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Short Review of Electromagnetic Principles

Electro-Optics & Applications

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MAXWELL EQUATIONS

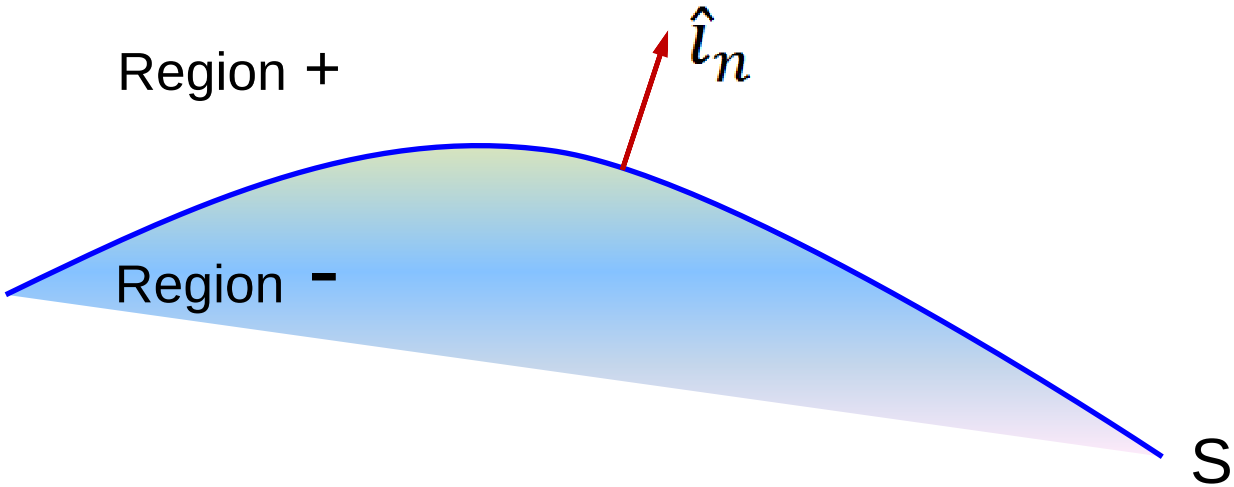
Differential Form

$$
\begin{aligned}
\vec{\nabla} \times \vec{\mathcal{E}} &= -\frac{\partial \vec{\mathcal{B}}}{\partial t}, \\
\vec{\nabla} \times \vec{\mathcal{H}} &= \vec{\mathcal{J}} + \frac{\partial \vec{\mathcal{D}}}{\partial t}, \\
\vec{\nabla} \cdot \vec{\mathcal{D}} &= \rho, \\
\vec{\nabla} \cdot \vec{\mathcal{B}} &= 0,
\end{aligned}
$$

Integral Form

$$
\begin{aligned}
\oint_C \vec{\mathcal{E}} \cdot d\vec{\ell} &= -\frac{d}{dt} \iint_S \vec{\mathcal{B}} \cdot d\vec{S}, \quad \text{(Faraday Law)}, \\
\oint_C \vec{\mathcal{H}} \cdot d\vec{\ell} &= \int_S \vec{\mathcal{J}} \cdot d\vec{S} + \frac{d}{dt} \iint_S \vec{\mathcal{D}} \cdot d\vec{S}, \quad \text{(Ampere Law)}, \\
\oiint_S \vec{\mathcal{D}} \cdot d\vec{S} &= \iiint_V \rho dV, \quad \text{(Gauss Law)}, \\
\oiint_S \vec{\mathcal{B}} \cdot d\vec{S} &= 0, \quad \text{(Absence of Magnetic Monopoles)},
\end{aligned}
$$

BOUNDARY CONDITIONS



$$
\begin{aligned}
\hat{i}_n \times (\vec{\mathcal{E}}_+ - \vec{\mathcal{E}}_-)_S &= 0, \\
\hat{i}_n \times (\vec{\mathcal{H}}_+ - \vec{\mathcal{H}}_-)_S &= \vec{\mathcal{K}}, \\
\hat{i}_n \cdot (\vec{\mathcal{D}}_+ - \vec{\mathcal{D}}_-)_S &= \sigma, \\
\hat{i}_n \cdot (\vec{\mathcal{B}}_+ - \vec{\mathcal{B}}_-)_S &= 0,
\end{aligned}
$$

Use of Phasors in Electromagnetics

Field Phasors

$$\vec{\mathcal{E}}(\vec{r},t) = \text{Re}\left\{\vec{E}(\vec{r},\omega)e^{j\omega t}\right\} = \frac{1}{2}\left[\vec{E}(\vec{r},\omega)e^{j\omega t} + \vec{E}^*(\vec{r},\omega)e^{-j\omega t}\right]$$

$$\vec{\mathcal{D}}(\vec{r},t) = \text{Re}\left\{\vec{D}(\vec{r},\omega)e^{j\omega t}\right\} = \frac{1}{2}\left[\vec{D}(\vec{r},\omega)e^{j\omega t} + \vec{D}^*(\vec{r},\omega)e^{-j\omega t}\right]$$

$$\vec{\mathcal{H}}(\vec{r},t) = \text{Re}\left\{\vec{H}(\vec{r},\omega)e^{j\omega t}\right\} = \frac{1}{2}\left[\vec{H}(\vec{r},\omega)e^{j\omega t} + \vec{H}^*(\vec{r},\omega)e^{-j\omega t}\right]$$

$$\vec{\mathcal{B}}(\vec{r},t) = \text{Re}\left\{\vec{B}(\vec{r},\omega)e^{j\omega t}\right\} = \frac{1}{2}\left[\vec{B}(\vec{r},\omega)e^{j\omega t} + \vec{B}^*(\vec{r},\omega)e^{-j\omega t}\right]$$

$$\vec{\mathcal{J}}(\vec{r},t) = \text{Re}\left\{\vec{J}(\vec{r},\omega)e^{j\omega t}\right\} = \frac{1}{2}\left[\vec{J}(\vec{r},\omega)e^{j\omega t} + \vec{J}^*(\vec{r},\omega)e^{-j\omega t}\right]$$

$$\varrho(\vec{r},t) = \text{Re}\left\{\rho(\vec{r},\omega)e^{j\omega t}\right\} = \frac{1}{2}\left[\rho(\vec{r},\omega)e^{j\omega t} + \rho^*(\vec{r},\omega)e^{-j\omega t}\right]$$

Apply Phasors for *Faraday Law*

$$\vec{\nabla} \times \vec{\mathcal{E}} = -\frac{\partial \vec{\mathcal{B}}}{\partial t} \quad \Longrightarrow$$

$$\vec{\nabla} \times \text{Re}\left\{\vec{E}(\vec{r},\omega)e^{j\omega t}\right\} = -\frac{\partial}{\partial t}\left(\text{Re}\left\{\vec{B}(\vec{r},\omega)e^{j\omega t}\right\}\right) \quad \Longrightarrow$$

$$\text{Re}\left\{\vec{\nabla} \times \vec{E}(\vec{r},\omega)e^{j\omega t}\right\} = \text{Re}\left\{\left(-j\omega\vec{B}(\vec{r},\omega)e^{j\omega t}\right)\right\} \quad \Longrightarrow$$

$$\text{Re}\left\{\left[\vec{\nabla} \times \vec{E}(\vec{r},\omega) + j\omega\vec{B}(\vec{r},\omega)\right]e^{j\omega t}\right\} = 0, \quad \forall t \quad \Longrightarrow$$

$$\boxed{\vec{\nabla} \times \vec{E}(\vec{r},\omega) = -j\omega\vec{B}(\vec{r},\omega)}$$

MAXWELL EQUATIONS

Time Harmonic Form

$$
\begin{aligned}
\vec{\nabla} \times \vec{E}(\vec{r}, \omega) &= -j\omega \vec{B}(\vec{r}, \omega), \\
\vec{\nabla} \times \vec{H}(\vec{r}, \omega) &= \vec{J}(\vec{r}, \omega) + j\omega \vec{D}(\vec{r}, \omega), \\
\vec{\nabla} \cdot \vec{D}(\vec{r}, \omega) &= \rho(\vec{r}, \omega), \\
\vec{\nabla} \cdot \vec{B}(\vec{r}, \omega) &= 0.
\end{aligned}
$$

Constitutive Relations

$$
\begin{aligned}
\vec{\mathcal{P}}(\vec{r}, t) = \epsilon_0 \int_0^\infty G_e(\tau) \vec{\mathcal{E}}(\vec{r}, t - \tau) d\tau \quad &\Longleftrightarrow \quad \vec{P}(\vec{r}, \omega) = \epsilon_0 \chi_e(\omega) \vec{E}(\vec{r}, \omega), \\
\vec{\mathcal{M}}(\vec{r}, t) = \int_0^\infty G_m(\tau) \vec{\mathcal{H}}(\vec{r}, t - \tau) d\tau \quad &\Longleftrightarrow \quad \vec{M}(\vec{r}, \omega) = \chi_m(\omega) \vec{H}(\vec{r}, \omega), \\
\vec{\mathcal{J}}(\vec{r}, t) = \int_0^\infty G_c(\tau) \vec{\mathcal{E}}(\vec{r}, t - \tau) d\tau \quad &\Longleftrightarrow \quad \vec{J}(\vec{r}, \omega) = \sigma(\omega) \vec{E}(\vec{r}, \omega),
\end{aligned}
$$

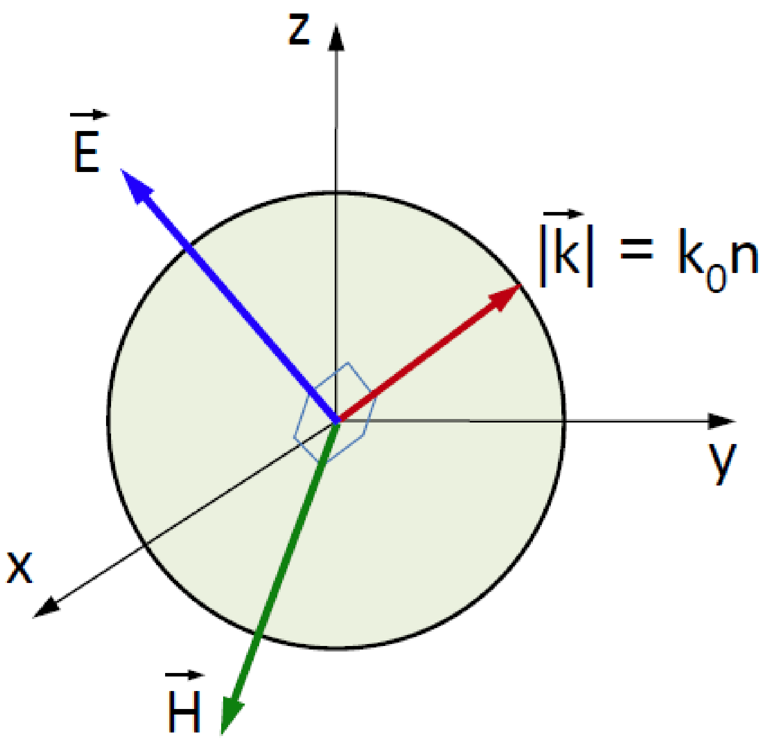
$$
\begin{aligned}
\vec{D}(\vec{r}, \omega) &= \epsilon_0 \vec{E}(\vec{r}, \omega) + \vec{P}(\vec{r}, \omega) = \epsilon_0 [1 + \chi_e(\omega)] \vec{E}(\vec{r}, \omega) = \epsilon(\omega) \vec{E}(\vec{r}, \omega), \\
\vec{B}(\vec{r}, \omega) &= \mu_0 [\vec{H}(\vec{r}, \omega) + \vec{M}(\vec{r}, \omega)] = \mu_0 [1 + \chi_m(\omega)] \vec{H}(\vec{r}, \omega) = \mu(\omega) \vec{H}(\vec{r}, \omega), \\
\vec{J}(\vec{r}, \omega) &= \sigma(\omega) \vec{E}(\vec{r}, \omega),
\end{aligned}
$$

MAXWELL EQUATIONS

Plane Wave Solutions – Isotropic Case

$$\vec{E}(\vec{r},\omega) = \vec{E}_0 \exp(-j\vec{k}\cdot\vec{r})$$

$$\vec{\mathcal{E}}(\vec{r},t) = \mathrm{Re}\{\vec{E}_0 \exp(-j\vec{k}\cdot\vec{r})\exp(j\omega t)\}$$

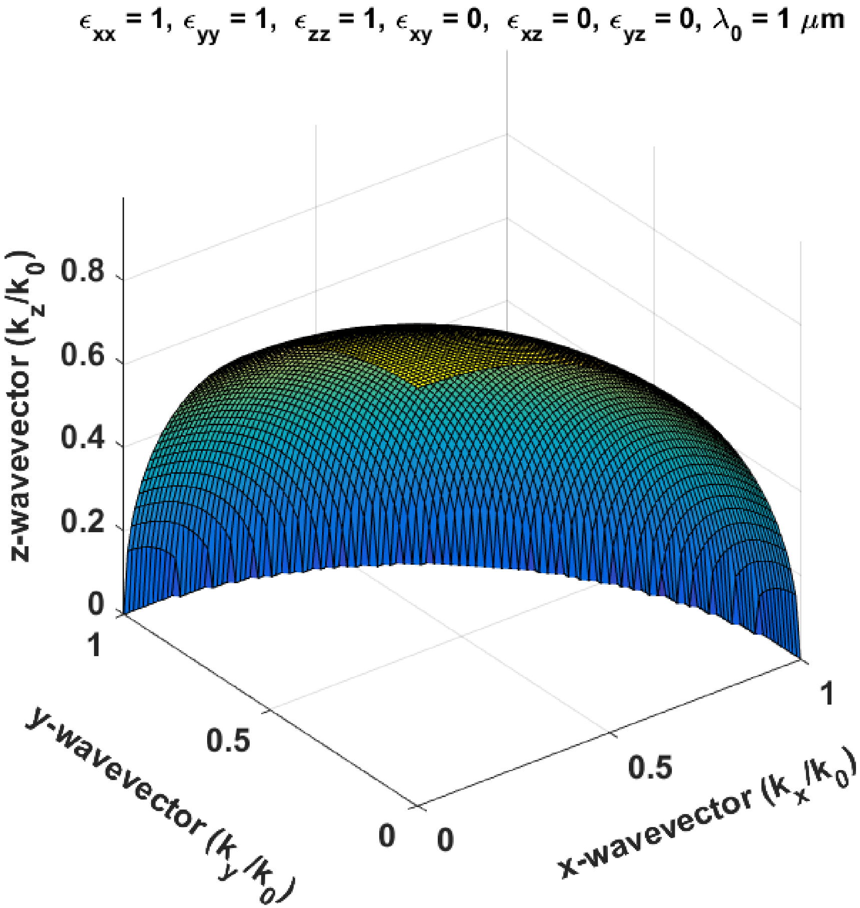


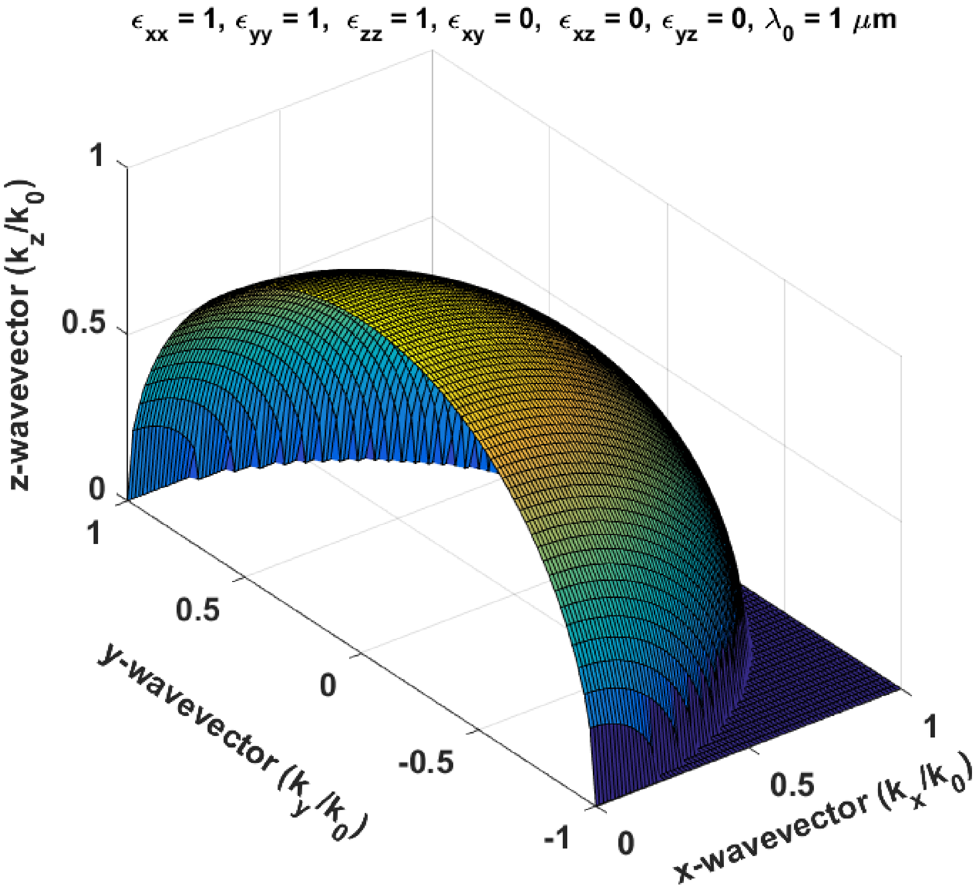
$$\begin{aligned}\vec{k}\times\vec{E} &= \omega\mu_0\vec{H},\\ \vec{k}\times\vec{H} &= -\omega\epsilon_0\varepsilon_r\vec{E},\\ \vec{k}\cdot\vec{E} &= 0,\\ \vec{k}\cdot\vec{H} &= 0.\end{aligned}$$

$$[\vec{k}\cdot\vec{k} - \omega^2\epsilon_0\mu_0 n^2]\vec{E} = 0$$

$$\vec{k}\cdot\vec{k} - \omega^2\epsilon_0\mu_0 n^2 = \vec{k}\cdot\vec{k} - k_0^2 n^2 = 0$$

MAXWELL EQUATIONS
Plane Wave Solutions – Isotropic Case
Wavevector Surfaces (Isotropic)





MAXWELL EQUATIONS

Plane Wave Solutions – Anisotropic Case

$$\begin{aligned} \vec{k} \times \vec{E} &= \omega\mu_0 \vec{H}, \\ \vec{k} \times \vec{H} &= -\omega\epsilon_0 [\tilde{\varepsilon}_r] \vec{E}, \\ \vec{k} \cdot \vec{D} &= \epsilon_0 \vec{k} \cdot [\tilde{\varepsilon}_r] \vec{E} = 0, \\ \vec{k} \cdot \vec{H} &= 0. \end{aligned}$$

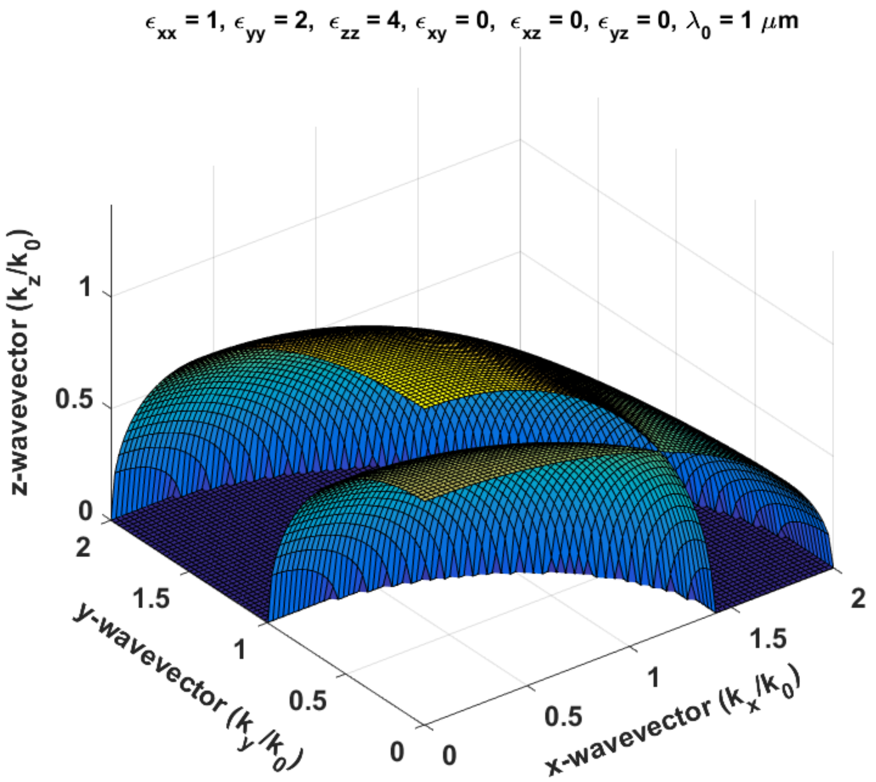
$$\vec{k}(\vec{k} \cdot \vec{E}) - (\vec{k} \cdot \vec{k})\vec{E} = -k_0^2 [\tilde{\varepsilon}_r] \vec{E}$$

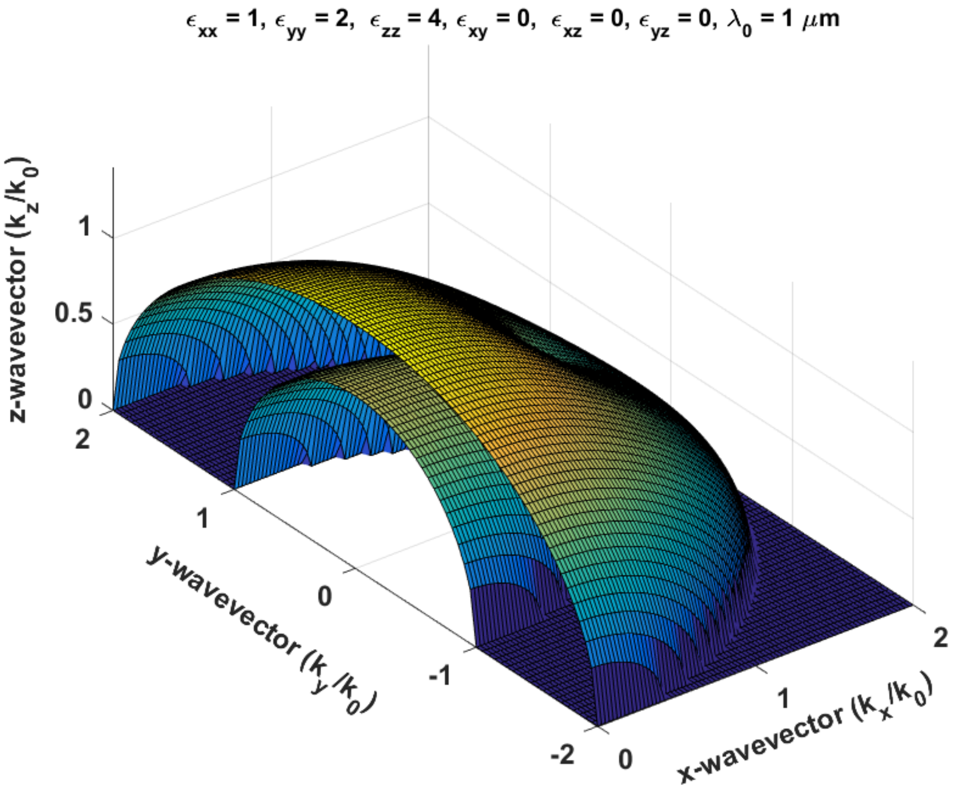
$$\begin{bmatrix} k_0^2 \varepsilon_{r,xx} - (k_y^2 + k_z^2) & k_x k_y + k_0^2 \varepsilon_{r,xy} & k_x k_z + k_0^2 \varepsilon_{r,xz} \\ k_y k_x + k_0^2 \varepsilon_{r,yx} & k_0^2 \varepsilon_{r,yy} - (k_x^2 + k_z^2) & k_y k_z + k_0^2 \varepsilon_{r,yz} \\ k_z k_x + k_0^2 \varepsilon_{r,zx} & k_z k_y + k_0^2 \varepsilon_{r,zy} & k_0^2 \varepsilon_{r,zz} - (k_x^2 + k_y^2) \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$

$$\Longrightarrow [\tilde{A}(k_x, k_y, k_z)] \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = 0$$

$$det[\tilde{A}(k_x, k_y, k_z)] = det\left[k_0^2 [\tilde{\varepsilon}_r] - k^2 \tilde{I} + \vec{k}\vec{k}\right] = 0$$

MAXWELL EQUATIONS
Plane Wave Solutions – Anisotropic Case
Wavevector Surfaces (Biaxial)





MAXWELL EQUATIONS

Plane Wave Solutions – Anisotropic Case

Propagation In a Biaxial Material

$$\vec{k} = k_0 n(a_x\hat{x} + a_y\hat{y} + a_z\hat{z}) = k_0 n(\sin\theta\cos\phi\ \hat{x} + \sin\theta\sin\phi\ \hat{y} + \cos\theta\ \hat{z})$$

$$det\left[\tilde{A}(k_x, k_y, k_z)\right] = det\left[k_0^2\tilde{\varepsilon}_r - k^2\tilde{I} + \vec{k}\vec{k}\right] = 0$$

$$\boxed{An^4 + Bn^2 + C = 0, \qquad \text{where,}}$$

$$\begin{aligned}
A &= a_x^4\varepsilon_{r,xx} + a_y^4\varepsilon_{r,yy} + a_z^4\varepsilon_{r,zz} + a_x^2a_y^2\varepsilon_{r,xx} + a_x^2a_z^2\varepsilon_{r,xx} + a_x^2a_y^2\varepsilon_{r,yy} + a_y^2a_z^2\varepsilon_{r,yy} \\
&\quad a_x^2a_z^2\varepsilon_{r,zz} + a_y^2a_z^2\varepsilon_{r,zz} + 2a_xa_y^3\varepsilon_{r,xy} + 2a_x^3a_y\varepsilon_{r,xy} + 2a_xa_z^3\varepsilon_{r,xz} + 2a_x^3a_z\varepsilon_{r,xz} \\
&\quad +2a_ya_z^3\varepsilon_{r,yz} + 2a_y^3a_z\varepsilon_{r,yz} + 2a_xa_ya_z^2\varepsilon_{r,xy} + 2a_xa_y^2a_z\varepsilon_{r,xz} + 2a_x^2a_ya_z\varepsilon_{r,yz}, \\
B &= -a_x^2\varepsilon_{r,xx}\varepsilon_{r,yy} + a_x^2\varepsilon_{r,xz}^2 + a_y^2\varepsilon_{r,xy}^2 + a_y^2\varepsilon_{r,yz}^2 + a_z^2\varepsilon_{r,xz}^2 + a_z^2\varepsilon_{r,yz}^2 + a_x^2\varepsilon_{r,xy}^2 - a_y^2\varepsilon_{r,xx}\varepsilon_{r,yy} \\
&\quad -a_x^2\varepsilon_{r,xx}\varepsilon_{r,zz} - a_z^2\varepsilon_{r,xx}\varepsilon_{r,zz} - a_y^2\varepsilon_{r,yy}\varepsilon_{r,zz} - a_z^2\varepsilon_{r,yy}\varepsilon_{r,zz} + 2a_xa_y\varepsilon_{r,xz}\varepsilon_{r,yz} - 2a_xa_y\varepsilon_{r,xy}\varepsilon_{r,zz} \\
&\quad +2a_xa_z\varepsilon_{r,xy}\varepsilon_{r,yz} - 2a_xa_z\varepsilon_{r,xz}\varepsilon_{r,yy} + 2a_ya_z\varepsilon_{r,xy}\varepsilon_{r,xz} - 2a_ya_z\varepsilon_{r,xx}\varepsilon_{r,yz}, \\
C &= \varepsilon_{r,xx}\varepsilon_{r,yy}\varepsilon_{r,zz} - \varepsilon_{r,zz}\varepsilon_{r,xy}^2 + 2\varepsilon_{r,xy}\varepsilon_{r,xz}\varepsilon_{r,yz} - \varepsilon_{r,yy}\varepsilon_{r,xz}^2 - \varepsilon_{r,xx}\varepsilon_{r,yz}^2.
\end{aligned}$$

MAXWELL EQUATIONS
Plane Wave Solutions – Anisotropic Case
Propagation In a Biaxial Material

$$a_x^2 n^2 (n^2 - \varepsilon_{r,yy})(n^2 - \varepsilon_{r,zz}) \quad + \quad a_y^2 n^2 (n^2 - \varepsilon_{r,xx})(n^2 - \varepsilon_{r,zz}) + a_z^2 n^2 (n^2 - \varepsilon_{r,xx})(n^2 - \varepsilon_{r,yy})$$

$$= \quad (n^2 - \varepsilon_{r,xx})(n^2 - \varepsilon_{r,yy})(n^2 - \varepsilon_{r,zz})$$

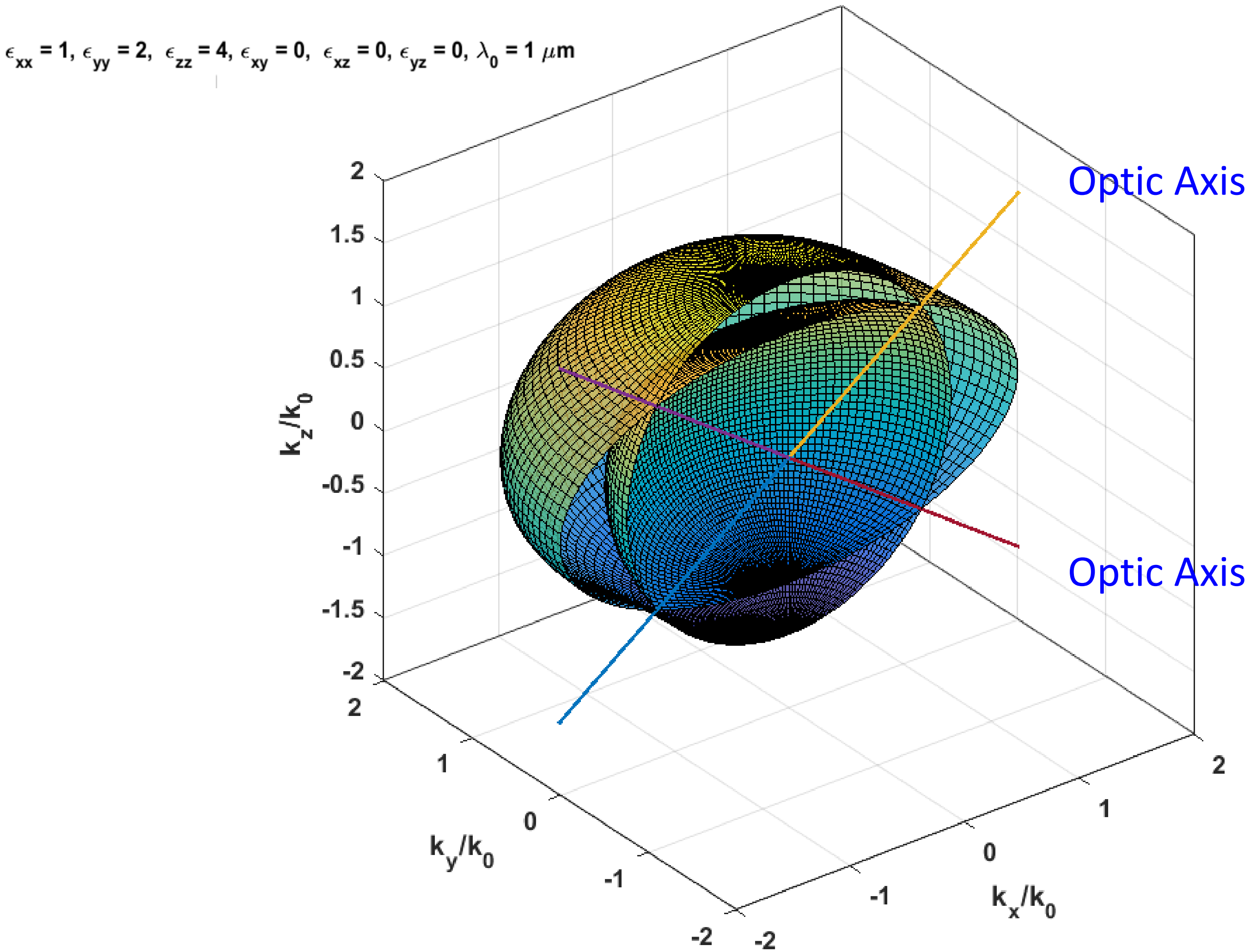
$$(\varepsilon_{r,xy} = \varepsilon_{r,xz} = \varepsilon_{r,yz} = 0)$$

$$\boxed{\frac{a_x^2}{n^2 - \varepsilon_{r,xx}} + \frac{a_y^2}{n^2 - \varepsilon_{r,yy}} + \frac{a_z^2}{n^2 - \varepsilon_{r,zz}} = \frac{1}{n^2}}$$

MAXWELL EQUATIONS

Plane Wave Solutions – Anisotropic Case

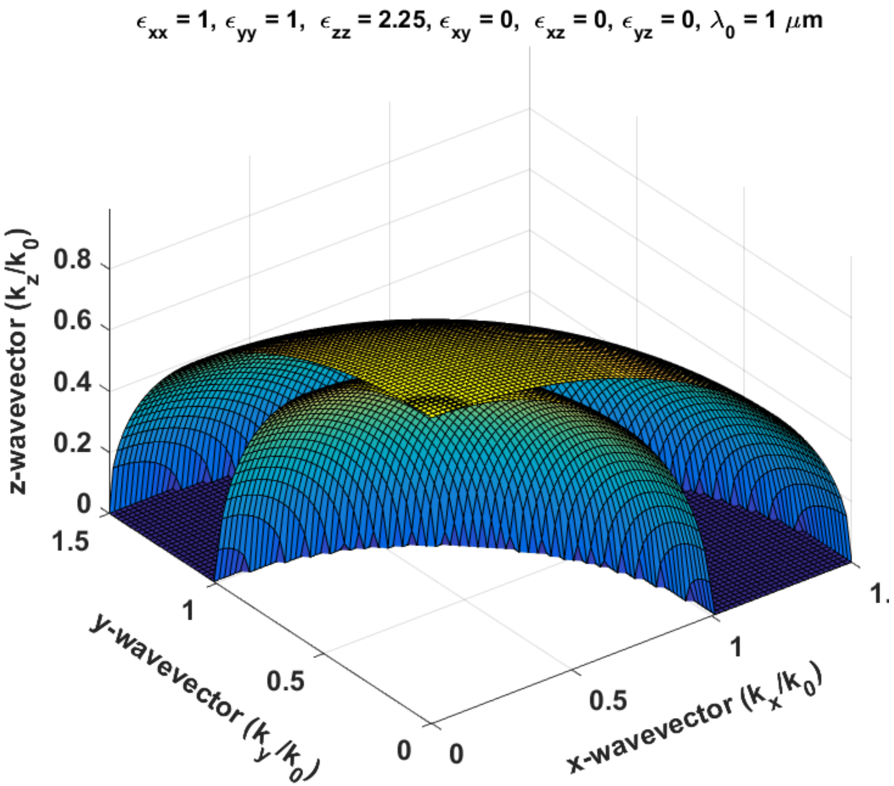
Wavevector Surfaces (Biaxial)

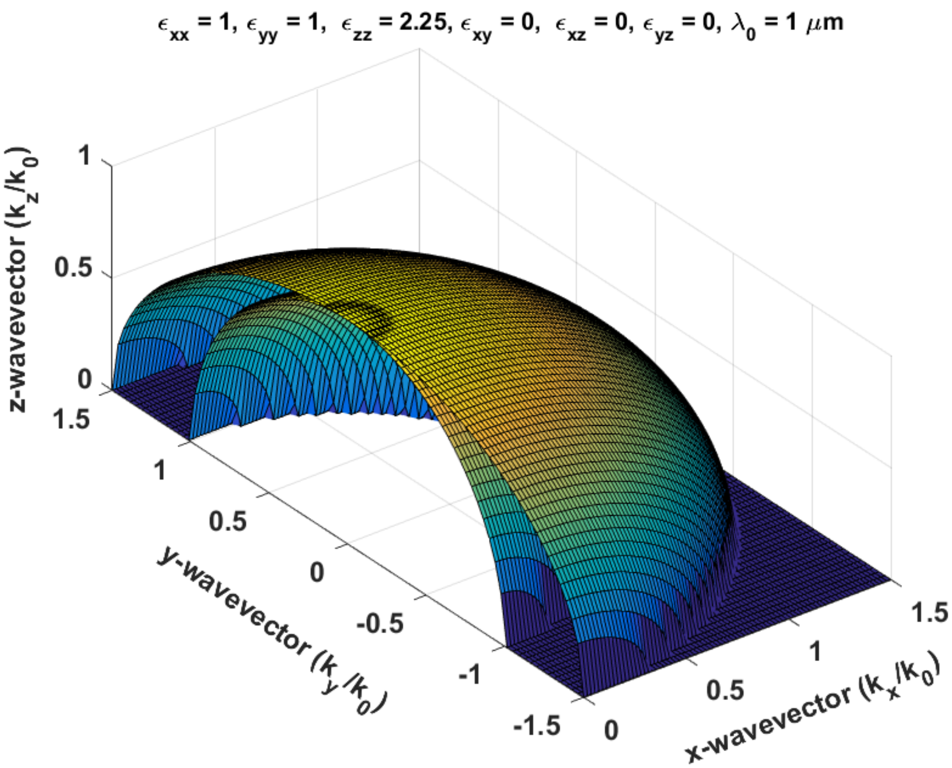


MAXWELL EQUATIONS

Plane Wave Solutions – Uniaxial Anisotropic Case

Wavevector Surfaces (Uniaxial)

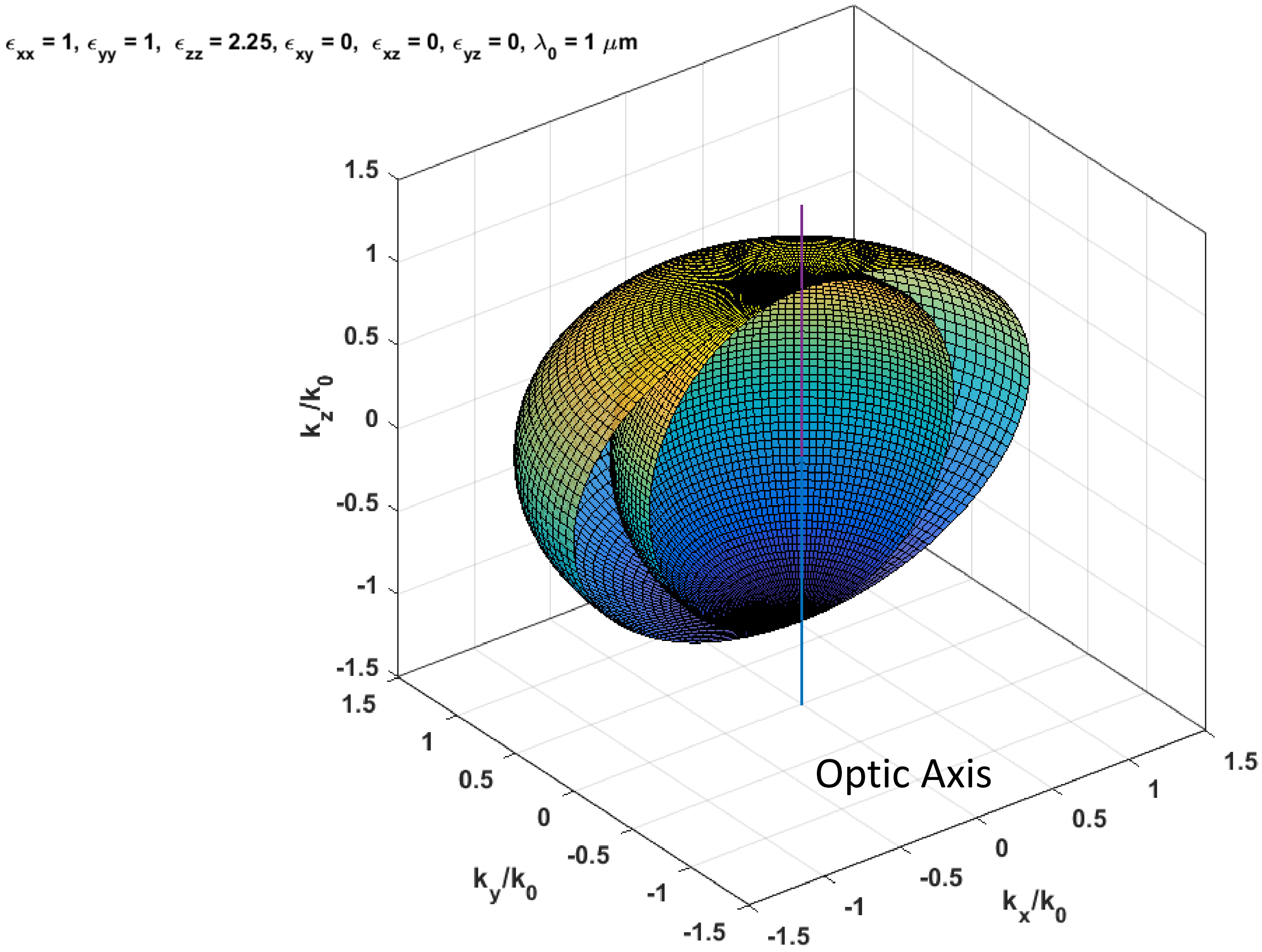




MAXWELL EQUATIONS

Plane Wave Solutions – Uniaxial Anisotropic Case

Wavevector Surfaces (Uniaxial)



MAXWELL EQUATIONS

Plane Wave Solutions – Uniaxial Anisotropic Case

Wavevector Surfaces (Uniaxial)

For principal axis system:

$$\tilde{\epsilon} = \epsilon_0 diag[\varepsilon_O, \varepsilon_O, \varepsilon_E] = \epsilon_0 diag[n_O^2, n_O^2, n_E^2]$$

For optic axis direction:

$$\hat{c} = c_x \hat{x} + c_y \hat{y} + c_z \hat{z}$$

Plane wave solutions:

$$\begin{aligned}
(\vec{k}_t \times \hat{c})E_c + k_c(\hat{c} \times \vec{E}_t) &= \omega\mu_0 \vec{H}_t, \\
\vec{k}_t \times \vec{E}_t &= \omega\mu_0 H_c \hat{c}, \\
(\vec{k}_t \times \hat{c})H_c + k_c(\hat{c} \times \vec{H}_t) &= -\omega\epsilon_0 n_O^2 \vec{E}_t, \\
\vec{k}_t \times \vec{H}_t &= -\omega\epsilon_0 n_E^2 E_c \hat{c}, \\
n_O^2 \vec{k}_t \cdot \vec{E}_t + n_E^2 k_c E_c &= 0, \\
\vec{k}_t \cdot \vec{H}_t + k_c H_c &= 0.
\end{aligned}$$

MAXWELL EQUATIONS
Plane Wave Solutions – Uniaxial Anisotropic Case
Wavevector Surfaces (Uniaxial)

Dispersion Equation:

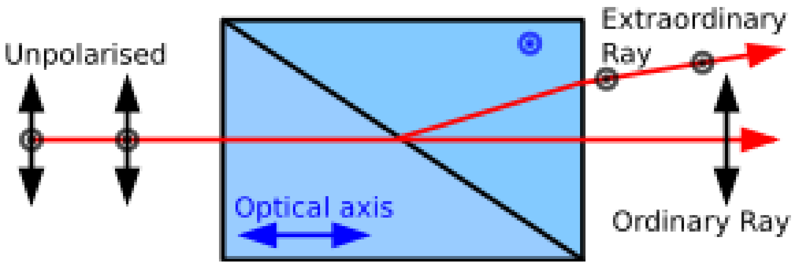
$$[\vec{k}\cdot\vec{k} - k_0^2 n_O^2][n_O^2\vec{k}\cdot\vec{k} + (n_E^2 - n_O^2)(\vec{k}\cdot\hat{c})^2 - k_0^2 n_O^2 n_E^2] = 0$$

$$\boxed{\vec{k}\cdot\vec{k} - k_0^2 n_O^2 = 0 \quad \Longrightarrow \quad \vec{E}\cdot\hat{c} = 0,} \quad \text{ordinary wave,}$$

$$\boxed{n_O^2\vec{k}\cdot\vec{k} + (n_E^2 - n_O^2)(\vec{k}\cdot\hat{c})^2 - k_0^2 n_O^2 n_E^2 = 0, \quad \Longrightarrow \quad \vec{H}\cdot\hat{c} = 0} \quad \text{extraordinary wave.}$$

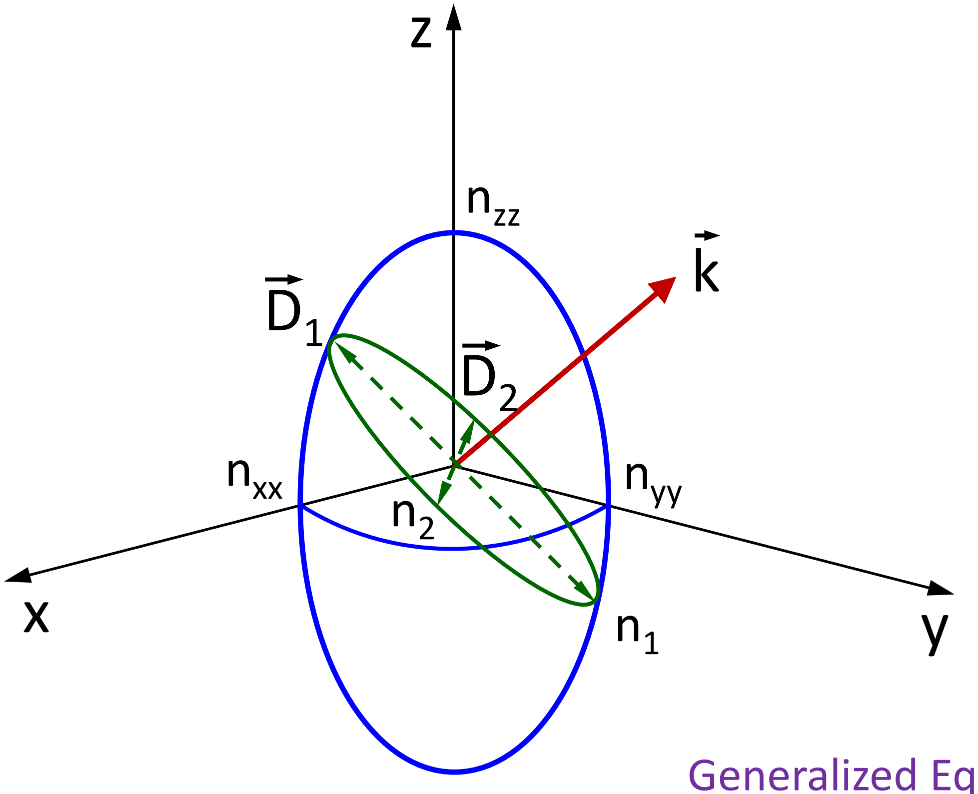
Rochon Prism

for $n_E - n_O > 0$



https://en.wikipedia.org/wiki/Rochon_prism

INDEX ELLIPSOID



$$\frac{x^2}{n_{xx}^2} + \frac{y^2}{n_{yy}^2} + \frac{z^2}{n_{zz}^2} = 1$$

Generalized Equation of Index Ellipsoid:

$$\begin{bmatrix} x & y & z \end{bmatrix}^T [\mathcal{A}] \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix}^T \begin{bmatrix} \frac{1}{n_{xx}^2} & \frac{1}{n_{xy}^2} & \frac{1}{n_{xz}^2} \\ \frac{1}{n_{yx}^2} & \frac{1}{n_{yy}^2} & \frac{1}{n_{yz}^2} \\ \frac{1}{n_{zx}^2} & \frac{1}{n_{zy}^2} & \frac{1}{n_{zz}^2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1$$

POYNTING's THEOREM

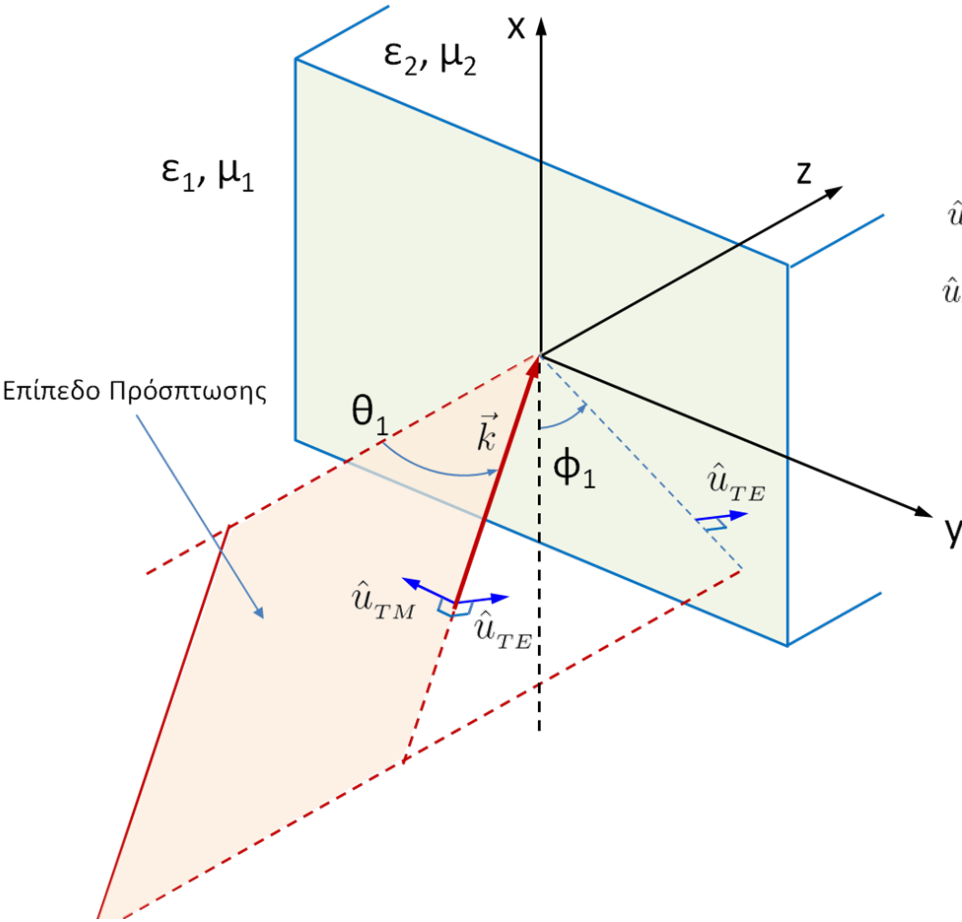
$$-\oint\!\!\!\!\oint_S (\vec{\mathcal{E}} \times \vec{\mathcal{H}}) \cdot d\vec{S} = \iiint_V \vec{\mathcal{E}} \cdot \vec{\mathcal{J}} dV + \iiint_V \frac{\partial}{\partial t}\left(\frac{\epsilon_0}{2}\vec{\mathcal{E}} \cdot \vec{\mathcal{E}} + \frac{\mu_0}{2}\vec{\mathcal{H}} \cdot \vec{\mathcal{H}}\right) dV + \iiint_V \vec{\mathcal{E}} \cdot \frac{\partial \vec{\mathcal{P}}}{\partial t} dV + \iiint_V \mu_0 \vec{\mathcal{H}} \cdot \frac{\partial \vec{\mathcal{M}}}{\partial t} dV,$$

For Dispersionless Isotropic Media

$$w_{em} = w_e + w_m = \frac{1}{2}\left(\vec{\mathcal{E}} \cdot \vec{\mathcal{D}} + \vec{\mathcal{H}} \cdot \vec{\mathcal{B}}\right) = \frac{1}{2}\left(\epsilon|\vec{\mathcal{E}}|^2 + \mu|\vec{\mathcal{H}}|^2\right)$$

$$-\oint\!\!\!\!\oint_S (\vec{\mathcal{E}} \times \vec{\mathcal{H}}) \cdot d\vec{S} = \iiint_V \left[\vec{\mathcal{E}} \cdot \vec{\mathcal{J}} + \frac{\partial w_{em}}{\partial t}\right] dV.$$

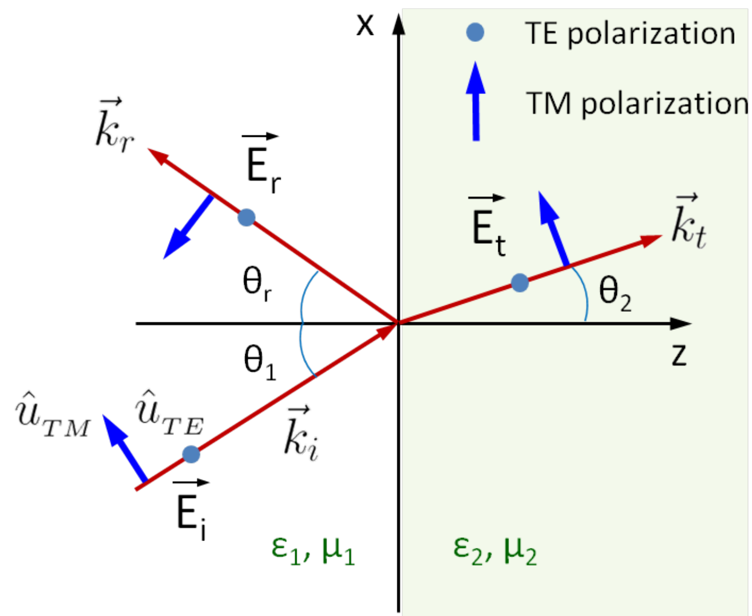
Plane Wave Reflection and Transmission



$$\vec{E} = \vec{E}_0 \ \exp(-j\vec{k}\cdot\vec{r}) = [E_{TE}\hat{u}_{TE} + E_{TM}\hat{u}_{TM}] \ \exp(-j\vec{k}\cdot\vec{r})$$

$$\hat{u}_{TE} = \hat{i}_y$$

$$\hat{u}_{TM} = \cos\theta_1 \ \hat{i}_x - \sin\theta_1 \ \hat{i}_z$$

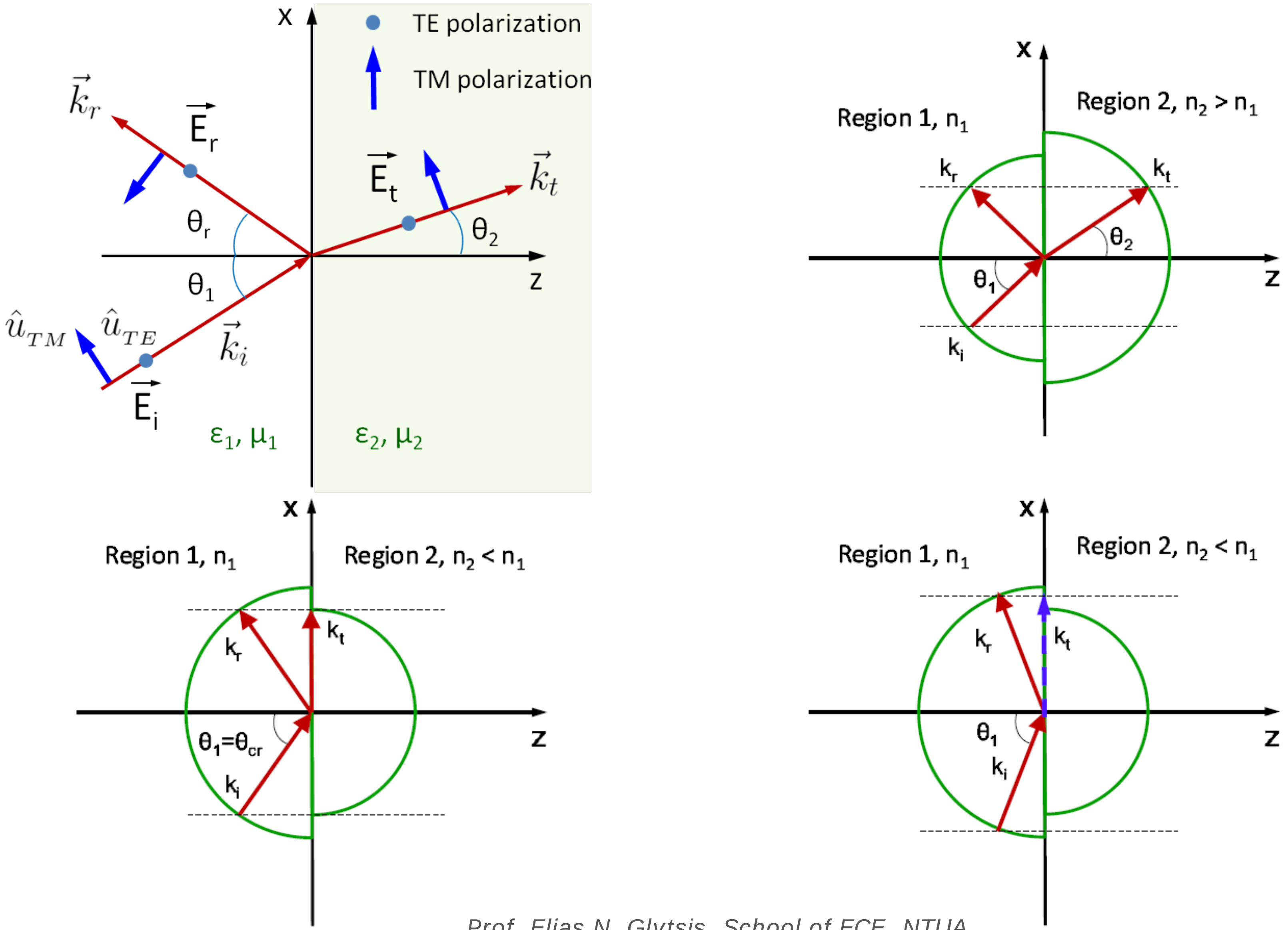


$$\vec{E} = \vec{E}_0 \ \exp(-j\vec{k}\cdot\vec{r}) = [E_{TE}\hat{u}_{TE} + E_{TM}\hat{u}_{TM}] \ \exp(-j\vec{k}\cdot\vec{r})$$

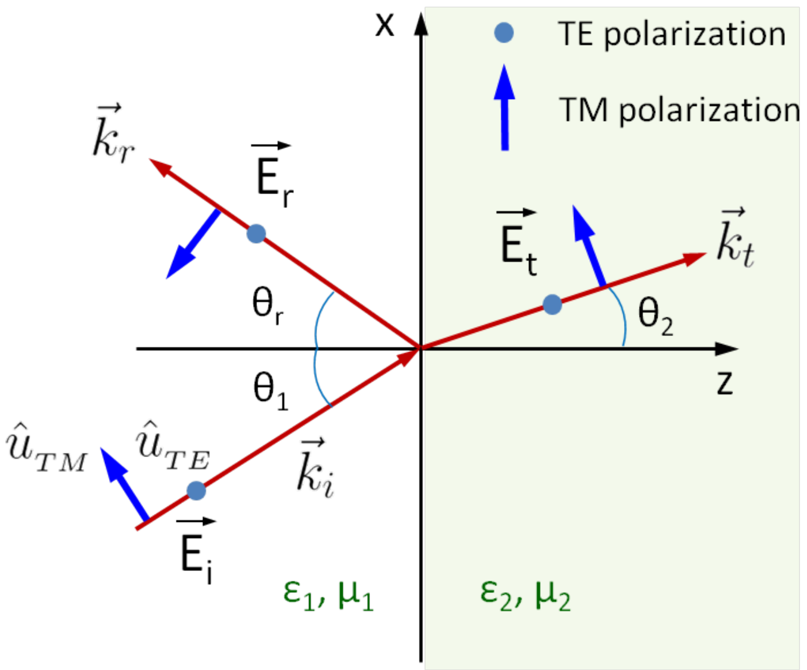
$$\hat{u}_{TE} = \sin\phi_1 \ \hat{i}_x + \cos\phi_1 \ \hat{i}_y$$

$$\hat{u}_{TM} = \cos\phi_1\cos\theta_1 \ \hat{i}_x - \sin\phi_1\cos\theta_1 \ \hat{i}_y - \sin\theta_1 \ \hat{i}_z$$

Planar Interface - Wavevector Diagrams



Fresnel Equations



$$r_{TE} = r_\perp = \frac{E_r}{E_i} = \frac{n_1\cos\theta_1 - n_2\cos\theta_2}{n_1\cos\theta_1 + n_2\cos\theta_2},$$

$$t_{TE} = t_\perp = \frac{E_t}{E_i} = \frac{2n_1\cos\theta_1}{n_1\cos\theta_1 + n_2\cos\theta_2},$$

$$r_{TM} = r_\parallel = \frac{E_r}{E_i} = \frac{n_2\cos\theta_1 - n_1\cos\theta_2}{n_2\cos\theta_1 + n_1\cos\theta_2},$$

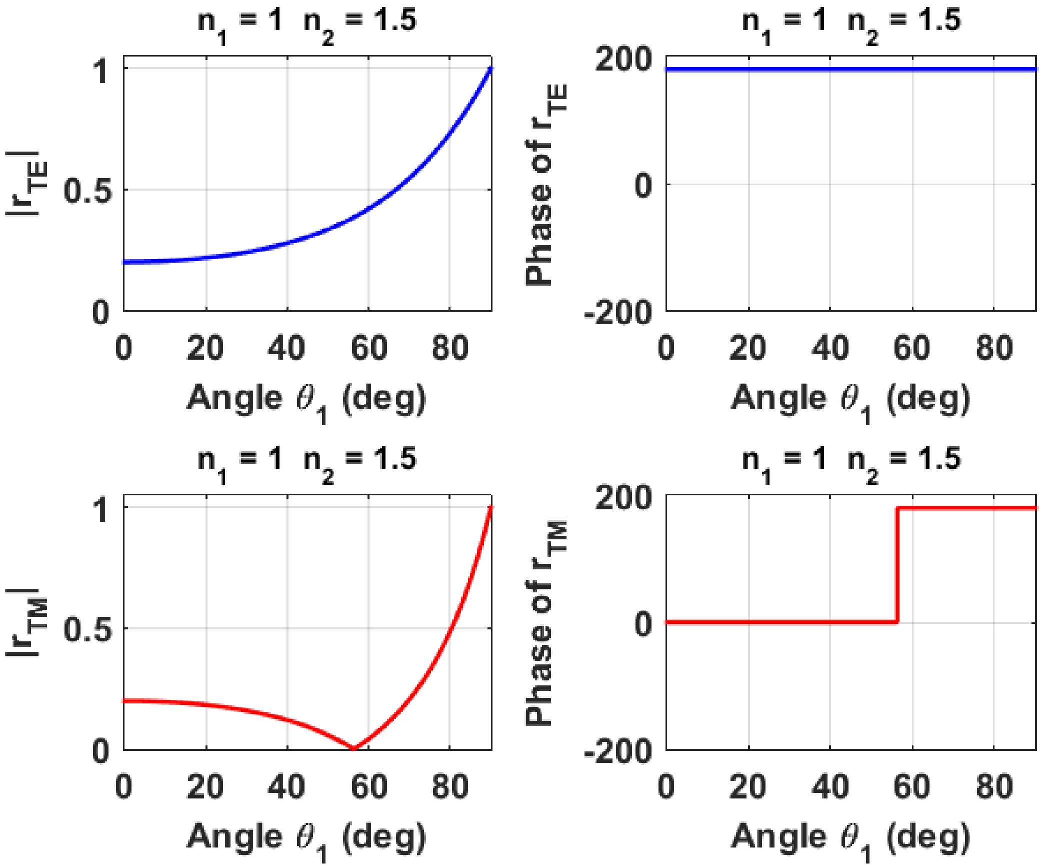
$$t_{TM} = t_\parallel = \frac{E_t}{E_i} = \frac{2n_1\cos\theta_1}{n_2\cos\theta_1 + n_1\cos\theta_2}.$$

$$\frac{P_r}{P_i} + \frac{P_t}{P_i} = |r_{TE}|^2 + |t_{TE}|^2\frac{\mathrm{Re}\{n_2\cos\theta_2\}}{n_1\cos\theta_1} = 1, \qquad \text{TE Polarization}$$

$$\frac{P_r}{P_i} + \frac{P_t}{P_i} = |r_{TM}|^2 + |t_{TM}|^2\frac{\mathrm{Re}\{n_2^*\cos\theta_2\}}{n_1\cos\theta_1} = 1, \qquad \text{TM Polarization}$$

Example of Fresnel Equations

$n_1 = 1,\ n_2 = 1.5$



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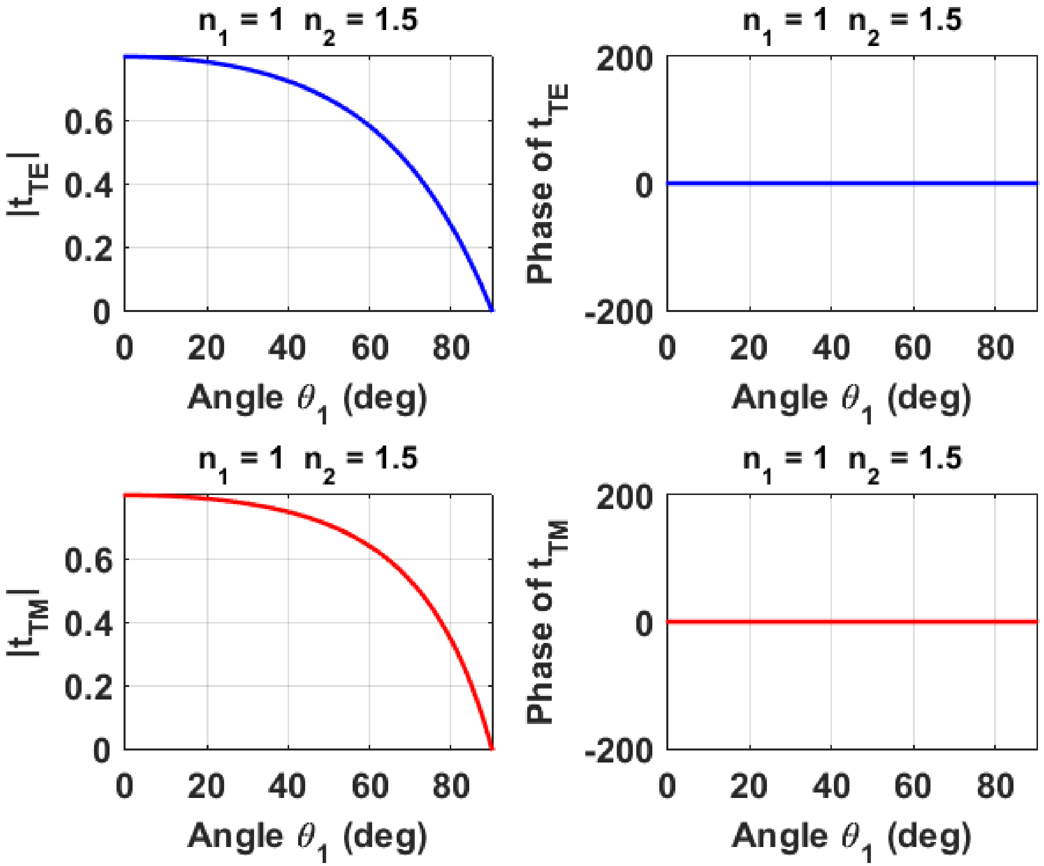

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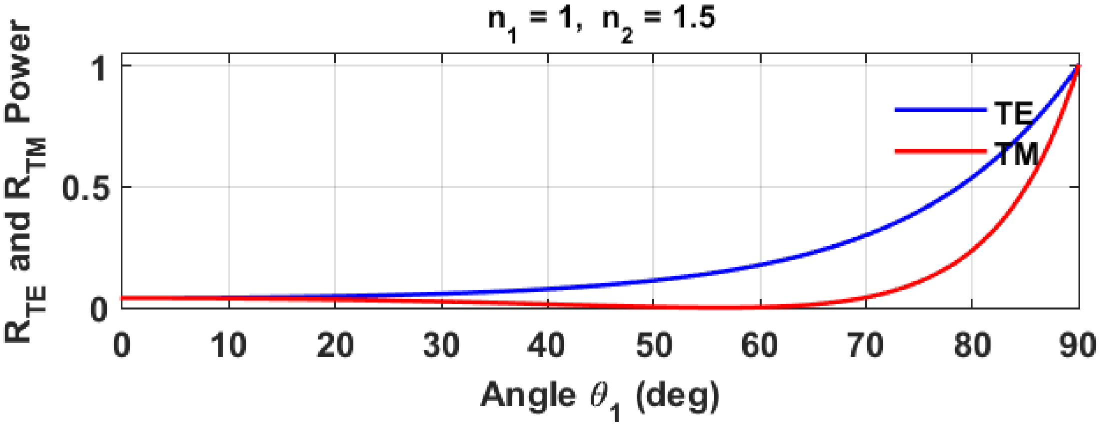
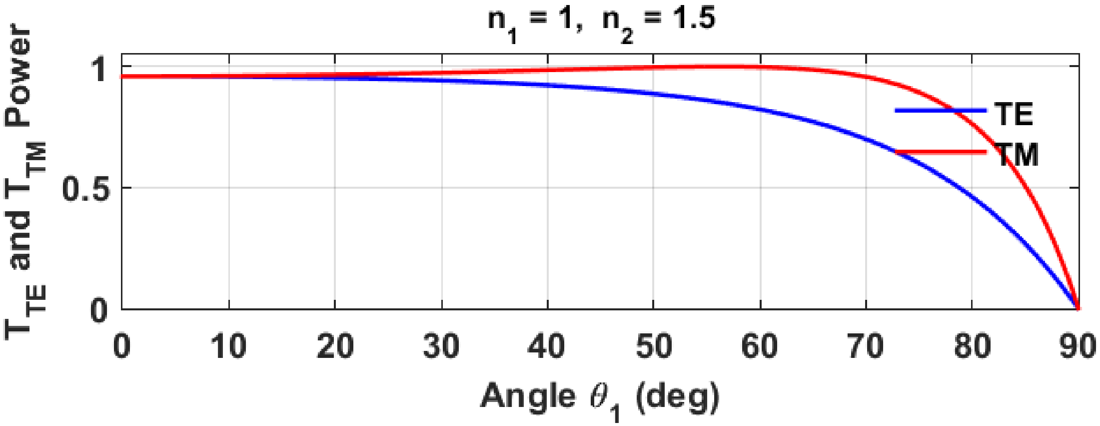
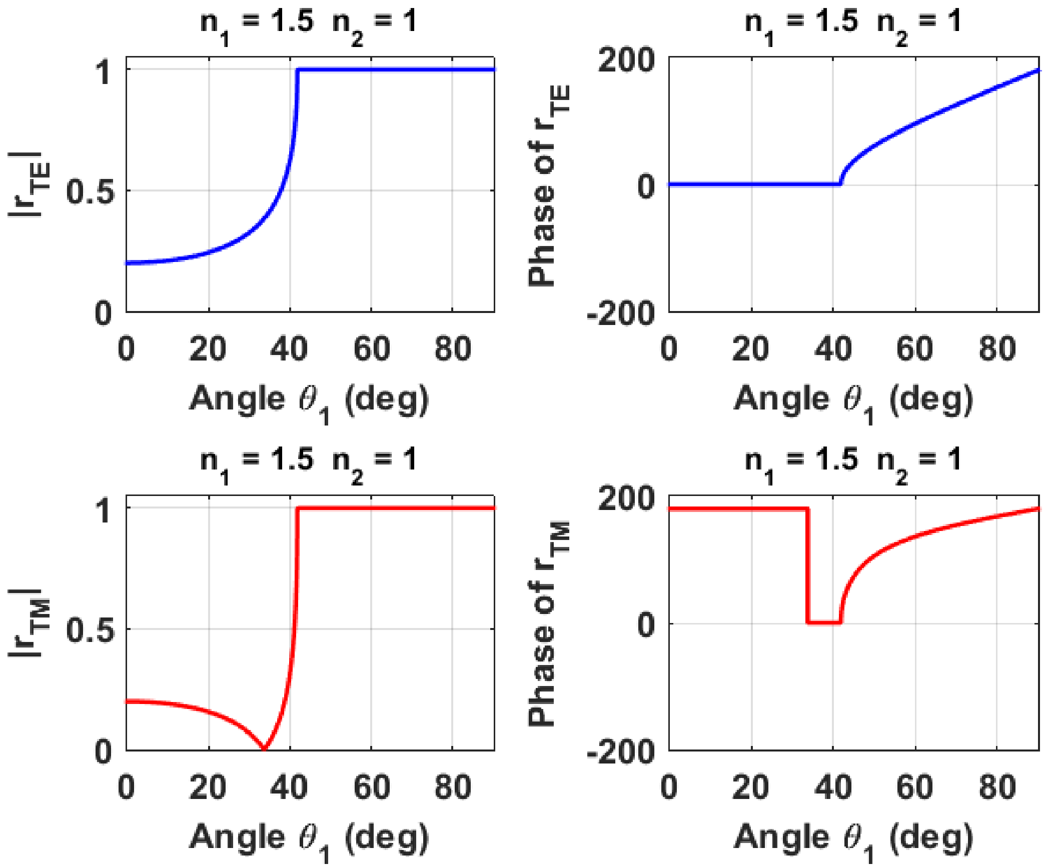




Example of Fresnel Equations

$n_1 = 1.5$, $n_2 = 1$



$$r_{TE} = \frac{E_r}{E_i} = 1 e^{j2\phi_{TE}(\theta_1)} = 1 \exp\left[j2\tan^{-1}\left\{ \frac{\sqrt{n_1^2 \sin^2\theta_1 - n_2^2}}{n_1\cos\theta_1} \right\} \right],$$

$$r_{TM} = \frac{E_r}{E_i} = 1 e^{j2\phi_{TM}(\theta_1)} = 1 \exp\left[j2\tan^{-1}\left\{ \frac{n_1^2}{n_2^2} \frac{\sqrt{n_1^2 \sin^2\theta_1 - n_2^2}}{n_1\cos\theta_1} \right\} \right]$$

Example of Fresnel Equations

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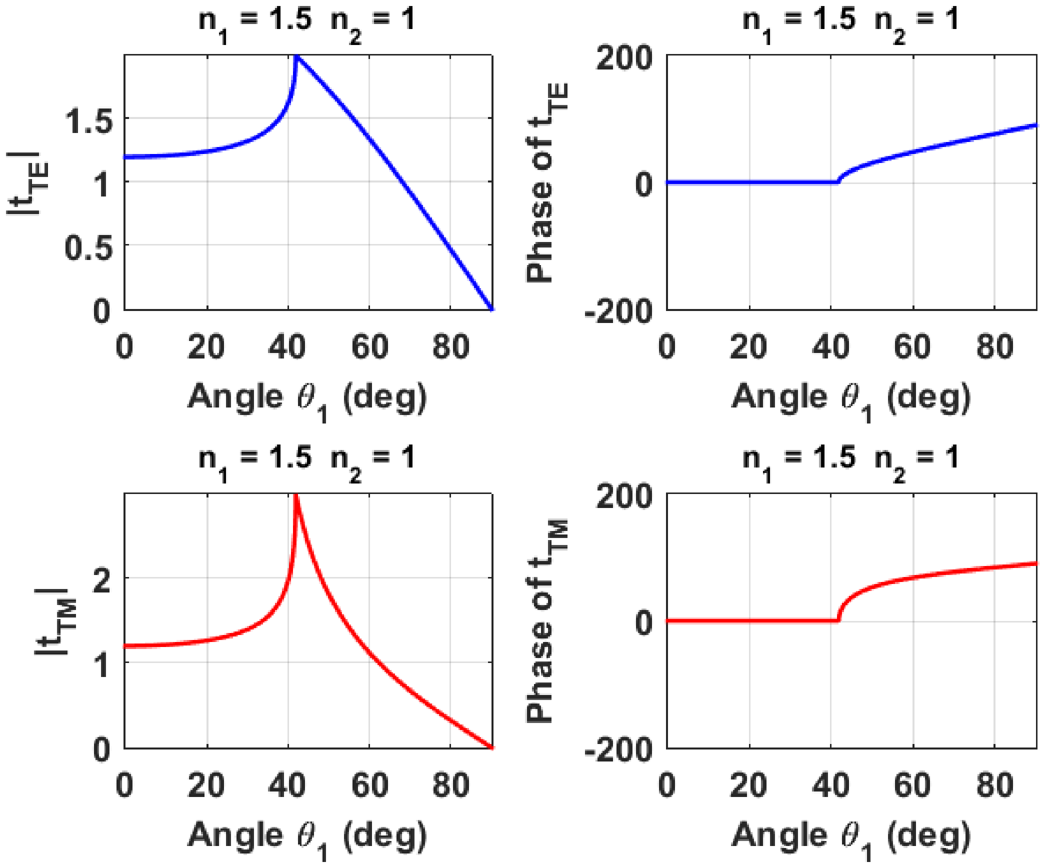


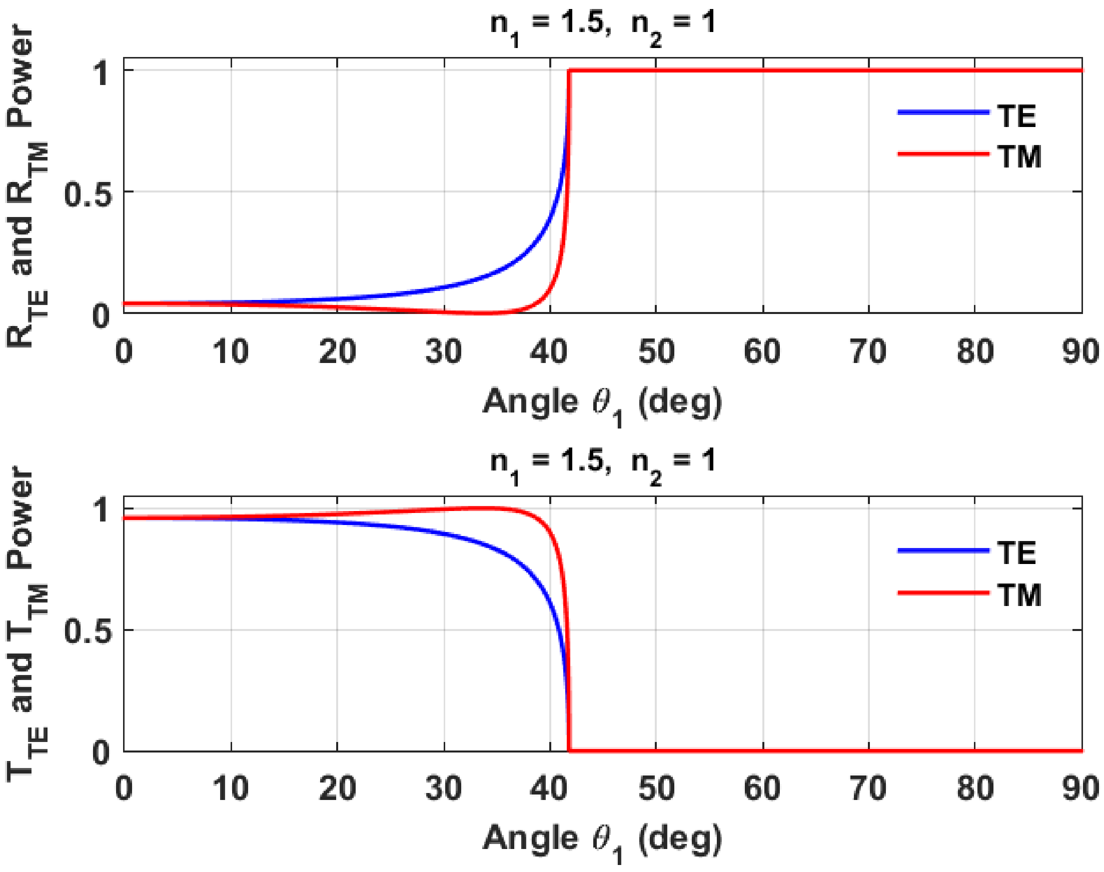
$$t_{TE} = \frac{E_t}{E_i} = |t_{TE}| e^{j\phi_{TE}(\theta_1)} = \frac{2n_1\cos\theta_1}{\sqrt{n_1^2 - n_2^2}} \exp\left[j\tan^{-1}\left\{ \frac{\sqrt{n_1^2\sin^2\theta_1 - n_2^2}}{n_1\cos\theta_1} \right\} \right],$$

$$t_{TM} = \frac{E_t}{E_i} = |t_{TM}| e^{j\phi_{TM}(\theta_1)} =$$

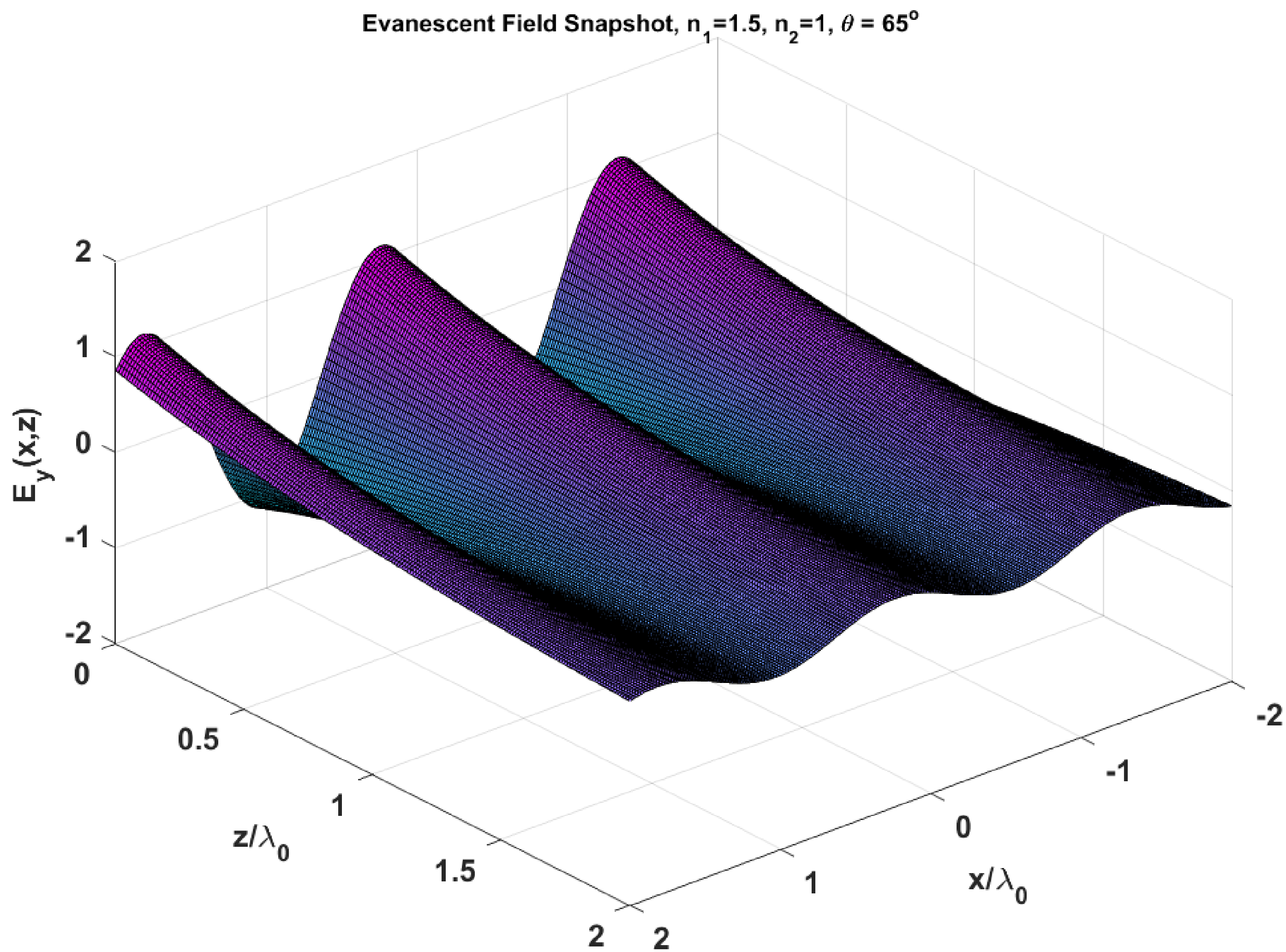
$$\frac{2n_1 n_2\cos\theta_1}{\sqrt{n_2^4\cos^2\theta_1 + n_1^4\sin^2\theta_1 - n_1^2 n_2^2}} \exp\left[j\tan^{-1}\left\{ \frac{n_1^2}{n_2^2}\frac{\sqrt{n_1^2\sin^2\theta_1 - n_2^2}}{n_1\cos\theta_1} \right\} \right]$$

Example of Fresnel Equations

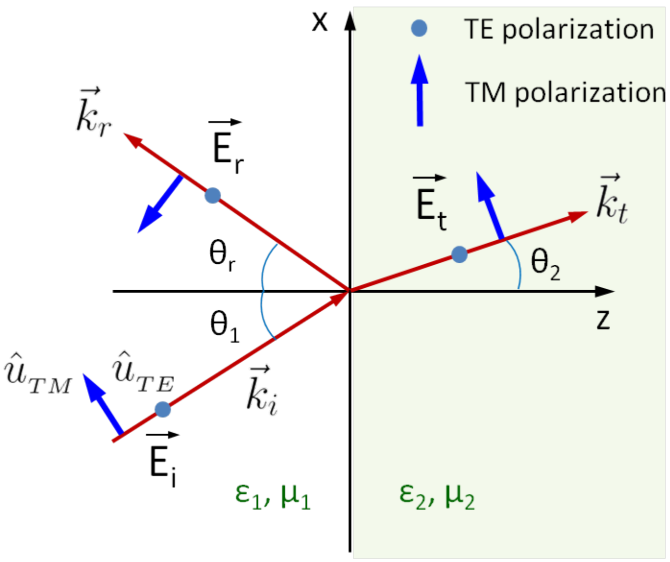
$n_1 = 1.5,\ n_2 = 1$



Example of Evanescent Field



Fresnel Equations Generalization



$$\vec{E}_i = E_{TE}\hat{u}_{TE}\exp(-j\vec{k}_i\cdot\vec{r}) + E_{TM}\hat{u}_{TM}\exp(-j\vec{k}_i\cdot\vec{r}), \quad \text{where}$$

$$\hat{u}_{TE} = \hat{y},$$

$$\hat{u}_{TM} = \cos\theta_1\ \hat{x} - \sin\theta_1\ \hat{z},$$

$$\vec{k}_i = k_0\sqrt{\epsilon_{r1}\mu_{r1}}\left(\sin\theta_1\ \hat{x} + \cos\theta_1\ \hat{z}\right),$$

$$\vec{E}_r = E_{TE}r_{TE}\hat{u}^r_{TE}\exp(-j\vec{k}_r\cdot\vec{r}) + E_{TM}r_{TM}\hat{u}^r_{TM}\exp(-j\vec{k}_r\cdot\vec{r}), \quad \text{where}$$

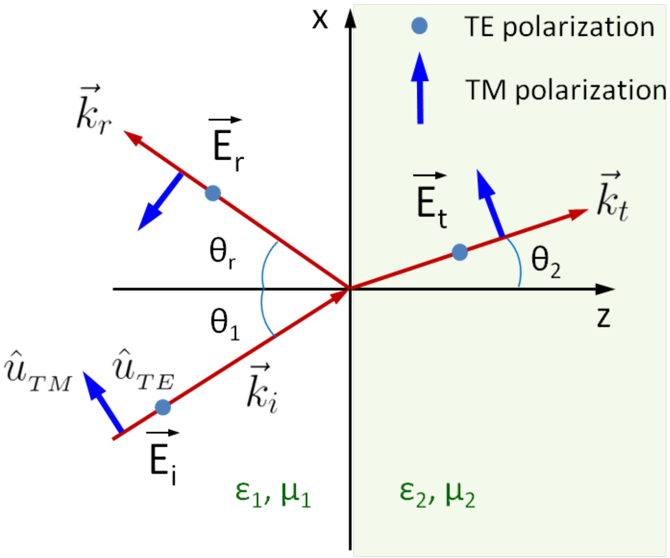
$$\hat{u}^r_{TE} = \hat{u}_{TE} = \hat{y},$$

$$\hat{u}^r_{TM} = -\cos\theta_1\ \hat{x} - \sin\theta_1\ \hat{z},$$

$$\vec{k}_r = k_0\sqrt{\epsilon_{r1}\mu_{r1}}\left(\sin\theta_1\ \hat{x} - \cos\theta_1\ \hat{z}\right),$$

$$\begin{aligned}\vec{E}_1 &= E_{TE}\hat{u}_{TE}\exp(-j\vec{k}_i\cdot\vec{r}) + E_{TM}\hat{u}_{TM}\exp(-j\vec{k}_i\cdot\vec{r}) + \\ &\quad E_{TE}r_{TE}\hat{u}_{TE}\exp(-j\vec{k}_r\cdot\vec{r}) + E_{TM}r_{TM}\hat{u}^r_{TM}\exp(-j\vec{k}_r\cdot\vec{r}) \\ \vec{E}_2 &= t_{TE}E_{TE}\hat{u}_{TE}\exp(-j\vec{k}_t\cdot\vec{r}) + t_{TM}E_{TM}\hat{u}^t_{TM}\exp(-j\vec{k}_t\cdot\vec{r})\end{aligned}$$

Fresnel Equations Generalization



$$r_{TE} = \frac{Z_2\cos\theta_1 - Z_1\cos\theta_2}{Z_2\cos\theta_1 + Z_1\cos\theta_2}$$

$$t_{TE} = \frac{2Z_2\cos\theta_1}{Z_2\cos\theta_1 + Z_1\cos\theta_2}$$

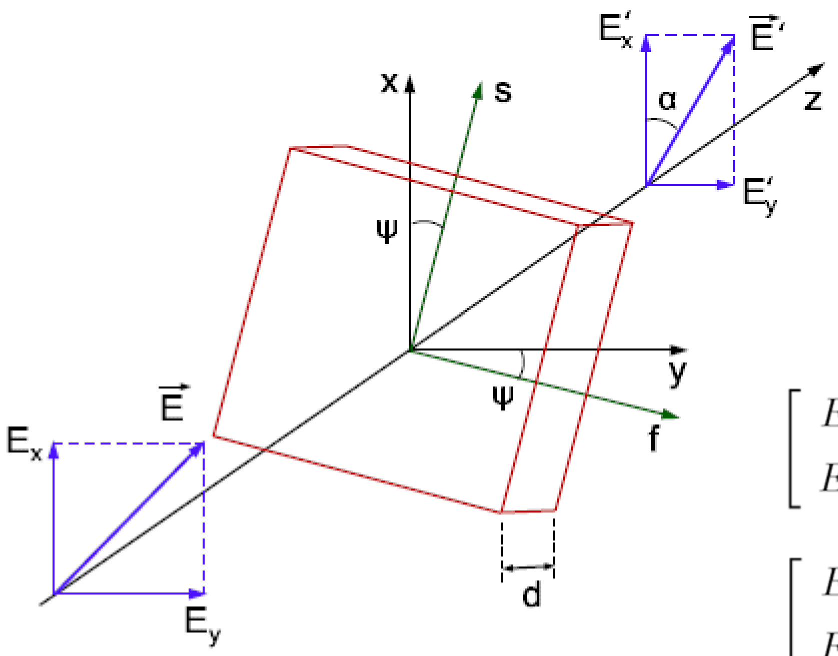
$$r_{TM} = \frac{Z_1\cos\theta_1 - Z_2\cos\theta_2}{Z_1\cos\theta_1 + Z_2\cos\theta_2}$$

$$t_{TM} = \frac{2Z_2\cos\theta_1}{Z_1\cos\theta_1 + Z_2\cos\theta_2}$$

$$\frac{P_r}{P_i} = \frac{|r_{TE}|^2|E_{TE}|^2 + |r_{TM}|^2|E_{TM}|^2}{|E_{TE}|^2 + |E_{TM}|^2}$$

$$\frac{P_t}{P_i} = \frac{Z_1}{\cos\theta_1}\,\frac{|t_{TE}|^2|E_{TE}|^2\mathrm{Re}\{(\cos\theta_2)^*/Z_2^*)\} + |t_{TM}|^2|E_{TM}|^2\mathrm{Re}\{\cos\theta_2/Z_2^*)\}}{|E_{TE}|^2 + |E_{TM}|^2}$$

JONES CALCULUS



Input Wave

$$\vec{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$\begin{bmatrix} E_s \\ E_f \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \mathcal{R}(\psi) \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$\begin{bmatrix} E'_s \\ E'_f \end{bmatrix} = \begin{bmatrix} \exp(-jk_0 n_s d) & 0 \\ 0 & \exp(-jk_0 n_f d) \end{bmatrix} \begin{bmatrix} E_s \\ E_f \end{bmatrix}$$

$$\Gamma = k_0(n_s - n_f)d \qquad \Phi = (1/2)k_0(n_s + n_f)d$$

$$\begin{bmatrix} E'_s \\ E'_f \end{bmatrix} = \exp(-j\Phi) \begin{bmatrix} \exp(-j\Gamma/2) & 0 \\ 0 & \exp(+j\Gamma/2) \end{bmatrix} \begin{bmatrix} E_s \\ E_f \end{bmatrix}$$

JONES CALCULUS

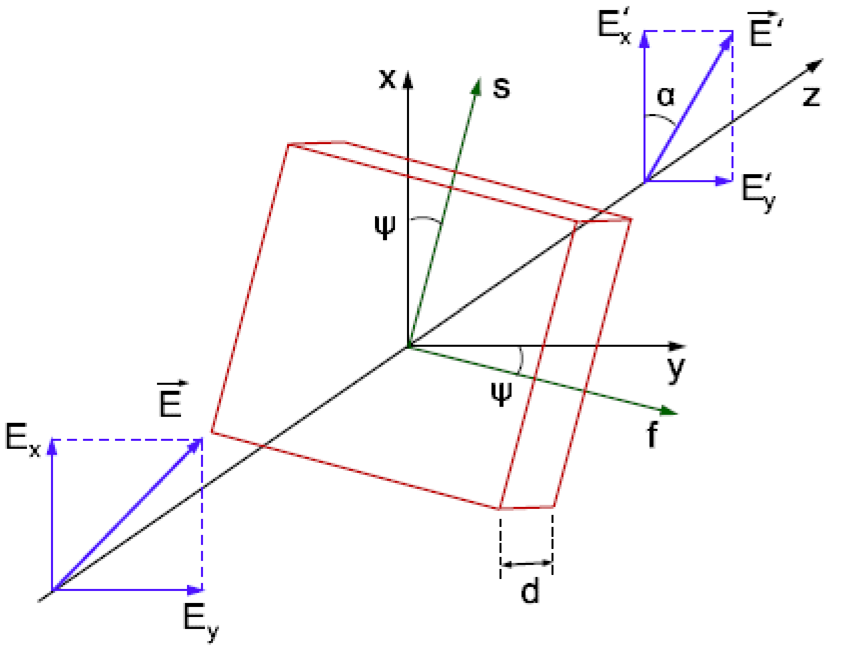
$$\begin{bmatrix} E'_x \\ E'_y \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} E'_s \\ E'_f \end{bmatrix} = \mathcal{R}(-\psi) \begin{bmatrix} E'_s \\ E'_f \end{bmatrix}$$

$$\begin{bmatrix} E'_x \\ E'_y \end{bmatrix} = \mathcal{R}(-\psi)\exp(-j\Phi) \begin{bmatrix} \exp(-j\Gamma/2) & 0 \\ 0 & \exp(+j\Gamma/2) \end{bmatrix} \mathcal{R}(\psi) \begin{bmatrix} E_x \\ E_y \end{bmatrix} = W(\psi,\Gamma) \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$W(\psi,\Gamma) = e^{-j\Phi} \begin{bmatrix} e^{-j\frac{\Gamma}{2}}\cos^2\psi + e^{+j\frac{\Gamma}{2}}\sin^2\psi & -j\sin\frac{\Gamma}{2}\sin 2\psi \\ -j\sin\frac{\Gamma}{2}\sin 2\psi & e^{-j\frac{\Gamma}{2}}\sin^2\psi + e^{+j\frac{\Gamma}{2}}\cos^2\psi \end{bmatrix}$$

JONES CALCULUS

Example 1



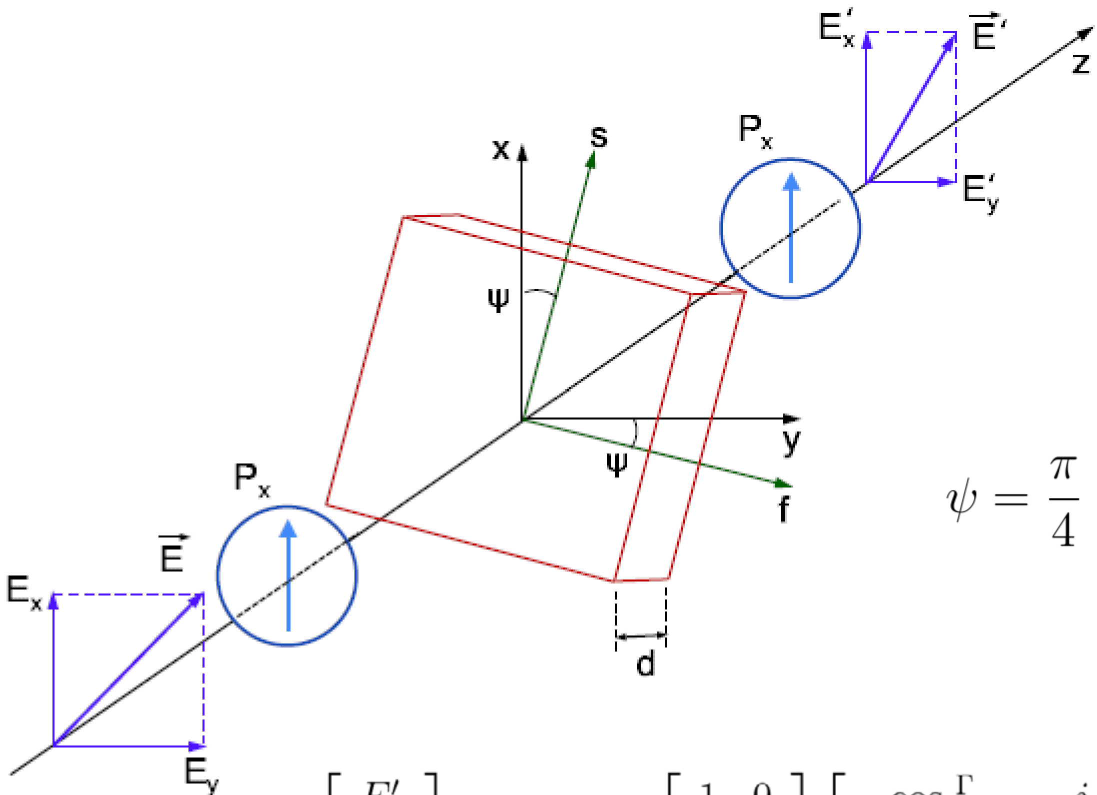
Input Wave

$$\vec{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} 0 \\ E_0 \end{bmatrix}$$

$$W(\Gamma = \pi, \psi = \frac{\pi}{4}) = e^{-j\Phi} \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix}$$

$$\begin{bmatrix} E'_x \\ E'_y \end{bmatrix} = e^{-j\Phi} \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix} \begin{bmatrix} 0 \\ E_0 \end{bmatrix} = e^{-j\Phi} \begin{bmatrix} -jE_0 \\ 0 \end{bmatrix} = -je^{-j\Phi} \begin{bmatrix} E_0 \\ 0 \end{bmatrix}$$

JONES CALCULUS
Example 2



Polarizer

$$P_x = e^{-j\Theta}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\psi = \frac{\pi}{4}$$

$$\begin{bmatrix} E_x' \\ E_y' \end{bmatrix} = e^{-j(\Phi+\Theta)}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \cos\frac{\Gamma}{2} & -j\sin\frac{\Gamma}{2} \\ -j\sin\frac{\Gamma}{2} & \cos\frac{\Gamma}{2} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = e^{-j(\Phi+\Theta)}\begin{bmatrix} \cos\frac{\Gamma}{2} \\ 0 \end{bmatrix}$$

$$\boxed{\frac{I_{out}}{I_{in}} = \frac{|E_x'|^2 + |E_y'|^2}{|E_x|^2 + |E_y|^2} = \cos^2\frac{\Gamma}{2}}$$