ΠΕΔΙΑ B 01/03/2011

Θεμα 1

(α)

$$\Phi(x,y,z) = \frac{\lambda_0}{4\pi\epsilon_0}\ln\Big[\frac{x+L+[(x+L)^2+y^2+(z-h)^2]^{1/2}}{x-L+[(x-L)^2+y^2+(z-h)^2]^{1/2}}\;\frac{x-L+[(x-L)^2+y^2+(z+h)^2]^{1/2}}{x+L+[(x+L)^2+y^2+(z+h)^2]^{1/2}}\Big],$$

for $z > 0$,

$$\Phi(x,y,z) = 0,$$

for $z < 0$.

(β)

$$\vec{E}(z) = \hat{i}_z\frac{\lambda_0}{4\pi\epsilon_0}\Big[\frac{2L}{(z-h)[L^2+(z-h)^2]^{1/2}} - \frac{2L}{(z+h)[L^2+(z+h)^2]^{1/2}}\Big],$$

for $z > 0$ and $z \neq h$,

$$\vec{E}(z) = 0,$$

for $z < 0$.

(γ)

$$[\lambda_j] = \frac{Q}{\sum_j \hat{\lambda}_j \Delta x}[A_{ij}]^{-1}[1]$$

where $[\lambda_j] = [\lambda(x_1), \cdots, \lambda(x_N)]^T$, $A_{ij} = (\Delta x/4\pi\epsilon_0)(1/R_{ji} - 1/R_{j'i})$,

$R_{ji} = |x_j - x_i|$, $\qquad R_{j'i} = [(x_j - x_i)^2 + 4h^2]^{1/2}$,

and $[1] = [1, 1, \cdots, 1]^T$

$$\Phi(x,y,z) = \sum_{j=1}^{N}\frac{\lambda_j \Delta x}{4\pi\epsilon_0}[\frac{1}{R_j} - \frac{1}{R_{j'}}],$$

where $R_j = [(x-x_j)^2 + y^2 + (z-h)^2]^{1/2}$, and $R_{j'} = [(x-x_j)^2 + y^2 + (z+h)^2]^{1/2}$.

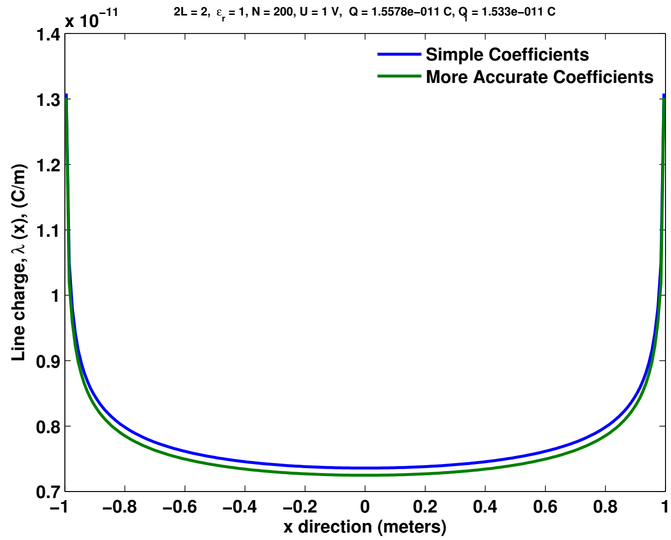

Figure 1: Characteristic Distribution of $\lambda(x)$

<u>Θεμα 2</u>

(α)

$$\Phi(\phi) = \frac{-V_0}{\ln[(\sigma_0 + \sigma_1(\pi/2))/\sigma_0]} \ln\left[\frac{\sigma_0 + \sigma_1\phi}{\sigma_0 + \sigma_1(\pi/2)}\right]$$

(β)

$$R = \frac{\ln[(\sigma_0 + \sigma_1(\pi/2))/\sigma_0]}{\sigma_1 \ln(b/a) h}$$

(γ)

$$\Phi(\phi = 0) = V_0, \qquad \Phi(\phi = \frac{\pi}{2}) = 0, \qquad \frac{\partial\Phi}{\partial r_T}\Big|_{r_T=\mathrm{a}} = 0, \qquad \frac{\partial\Phi}{\partial r_T}\Big|_{r_T=\mathrm{b}} = 0.$$

<u>Θεμα 3</u>

(α)

$$\vec{H} = \frac{J_0}{4\pi}\int_{z'=-L}^{+L}\int_{r_T=a}^{b}\int_{\phi'=0}^{2\pi}\frac{1}{R^3}[(z-z')\cos\phi'\hat{i}_x + (z-z')\sin\phi'\hat{i}_y - (r_T\cos(\phi-\phi') - r'_T)\hat{i}_z]r'_T dr'_T d\phi' dz'$$

where $R = [r_T^2 + r_T'^2 - 2r_T r'_T\cos(\phi-\phi') + (z-z')^2]^{1/2}$.

(β)

$$\vec{H}(z) = \hat{i}_z\frac{J_0}{2}\int_{z'=-L}^{+L}\int_{r_T=a}^{b}\frac{r_T'^2}{[r_T'^2 + (z-z')^2]^{3/2}}dr'_T dz'$$

(γ)

$$\vec{m} \;=\; \hat{i}_z 2J_0\pi L\frac{(b^3 - \mathrm{a}^3)}{3}$$

$$\vec{H}(x) = -\hat{i}_z \frac{J_0 L(b^3 - \mathrm{a}^3)}{6x^3}$$

Θεμα 4

(α)

$$\Phi(x,y) = \sum_{m=0}^{\infty} \frac{4V_0}{(2m+1)\pi \sinh(k_m \mathrm{a})}[\sinh(k_m y) - \sinh(k_m(y-\mathrm{a}))]\sin(k_m x) - \sum_{m=0}^{\infty} \frac{4V_0}{(2m+1)\pi \sinh(k_m \mathrm{a})}[\sinh(k_m x) - \sinh(k_m(x-\mathrm{a}))]\sin(k_m y)$$

where $k_m = (2m+1)(\pi/\mathrm{a})$.

(β)

$$E_x(x,y) = -\sum_{m=0}^{\infty} \frac{4V_0}{(2m+1)\pi \sinh(k_m \mathrm{a})}[\sinh(k_m y) - \sinh(k_m(y-\mathrm{a}))]k_m \cos(k_m x) + \sum_{m=0}^{\infty} \frac{4V_0}{(2m+1)\pi \sinh(k_m \mathrm{a})}[\cosh(k_m x) - \cosh(k_m(x-\mathrm{a}))]k_m \sin(k_m y)$$

$$E_y(x,y) = -\sum_{m=0}^{\infty} \frac{4V_0}{(2m+1)\pi \sinh(k_m \mathrm{a})}[\cosh(k_m y) - \cosh(k_m(y-\mathrm{a}))]k_m \sin(k_m x) + \sum_{m=0}^{\infty} \frac{4V_0}{(2m+1)\pi \sinh(k_m \mathrm{a})}[\sinh(k_m x) - \sinh(k_m(x-\mathrm{a}))]k_m \cos(k_m y)$$

where $k_m = (2m+1)(\pi/\mathrm{a})$.

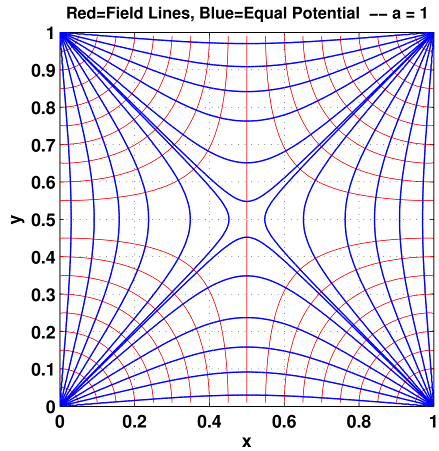

Figure 2: Electric field Lines (red) and Equipotential Lines (blue)

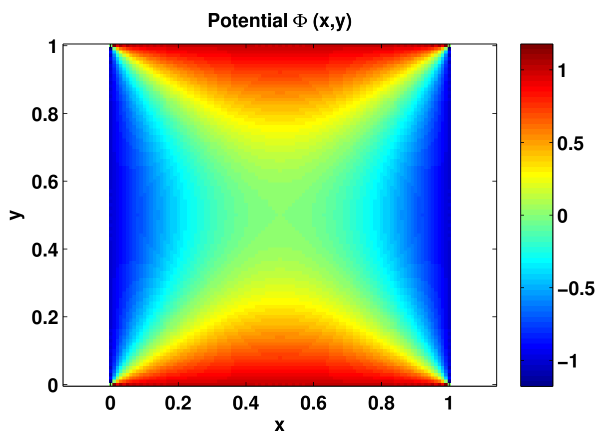

Figure 3: Colormap of the Electric Potential Distribution

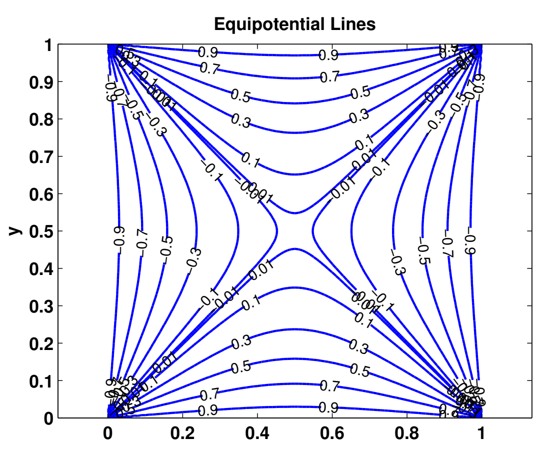

Figure 4: Equipotential Lines at Specific Potential Levels