Θεμα 1

(α)

$$\Phi(x_0,y_0,z_0)=\frac{1}{4\pi\epsilon_0}\Bigg\{\int_0^a\int_0^{2\pi}\frac{\sigma_1 r'_T dr'_T d\phi'}{[r_{T0}^2+r_T'^2-2r_{T0}r'_T\cos(\phi_0-\phi')+z_0^2]^{1/2}}+\int_b^c\int_0^{2\pi}\frac{\sigma_2 r'_T dr'_T d\phi'}{[r_{T0}^2+r_T'^2-2r_{T0}r'_T\cos(\phi_0-\phi')+z_0^2]^{1/2}}\Bigg\}$$

(β)

$$\Phi(x_0,y_0,z_0)=\frac{\sigma_1}{2\epsilon_0}\Big[\sqrt{z_0^2+a^2}-|z_0|\Big]+\frac{\sigma_2}{2\epsilon_0}\Big[\sqrt{z_0^2+c^2}-\sqrt{z_0^2+b^2}\Big]$$

(γ)

$$\frac{q_1}{q_2}=-\frac{a}{b+c}$$

(δ)

$$\vec{p}=-sq\hat{i}_x=-s\sigma_1\pi a^2\hat{i}_x=-s\sigma_2\pi(c^2-b^2)\hat{i}_x$$

Θεμα 2

(α)

$$\Phi(x)=-\frac{U}{\ln(\sigma_2/\sigma_1)}\ln\Big[\frac{\sigma_1}{\sigma_2}+\big(1-\frac{\sigma_1}{\sigma_2}\big)\frac{x}{s}\Big]$$

(β)

$$\vec{E}=\frac{U}{\ln(\sigma_2/\sigma_1)}\ \frac{1}{\sigma_1+(\sigma_2-\sigma_1)(x/s)}\ \frac{\sigma_2-\sigma_1}{s}\ \hat{i}_x$$

(γ)

$$\vec{J}=\frac{U}{\ln(\sigma_2/\sigma_1)}\ \frac{\sigma_2-\sigma_1}{s}\ \hat{i}_x$$

(δ)

$$R=\frac{s\ln(\sigma_2/\sigma_1)}{(\sigma_2-\sigma_1)Ld}$$

(ϵ)

$$\rho(x)=-\frac{\rho_0}{[\sigma_1+(\sigma_2-\sigma_1)(x/s)]^2}$$

$$q=-\frac{(\sigma_2-\sigma_1)^2}{\sigma_1\sigma_2 s}\frac{U\epsilon}{\ln(\sigma_2/\sigma_1)}Ld$$

$$\sigma(x=0) = \frac{\epsilon}{\sigma_1}\frac{U(\sigma_2-\sigma_1)}{s\ln(\sigma_2/\sigma_1)}$$

$$\sigma(x=s) = -\frac{\epsilon}{\sigma_2}\frac{U(\sigma_2-\sigma_1)}{s\ln(\sigma_2/\sigma_1)}$$

$$\sigma_{total} = \sigma(x=0) + \sigma(x=s) = -q$$

<u>$\Theta\epsilon\mu\alpha$ 3</u>

(α)

$$A_{1z}(x,y) = \frac{K_0}{a}\ \frac{\mu_1\mu_2}{\mu_1+\mu_2}\ e^{ax}\ \cos(ay) \qquad (-\infty < x \leq 0)$$
$$A_{2z}(x,y) = \frac{K_0}{a}\ \frac{\mu_1\mu_2}{\mu_1+\mu_2}\ e^{-ax}\ \cos(ay) \qquad (0 \leq x < +\infty)$$

(β)

$$\vec{H}_1(x,y) = -\,K_0\frac{\mu_2}{\mu_1+\mu_2}\ e^{ax}\ [\sin(ay)\hat{i}_x + \cos(ay)\hat{i}_y] \qquad (-\infty < x \leq 0)$$
$$\vec{H}_1(x,y) = K_0\frac{\mu_1}{\mu_1+\mu_2}\ e^{-ax}\ [-\sin(ay)\hat{i}_x + \cos(ay)\hat{i}_y] \qquad (0 \leq x < +\infty)$$

<u>$\Theta\epsilon\mu\alpha$ 4</u>

(α)

$$\vec{m} = (K_0\pi\frac{a^3-b^3}{3})\hat{i}_z = m_0\hat{i}_z$$

(β)

$$\vec{H}(x_0,y_0,z_0) = \frac{1}{4\pi R^3}\Big[\frac{3m_0(z_0-h)}{R}\ \frac{x_0\hat{i}_x + y_0\hat{i}_y + (z_0-h)\hat{i}_z}{R} - m_0\hat{i}_z\Big] +$$
$$\frac{1}{4\pi R'^3}\Big[-\frac{3m_0(z_0+h)}{R'}\ \frac{x_0\hat{i}_x + y_0\hat{i}_y + (z_0+h)\hat{i}_z}{R'} + m_0\hat{i}_z\Big]$$

(γ)

$$\vec{H}(0,0,z_0) = \frac{m_0}{4\pi}\Big[\frac{2}{(z_0-h)^3} - \frac{2}{(z_0+h)^3}\Big]\ \hat{i}_z$$

(δ)

$$\vec{K}_p = -\frac{3m_0h}{2\pi}\ \frac{r_T}{(r_T^2+h^2)^{5/2}}\ \hat{i}_\phi$$