

Θεμα 1

(α)

$$\Phi(r, \theta, \phi) = \frac{p_0 r \cos \theta}{4\pi\epsilon_0 [r^2 + d^2 - 2rd \sin \theta \cos \phi]^{3/2}} + \frac{-p_0 (a/d) r \cos \theta}{4\pi\epsilon_0 [r^2 + d'^2 - 2rd' \sin \theta \cos \phi]^{3/2}}$$

$$d' = \frac{a^2}{d}$$

(α) (more accurate)

$$\Phi(r, \theta, \phi) = \frac{p_0 r \cos \theta}{4\pi\epsilon_0 [r^2 + d^2 - 2rd \sin \theta \cos \phi]^{3/2}} + \frac{-p_0 (a/d)^3 r \cos \theta}{4\pi\epsilon_0 [r^2 + d'^2 - 2rd' \sin \theta \cos \phi]^{3/2}}$$

$$d' = \frac{a^2}{d}$$

(β)

$$\sigma(\theta, \phi) = \frac{3p_0 a \cos \theta (a - d \sin \theta \cos \phi) - p_0 a \cos \theta R^2}{4\pi\epsilon_0 R^5} + \frac{-3p_0 d' \cos \theta (a - d' \sin \theta \cos \phi) + d' p_0 \cos \theta R'^2}{4\pi\epsilon_0 R'^5}$$

$$d' = \frac{a^2}{d},$$

$$R = [a^2 + d^2 - 2da \sin \theta \cos \phi]^{1/2},$$

$$R' = [a^2 + d'^2 - 2d'a \sin \theta \cos \phi]^{1/2}.$$

(β) (more accurate)

$$\sigma(\theta, \phi) = \frac{3p_0 a \cos \theta (a - d \sin \theta \cos \phi) - p_0 a \cos \theta R^2}{4\pi\epsilon_0 R^5} + \frac{\frac{a^2}{d^2} - 3p_0 d' \cos \theta (a - d' \sin \theta \cos \phi) + d' p_0 \cos \theta R'^2}{4\pi\epsilon_0 R'^5}$$

$$d' = \frac{a^2}{d},$$

$$R = [a^2 + d^2 - 2da \sin \theta \cos \phi]^{1/2},$$

$$R' = [a^2 + d'^2 - 2d'a \sin \theta \cos \phi]^{1/2}.$$

Θεμα 2

(α)

$$\begin{aligned}\vec{H} &= \hat{i}_z \frac{M_0}{2} \left[\frac{(d/2) - z}{[a^2 + (z - (d/2))^2]^{1/2}} + \frac{(d/2) + z}{[a^2 + (z + (d/2))^2]^{1/2}} \right] \quad |z| > d/2 \\ \vec{H} &= \hat{i}_z \frac{M_0}{2} \left[\frac{(d/2) - z}{[a^2 + (z - (d/2))^2]^{1/2}} + \frac{(d/2) + z}{[a^2 + (z + (d/2))^2]^{1/2}} \right] - M_0 \hat{i}_z \quad |z| < d/2\end{aligned}$$

(β)

$$\Psi_m = \frac{\mu_0}{2} \frac{\pi a^2 d M_0}{b}$$

Θεμα 3

$$L_{12} = L_{21} = -\frac{\mu_0 a^2}{8\pi} \left[\frac{1}{(d - a/2)^2} - \frac{1}{(d + a/2)^2} \right]$$

Θεμα 4

(α)

$$\Phi_1(x) = \frac{V}{a}x, \quad \vec{E}_1 = -\hat{i}_x \frac{V}{a}, \quad \vec{J}_1 = -\hat{i}_x \sigma \frac{V}{a},$$

$$\Phi_2(y) = \frac{V}{a}y, \quad \vec{E}_2 = -\hat{i}_y \frac{V}{a}, \quad \vec{J}_2 = -\hat{i}_y \sigma \frac{V}{a},$$

(β)

$$\Phi(x, y) = \frac{V}{a^2}xy, \quad \text{for } 0 \leq x \leq a, \quad 0 \leq y \leq a,$$

$$\vec{E} = -\frac{V}{a^2}(y\hat{i}_x + x\hat{i}_y), \quad \text{for } 0 \leq x \leq a, \quad 0 \leq y \leq a.$$

(γ)

$$\sigma(x = a, y) = \frac{V}{a^2}\epsilon_0 y,$$

$$\sigma(x, y = a) = \frac{V}{a^2}\epsilon_0 x.$$