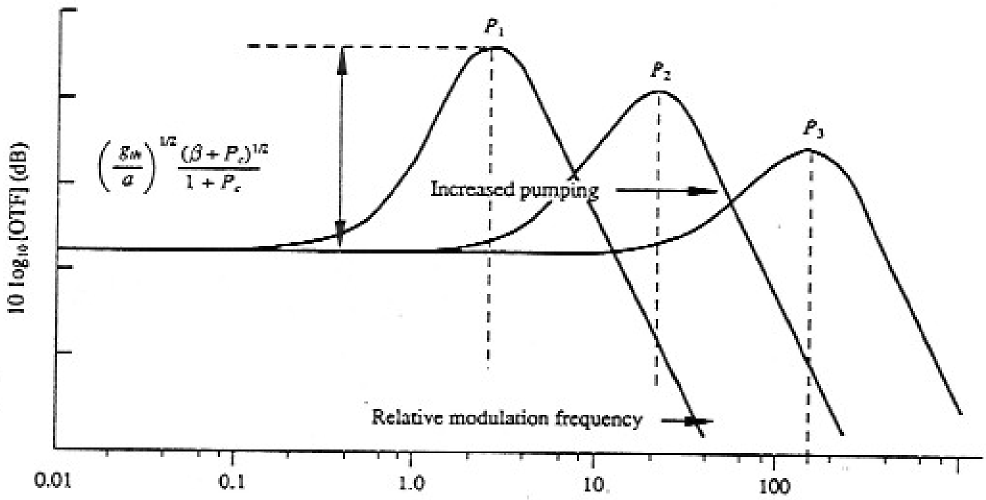


FIGURE 9.7. The relative response of a laser to a sinusoidal modulation of the pump. As the pumping is increased, the power, the resonant frequency, and the resonant width all increase. The "standard" method of plotting the modulation "gain" (in dB relative to that at a low frequency) as a function of the modulation frequency on a logarithmic frequency scale was used.

J. T. Verdeyen, "Laser Electronics," 3rd Ed., Prentice Hall, 1995

Case 4: A sudden step change in excitation

Now let $R = R_o + \Delta r\, u(\tau)$ with $R_o > R_{th}$. Then $g(\tau) = g_{th} + \Delta g(\tau)$ and $P(\tau) = P_o + \Delta p(\tau)$ where $P_o = \frac{R_o}{g_{th}} - 1$.

Assuming again small $\Delta r, \Delta p, \Delta g$ we can use the previous equations (case 3):

$$\frac{d(\Delta p)}{d\tau} = \Delta g\,(P_o + \beta)$$

$$\frac{d(\Delta g)}{d\tau} = a\,[\,\Delta r - \Delta g\,(1 + P_o) - g_{th}\,\Delta p\,]$$

Using Laplace's transform with $\Delta g(\tau=0) = \Delta p(\tau=0) = 0$ we get:

$$\begin{bmatrix} a\,g_{th} & s + a(1+P_o) \\ s & -(\beta + P_o) \end{bmatrix} \begin{bmatrix} \Delta p(s) \\ \Delta g(s) \end{bmatrix} = \begin{bmatrix} a\,\Delta r/s \\ 0 \end{bmatrix}$$

$$\Delta p(s) = \frac{a(\beta + P_o)\,\Delta r}{s\,[\,s^2 + a(1+P_o)s + a g_{th}(\beta + P_o)\,]}$$

$$\Delta g(s) = \frac{a\,\Delta r}{[\,s^2 + a(1+P_o)s + a g_{th}(\beta + P_o)\,]}$$

For $\Delta p(s)$ the poles of the response are at:

$$s = \frac{-a(1+P_o)}{2} \pm j\left[\,a g_{th}(\beta + P_o) - \left(\frac{a(1+P_o)}{2}\right)^2\right]^{1/2} =$$

$$= -\frac{1}{\tau_d} \pm j\,\omega_o \qquad (\omega_o \simeq \omega_{mr})$$

Then $\Delta p(\tau) \simeq \frac{\Delta r}{g_{th}}\left\{u(\tau) - e^{-\tau/\tau_d}\left(\cos(\omega_o\tau) + \frac{1}{\omega_o\tau_d}\sin(\omega_o\tau)\right)\right\}$

Now if $l = 400\,\mu m$, $n = 3.6$, $\tau_2 = 1\,ns$, $\beta \simeq 10^{-3}$, $g_{th} \simeq L = 0.64$

$P_o = 2$, $\Delta r = 0.128$.

$\tau_{RT} = \frac{2\ell}{c/n} = 9.6\,ps$, $a = \tau_{RT}/\tau_2 = 9.6 \cdot 10^{-3}$

$a g_{th} (\beta + P_0) = 1.23 \cdot 10^{-2}$

$[a(1+P_0)/2]^2 = 2.07 \cdot 10^{-4}$

$\omega_0^2 \simeq a g_{th} (\beta + P_0) = 1.23 \cdot 10^{-2} \rightarrow \omega_0 = 0.111$

(real frequency $\omega_0' = \omega_0 / \tau_{RT} = (0.111/9.6) \cdot 10^{12}\,Hz = 1.156 \cdot 10^{10}\,Hz$

$\tau_d = \frac{2}{a(1+P_0)} = 69.4 \sim \omega_0 \tau_p \simeq 7.7$

$$\Delta p(\tau) \simeq \frac{\Delta r}{g_{th}} \left(1 - e^{-\tau/\tau_d} \cos(\omega_0 \tau)\right)$$

which is shown in the next page.

SUDDEN-STEP EXCITATION

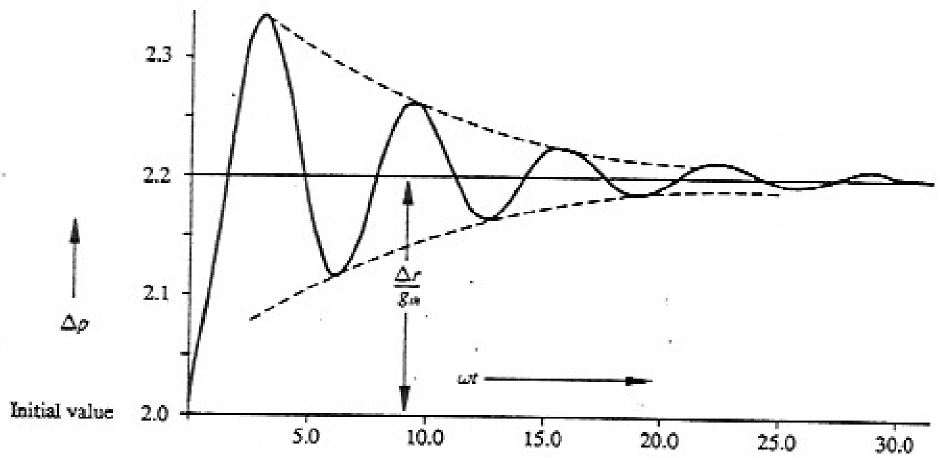


FIGURE 9.8. Damped oscillation while a laser approaches a new steady state. ($\Delta r = 0.2 \times 0.64 = 0.128$)

J. T. Verdeyen, "Laser Electronics," 3rd Ed., Prentice Hall, 1995

Case 5: Pulse excitation – gain switching

Assume a laser system excited with a strong pump amplitude which is well above the threshold value for steady-state.

Initially, before P becomes appreciable, we can solve the system of the coupled equations analytically.

$$\frac{dg}{d\tau} = a(R - g)$$

$$\frac{dP}{d\tau} = (Se^{g} - 1)P + \beta g$$

which results in

$$g(\tau) = R(1 - e^{-a\tau}) \simeq R \quad \text{for large } \tau >> 1/a$$

and

$$P(\tau) \simeq \delta P_o \, e^{(Se^{g}-1)t}$$

If the gain increases fast then it takes sometime before P becomes appreciable. This can lead to large inversions before the two equations become coupled nonlinear equations. Then the term P can not be ignored from the inversion equation. In this case the equations must be solved numerically starting with a small seed number of photons. This approach result in the generation of pulse, and it is the principle of operation of the Q-switch/giant pulse lasers.

An example solution is shown in the next figure where now time has been normalized to the cavity lifetime.

PULSE EXCITATION

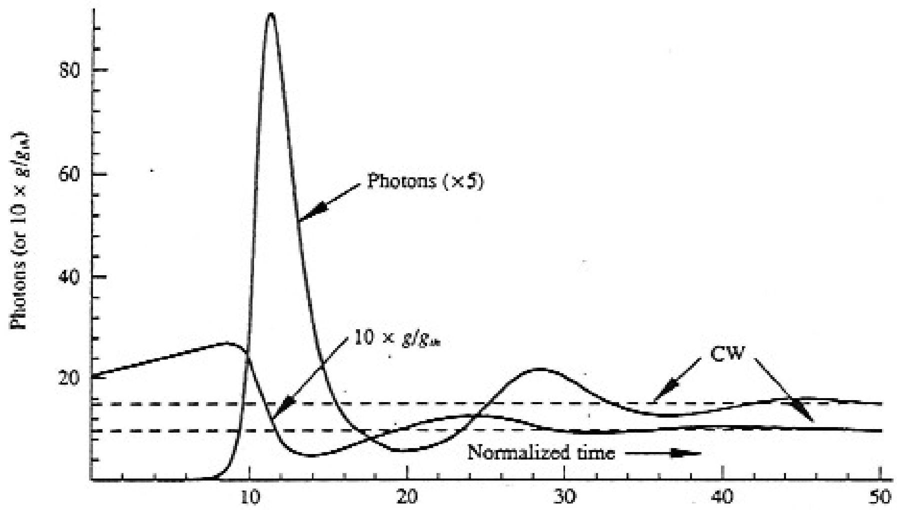
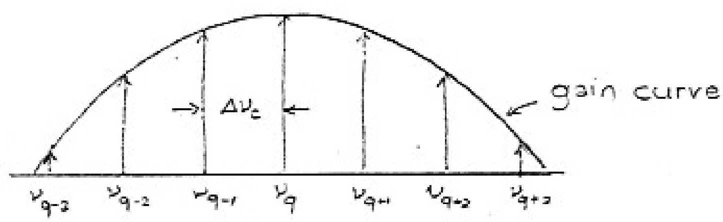


FIGURE 9.9. The time evolution of a "gain switched" pulse. The gain and photon number have been multiplied by 10 and 5, respectively, so as to separate the curves.

J. T. Verdeyen, "Laser Electronics," 3rd Ed., Prentice Hall, 1995

Mode Locking:

As we have just mentioned previously, in a typical inhomogeneously broadened laser there can be several modes that can oscillate simultaneously. The total electric field depends on the frequencies of these modes, their amplitudes, and their phases. If the modes are forced to maintain fixed phases as a function of time relative to each other the laser output is periodic in time and the laser is said to be "mode-locked." Assume that the laser supports a number of longitudinal modes which are separated in frequency by $\Delta\nu_c = \frac{c/n}{2\ell}$. For simplicity assume $n = 1.0$



The total electric field $e(t)$ can be written as

$$e(t) = \sum_n E_n e^{j[(\omega_0 + n\omega_c)t + \phi_n]} \qquad \text{(real part is assumed).}$$

If $T = \frac{2\pi}{\omega_c} = \frac{2\ell}{c}$ = round-trip transit time, $\omega_c = \frac{\pi c}{\ell} = 2\pi\Delta\nu_c$ then it is straightforward to show that $e(t+T) = e(t)$ assuming that E_n and ϕ_n are constants (independent of time).

Usually $\phi_n = \phi_n(t)$ = randomly varying phases and cause the laser output to fluctuate randomly.

Now assume that "mode locking" is enforced and ϕ_n maintain their relative values. The $e(t)$ and consequently the output power can be maximized if ϕ_n are all the same. For simplicity we can set $\phi_n = 0$ and $E_n = 1$. Then, assuming that ω_0 is the central $n=0$ mode we can write:

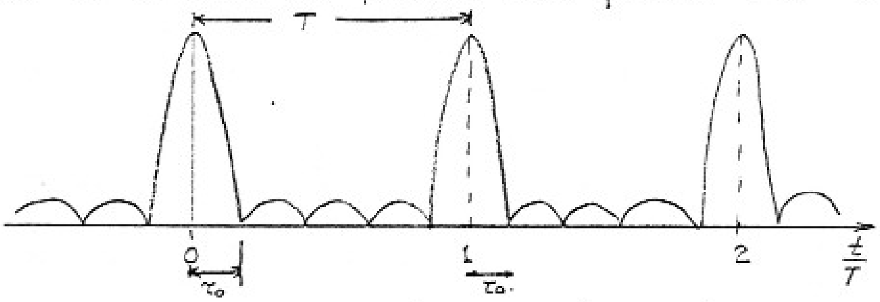
$$e(t) = \sum_{-(N-1)/2}^{(N-1)/2} e^{\pm j(\omega_0 + n\omega_c)t} = e^{j\omega_0 t} \frac{\sin(N\omega_c t/2)}{\sin(\omega_c t/2)}$$

Then the laser power is proportional to $|e(t)|^2$, i.e.

$$p(t) \propto \frac{\sin^2(N\omega_c t/2)}{\sin^2(\omega_c t/2)} \rightsquigarrow p(t) = p_0 \frac{\sin^2(N\omega_c t/2)}{\sin^2(\omega_c t/2)}$$

Properties of $p(t)$:

(1) $p(t)$ is a train of pulses with period $T = 2\ell/c$.



(2) The peak power $p(sT)$ $(s = 0, 1, 2, \dots)$ is equal to N times the average power. (N is the number of modes locked together).

(3) The peak field amplitude is equal to N times the amplitude of a single mode.

(4) The individual pulse width τ_0 (as defined in the figure) is T/N.
Since approximately $N \simeq \dfrac{\Delta\nu \text{ (lineshape)}}{1/T = \Delta\nu_c} \rightsquigarrow \tau_0 \simeq \dfrac{1}{\Delta\nu}$

For a He-Ne laser $\Delta\nu = 1500\,\text{MHz} \rightsquigarrow \tau_0 \simeq 0.6\,\text{ns}$ (pulse occupy $\sim 20\,\text{cm}$ of free space). For a Nd-glass laser $\Delta\nu = 3 \cdot 10^{12}\,\text{Hz} \Rightarrow \tau_0 \simeq 0.3\,\text{ps} \rightsquigarrow$ (0.1mm of free space). An enormous number of photons are packed into a small portion of space.

$$L_p = \tau_0 c = \frac{T}{N} \cdot c = \frac{2\ell}{cN} c = \frac{2\ell}{N}$$

MODE COMBINATIONS EXAMPLES

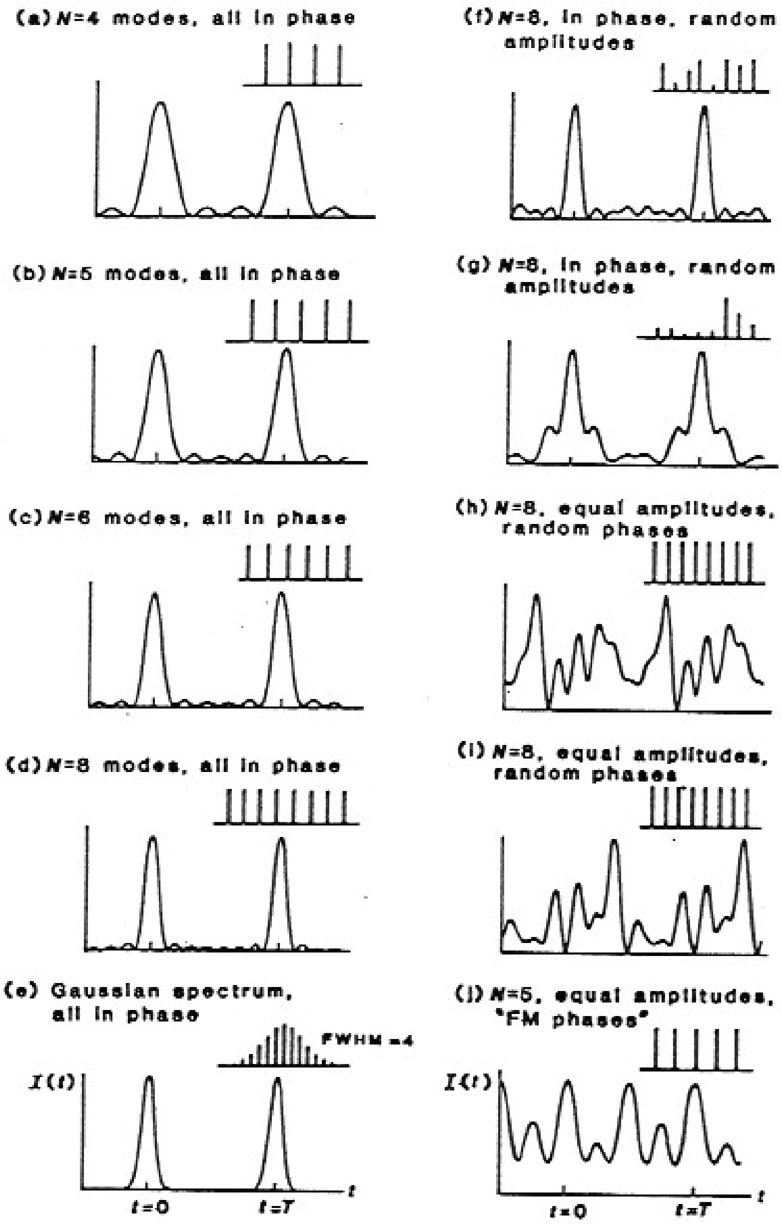
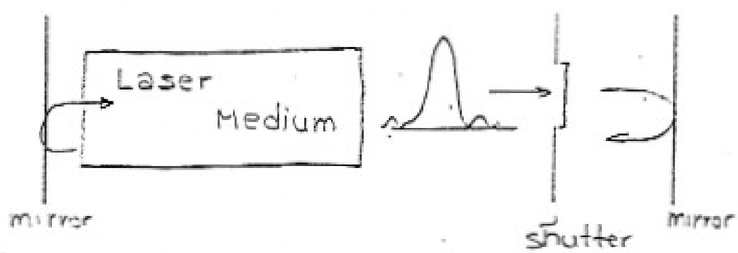


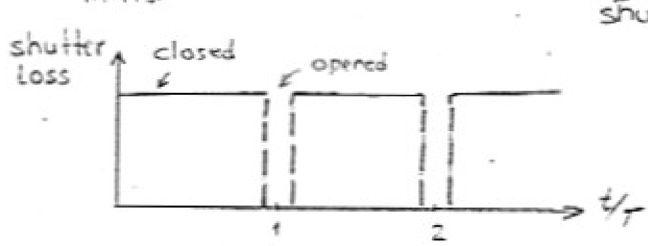
FIGURE 27.7
Examples of the different intensity patterns in time that can be synthesized using N equally spaced frequency components with different relative amplitudes and phase angles.

A. E. Siegman, "Lasers", University Science Books, Mill Valey, CA , 1986

Methods of Mode Locking:

Mode locking can be achieved by modulating the losses (or gain) of the laser at a radian frequency $\omega_c = 2\pi\Delta\nu_c = \pi c/\ell$. This can be done by placing a shutter inside the laser cavity. The shutter opens every $T = \frac{2\pi}{\omega_c} = \frac{2\ell}{c}$ seconds and remains open for a small amount of time in order for the mode-locked pulse to pass through. When the shutter is open the pulse is not attenuated and consequently does not "feel" the presence of the shutter. On the other hand, light that arrives at different times is attenuated and does not oscillate.





Therefore, through the loss modulation provided by the shutter we can achieve mode-locking.

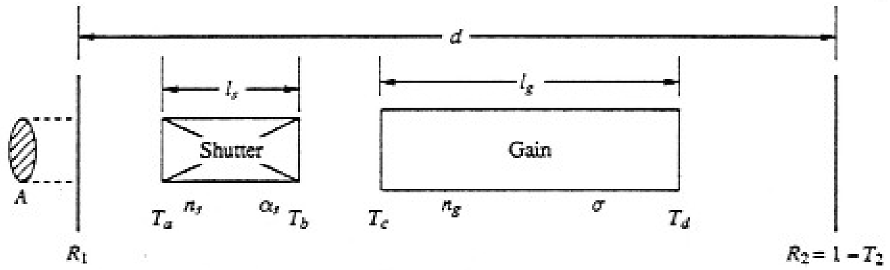
An active implementation of the shutter is through an acousto-optic Bragg cell. When a signal is applied to the acousto-optic cell an acoustic wave is lauched that can diffract the light. This creates a loss mechanism in the cavity. When the signal is off the light passes through the cell undiffracted and does not experience any loss.

Passive mode locking can be achieved via saturable absorber. This is a medium that exhibits loss (absorption) which decreases when the light intensity increases. In this case only the mode-locked pulse will exhibit enough gain (due to the smaller loss) and will oscillate.

Q-Switching or Giant Pulse Lasers:

An alternate method of producing high power laser pulses is the "Q-switching". In this technique a giant laser pulse can be created by using energy stored via a large population inversion. Of course, spontaneous emission out of the upper state represents a drain of the stored energy. At least, we can avoid amplification of the spontaneous emissio photons by imposing large losses in the cavity. In other words, the Q factor of the cavity is degraded. Under these conditions a large inversion can be achieved. This is the equivalent of storing energy. This stored energy can be released when the cavity factor Q is restored to its high value (loss is decreased). Under these circumstances, the gain is much higher than the loss and this results in a rapid buildup of photons via stimulated emission. This phenomenon is a transient phenomenon and produces a large output laser pulse. The duration of this pulse is of the order of the cavity lifetime t_c which is of the order of 10^{-8} sec. Of course this large and rapid production of photon is going to deplete fast the population inversion. In the following analysis it is assumed that the photon and population densities are independent of position. Also it is assumed that during the buildup of the giant pulse pumping effects are neglected. In addition, any other relaxation mechanism of the excited atoms except stimulated emission is neglected.

An example cavity for a Q-switched laser is shown next. It contains the gain medium, the shutter (that produces the Q switching)



and the two mirrors of the cavity. T_a, T_b, T_c, and T_d denote the transmitter through the various interfaces. If N_p is the number of photons in the cavity then the rate of change of N_p is given by:

$$\frac{dN_p}{dt} = \left(\frac{Se^{2\gamma \ell_g} - 1}{\tau_{RT}} \right) N_p$$

where $S = R_1 R_2 (T_a T_b T_c T_d)^2 e^{-2\alpha_s \ell_s}$ = survival factor. $Se^{2\gamma_{th}\ell_g} - 1 \simeq 0$ when threshold is reached. Also the cavity lifetime can be written as $t_c = \tau_{RT}/1-S = \tau_{RT}/1-e^{-2\gamma_{th}\ell_g}$. Using the last two expressions and assuming that $e^x \simeq 1+x$ (small changes of N_p per round trip), the photon rate equation can be written as:

$$\frac{dN_p}{dt} = \frac{N_p}{t_c}\left(\frac{\gamma}{\gamma_{th}} - 1\right) = \frac{N_p}{t_c}\left(\frac{n}{n_{th}} - 1\right)$$

where $n = (N_2 - N_1) A \ell_g$ = total inversion and

$n_{th} = (N_2 - N_1)_{th} A \ell_g$ = total threshold inversion.

Of course, as the number of photons increases via stimulated emission the inversion decreases. Since $\frac{N_p}{t_c}\frac{n}{n_{th}}$ is the rate of photon production due to stimulated emission we can write the rate of decrease of the inversion n as

$$\frac{dn}{dt} = -2 \frac{n}{n_{th}} \frac{N_p}{t_c}$$

since for every photon produced the inversion decreases by 2 (assuming equal degeneracies).

The two rate equation can be written in normalized time $T = t/t_c$ as:

$$\frac{dN_p}{dT} = N_p \cdot \left(\frac{n}{n_{th}} - 1\right)$$

$$\frac{dn}{dT} = -2\,\frac{n}{n_{th}}\,N_p$$

The above equations are nonlinear coupled differential equations and can be solved only numerically. However, a relationship between N_p, n can be determined by dividing the two differential equations:

$$\frac{dN_p}{dn} = \frac{1}{2}\left(\frac{n_{th}}{n} - 1\right) \implies$$

$$dN_p = \frac{1}{2}\left(\frac{n_{th}}{n} - 1\right) dn \implies \int_{N_{p\,initial}}^{N_p} dN_p = \int_{n_i}^{n} \frac{1}{2}\left(\frac{n_{th}}{n} - 1\right) dn \implies$$

$$N_p - \underset{\overset{\wr\wr}{0}}{N_{p\,initial}} = \frac{1}{2}(n_i - n) - \frac{n_{th}}{2}\ln\left(\frac{n_i}{n}\right) \implies$$

$$N_p = \frac{n_i - n}{2} - \frac{n_{th}}{2}\ln\left(\frac{n_i}{n}\right)$$

The peak in N_p occurs when $n = n_{th}$ since then $dN_p/dn = 0$. Therefore, the peak $N_{p\,max}$ is given by

$$N_{p\,max} = \frac{n_i - n_{th}}{2} - \frac{n_{th}}{2}\ln\left(\frac{n_i}{n_{th}}\right)$$

The useful optical output power is given by:

$$P(t) = h\nu\, N_p(t)\, \frac{\text{out coupling loss / round trip}}{\text{total loss / round trip}} \cdot \frac{\text{total loss / round trip}}{\text{time for round trip}}$$

$$= \left(\eta_{cpl} \simeq \frac{\langle\alpha_{ext}\rangle}{\langle\alpha_{Total}\rangle}\right) \frac{h\nu\, N_p(t)}{t_c}$$

where the outcoupling efficiency η_{cpl} is:

$$\eta_{cpl} = \frac{1 - R_2 = T_2}{1 - S} = \frac{1 - e^{-2\langle\alpha_{ext}\rangle d}}{1 - e^{-2\langle\alpha_{Total}\rangle d}} \simeq \frac{\langle\alpha_{ext}\rangle}{\langle\alpha_{total}\rangle}$$

where the "spatially averaged" attenuation coefficients are defined by

$$\exp\{-2\langle\alpha_{ext}\rangle d\} = R_2$$

$$\exp\{-2\langle\alpha_{total}\rangle d\} = S = (T_a T_b T_c T_d)^2 R_1 R_2 e^{-2\alpha_s l_s}$$

Then the peak optical power is given by:

$$P_{max} = \eta_{cpl} \frac{h\nu N_{p\,max}}{t_c}$$

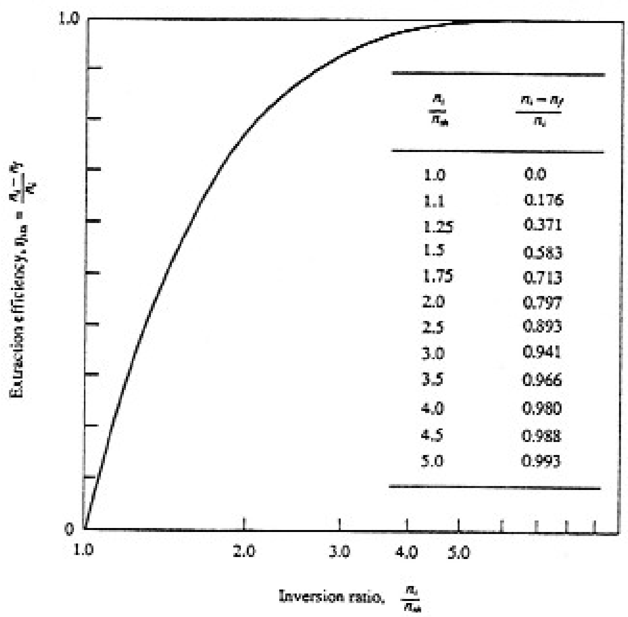
For times $t \gg t_c$ there are no more photons left, i.e. $N_p \to 0$.

Then the final inversion n_f can be calculated as follows:

$$0 = \frac{n_i - n_f}{2} - \frac{n_{th}}{2} \ln\left(\frac{n_i}{n_f}\right) \Longrightarrow$$

$$\frac{n_f}{n_i} = \exp\left\{-\frac{n_i - n_f}{n_{th}}\right\}$$

The last equation can be solved numerically only and the result is shown in the figure below:



The term $n_i - n_f / n_i$ is the fraction of the initial inversion that has been converted into photons.

Then the total photon energy of the pulse is:

$$W = h\nu \left(\frac{n_i - n_f}{2}\right) = \eta_{xtn} \frac{h\nu n_i}{2}$$

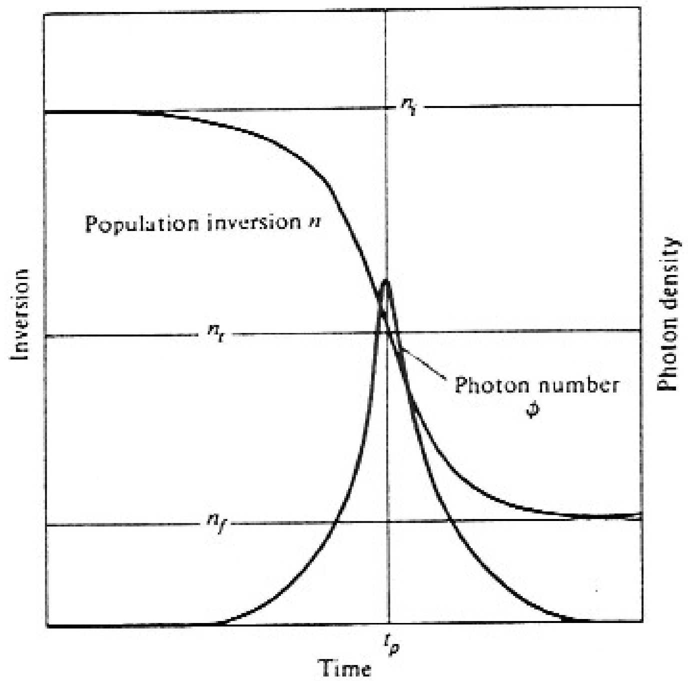
$$\eta_{xtn} = \frac{n_i - n_f}{n_i} = \text{extraction efficiency.}$$

The actual output optical energy though is given by:

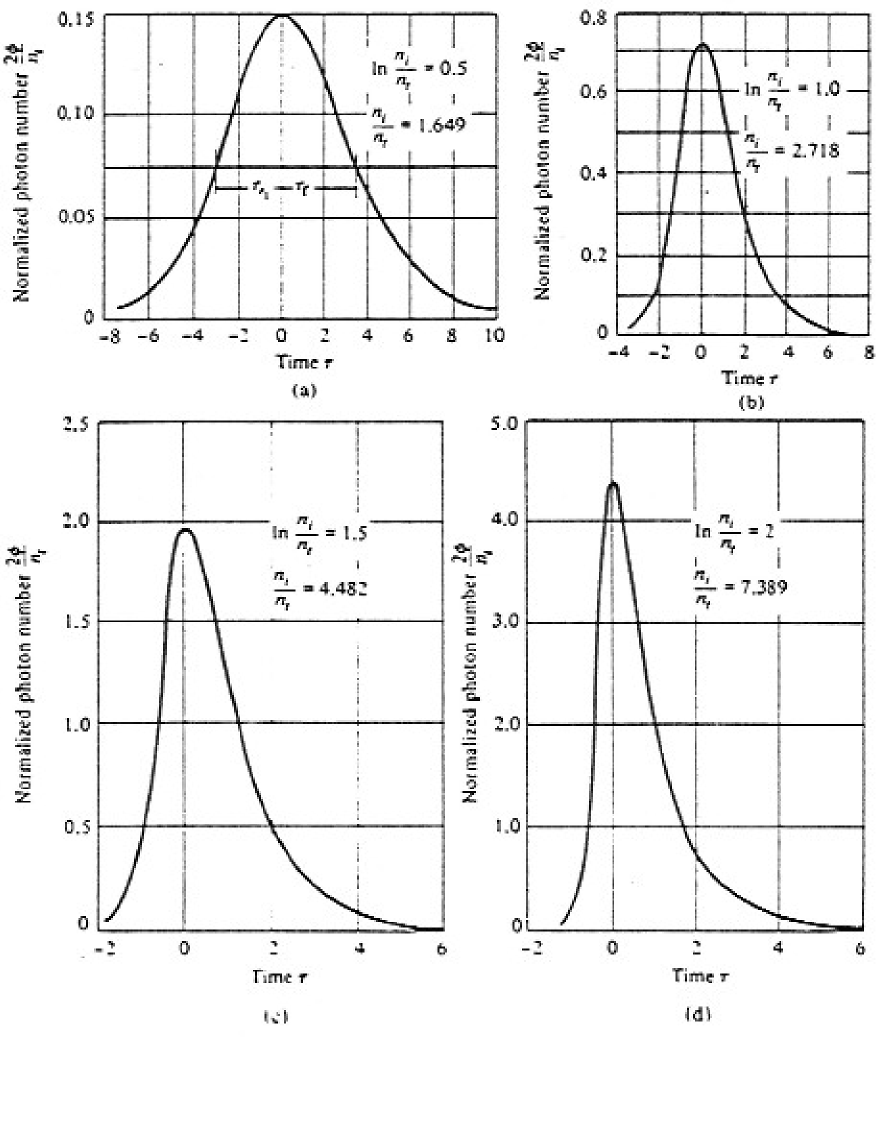
$$W_{out} = \eta_{cpl} W = \eta_{clp} \eta_{xtn} \frac{h\nu n_i}{2}$$

Finally, the pulse width can be estimated by:

$$\tau_o = \Delta t_p \simeq W_{out} / P_{max}$$



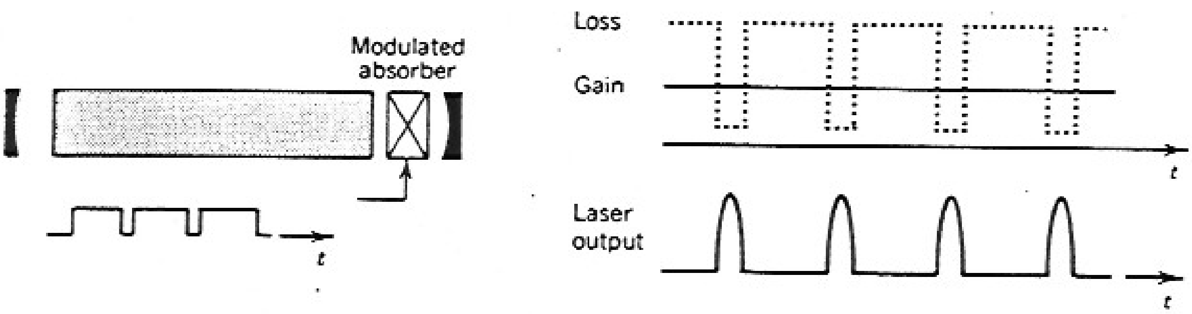
n_i
Population inversion n
Inversion
n_t
Photon number ϕ
Photon density
n_f
t_p
Time



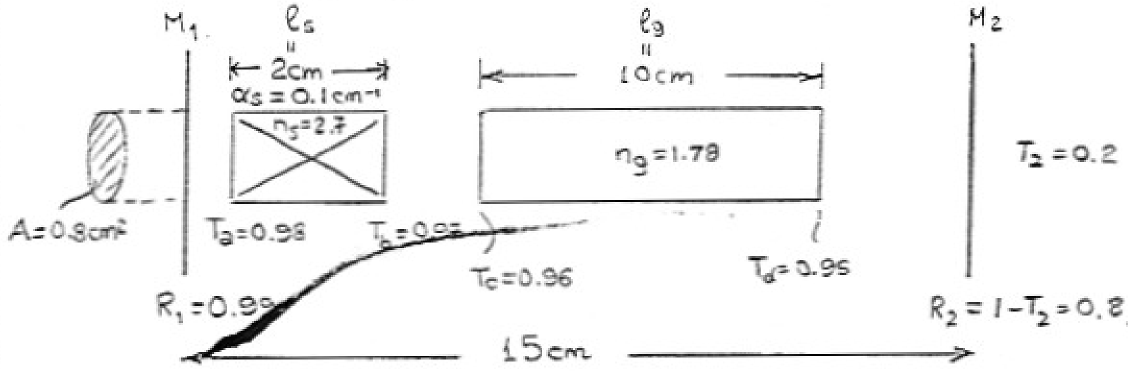
Normalized photon number $\frac{2\phi}{n_t}$
$\ln \frac{n_i}{n_t} = 0.5$
$\frac{n_i}{n_t} = 1.649$
$\tau_{r_1} - \tau_f$
Time τ
(a)
$\ln \frac{n_i}{n_t} = 1.0$
$\frac{n_i}{n_t} = 2.718$
Time τ
(b)
$\ln \frac{n_i}{n_t} = 1.5$
$\frac{n_i}{n_t} = 4.482$
Time τ
(c)
$\ln \frac{n_i}{n_t} = 2$
$\frac{n_i}{n_t} = 7.389$
Time τ
(d)

All Q-switching methods are based on the formation of a fast efficient shutter. Some common methods are:

1. Rotating Reflector
2. Saturable Absorber
3. Electro-optic crystal
4. Acousto-optic crystal



Example: Assume a ruby laser system ($\lambda_0 = 0.6943\ \mu m$). It is pumped to an initial inversion $n_i = 4 n_{th}$. The system is depicted in the following figure:



(a) Find the threshold gain:

Threshold condition: (for a round trip) $\quad T_a T_b T_c T_d R_2 T_d T_c T_b T_a R_1 e^{-2\alpha_s \ell_s} e^{2\gamma_{th}\ell_g} = 1$

$$\rightarrow \quad R_1 R_2 (T_a T_b T_c T_d)^2 e^{-2\alpha_s \ell_s} e^{2\gamma_{th}\ell_g} = 1 \quad \Rightarrow$$

$$\gamma_{th} = \frac{1}{2\ell_g} \ln\left[e^{2\alpha_s \ell_s} \frac{1}{R_1 R_2 (T_a T_b T_c T_d)^2} \right] =$$

$$= \underbrace{\frac{1}{2\ell_g} \ln\left[\frac{1}{R_1 (T_a T_b T_c T_d)^2} \right] + \frac{\ell_s}{\ell_g}\alpha_s}_{\text{internal cavity \& switching losses}} + \underbrace{\frac{1}{2\ell_g} \ln\left(\frac{1}{R_2}\right)}_{\text{external coupling loss}}$$

$$\gamma_{th} = \frac{1}{2\cdot 10} \cdot \ln\left(\frac{1}{0.99(0.98 \cdot 0.97 \cdot 0.96 \cdot 0.95)^2} \right) + \frac{2}{10} 0.1 + \frac{1}{2\cdot 10} \ln\left(\frac{1}{0.8}\right) \ \text{cm}^{-1}$$

$$= 1.478 \cdot 10^{-2}\,\text{cm}^{-1} + 2\cdot 10^{-2}\,\text{cm}^{-1} + 1.116 \cdot 10^{-2}\,\text{cm}^{-1} = 4.594 \cdot 10^{-2}\,\text{cm}^{-1}$$

(b) Assume that the stimulated emission cross-section $\sigma(\nu) = 2.5 \cdot 10^{-20}\,\text{cm}^2$

Find threshold inversion:

$$\left.\begin{array}{l} n_{th} = N_{th} \cdot V = \dfrac{\gamma_{th}}{\sigma} V \\ V = A \cdot \ell_g = 0.8 \cdot 10\,\text{cm}^3 = 8\,\text{cm}^3 \end{array}\right\} \quad n_{th} = \frac{4.594 \cdot 10^{-2}\,\text{cm}^{-1}}{2.5 \cdot 10^{-20}\,\text{cm}^2} 8\,\text{cm}^3 = 1.47 \cdot 10^{19}\ \text{atoms}$$

Therefore the initial inversion is $n_i = 4n_{th} = 5.88 \cdot 10^{19}$ atoms

(c) Find cavity lifetime:

$$t_c = \frac{\tau_{RT}}{1 - R_1R_2(T_aT_bT_cT_d)^2 e^{-2\alpha_s \ell_s}}$$

$$\tau_{RT} = \frac{2\ell_g}{c/n_g} + \frac{2\ell_s}{c/n_s} + \frac{2(d-\ell_s-\ell_g)}{c}$$

$$R_1R_2(T_aT_bT_cT_d)^2 e^{-2\alpha_s \ell_s} = 0.99 \cdot 0.8\,(0.98 \cdot 0.97 \cdot 0.96 \cdot 0.95)^2 e^{-0.4} = 0.399018$$

$$\tau_{RT} = \frac{2 \cdot 10 \cdot 10^{-2} \cdot 1.78}{3 \cdot 10^8} + \frac{2 \cdot 2 \cdot 10^{-2}\, 2.7}{3 \cdot 10^8} + \frac{2 \cdot 3 \cdot 10^{-2}}{3 \cdot 10^8} = \frac{52.4 \quad 10^{-2}}{3 \cdot 10^8} = 1.7467\ \text{ns}$$

$$t_c = \frac{1.7467\ \text{ns}}{1 - 0.399018} = 2.906\ \text{ns} \simeq 2.91\ \text{ns}$$

(d) Find the peak power of the Q-switched pulse: $(n_i = 4n_{th})$

$$P_{peak} = \frac{h\nu}{2t_c}\left[n_{th} \ln\left(\frac{n_{th}}{n_i}\right) - (n_{th} - n_i)\right] =$$

$$= \frac{hc}{2t_c\lambda_0}\left[n_{th} \ln\left(\frac{1}{4}\right) + 3n_{th}\right] = \frac{hc\,n_{th}}{2t_c\lambda_0}\left(3 + \ln(1/4)\right) =$$

$$= \frac{6.626 \cdot 10^{-34} \cdot 3 \cdot 10^8 \cdot 1.47 \cdot 10^{19}}{2 \cdot 2.906 \cdot 10^{-9} \cdot 0.6943 \cdot 10^{-6}} (3 + \ln(1/4)) = 11.68 \cdot 10^8\ \text{W} =$$

$$= 1168\ \text{MW}$$

The output peak power is $P_{peak}^{out} = \frac{\langle \alpha_{ext} \rangle}{\langle \alpha_{total} \rangle} P_{peak} = \frac{1.116}{4.594} \cdot 1168\,\text{MW} =$
$= 283.7\ \text{MW}$

$$\left.\begin{array}{l} \text{The total energy } W = h\nu\left(\dfrac{n_i - n_f}{2}\right) \\ \text{But for } 4n_{th} = n_i \rightarrow \dfrac{n_i - n_f}{n_i} = 0.98 \end{array}\right\} \Rightarrow W = \frac{h\nu}{2}\, 0.98\, n_i \Rightarrow$$

$$W = \frac{hc}{2\lambda_0} 0.98\, n_i = \frac{6.626 \cdot 10^{-34} \cdot 3 \cdot 10^8 \cdot 5.88 \cdot 10^{19}\ 0.98}{2 \cdot 0.6943 \cdot 10^{-6}}\ \text{J} = 82.49 \cdot 10^{-1}\ \text{J} \Rightarrow$$

$W = 8.249$ Joules

The output energy is $W_{out} = \frac{\langle \alpha_{ext} \rangle}{\langle \alpha_{total} \rangle} W = \frac{1.116}{4.594}\, 8.249\ \text{J} = 2.004$ Joules

(e) Estimate of pulse width:

$$\tau \simeq \frac{W_{out}}{P_{peak}^{out}} = \frac{W}{P_{peak}} = \frac{8.249\ \text{J}}{1168 \cdot 10^6\ \text{W}} = 7.06\ \text{ns}$$