ΠΕΔΙΑ Β 10/09/2010

Θεμα 1

(α)

$$\Phi(x,y,z) = \frac{\sigma}{4\pi\epsilon_1}\int_{r'_T=0}^{\mathrm{a}}\int_{\phi'=0}^{2\pi}\frac{r'_T dr'_T d\phi'}{[r_T^2+r'^2_T-2r_T r'_T\cos(\phi-\phi')+(z-h)^2]^{1/2}} + \frac{\sigma}{4\pi\epsilon_1}\frac{\epsilon_1-\epsilon_2}{\epsilon_1+\epsilon_2}\int_{r'_T=0}^{\mathrm{a}}\int_{\phi'=0}^{2\pi}\frac{r'_T dr'_T d\phi'}{[r_T^2+r'^2_T-2r_T r'_T\cos(\phi-\phi')+(z+h)^2]^{1/2}} \quad \text{for } z>0$$

$$\Phi(x,y,z) = \frac{\sigma}{4\pi\epsilon_2}\frac{2\epsilon_2}{\epsilon_1+\epsilon_2}\int_{r'_T=0}^{\mathrm{a}}\int_{\phi'=0}^{2\pi}\frac{r'_T dr'_T d\phi'}{[r_T^2+r'^2_T-2r_T r'_T\cos(\phi-\phi')+(z-h)^2]^{1/2}} \quad \text{for } z<0$$

(β)

$$\Phi(z) = \frac{\sigma}{2\epsilon_1}\Big[\sqrt{\mathrm{a}^2+(z-h)^2}-|z-h|\Big] + \frac{\sigma}{2\epsilon_1}\frac{\epsilon_1-\epsilon_2}{\epsilon_1+\epsilon_2}\Big[\sqrt{\mathrm{a}^2+(z+h)^2}-|z+h|\Big] \quad \text{for } z>0$$

$$\Phi(z) = \frac{\sigma}{\epsilon_1+\epsilon_2}\Big[\sqrt{\mathrm{a}^2+(z-h)^2}-|z-h|\Big] \quad \text{for } z<0$$

(γ)

$$\vec{E}(z) = \hat{i}_z\Big\{-\frac{\sigma}{2\epsilon_1}\Big[\frac{z-h}{[\mathrm{a}^2+(z-h)^2]^{1/2}}+1\Big] - \frac{\sigma}{2\epsilon_1}\frac{\epsilon_1-\epsilon_2}{\epsilon_1+\epsilon_2}\Big[\frac{z+h}{[\mathrm{a}^2+(z+h)^2]^{1/2}}-1\Big]\Big\} \quad \text{for } 0<z<h$$

$$\vec{E}(z) = \hat{i}_z\Big\{-\frac{\sigma}{2\epsilon_1}\Big[\frac{z-h}{[\mathrm{a}^2+(z-h)^2]^{1/2}}-1\Big] - \frac{\sigma}{2\epsilon_1}\frac{\epsilon_1-\epsilon_2}{\epsilon_1+\epsilon_2}\Big[\frac{z+h}{[\mathrm{a}^2+(z+h)^2]^{1/2}}-1\Big]\Big\} \quad \text{for } z>h$$

$$\vec{E}(z) = \hat{i}_z\Big\{-\frac{\sigma}{\epsilon_1+\epsilon_2}\Big[\frac{z-h}{[\mathrm{a}^2+(z-h)^2]^{1/2}}+1\Big]\Big\} \quad \text{for } z<0$$

Θεμα 2

(α)

$$\vec{J} = \frac{I}{4\pi}\Big[\frac{x\hat{i}_x+y\hat{i}_y+(z-h)\hat{i}_z}{[x^2+y^2+(z-h)^2]^{3/2}} + \frac{x\hat{i}_x+y\hat{i}_y+(z+h)\hat{i}_z}{[x^2+y^2+(z+h)^2]^{3/2}}\Big]$$

(β)

$$
\begin{aligned}
V_{sphere} &= \frac{I}{4\pi\sigma_0}\Big[\frac{1}{2h+\mathrm{a}}+\frac{1}{\mathrm{a}}\Big] \\
R &= \frac{1}{4\pi\sigma_0}\Big[\frac{1}{2h+\mathrm{a}}+\frac{1}{\mathrm{a}}\Big]
\end{aligned}
$$

$\underline{\Theta\epsilon\mu\alpha\ \ 3}$

(α)

$$
\vec{H} = \frac{\sigma\omega\mathrm{a}}{4\pi}\int_{z'=-L/2}^{+L/2}\int_{\phi'=0}^{2\pi}\frac{1}{R^3}[(z-z')\cos\phi'\hat{i}_x+(z-z')\sin\phi'\hat{i}_y+(r_T\cos(\phi-\phi')-\mathrm{a})\hat{i}_z]\mathrm{a}d\phi'dz'
$$

where $R = [r_T^2+\mathrm{a}^2-2r_T\mathrm{a}\cos(\phi-\phi')+(z-z')^2]^{1/2}$.

(β)

$$
\vec{H}(z) = \hat{i}_z\frac{\sigma\omega\mathrm{a}}{2}\Big[\frac{(L/2)-z}{[(z-(L/2))^2+\mathrm{a}^2]^{1/2}}+\frac{(L/2)+z}{[(z+(L/2))^2+\mathrm{a}^2]^{1/2}}\Big]
$$

(γ)

$$
\begin{aligned}
\vec{H}(x) &= \hat{i}_z\frac{\sigma\omega\mathrm{a}^3}{4}\int_{z'=-L/2}^{+L/2}(\frac{3z'^2}{R^5}-\frac{1}{R^3})dz' \\
\text{where}\quad R = &\ [x^2+z'^2]^{1/2} \\
\vec{H}(x) &= -\hat{i}_z\frac{\sigma\omega\mathrm{a}^3L}{4[x^2+(L/2)^2]^{3/2}}
\end{aligned}
$$

$\underline{\Theta\epsilon\mu\alpha\ \ 4}$

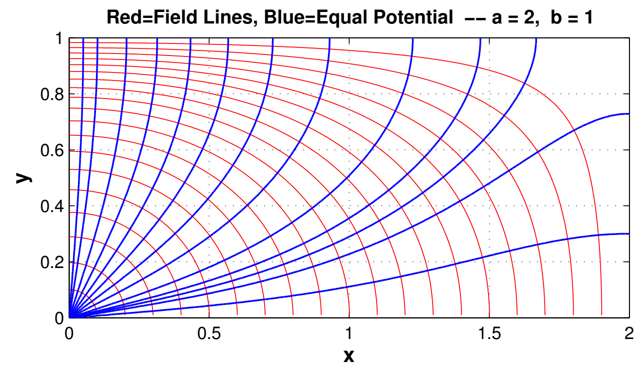
(α)

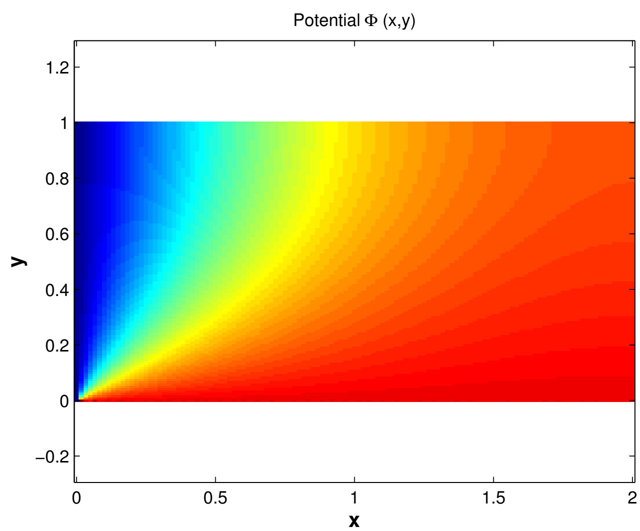
$$
\Phi(x,y) = \sum_{n=0}^{\infty}\frac{4V_0}{\pi}\frac{1}{2n+1}\frac{1}{\cosh(k_nb)}\sin(k_nx)\cosh(k_n(y-b))
$$

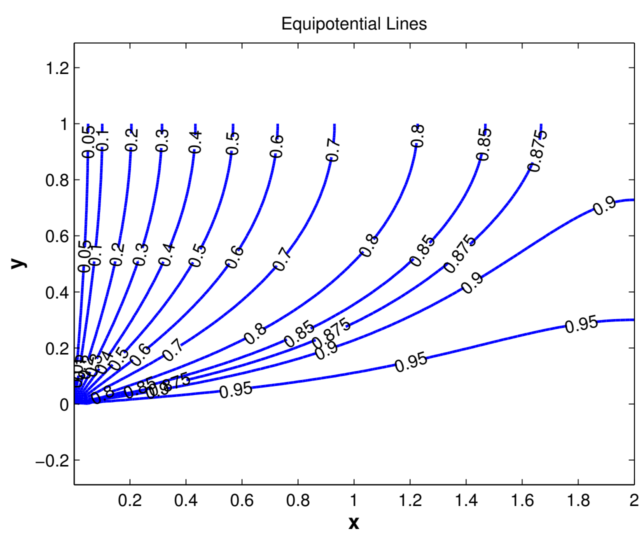
where $k_n = (2n+1)(\pi/2\mathrm{a})$.

(β)

$$
\begin{aligned}
J_x(x,y) &= -\sigma\sum_{n=0}^{\infty}\frac{4V_0}{\pi}\frac{k_n}{2n+1}\frac{1}{\cosh(k_nb)}\cos(k_nx)\cosh(k_n(y-b)) \\
J_y(x,y) &= -\sigma\sum_{n=0}^{\infty}\frac{4V_0}{\pi}\frac{k_n}{2n+1}\frac{1}{\cosh(k_nb)}\sin(k_nx)\sinh(k_n(y-b))
\end{aligned}
$$


Red=Field Lines, Blue=Equal Potential -- a = 2, b = 1
y
x


Potential Φ (x,y)
y
x


Equipotential Lines
y
x

where $k_n = (2n+1)(\pi/2\text{a})$.

(γ)

$$R/dz = \frac{\pi}{4\sigma}\Big[\frac{1}{\sum_n \frac{1}{2n+1}\tanh(k_n b)}\Big]$$

where $k_n = (2n+1)(\pi/2\text{a})$.