

Θεµα 1

(α)

$$\Phi(r, \theta, \phi) = \frac{\sigma a^2}{4\pi\epsilon_0} \int_{\phi'=0}^{2\pi} \int_{\theta'=0}^{\pi/2} \frac{\sin \theta'}{R(r, \theta, \phi, a, \theta', \phi')} d\theta' d\phi'$$

$$R(r, \theta, \phi, a, \theta', \phi') = [r^2 + a^2 - 2ra(\cos \theta \cos \theta' + \sin \theta \sin \theta' \cos(\phi_0 - \phi'))]^{1/2}$$

(β)

$$\Phi(z) = \begin{cases} \frac{\sigma a^2}{2\epsilon_0} \left[ \frac{\sqrt{z^2 + a^2} - |z - a|}{az} \right] & z > 0 \\ \frac{\sigma a^2}{2\epsilon_0} \left[ \frac{\sqrt{z^2 + a^2} - |z + a|}{-az} \right] & z < 0 \end{cases}$$

(γ)

$$\Phi(z = a) - \Phi(z = 0) = \frac{\sigma a}{2\epsilon_0} [\sqrt{2} - 1]$$

Θεµα 2

(α)

$$\Phi_1 = \frac{2V}{\pi} \frac{1}{1 + \sigma_1/\sigma_2} \phi + V \frac{-1 + \sigma_1/\sigma_2}{1 + \sigma_1/\sigma_2}, \quad \frac{\pi}{2} < \phi < \pi$$

$$\Phi_2 = \frac{2V}{\pi} \frac{\sigma_1/\sigma_2}{1 + \sigma_1/\sigma_2} \phi, \quad 0 < \phi < \frac{\pi}{2}$$

(β)

$$R = \frac{\pi(1 + \sigma_1/\sigma_2)}{2\sigma_1 d \ln(a/b)}$$

Θεµα 3

(α)

$$\vec{K} = r_T \sigma \omega \hat{i}_\phi$$

(β)

$$\vec{H} = \frac{\sigma \omega}{4\pi} \int_{\phi'=0}^{2\pi} \int_{r'_T=0}^a \frac{r'^2_T}{R^2} (\hat{i}_\phi \times \hat{i}_R) dr'_T d\phi',$$

$$R = [r_2 + r'^2_T - 2rr'_T \sin \theta \cos(\phi - \phi')]^{1/2},$$

$$\hat{i}_R = \frac{1}{R} [(r \sin \theta \cos \phi - r'_T \cos \phi') \hat{i}_x + (r \sin \theta \sin \phi - r'_T \sin \phi') \hat{i}_y + r \cos \theta \hat{i}_z],$$

$$\hat{i}_\phi = -\sin \phi' \hat{i}_x + \cos \phi' \hat{i}_y$$

(γ)

$$\vec{H}(0, 0, z) = \hat{i}_z \frac{\sigma\omega}{2} \left[ \frac{a^2 + 2z^2}{\sqrt{z^2 + a^2}} - 2|z| \right]$$

(δ)

$$\vec{H}(0, 0, z) = \hat{i}_z \frac{\rho a \omega}{2} \int_{\theta=0}^{\pi} \sin \theta \left[ \frac{a^2 \sin^2 \theta + 2(z - a \cos \theta)^2}{\sqrt{(z - a \cos \theta)^2 + a^2 \sin^2 \theta}} - 2|z - a \cos \theta| \right] d\theta$$

Θεµα 4

$$\Phi(r_T, \phi) = \begin{cases} \frac{4V_0}{\pi} \sum_{m=1,3,5,\dots}^{\infty} \frac{1}{m} \left( \frac{r_T}{a} \right)^m \sin m\phi & 0 < r_T < a \\ \frac{4V_0}{\pi} \sum_{m=1,3,5,\dots}^{\infty} \frac{1}{m} \frac{r_T^m - b^{2m} r_T^{-m}}{a^m (1 - (b/a)^{2m})} \sin m\phi & a < r_T < b \end{cases}$$