

Θεμα 1

(α)

$$\begin{aligned} \Phi(x, y, z) = & \frac{\lambda}{4\pi\epsilon_1} \ln \left[\frac{z + \sqrt{x^2 + y^2 + z^2}}{z - L_1 + \sqrt{x^2 + y^2 + (z - L_1)^2}} \right] + \\ & \frac{\lambda}{4\pi\epsilon_1} \frac{\epsilon_1 - \epsilon_2}{\epsilon_1 + \epsilon_2} \ln \left[\frac{z + L_1 + \sqrt{x^2 + y^2 + (z + L_1)^2}}{z + \sqrt{x^2 + y^2 + z^2}} \right] + \\ & \frac{\lambda}{4\pi\epsilon_1} \frac{2\epsilon_1}{\epsilon_1 + \epsilon_2} \ln \left[\frac{z + L_2 + \sqrt{x^2 + y^2 + (z + L_2)^2}}{z + \sqrt{x^2 + y^2 + z^2}} \right], \quad \text{for } z > 0, \end{aligned}$$

$$\begin{aligned} \Phi(x, y, z) = & \frac{\lambda}{4\pi\epsilon_2} \frac{2\epsilon_2}{\epsilon_1 + \epsilon_2} \ln \left[\frac{z + \sqrt{x^2 + y^2 + z^2}}{z - L_1 + \sqrt{x^2 + y^2 + (z - L_1)^2}} \right] + \\ & \frac{\lambda}{4\pi\epsilon_1} \frac{\epsilon_2 - \epsilon_1}{\epsilon_1 + \epsilon_2} \ln \left[\frac{z + \sqrt{x^2 + y^2 + z^2}}{z - L_2 + \sqrt{x^2 + y^2 + (z - L_2)^2}} \right] + \\ & \frac{\lambda}{4\pi\epsilon_1} \ln \left[\frac{z + L_2 + \sqrt{x^2 + y^2 + (z + L_2)^2}}{z + \sqrt{x^2 + y^2 + z^2}} \right], \quad \text{for } z < 0. \end{aligned}$$

(β)

$$E_x(x, z = 0^+) = E_x(x, z = 0^-) = \frac{\lambda}{4\pi(\epsilon_1 + \epsilon_2)} \frac{1}{x} \left[\frac{L_1}{\sqrt{x^2 + L_1^2}} + \frac{L_2}{(\sqrt{x^2 + L_2^2})} \right],$$

$$E_z(x, z = 0^+) = \frac{\epsilon_2}{\epsilon_1} E_z(x, z = 0^-) = \frac{\lambda}{4\pi\epsilon_1} \left[\frac{2(\epsilon_2 - \epsilon_1)}{\epsilon_2 + \epsilon_1} \frac{1}{|x|} + \frac{-2\epsilon_2}{\epsilon_2 + \epsilon_1} \frac{1}{\sqrt{x^2 + L_1^2}} + \frac{2\epsilon_1}{\epsilon_2 + \epsilon_1} \frac{1}{\sqrt{x^2 + L_2^2}} \right].$$

Θεμα 2

(α)

$$p_{11} = p_{22} = \frac{1}{2\pi\epsilon_0} \ln \left(\frac{2h}{a} \right),$$

$$p_{12} = p_{21} = \frac{1}{2\pi\epsilon_0} \ln \left(\frac{\sqrt{d^2 + 4h^2}}{d} \right).$$

(β)

$$C_{11} = C_{22} = \frac{2\pi\epsilon_0}{\ln\left(\frac{2h}{a} \frac{\sqrt{d^2 + 4h^2}}{d}\right)},$$
$$C_{12} = C_{21} = \frac{2\pi\epsilon_0 \ln\left(\frac{\sqrt{d^2 + 4h^2}}{d}\right)}{\ln\left(\frac{2h}{a} \frac{\sqrt{d^2 + 4h^2}}{d}\right) \ln\left(\frac{2h}{a} \frac{d}{\sqrt{d^2 + 4h^2}}\right)}.$$

Θεµα 3

(α)

$$\vec{A}(x, y, z) = \frac{\mu_0 I_1}{4\pi} \int_0^{2\pi} \frac{a(\cos\phi_1 \hat{i}_x - \sin\phi_1 \hat{i}_z)}{\sqrt{(x - a \sin\phi_1)^2 + y^2 + (z - a \cos\phi_1)^2}} d\phi_1 +$$
$$\frac{\mu_0 I_2}{4\pi} \int_0^{2\pi} \frac{a(-\sin\phi_2 \hat{i}_x - \cos\phi_2 \hat{i}_z)}{\sqrt{(x - x_0 - a \cos\phi_2)^2 + (y - y_0 - a \sin\phi_2)^2 + z^2}} d\phi_2$$

(β)

$$\vec{A}(x, y, z) = \frac{\mu_0 \pi a^2 I_1}{4\pi} \frac{z \hat{i}_x - x \hat{i}_z}{(x^2 + y^2 + z^2)^{3/2}} + \frac{\mu_0 \pi a^2 I_2}{4\pi} \frac{-(y - y_0) \hat{i}_x + (x - x_0) \hat{i}_y}{[(x - x_0)^2 + (y - y_0)^2 + z^2]^{3/2}}$$

(γ)

$$\vec{H} = \frac{I_1 a^2}{2} \frac{1}{y^3} \hat{i}_y + \frac{I_2 a^2}{4} \frac{1}{[x_0^2 + (y - y_0)^2]^{3/2}} \hat{i}_z$$

Θεµα 4

(α)

$$\Phi(r_T, \phi) = \left[-E_0 r_T + \frac{B_1}{r_T} \right] \cos\phi \quad r_T \geq b,$$
$$\Phi(r_T, \phi) = \left[A_2 r_T + \frac{B_2}{r_T} \right] \cos\phi \quad a \leq r_T \leq b,$$
$$\Phi(r_T, \phi) = A_3 r_T \cos\phi \quad r_T \leq a,$$

$$B_1 = K \frac{E_0 b^2 [1 - (a/b)^2]}{1 - K^2 (a/b)^2},$$
$$A_3 = E_0 \frac{K^2 - 1}{1 - K^2 (a/b)^2},$$
$$A_2 = \frac{E_0}{2} \frac{K^2 - 1}{1 - K^2 (a/b)^2} \left(1 + \frac{\epsilon_1}{\epsilon_2} \right),$$

$$B_2 = \frac{E_0}{2} a^2 \frac{K^2 - 1}{1 - K^2(a/b)^2} \left(1 - \frac{\epsilon_1}{\epsilon_2}\right),$$

$$K = \frac{\epsilon_2 - \epsilon_1}{\epsilon_2 + \epsilon_1}.$$

(β)

$$\frac{|E_3|}{E_0} = \frac{16(\epsilon_2/\epsilon_1)}{[3(\epsilon_2/\epsilon_1) + 1][(\epsilon_2/\epsilon_1) + 3]}$$

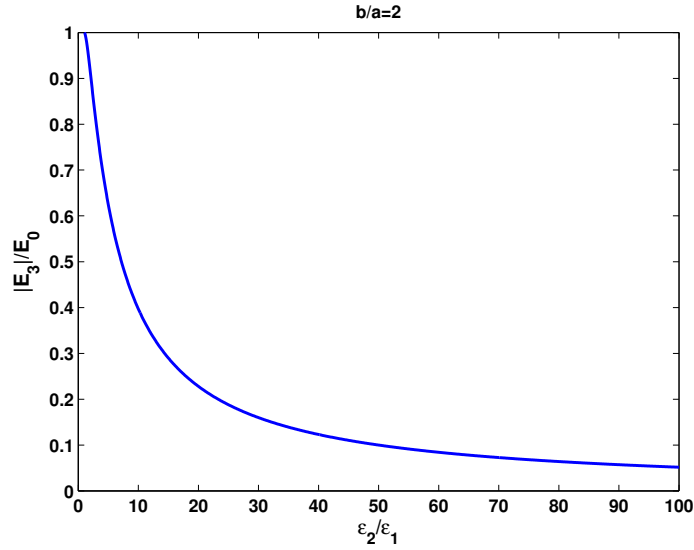


Figure 1: $|E_3|/E_0$ dependence on ϵ_2/ϵ_1 ratio. The high dielectric hollow cylinder protects its interior from the externally applied field E_0 .