ΠΕΔΙΑ B 27/01/2010

Θεμα 1

(α)

$$\begin{aligned}
\Phi(x,y,z) &= \frac{1}{4\pi\epsilon_1}\Big[\frac{q_1}{R_1}+\frac{q_1'}{R_1'}+\frac{q_2''}{R_2''}\Big] \qquad (z\geq 0)\\
& q_1' = \frac{\epsilon_1-\epsilon_2}{\epsilon_1+\epsilon_2}q_1\\
& q_2'' = \frac{2\epsilon_1}{\epsilon_1+\epsilon_2}q_2\\
& R_1 = [x^2+y^2+(z-h_1)^2]^{1/2}\\
& R_1' = [x^2+y^2+(z+h_1)^2]^{1/2}\\
& R_2'' = [x^2+y^2+(z+h_2)^2]^{1/2}\\
\Phi(x,y,z) &= \frac{1}{4\pi\epsilon_2}\Big[\frac{q_1''}{R_1''}+\frac{q_2}{R_2}+\frac{q_2'}{R_2'}\Big] \qquad (z\leq 0)\\
& q_1'' = \frac{2\epsilon_2}{\epsilon_1+\epsilon_2}q_1\\
& q_2' = \frac{\epsilon_2-\epsilon_1}{\epsilon_1+\epsilon_2}q_2\\
& R_1'' = [x^2+y^2+(z-h_1)^2]^{1/2}\\
& R_2 = [x^2+y^2+(z+h_2)^2]^{1/2}\\
& R_2' = [x^2+y^2+(z-h_2)^2]^{1/2}
\end{aligned}$$

(β)

$$\begin{aligned}
\vec{E}(x,y,z) &= \frac{1}{4\pi\epsilon_1}\Big[\frac{q_1}{R_1^2}\hat{i}_{R_1}+\frac{q_1'}{R_1'^2}\hat{i}_{R_1'}+\frac{q_2''}{(R_2'')^2}\hat{i}_{R_2''}\Big] \qquad (z\geq 0)\\
& \hat{i}_{R_1} = \frac{\vec{r}-h_1\hat{i}_z}{R_1} \qquad \vec{r}=x\hat{i}_x+y\hat{i}_y+z\hat{i}_z\\
& \hat{i}_{R_1'} = \frac{\vec{r}+h_1\hat{i}_z}{R_1'}\\
& \hat{i}_{R_2''} = \frac{\vec{r}+h_2\hat{i}_z}{R_2''}\\
\vec{E}(x,y,z) &= \frac{1}{4\pi\epsilon_2}\Big[\frac{q_1''}{(R_1'')^2}\hat{i}_{R_1''}+\frac{q_2}{R_2^2}\hat{i}_{R_2}+\frac{q_2'}{(R_2')^2}\hat{i}_{R_2'}\Big] \qquad (z\leq 0)\\
& \hat{i}_{R_1''} = \frac{\vec{r}-h_1\hat{i}_z}{R_1''}
\end{aligned}$$

$$\hat{i}_{R_2} = \frac{\vec{r} + h_2 \hat{i}_z}{R_2}$$
$$\hat{i}_{R'_2} = \frac{\vec{r} - h_2 \hat{i}_z}{R'_2}$$

(γ)

$$\frac{q_1}{q_2} = \frac{h_1^2}{h_2^2}\frac{\epsilon_1}{\epsilon_2}$$

(δ)

$$\vec{F} = \Big[\frac{q_1 q_2}{2\pi(\epsilon_1+\epsilon_2)}\frac{1}{(h_1+h_2)^2} + \frac{q_1^2}{4\pi\epsilon_1}\frac{\epsilon_1-\epsilon_2}{\epsilon_1+\epsilon_2}\frac{1}{4h_1^2}\Big]\hat{i}_z$$

<u>$\Theta\epsilon\mu\alpha$ 2</u>

(α)

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \end{bmatrix} = \frac{1}{4\pi\epsilon_0}\begin{bmatrix} \frac{1}{R} & \frac{1}{d} & \frac{1}{d} \\ \frac{1}{d} & \frac{1}{R} & \frac{1}{d} \\ \frac{1}{d} & \frac{1}{d} & \frac{1}{R} \end{bmatrix}\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}$$

(β)

$$Q_1 = -\frac{2R}{d}Q$$

$$Q_2 = \Big(\frac{2R^2}{d^2} - \frac{R}{d}\Big)Q \simeq -\frac{R}{d}Q$$

$$Q_3 = \Big(-2\frac{2R^3}{d^3} + 3\frac{R^2}{d^2}\Big)Q \simeq 3\frac{R^2}{d^2}Q \simeq 0$$

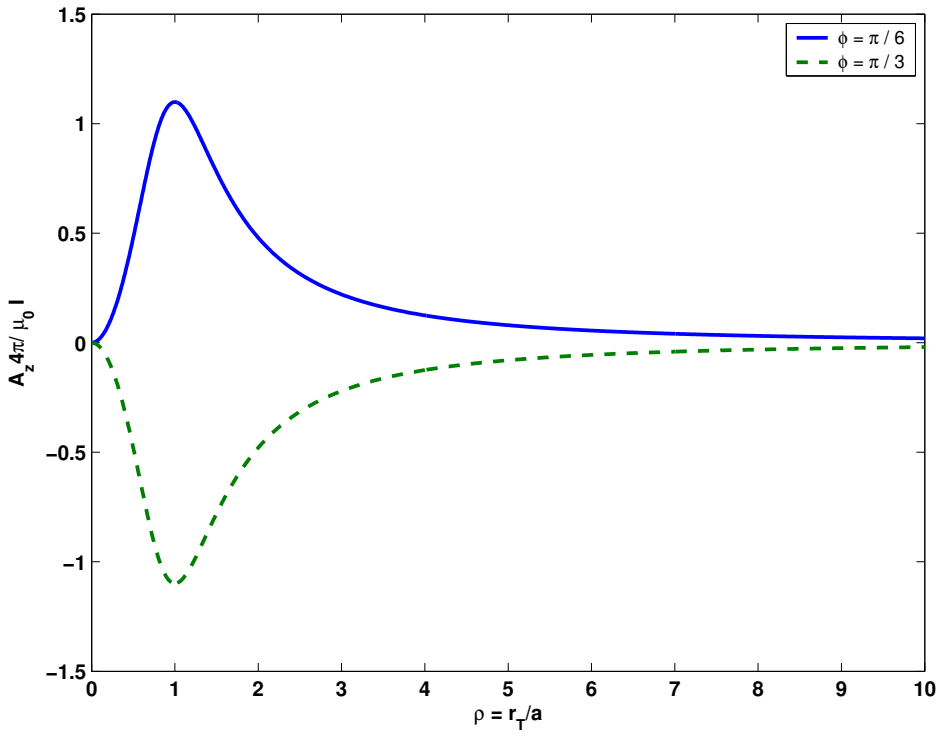
<u>$\Theta\epsilon\mu\alpha$ 3</u>

(α)

$$\vec{A} = \hat{i}_z \frac{\mu_0 I}{4\pi}\ln\Big[\frac{a^4 + r_T^4 + 2a^2 r_T^2\cos 2\phi}{a^4 + r_T^4 - 2a^2 r_T^2\cos 2\phi}\Big]$$

(β)

$$H_{r_T} = -\frac{2I}{\pi r_T}\frac{\rho^2(1+\rho^4)}{(1+\rho^4)^2 - 4\rho^4\cos^2 2\phi}\sin 2\phi \qquad (\rho = \frac{r_T}{a})$$

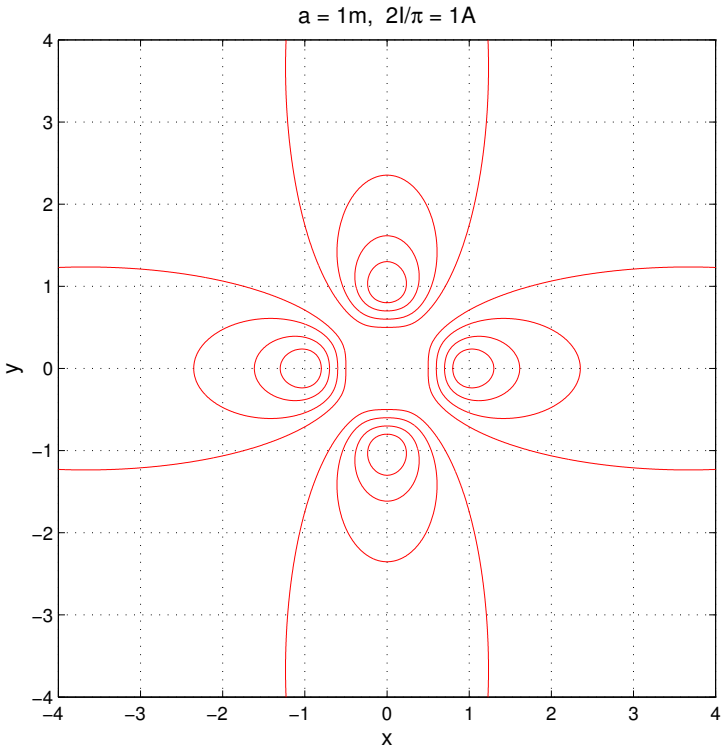


$$H_\phi \quad = \quad -\frac{2I}{\pi r_T}\frac{\rho^2(1-\rho^4)}{(1+\rho^4)^2-4\rho^4\cos^2 2\phi}\cos 2\phi \qquad (\rho = \frac{r_T}{a})$$

(γ)

$$H_{r_T} = 0 \quad \text{for} \quad \phi = 0,\ \pi/2,\ \pi,\ 3\pi/2$$

$$H_\phi = 0 \quad \text{for} \quad \phi = \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4$$



$\underline{\Theta\epsilon\mu\alpha \;\; 4}$

(α)

$$\Phi(x,y) = \exp(-a|y|)\frac{\sigma_0}{2\epsilon_0 a}\cos(ax)$$

(β)

$$\Phi(x,y) = \frac{\sigma_0}{\epsilon_0 a}\exp(-ad)\cos(ax)\sinh(ay) \qquad 0 \le y \le d$$

(γ)

$$\sigma_c = -\sigma_0 \exp(-ad)\cos(ax)$$