

Θεµα 1

(α)

$$\Phi(x_0, y_0, z_0) = \frac{\sigma_0}{4\pi\epsilon_0 a^2} \int_{-a}^{+a} \int_{-a}^{+a} \frac{|x'y'| dx' dy'}{[(x_0 - x')^2 + (y_0 - y')^2 + (z_0 - z')^2]^{1/2}}$$

(β)

$$\Phi(0, 0, z_0) = \frac{\sigma_0}{3\pi\epsilon_0 a^2} [(2a^2 + z_0^2)^{3/2} - 2(a^2 + z_0^2)^{3/2} + (z_0^2)^{3/2}]$$

(γ)

$$\Phi(0, 0, 0) = \frac{2\sigma_0 a}{3\pi\epsilon_0} [\sqrt{2} - 1]$$

(δ)

$$\vec{E}(0, 0, z) = \frac{\sigma_0}{\pi\epsilon_0 a^2} z [2(a^2 + z_0^2)^{1/2} - (2a^2 + z_0^2)^{1/2} - |z|] \hat{i}_z$$

(ε)

$$\Phi(0, 0, z_0 \gg a) \simeq \frac{\sigma_0 a^2}{4\pi\epsilon_0 |z_0|}$$

Θεµα 2

(α)

$$\vec{P} = \left(-\rho_0 \frac{r_T}{2}\right) \hat{i}_r, \quad \sigma_b = -\rho_0 a/2$$

(β)

$$\vec{D} = \hat{i}_r \begin{cases} 0 & (r_T < a) \\ \frac{\sigma_0 a}{r_T} & (r_T > a) \end{cases}$$
$$\vec{E} = \hat{i}_r \begin{cases} \frac{\rho_0 r_T}{2\epsilon_0} & (r_T < a) \\ \frac{a\sigma_0}{\epsilon_0 r_T} & (r_T > a) \end{cases}$$
$$\Phi(r_T) = \begin{cases} \frac{\rho_0}{4\epsilon_0} (a^2 - r_T^2) & (r_T \leq a) \\ -\frac{a\sigma_0}{\epsilon_0} \ln(r_T/a) & (r_T \geq a) \end{cases}$$

Θεµα 3

(α)

$$\vec{H}(z) = \hat{i}_z \frac{K_0 R^3}{2} \int_{\theta=0}^{\pi} \frac{\sin^2 \theta}{[z^2 + R^2 - 2zR \cos \theta]^{3/2}} d\theta$$

(β)

$$\vec{H}(z = 0) = \hat{i}_z \frac{K_0 \pi}{4}$$

(γ)

$$\vec{H}(z = 0) = \hat{i}_z \frac{J_0 \pi}{4} (R_2 - R_1)$$

Θεμα 4

(α)

$$\Phi(x, y) = \sum_{n=1}^{\infty} \frac{4V_0}{\pi} \frac{1}{2n-1} \frac{1}{\sinh(k_n a)} \sinh(k_n x) \sin(k_n y)$$

$$k_n = (2n-1) \frac{\pi}{2b}$$

(β)

$$\begin{aligned} \vec{J} = & -\sigma \left[\sum_{n=1}^{\infty} k_n C_n \cosh(k_n x) \sin(k_n y) \right] \hat{i}_x + \\ & -\sigma \left[\sum_{n=1}^{\infty} k_n C_n \sinh(k_n x) \cos(k_n y) \right] \hat{i}_y + \end{aligned}$$

$$k_n = (2n-1) \frac{\pi}{2b}$$

$$C_n = \frac{4V_0}{\pi} \frac{1}{2n-1} \frac{1}{\sinh(k_n a)}$$

(γ)

$$R = \frac{\pi}{4\sigma \sum_{n=1}^{\infty} [1/(2n-1)] \coth(k_n a)}$$

$$k_n = (2n-1) \frac{\pi}{2b}$$