

### Θεμα 1

(α)

$$\Phi(x_0, z_0) = \frac{p}{4\pi\epsilon_0 R^3} [x_0 \sin \theta + (z_0 - h) \cos \theta] + \frac{p}{4\pi\epsilon_0 R'^3} [-x_0 \sin \theta + (z_0 + h) \cos \theta]$$

οπου

$$R = [x_0^2 + (z_0 - h)^2]^{1/2}, \quad R' = [x_0^2 + (z_0 + h)^2]^{1/2}$$

(β)

$$\sigma(x) = \frac{3ph(-x_0 \sin \theta + h \cos \theta) - (x_0^2 + h^2)p \cos \theta}{2\pi(x_0^2 + h^2)^{5/2}}$$

(γ)

$$T = -\hat{i}_y \frac{p^2 \sin 2\theta}{64\pi\epsilon_0 h^3}$$

### Θεμα 2

(α)

$$\begin{aligned} \Phi_1(\phi) &= A_1\phi + B_1 & (0 \leq \phi < \phi_1) \\ \Phi_2(\phi) &= A_2\phi + B_2 & (\phi_1 \leq \phi < \phi_0) \end{aligned}$$

οπου

$$A_1 = \frac{\epsilon_2 U}{(\epsilon_2 - \epsilon_1)\phi_1 + \epsilon_1\phi_0}, \quad B_1 = 0, \quad A_2 = \frac{\epsilon_1 U}{(\epsilon_2 - \epsilon_1)\phi_1 + \epsilon_1\phi_0}, \quad B_2 = \frac{(\epsilon_2 - \epsilon_1)U}{(\epsilon_2 - \epsilon_1)\phi_1 + \epsilon_1\phi_0}.$$

(β)

$$\vec{E}_1 = -\frac{1}{r_T} A_1 \hat{i}_\phi, \quad \vec{E}_2 = -\frac{1}{r_T} A_2 \hat{i}_\phi.$$

(γ)

$$\rho_{b1} = 0, \quad \rho_{b2} = 0, \quad \sigma_b(\phi_1) = \frac{(A_1 - A_2)\epsilon_0}{r_T}, \quad \sigma_b(\phi = 0^+) = \frac{(\epsilon_1 - \epsilon_0)A_1}{r_T}, \quad \sigma_b(\phi_0^-) = -\frac{(\epsilon_2 - \epsilon_0)A_2}{r_T},$$

(δ)

$$\sigma(\phi = 0^+) = -\frac{A_1\epsilon_1}{r_T}, \quad \sigma(\phi = \phi_0^-) = +\frac{A_2\epsilon_2}{r_T}.$$

Θεμα 3

(α)

$$\begin{aligned}\Phi_1(x, y) &= \frac{\sigma_0}{a(\epsilon_1 + \epsilon_2)} e^{ax} \sin(ay) & (-\infty < x \leq 0) \\ \Phi_2(x, y) &= \frac{\sigma_0}{a(\epsilon_1 + \epsilon_2)} e^{-ax} \sin(ay) & (0 \leq x < +\infty)\end{aligned}$$

(β)

$$\begin{aligned}\vec{E}_1(x, y) &= -\frac{\sigma_0}{(\epsilon_1 + \epsilon_2)} e^{ax} [\sin(ay)\hat{i}_x + \cos(ay)\hat{i}_y] & (-\infty < x \leq 0) \\ \vec{E}_2(x, y) &= -\frac{\sigma_0}{(\epsilon_1 + \epsilon_2)} e^{-ax} [-\sin(ay)\hat{i}_x + \cos(ay)\hat{i}_y] & (0 \leq x < +\infty)\end{aligned}$$

(γ)

$$\vec{E}(x, y) = -\hat{i}_y \frac{\sigma_0}{(\epsilon_1 + \epsilon_2)} e^{-a|x|} \quad (-\infty < x < +\infty)$$

Θεμα 4

(α)

$$\vec{H} = \frac{1}{4\pi} \int_{\phi'=0}^{2\pi} \int_{r'_T=b}^a \frac{\vec{K}(\phi') \times \hat{i}_R(r'_T, \phi')}{R^2} r'_T d\phi' dr'_T$$

οπου

$$\hat{i}_R = \frac{1}{R} [(r_{T0} \cos \phi_0 - r'_T \cos \phi')\hat{i}_x + (r_{T0} \sin \phi_0 - r'_T \sin \phi')\hat{i}_y + z_0\hat{i}_z] = a_x \hat{i}_x + a_y \hat{i}_y + a_z \hat{i}_z,$$

$$R = [r_{T0}^2 + r_T'^2 - 2r_{T0}r_T' \cos(\phi_0 - \phi') + z_0^2]^{1/2},$$

$$\vec{K} \times \hat{i}_R = \hat{i}_x (a_z K_0 \cos \phi') + \hat{i}_y (a_z K_0 \sin \phi') + \hat{i}_z (-K_0 \sin \phi' a_y - a_x K_0 \cos \phi').$$

(β)

$$\vec{H}(0, 0, z) = \hat{i}_z \frac{K_0}{2} \left[ -\frac{a}{\sqrt{a^2 + z^2}} + \frac{b}{\sqrt{b^2 + z^2}} + \ln \left( \frac{a + \sqrt{a^2 + z^2}}{b + \sqrt{b^2 + z^2}} \right) \right]$$