28/11/2023

Blackbody Radiation

Electro-Optics & Applications

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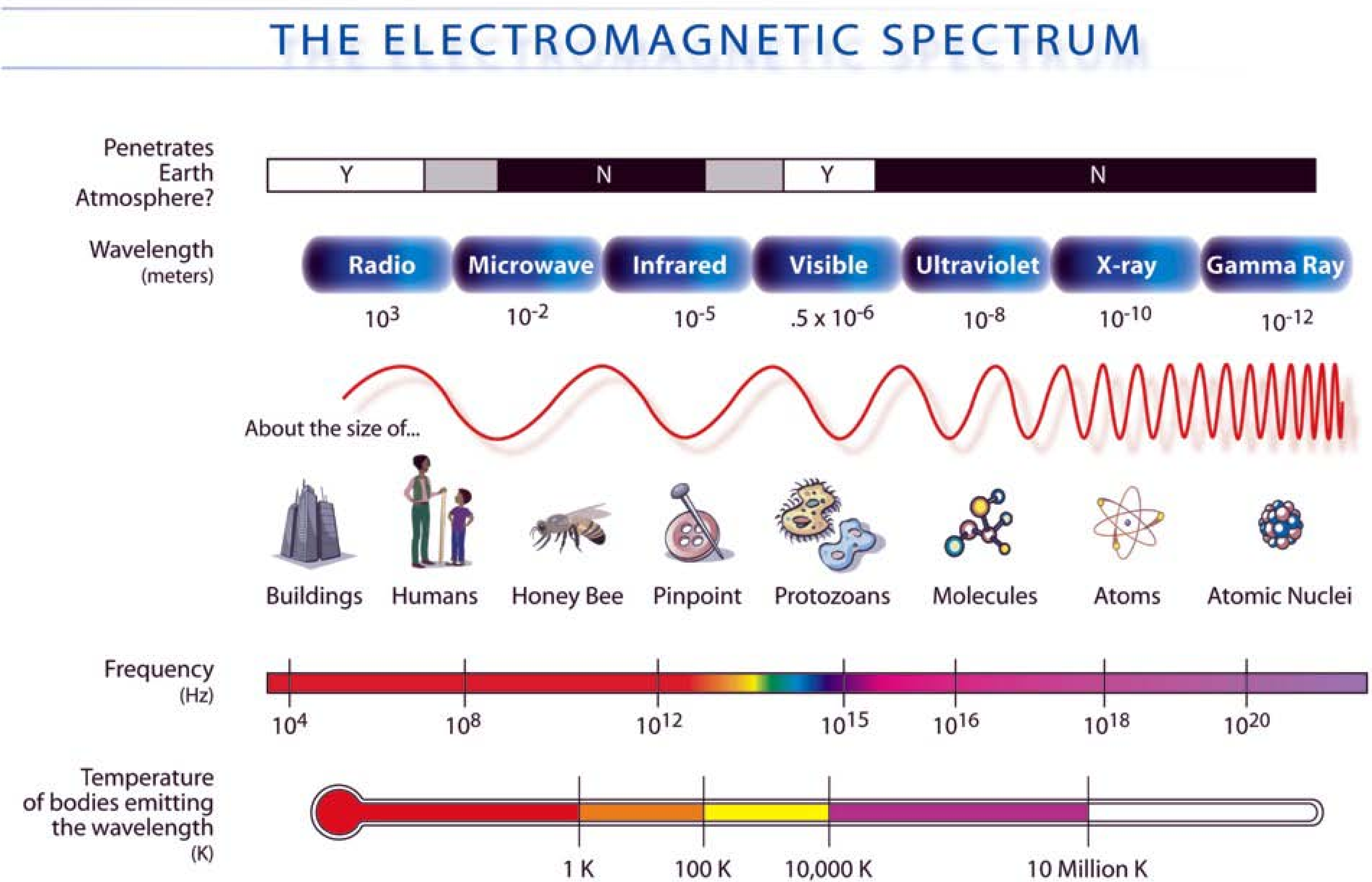
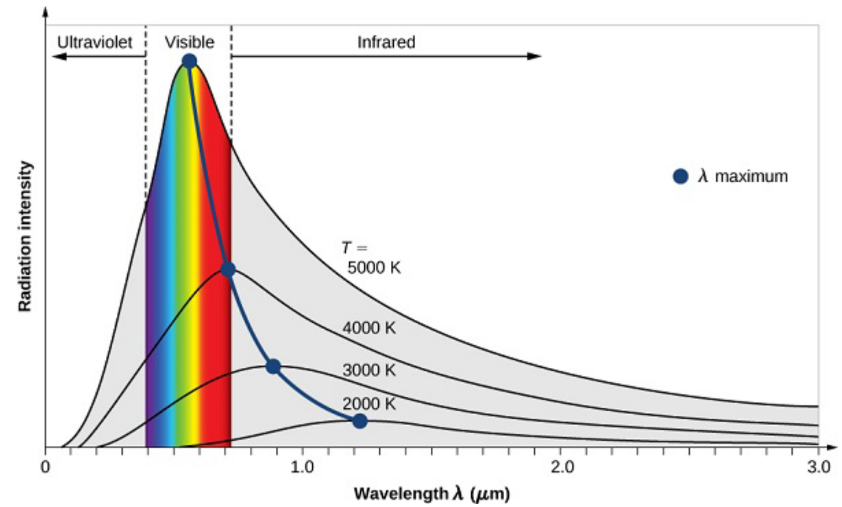
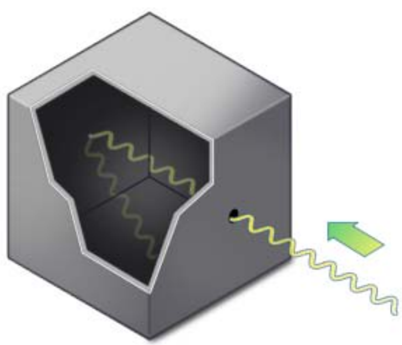

THE ELECTROMAGNETIC SPECTRUM
Penetrates Earth Atmosphere?
Y
N
Y
N
Wavelength (meters)
Radio
Microwave
Infrared
Visible
Ultraviolet
X-ray
Gamma Ray
10^3
10^{-2}
10^{-5}
$.5 \times 10^{-6}$
10^{-8}
10^{-10}
10^{-12}
About the size of...
Buildings
Humans
Honey Bee
Pinpoint
Protozoans
Molecules
Atoms
Atomic Nuclei
Frequency (Hz)
10^4
10^8
10^{12}
10^{15}
10^{16}
10^{18}
10^{20}
Temperature of bodies emitting the wavelength (K)
1 K
100 K
10,000 K
10 Million K

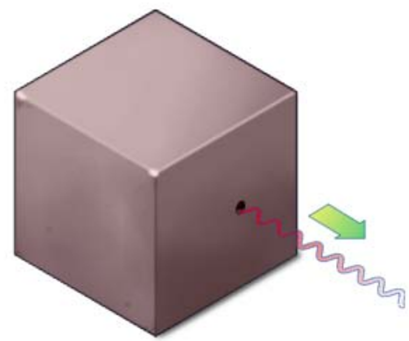
Blackbody Radiation



Blackbody Radiator



All incident radiation is absorbed



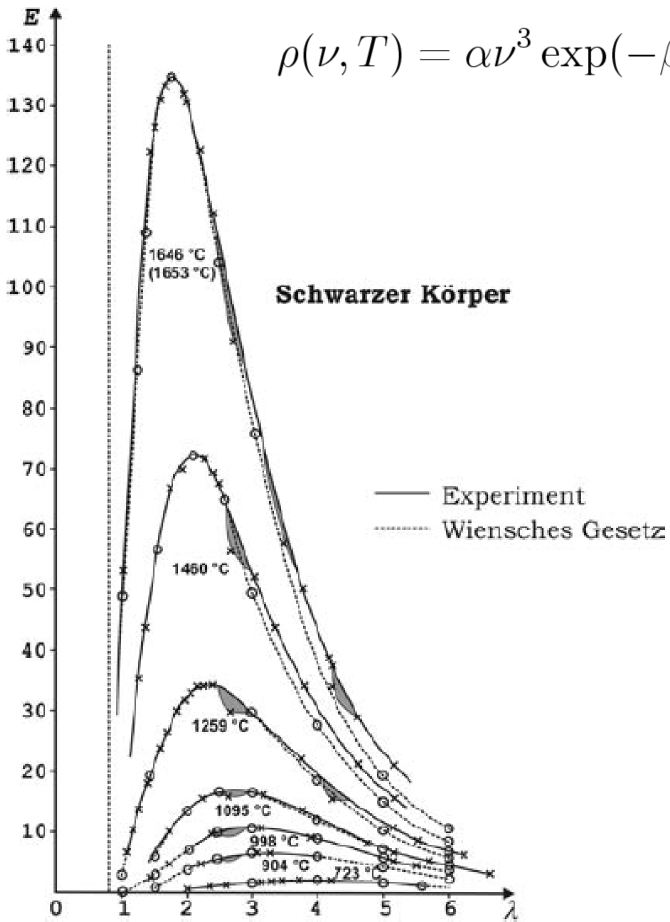
Emitted radiation is only a function of radiator's temperature (T)

https://chem.libretexts.org/Bookshelves/Physical_and_Theoretical_Chemistry_Textbook_Maps/Map%3A_Physical_Chemistry_(McQuarrie_and_Simon)/01%3A_The_Dawn_of_the_Quantum_Theory/1.01%3A_Blackbody_Radiation_Cannot_Be_Explained_Classically

Blackbody Radiation Measurements around 1900

Wien’s Formula

$$\rho(\nu, T) = \alpha \nu^3 \exp(-\beta \nu / T)$$



Spectrum of the thermal radiation emitted by a blackbody, measured by Lummer and Pringsheim in 1900 and compared to Wien’s radiation law

http://users.df.uba.ar/dmitnik/fisica4/articulos/cuantica/Lummer.pdf

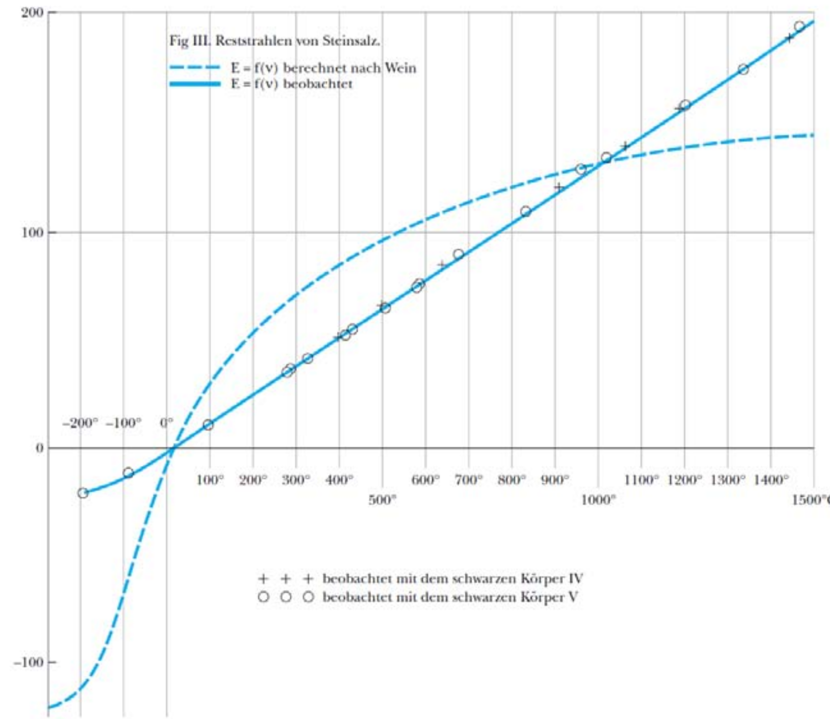
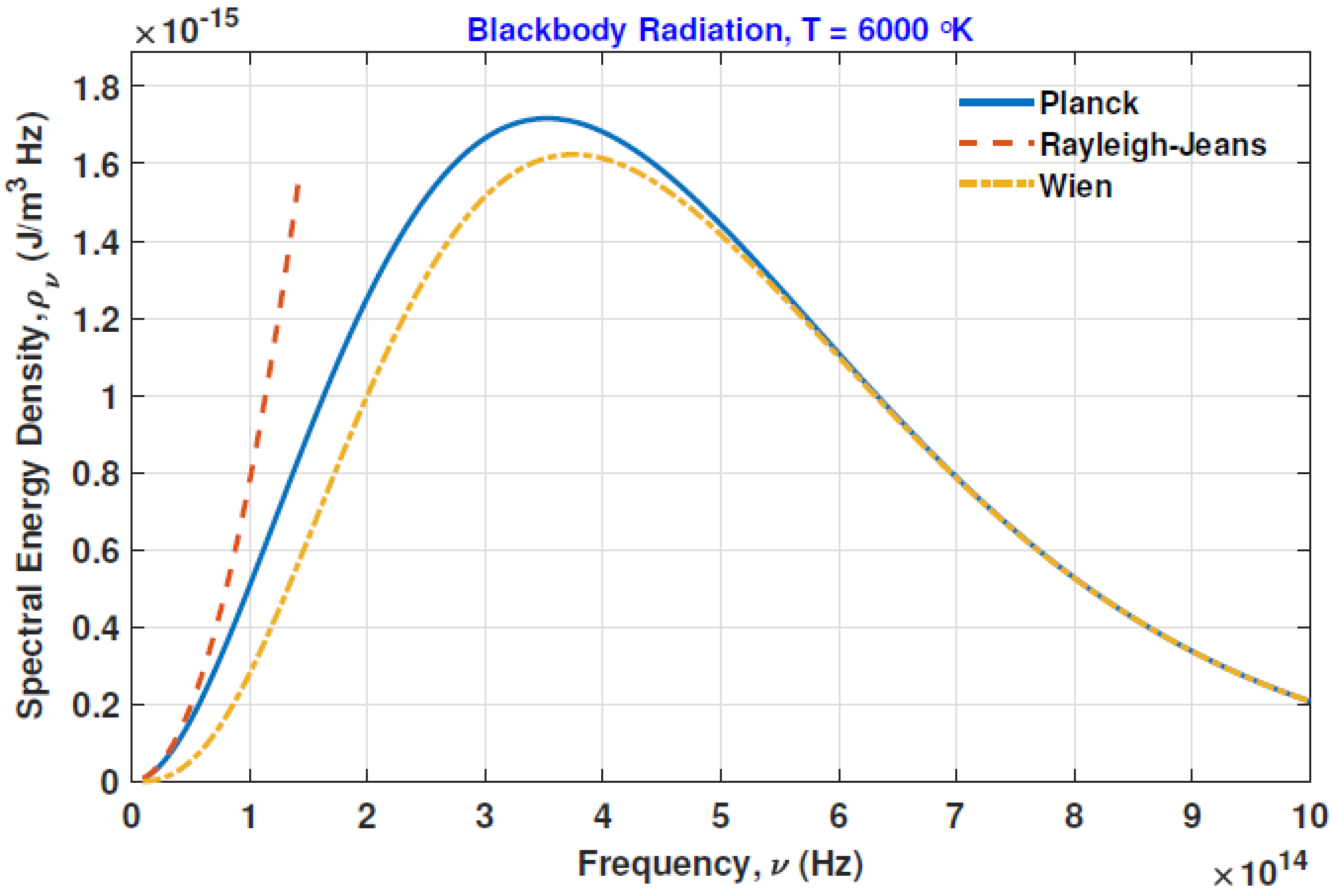


Figure 3.7 Comparison of theoretical and experimental blackbody emission curves at 51.2 μm and over the temperature range of −188° to 1500°C. The title of this modified figure is “Residual Rays from Rocksalt.” *Berechnet nach* means “calculated according to,” and *beobachtet* means “observed.” The vertical axis is emission intensity in arbitrary units. (*From H. Rubens and S. Kurlbaum,* Ann. Physik, *4:649, 1901.*)

https://physlab.lums.edu.pk/images/e/e5/Phys_ref2.pdf

Blackbody Radiation



Electromagnetic Cavity Modes

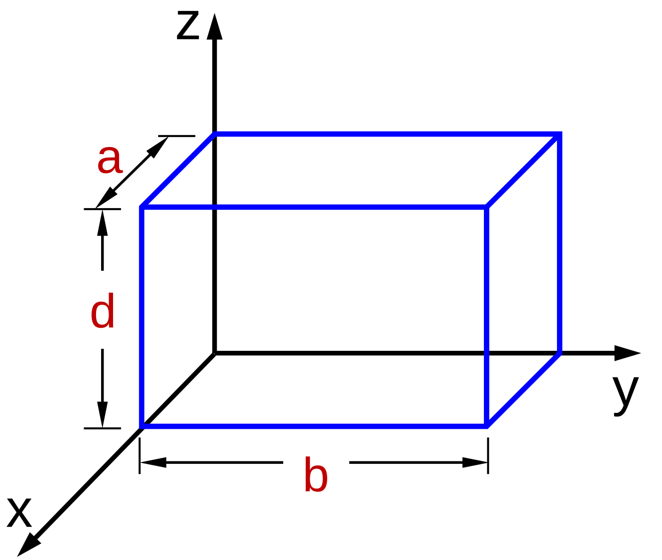
TE_{mpq} Modes

$$
\begin{aligned}
E_x &= C\frac{j\omega\mu_0}{k_c^2}\left(\frac{p\pi}{b}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{p\pi}{b}y\right)\sin\left(\frac{q\pi}{d}z\right),\\
E_y &= -C\frac{j\omega\mu_0}{k_c^2}\left(\frac{m\pi}{a}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{p\pi}{b}y\right)\sin\left(\frac{q\pi}{d}z\right),\\
E_z &= 0,\\
H_x &= -C\frac{1}{k_c^2}\left(\frac{m\pi}{a}\right)\left(\frac{q\pi}{d}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{p\pi}{b}y\right)\cos\left(\frac{q\pi}{d}z\right),\\
H_y &= -C\frac{1}{k_c^2}\left(\frac{p\pi}{b}\right)\left(\frac{q\pi}{d}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{p\pi}{b}y\right)\cos\left(\frac{q\pi}{d}z\right),\\
H_z &= C\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{p\pi}{b}y\right)\sin\left(\frac{q\pi}{d}z\right).
\end{aligned}
$$

TM_{mpq} Modes

$$
\begin{aligned}
E_x &= -D\frac{1}{k_c^2}\left(\frac{m\pi}{a}\right)\left(\frac{q\pi}{d}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{p\pi}{b}y\right)\sin\left(\frac{q\pi}{d}z\right),\\
E_y &= -D\frac{1}{k_c^2}\left(\frac{p\pi}{b}\right)\left(\frac{q\pi}{d}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{p\pi}{b}y\right)\sin\left(\frac{q\pi}{d}z\right),\\
E_z &= D\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{p\pi}{b}y\right)\cos\left(\frac{q\pi}{d}z\right),\\
H_x &= D\frac{j\omega\epsilon_0 n^2}{k_c^2}\left(\frac{p\pi}{b}\right)\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{p\pi}{b}y\right)\cos\left(\frac{q\pi}{d}z\right),\\
H_y &= -D\frac{j\omega\epsilon_0 n^2}{k_c^2}\left(\frac{m\pi}{a}\right)\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{p\pi}{b}y\right)\cos\left(\frac{q\pi}{d}z\right),\\
H_z &= 0.
\end{aligned}
$$

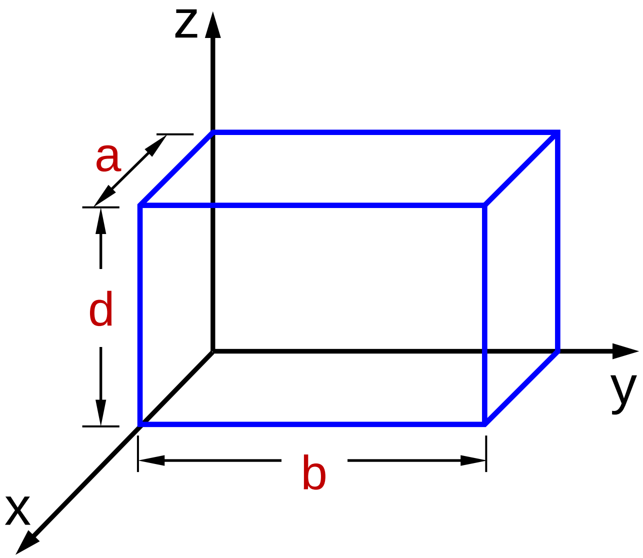
From Maxwell's equations
Electromagnetic Knowledge

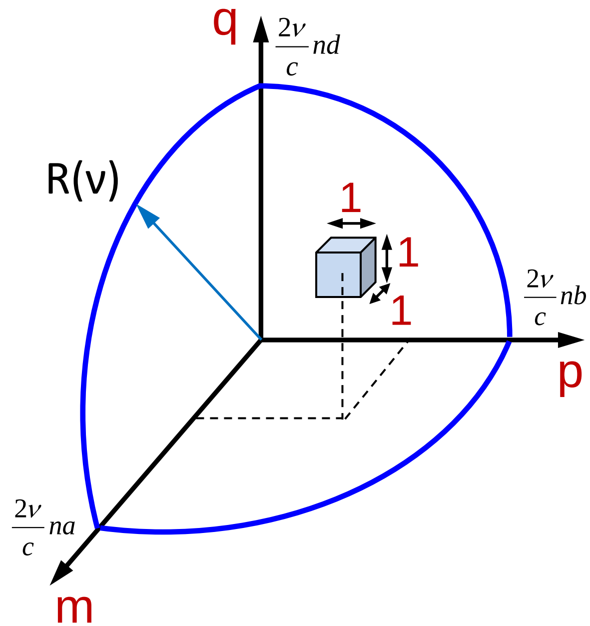


Dispersion Relation

$$
\begin{aligned}
k_0^2 n^2 &= \left(\frac{m\pi}{a}\right)^2 + \left(\frac{p\pi}{b}\right)^2 + \left(\frac{q\pi}{d}\right)^2,\\
\omega_{mnq} &= \frac{c}{n}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{p\pi}{b}\right)^2 + \left(\frac{q\pi}{d}\right)^2}.
\end{aligned}
$$

Blackbody Radiation





$$\left(\frac{m\pi}{a}\right)^2+\left(\frac{p\pi}{b}\right)^2+\left(\frac{q\pi}{d}\right)^2=\left(\frac{2\pi\nu}{c}\right)^2 n^2$$

$$m^2+p^2+q^2=\left(\frac{2\nu}{c}\right)^2 a^2 n^2=\left(\frac{2\nu n a}{c}\right)^2$$

$$N(\nu)=\frac{(1/8)\ \text{cavity volume}}{\text{volume of a mode}}=\frac{(1/8)(4/3)\pi(2\nu a n/c)^3}{1\times 1\times 1}=\frac{4}{3}\pi\frac{\nu^3 n^3 a^3}{c^3}$$

$$\mathcal{N}(\nu)=\frac{N(\nu)}{Volume=a^3}=\frac{8}{3}\pi\frac{\nu^3 n^3}{c^3}$$

Density of Electromagnetic Modes per Frequency:

$$\frac{d\mathcal{N}(\nu)}{d\nu}=\frac{8\pi\nu^2 n^3}{c^3}$$

Blackbody Radiation

From Boltzmann's statistics (energy of an *em* mode between E and E+dE):

$$p(E)dE = A\exp\left(-\frac{E}{k_BT}\right)dE$$

Boltzmann's Constant

$$k_B = 1.380649 \times 10^{-23}\, J/^\circ K$$

Normalization $$\int_0^\infty p(E)dE = 1 \Longrightarrow A = \frac{1}{\int_0^\infty \exp(-E/k_BT)dE} = \frac{1}{k_BT}$$

Average Energy per Electromagnetic Mode:

$$\langle E\rangle = \int_0^\infty Ep(E)dE = \int_0^\infty E\frac{1}{k_BT}\exp\left(-\frac{E}{k_BT}\right)dE = k_BT$$

Rayleigh-Jeans Equation

$$\rho(\nu, T) = \frac{dN(\nu)}{d\nu}k_BT = \frac{8\pi\nu^2 n^3}{c^3}k_BT$$

Blackbody Radiation

Planck's Equation

$$\rho(\nu, T) = \frac{8\pi\nu^2 n^3}{c^3} \frac{h\nu}{\exp(h\nu/k_B T) - 1}$$

Boltzmann Statistics – Discrete Energy States

$$p(E_i) = A \exp\left(-\frac{E_i}{k_B T}\right)$$

$$\sum_{i=0}^{\infty} p(E_i) = 1 \Longrightarrow A = \frac{1}{\sum_{i=0}^{\infty} \exp(-E_i/k_B T)} = 1 - \exp\left(-\frac{h\nu}{k_B T}\right)$$

Planck's Energy Quantization $\boxed{E_i = ih\nu}$ $\quad i = 0, 1, 2, \cdots$

$$\begin{aligned} \langle E \rangle &= A \sum_{i=0}^{\infty} E_i \exp\left(-\frac{E_i}{k_B T}\right) = A\left[1h\nu e^{-h\nu/k_B T} + 2h\nu e^{-2h\nu/k_B T} + \cdots\right] = \\ &= A \frac{h\nu \exp(-h\nu/k_B T)}{[1 - \exp(-h\nu/k_B T)]^2} = \frac{h\nu}{\exp(h\nu/k_B T) - 1}. \end{aligned}$$

Blackbody Radiation

$$\rho(\nu,T) = \underbrace{\frac{8\pi\nu^2 n^3}{c^3}}_{\text{Number of em modes}} \overbrace{h\nu}^{\text{photon energy}} \underbrace{\frac{1}{\exp(h\nu/k_BT)-1}}_{\text{Number of photons/mode}}$$

Spectral Power per Unit Area

$$dP_{avg} = \frac{8\pi n^2\nu^2}{c^2}\frac{h\nu}{\exp(h\nu/k_BT)-1}d\nu = P_{avg,\nu}d\nu,$$

$$dP_{avg} = \frac{8\pi n^2 c}{\lambda_0^4}\frac{hc/\lambda_0}{\exp(hc/\lambda_0 k_BT)-1}d\lambda_0 = P_{avg,\lambda_0}d\lambda_0$$

Spectral Radiant **Exitance** per Frequency (W/m^{2}Hz)

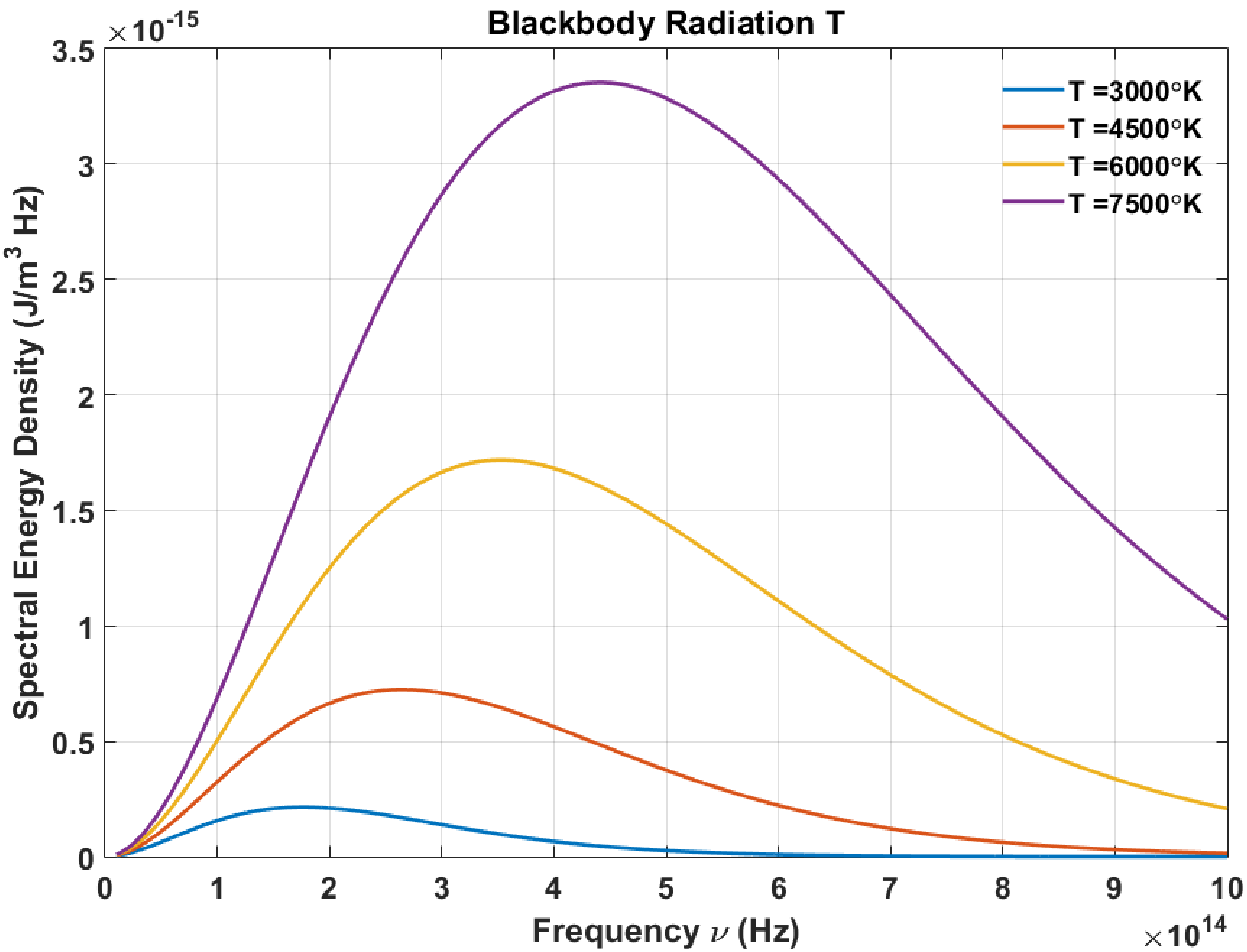
$$M_\nu(\nu) = \frac{2\pi n^2\nu^2}{c^2}\frac{h\nu}{\exp(h\nu/k_BT)-1}$$

$$M_{\lambda_0} = \frac{2\pi n^2 c}{\lambda_0^4}\frac{hc/\lambda_0}{\exp(hc/\lambda_0 k_BT)-1},$$

$$M_\lambda = \frac{2\pi c}{n^2\lambda^4}\frac{hc/\lambda}{\exp(hc/\lambda n k_BT)-1}.$$

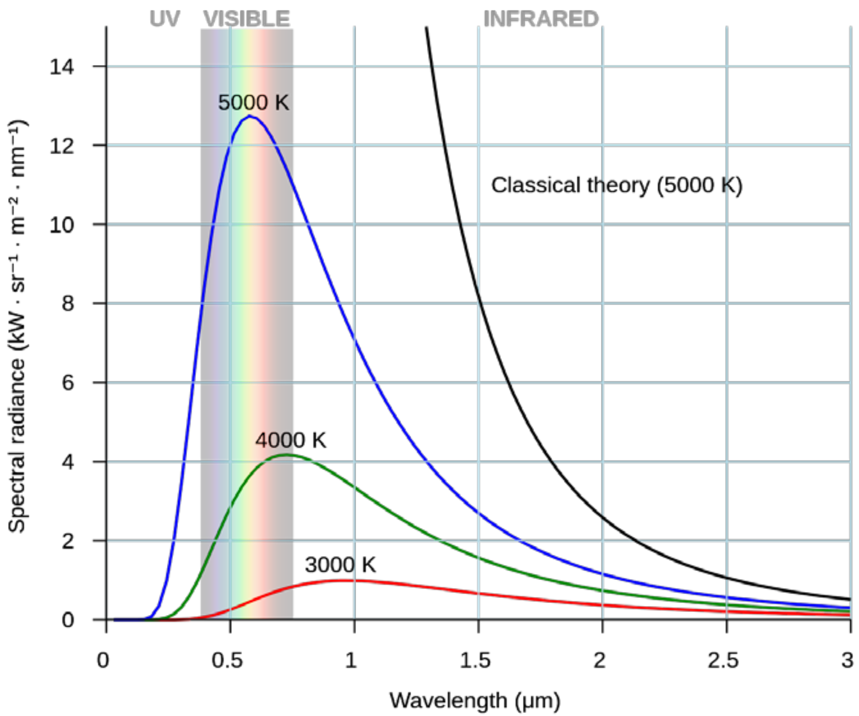
Blackbody Radiation

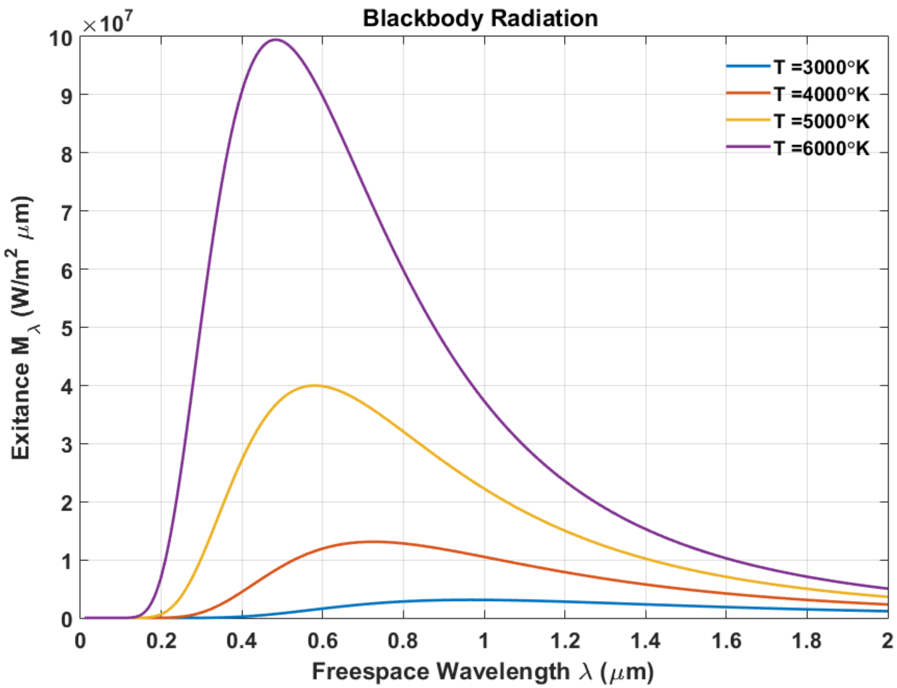
$$\rho(\nu, T) = \frac{8\pi\nu^2 n^3}{c^3} \frac{h\nu}{\exp(h\nu/k_B T) - 1}$$



Blackbody Radiation

$$M_\lambda = \frac{2\pi n^2 c}{\lambda^4} \frac{hc/\lambda}{\exp(hc/\lambda k_B T) - 1}$$





https://en.wikipedia.org/wiki/Black-body_radiation

Blackbody Radiation

Stefan-Boltzmann Law

$$M = \int_0^{\infty} M_{\lambda_0} d\lambda_0 = \left(\frac{2\pi^5 k_B^4}{15 h^3 c^2}\right) n^2 T^4 = \sigma n^2 T^4$$

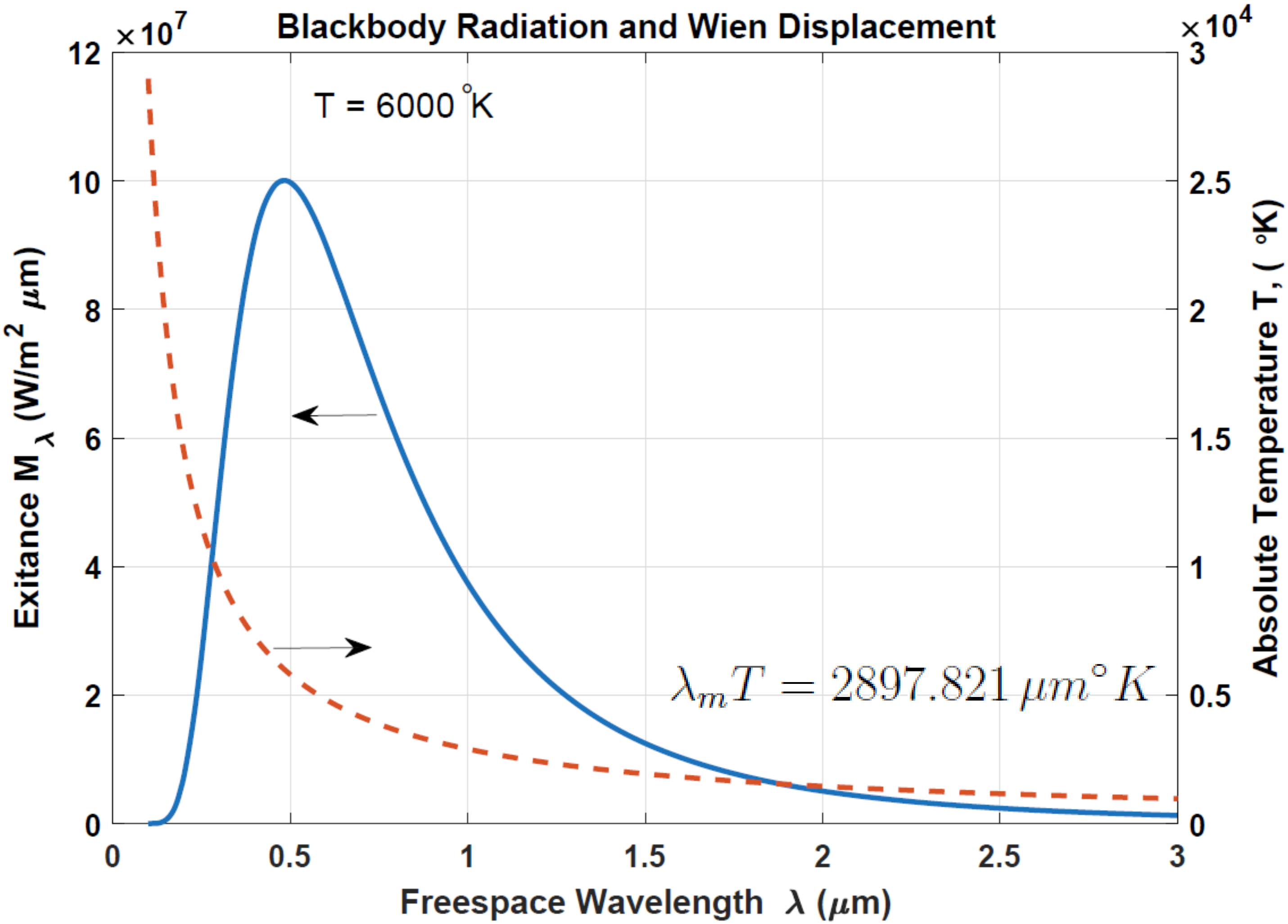
$$\sigma = 5.67 \times 10^{-8}\, W/m^2\, {}^\circ K^{-4}$$

Wien's Displacement Law

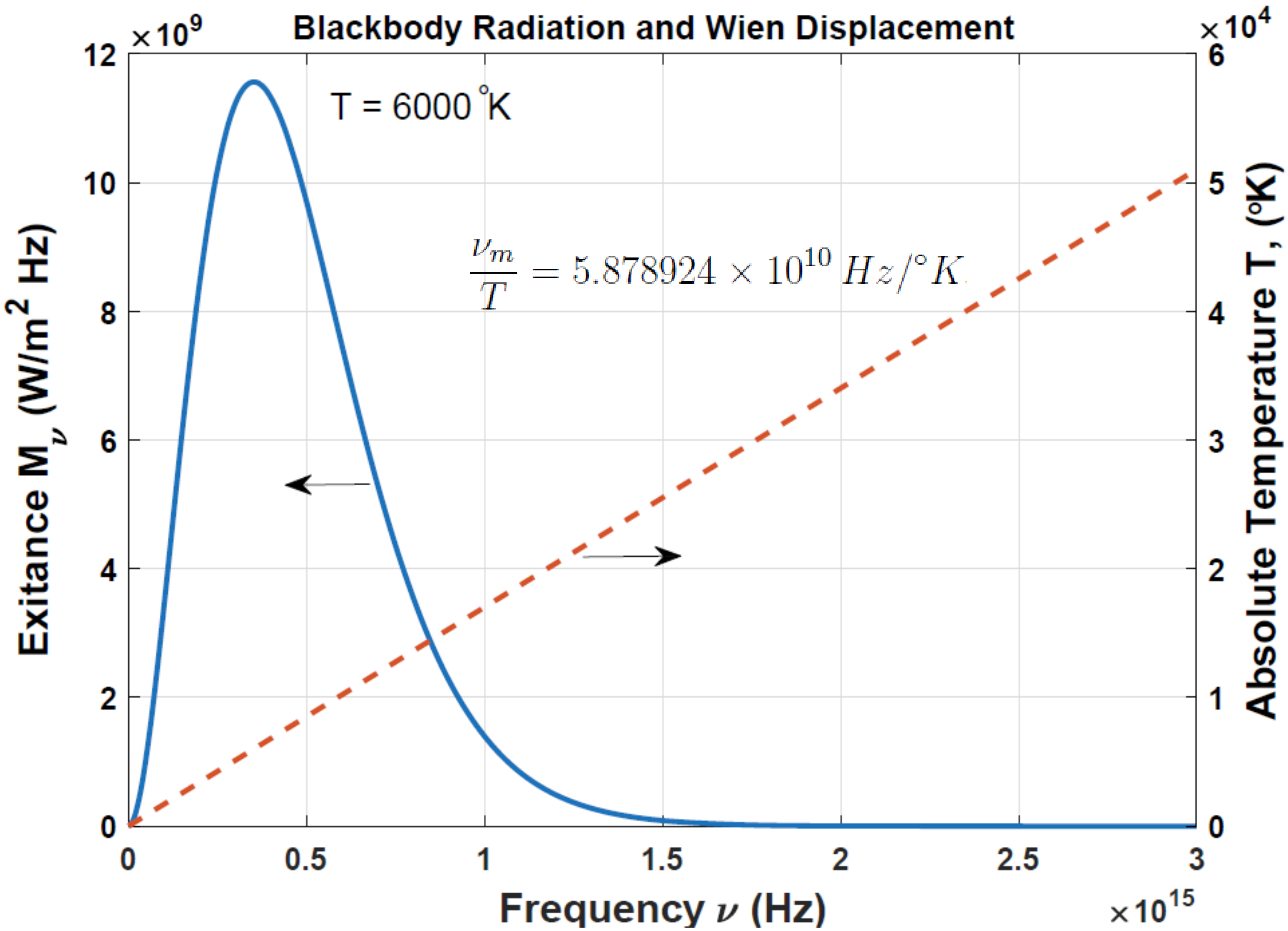
$$\frac{dM_{\lambda_0}(\lambda_{0,max})}{d\lambda_0} = 0 \Rightarrow \frac{hc}{\lambda_{0,max} k_B T} = 4.96511423 \Rightarrow \lambda_{0,max} T = 2897.821\, \mu m^\circ K$$

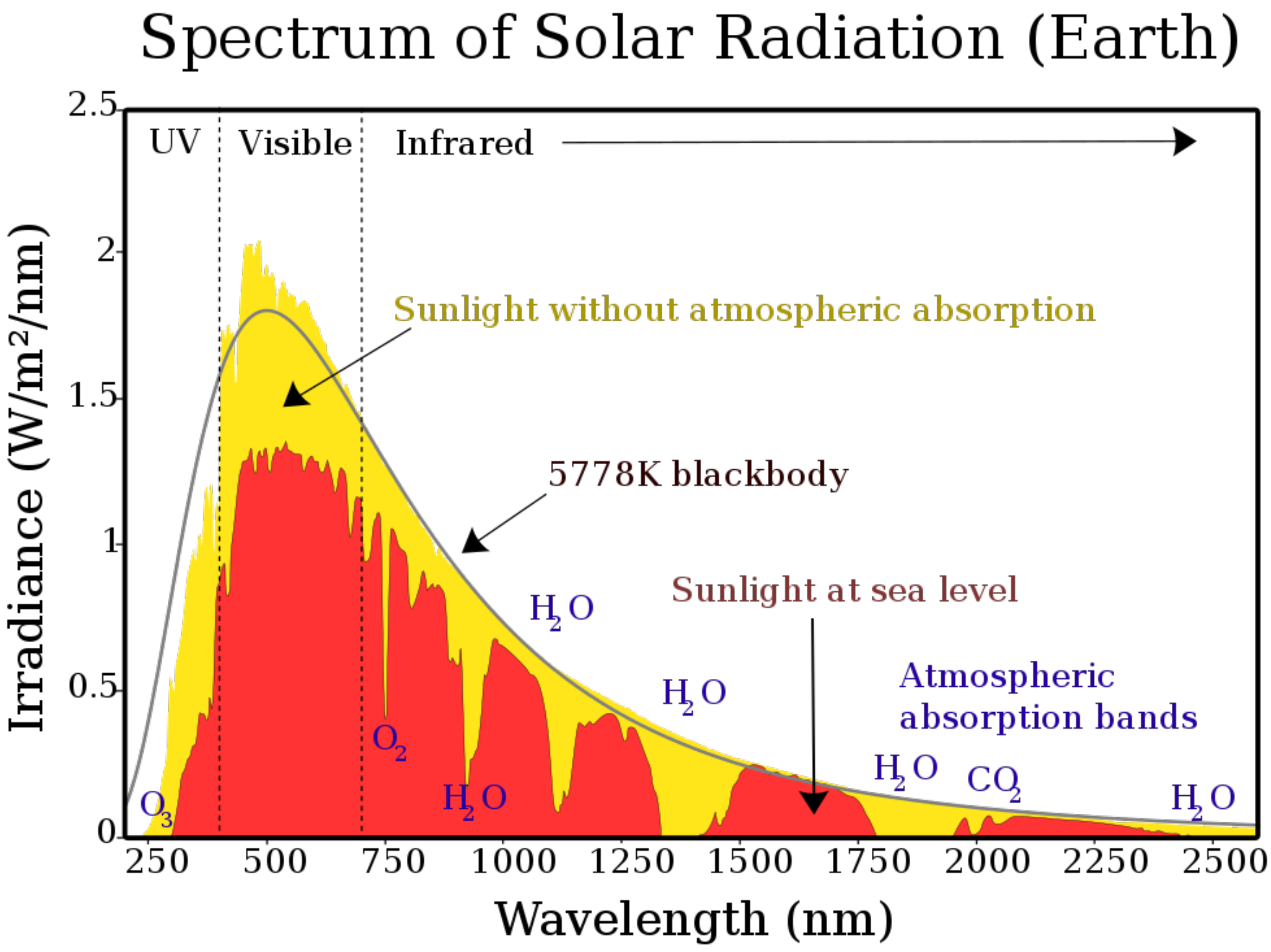
$$\frac{dM_{\nu}(\nu_{max})}{d\nu} = 0 \Rightarrow \frac{h\nu_{max}}{k_B T} = 2.82143937 \Rightarrow \frac{\nu_{max}}{T} = 5.878924 \times 10^{10}\, Hz/{}^\circ K$$

Blackbody Radiation Spectral Exitance



Blackbody Radiation Spectral Exitance

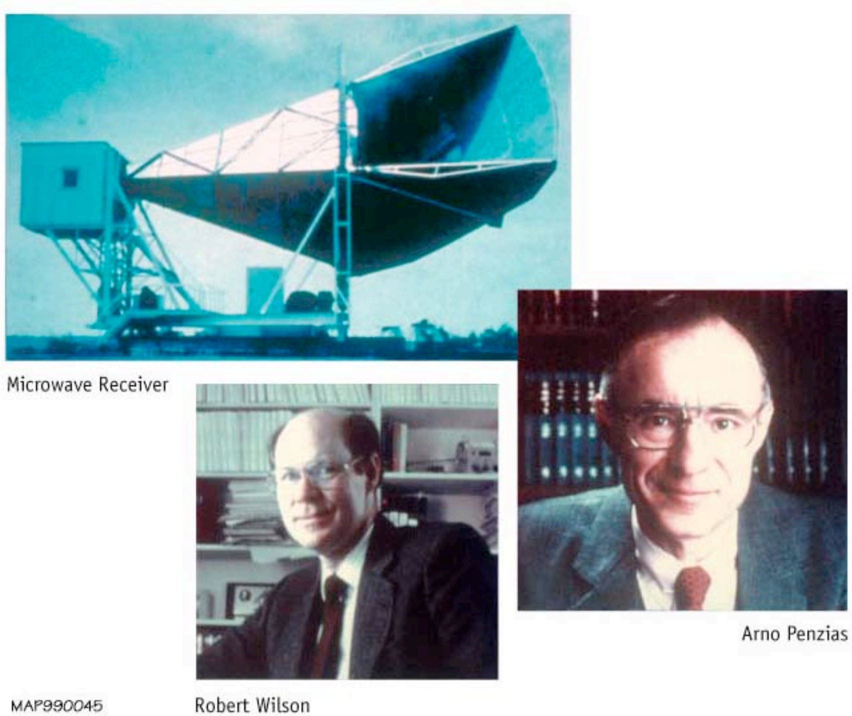


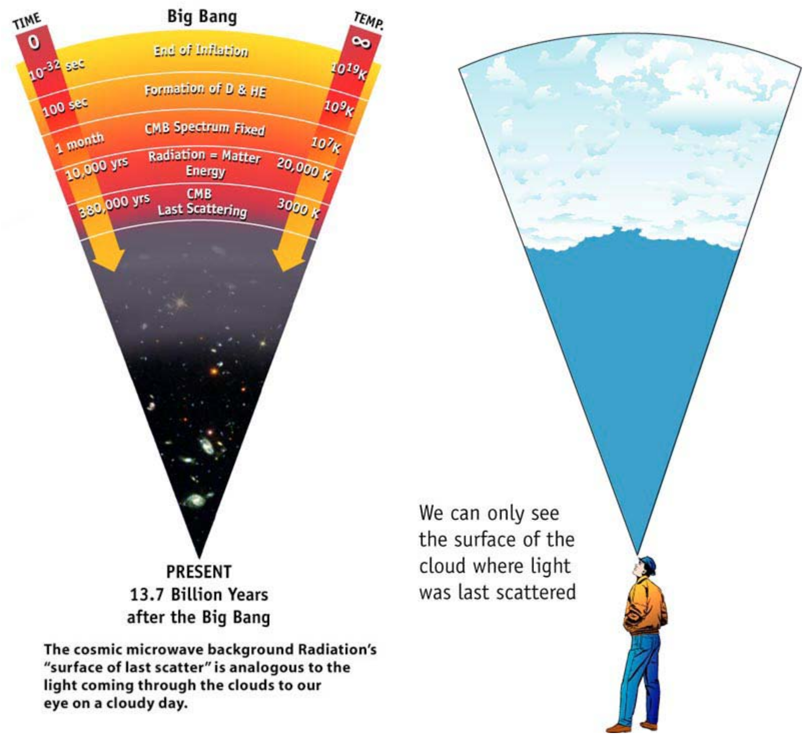


https://upload.wikimedia.org/wikipedia/commons/thumb/e/e7/Solar_spectrum_en.svg/1024px-Solar_spectrum_en.svg.png

Discovery of Cosmic Background Radiation

It was first observed inadvertently in 1965 by Arno Penzias and Robert Wilson at the Bell Telephone Laboratories in Murray Hill, New Jersey.



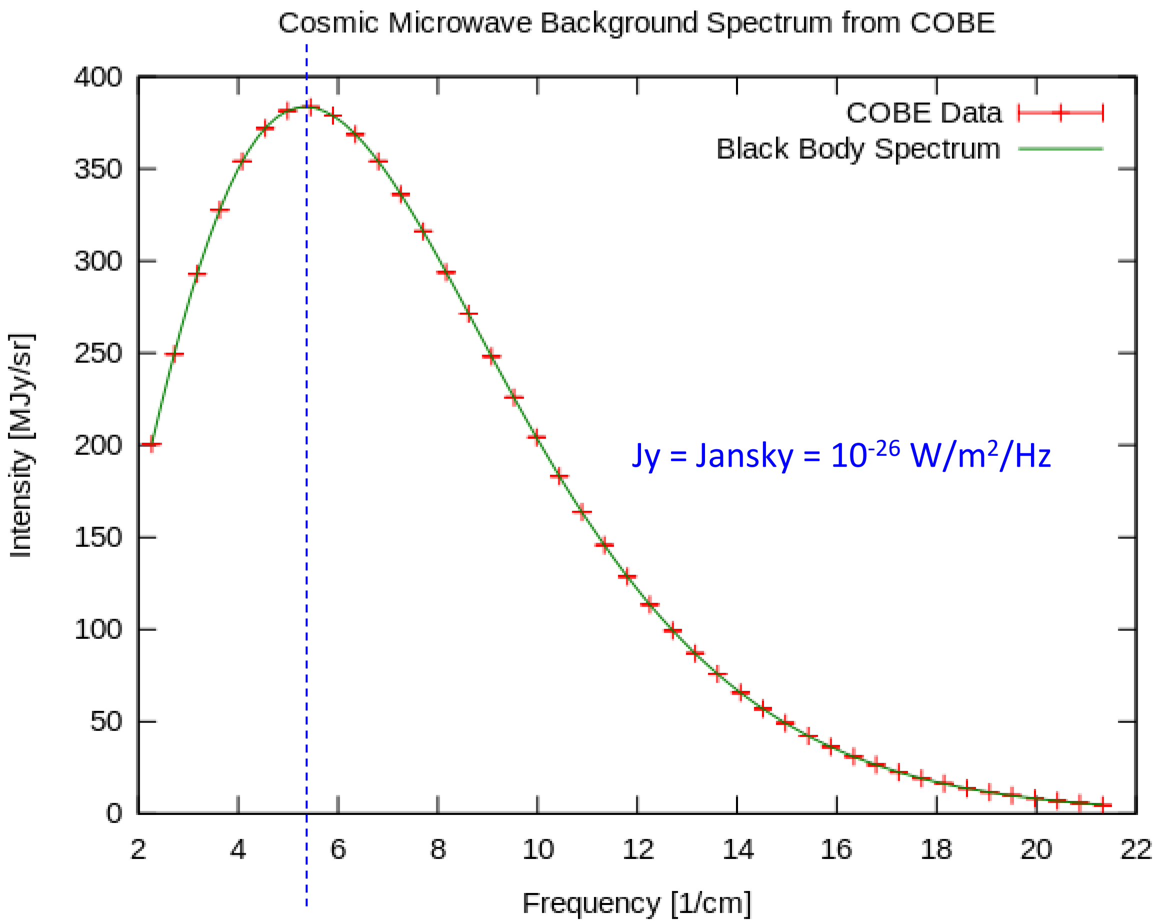


A. Penzias and R. Wilson shared the **1978 Nobel prize** in physics for their discovery.

http://map.gsfc.nasa.gov/universe/bb_tests_cmb.html

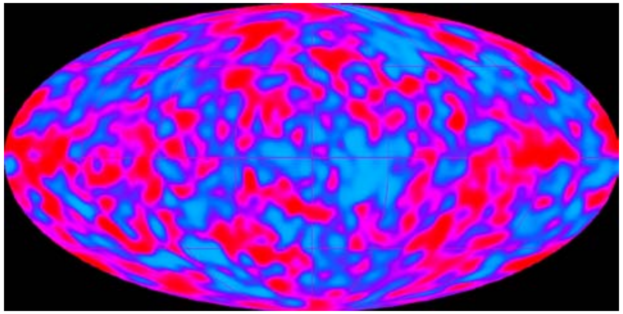
COsmic Background Explorer (COBE)

Data from COBE showed a perfect fit between the black body curve predicted by big bang theory and that observed in the microwave background. T = 2.725 K, ν_{max} = 160.1GHz (λ_0 = 1.873mm)

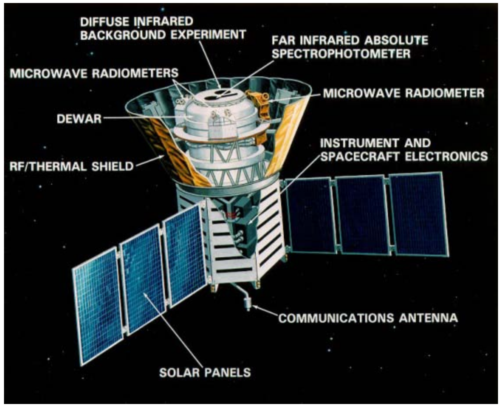


5.34 (1/cm)

https://en.wikipedia.org/wiki/Cosmic_Background_Explorer



The famous map of the CMB anisotropy formed from data taken by the COBE spacecraft.

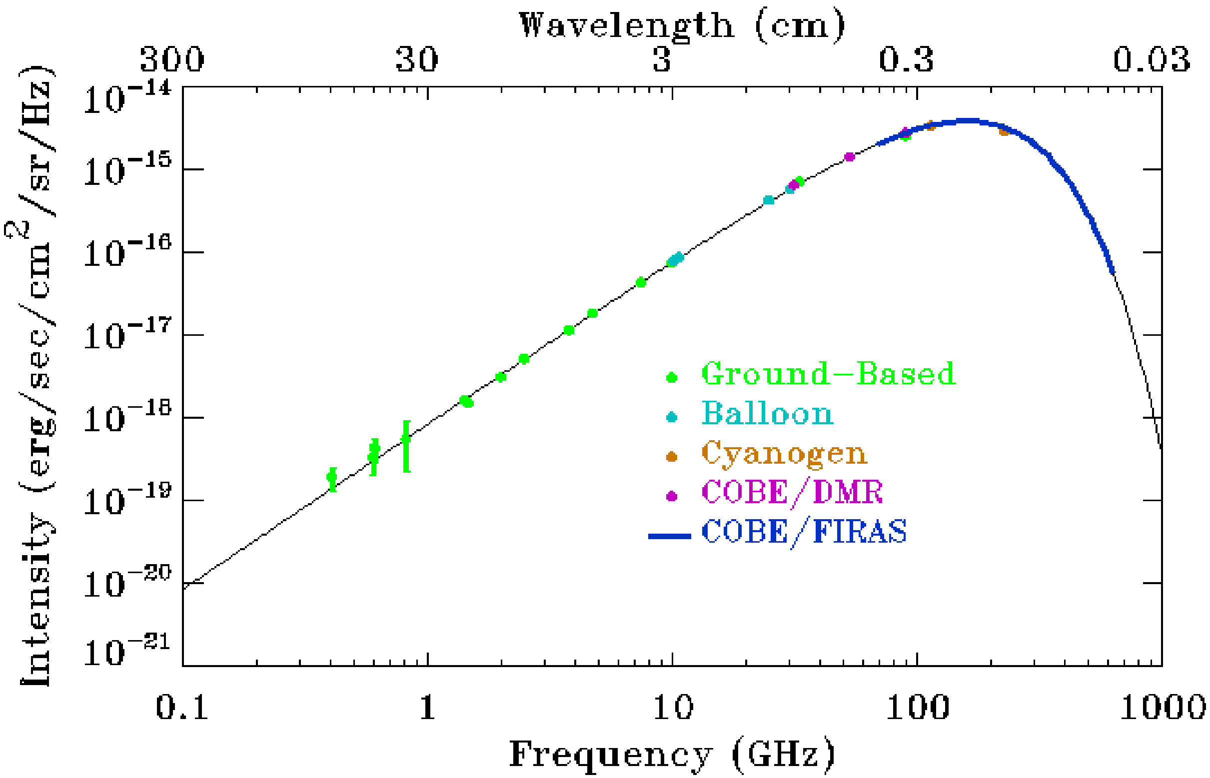


1989-1993

<u>CO</u>smic <u>B</u>ackground <u>E</u>xplorer (COBE)

Data from COBE showed a perfect fit between the blackbody curve predicted by Big-Bang theory and that observed in the microwave background.

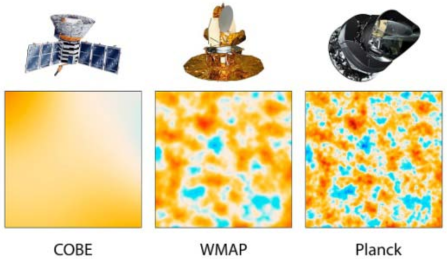
$T = 2.725$ K, $\nu_{max} = 160.1$GHz ($\lambda_0 = 1.873$mm)

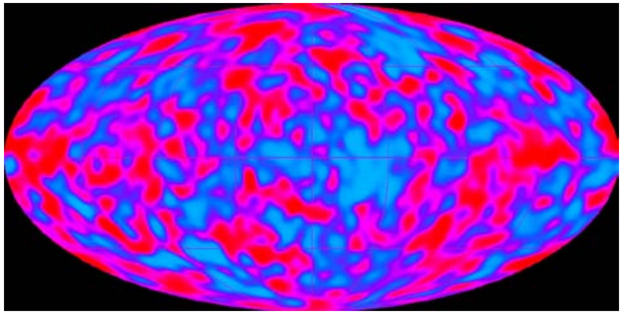


The most precise measurements of the CMB spectrum at the millimeter wavelengths near its peak were made by the Far Infrared Asbolute Spectrophotometer (FIRAS) instrument aboard the Cosmic Background Explorer (COBE) satellite. FIRAS determined the CMB temperature to be 2.725 +/- 0.001 K, with deviations from a perfect blackbody limited to less than 50 parts per million in intensity.

http://asd.gsfc.nasa.gov/archive/arcade/cmb_intensity.html

Cosmic Background Radiation





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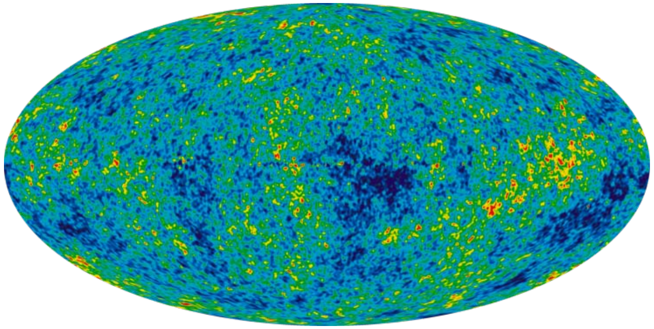
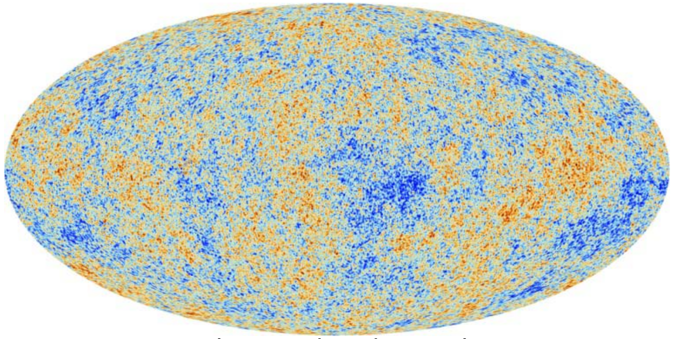


All-sky mollweide map of the CMB, created from 9 years of WMAP data

https://en.wikipedia.org/wiki/Cosmic_microwave_background



Cosmic Background Radiation by PLANK spacecraft (ESA) 2013.

https://www.esa.int/Science_Exploration/Space_Science/Planck/Planck_and_the_cosmic_microwave_background

Temperature Anisotropy
dT/T ~ 1/100000 or 1/1000000