

ΑΣΚΗΣΗ 1:

$$(α) z(t) = 3 \cos(2\pi 30t - \pi/4) \quad \omega = 2\pi 30$$

$$\leadsto Z = 3 e^{-j\pi/4} = 3 \cos(\pi/4) - j 3 \sin(\pi/4) = 3 \frac{1-j}{\sqrt{2}}$$

$$(β) z(x,t) = 4 e^{-3x} \sin(\omega t - \pi/6) =$$

$$= 4 e^{-3x} \cos(\omega t - \frac{\pi}{6} - \frac{\pi}{2}) = 4 e^{-3x} \cos(\omega t - \frac{2\pi}{3})$$

$$\leadsto Z = 4 e^{-3x} e^{-j2\pi/3}$$

$$(γ) z(t) = 2 \sin[\omega t + \pi/3] + 3 \cos[\omega t - \pi/6] \Rightarrow$$

$$z(t) = 2 \cos[\omega t + \frac{\pi}{3} - \frac{\pi}{2}] + 3 \cos[\omega t - \pi/6] =$$

$$= 2 \cos[\omega t - \pi/6] + 3 \cos[\omega t - \pi/6] = 5 \cos(\omega t - \pi/6)$$

$$\leadsto Z = 5 e^{-j\pi/6}$$

$$(δ) Z = \sqrt{j} 6 e^{j\pi/4} = (e^{j\pi/2})^{1/2} 6 e^{j\pi/4} = 6 e^{j\pi/4} e^{j\pi/4} = 6 e^{j\pi/2}$$

$$z(t) = \operatorname{Re}\{Z e^{j\omega t}\} = \operatorname{Re}\{6 e^{j\pi/2} e^{j\omega t}\} =$$

$$= 6 \cos(\omega t + \pi/2) = -6 \sin(\omega t)$$

$$(ε) Z = 3 - j4 = \sqrt{3^2 + 4^2} e^{-j \tan^{-1}(4/3)} = 5 e^{-j53.13^\circ}$$

$$z(t) = \operatorname{Re}\{Z e^{j\omega t}\} = \operatorname{Re}\{5 e^{-j53.13^\circ} e^{j\omega t}\} =$$

$$= 5 \cos(\omega t - 53.13^\circ)$$

$$(σ) Z = -3 e^{j\pi/3} = 3 e^{j\pi + j\pi/3} = 3 e^{j4\pi/3}$$

$$z(t) = \operatorname{Re}\{3 e^{j4\pi/3} e^{j\omega t}\} = 3 \cos(\omega t + 4\pi/3)$$

ΑΣΚΗΣΗ 2:

$$\vec{E} = E_0 (\hat{i}_x + \hat{i}_y - \hat{i}_z) e^{-j \frac{2\omega}{c} \left(\frac{x}{\sqrt{6}} + \frac{y}{\sqrt{6}} + \frac{2z}{\sqrt{6}} \right)}$$

$$\begin{aligned} \alpha) \quad \vec{k} \cdot \vec{r} &= \frac{2\omega}{c} \left(\frac{x}{\sqrt{6}} + \frac{y}{\sqrt{6}} + \frac{2z}{\sqrt{6}} \right) = \\ &= \underbrace{\frac{2\omega}{c} \left(\frac{1}{\sqrt{6}} \hat{i}_x + \frac{1}{\sqrt{6}} \hat{i}_y + \frac{2}{\sqrt{6}} \hat{i}_z \right)}_{\vec{k}} \cdot \underbrace{\left(x \hat{i}_x + y \hat{i}_y + z \hat{i}_z \right)}_{\vec{r}} \end{aligned}$$

$$\begin{aligned} \text{Επομένως, } \vec{k} &= \frac{2\omega}{c} \frac{1}{\sqrt{6}} (\hat{i}_x + \hat{i}_y + 2\hat{i}_z) = k \hat{k} = \\ &= \frac{2\omega}{c} \left[\frac{1}{\sqrt{6}} (\hat{i}_x + \hat{i}_y + 2\hat{i}_z) \right] \rightarrow k = \frac{2\omega}{c} \end{aligned}$$

$$\left. \begin{aligned} \text{Όπως } k &= k_0 \sqrt{\epsilon_r \mu_r} = \frac{\omega}{c} \sqrt{\mu_r \epsilon_r} \\ k &= \frac{2\omega}{c} \end{aligned} \right\} \Rightarrow \sqrt{\mu_r \epsilon_r} = 2$$

$\mu_r = 1$ (μη μαγνητικό υλικό)

οπότε $\epsilon_r = 4$

$$\beta) \text{ Το κυματοδιάνυσμα ήδη βρέθηκε } \vec{k} = \frac{2\omega}{c} \frac{1}{\sqrt{6}} (\hat{i}_x + \hat{i}_y + 2\hat{i}_z)$$

παρατηρούμε ότι $\vec{E}_0 = E_0 (\hat{i}_x + \hat{i}_y - \hat{i}_z)$ και

$$\vec{k} \cdot \vec{E}_0 = E_0 \left(\frac{2\omega}{c} \right) \frac{1}{\sqrt{6}} (1 + 1 - 2) = 0 \text{ όπως είναι αναγκαίο από}$$

τις εξισώσεις του Maxwell.

$$\gamma) \quad Z = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_r \epsilon_0}} = \frac{1}{\sqrt{\epsilon_r}} \sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{Z_0}{\sqrt{4}} = \frac{Z_0}{2} \approx 60\pi \text{ } (\Omega)$$

$$\delta) \quad \vec{P} = \frac{1}{2} \text{Re} \{ \vec{E} \times \vec{H}^* \}$$

$$\vec{H} = \frac{1}{Z} (\hat{k} \times \vec{E}) = \frac{1}{Z} \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ E_0 A & E_0 A & -E_0 A \end{vmatrix} \Rightarrow \text{(όπου } A = e^{-j \frac{2\omega}{c} \left(\frac{x}{\sqrt{6}} + \frac{y}{\sqrt{6}} + \frac{2z}{\sqrt{6}} \right)})$$

$$\begin{aligned} \vec{H} &= \frac{1}{Z} \left[\hat{i}_x \left(-\frac{3E_0}{\sqrt{6}} \right) A - \hat{i}_y \left(-\frac{3E_0}{\sqrt{6}} \right) A + \hat{i}_z (\emptyset) A \right] = \\ &= \frac{E_0}{Z} \frac{3}{\sqrt{6}} (-\hat{i}_x + \hat{i}_y) e^{-j \frac{2\omega}{c} \left(\frac{x}{\sqrt{6}} + \frac{y}{\sqrt{6}} + \frac{2z}{\sqrt{6}} \right)} \end{aligned}$$

$$\vec{H}^* = \frac{E_0}{Z} \frac{3}{\sqrt{6}} (-\hat{i}_x + \hat{i}_y) A^* = H_x^* \hat{i}_x + H_y^* \hat{i}_y$$

Επομένως:

$$\vec{P} = \frac{1}{2} \operatorname{Re} \left\{ \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ E_0 A & E_0 A & -E_0 A \\ H_x^* & H_y^* & 0 \end{vmatrix} \right\} =$$

$$= \frac{1}{2} \operatorname{Re} \left\{ \hat{i}_x \left(E_0 A \frac{E_0}{Z} \frac{3}{\sqrt{6}} A^* \right) - \hat{i}_y \left(E_0 A \left(\frac{-E_0}{Z} \right) \frac{3}{\sqrt{6}} A^* \right) + \hat{i}_z \left(E_0 A \frac{E_0}{Z} \frac{3}{\sqrt{6}} A^* - E_0 A \left(-\frac{E_0}{Z} \frac{3}{\sqrt{6}} A^* \right) \right) \right\} \Rightarrow (AA^* = 1)$$

$$\vec{P} = \frac{1}{2} \operatorname{Re} \left\{ \frac{E_0^2}{Z} \frac{3}{\sqrt{6}} \hat{i}_x + \frac{E_0^2}{Z} \frac{3}{\sqrt{6}} \hat{i}_y + 2 \frac{E_0^2}{Z} \frac{3}{\sqrt{6}} \hat{i}_z \right\} =$$

$$= \frac{1}{2} \frac{E_0^2}{Z} \frac{3}{\sqrt{6}} (\hat{i}_x + \hat{i}_y + 2\hat{i}_z) = \frac{E_0^2}{120\pi} \frac{3}{\sqrt{6}} (\hat{i}_x + \hat{i}_y + 2\hat{i}_z) \quad (\text{W/m}^2)$$

ΑΣΚΗΣΗ 3:

$$\vec{H}(y=0, z) = \hat{x} 0.1 \sin(10^{10}\pi t - \pi/3) \quad (\text{A/m})$$

$$\omega = 10^{10}\pi \text{ (rad/sec)} \quad \epsilon_r = 80, \mu_r = 1, \sigma = 4 \text{ S/m}$$

$$\begin{aligned} \alpha &= \omega \sqrt{\mu\epsilon} \left\{ \frac{1}{2} \left(-1 + \left(1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right)^{1/2} \right) \right\}^{1/2} = \omega \sqrt{\mu\epsilon} A \\ \beta &= \omega \sqrt{\mu\epsilon} \left\{ \frac{1}{2} \left(+1 + \left(1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right)^{1/2} \right) \right\}^{1/2} = \omega \sqrt{\mu\epsilon} B \end{aligned}$$

$$\frac{\sigma}{\omega\epsilon} = \frac{4}{10^{10}\pi \cdot 8.854 \cdot 10^{-12} \cdot 80} = 0.17975$$

$$A = \left\{ \frac{1}{2} \left[-1 + \left(1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right)^{1/2} \right] \right\}^{1/2} = 8.95194477 \cdot 10^{-2}$$

$$B = \left\{ \frac{1}{2} \left[+1 + \left(1 + \left(\frac{\sigma}{\omega\epsilon} \right)^2 \right)^{1/2} \right] \right\}^{1/2} = 1.00399887$$

$$\begin{aligned} \alpha &= \omega \sqrt{\mu\epsilon} A = 10^{10}\pi \left[4\pi \cdot 10^{-7} \cdot 80 \cdot 8.854 \cdot 10^{-12} \right]^{1/2} \cdot 8.95194477 \cdot 10^{-2} \text{ 1/m} \\ &= 10^{10}\pi \cdot 29.8345 \cdot 10^{-9} \cdot 8.95194477 \cdot 10^{-2} \text{ (1/m)} = 83.9 \text{ (1/m)} \end{aligned}$$

$$\begin{aligned} \beta &= \omega \sqrt{\mu\epsilon} B = 10^{10}\pi \left[29.8345 \cdot 10^{-9} \right] \cdot 1.00399887 = \\ &= 941.03 \text{ (1/m)} \end{aligned}$$

$$\begin{aligned} Z &= \left[\frac{j\omega\mu}{\sigma + j\omega\epsilon} \right]^{1/2} = \left[\frac{j\omega\mu}{j\omega\epsilon \left[1 + \frac{\sigma}{j\omega\epsilon} \right]} \right]^{1/2} = \frac{\sqrt{\mu/\epsilon}}{\left[1 - j\frac{\sigma}{\omega\epsilon} \right]^{1/2}} = \\ &= \frac{\sqrt{\mu_0/\epsilon_0} (1/\sqrt{\epsilon_r})}{\left(1 - j\frac{\sigma}{\omega\epsilon} \right)^{1/2}} = \frac{376.734 (1/\sqrt{80})}{(1 - j0.17975)^{1/2}} = 41.786 \angle +5.0952^\circ (\Omega) \end{aligned}$$

$$\rightarrow Z = 41.786 \angle +5.0952^\circ (\Omega) = 41.62 + j3.71 (\Omega)$$

$$\begin{aligned} u &= \frac{c}{\sqrt{\mu_r \epsilon_r}} = \frac{c}{\sqrt{80}} = 1.11803399 \cdot 10^{-1} c = 3.354 \cdot 10^7 \text{ m/s} \\ &\text{(χρησιμοποιώντας } c \approx 3 \cdot 10^8 \text{ m/s)} \end{aligned}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{941.03} \text{ m} = 6.677 \cdot 10^{-3} \text{ m} = 6.677 \text{ mm}$$

$$\delta = 1/\alpha = (1/83.9) \text{ m} = 1.1919 \cdot 10^{-2} \text{ m} = 1.1919 \text{ cm}$$

$$(\beta) \quad \vec{H}(y,t) = \hat{i}_x 0.1 e^{-\alpha y} \cos \left[10^{10} \pi t - \frac{\pi}{3} - \frac{\pi}{2} - \beta y \right]$$

$$|\vec{H}| = 0.01 \Rightarrow 0.01 = 0.1 e^{-\alpha y} \Rightarrow e^{-\alpha y} = 0.1 \Rightarrow$$

$$\Rightarrow -\alpha y = \ln(0.1) \Rightarrow y = -\frac{\ln(0.1)}{\alpha} = 2.7444 \cdot 10^{-2} \text{ m}$$

$$\begin{aligned} (\gamma) \quad \vec{H}(y=0.5 \text{ m}, t) &= \hat{i}_x 0.1 e^{-\alpha \cdot 0.5} \cos \left[10^{10} \pi t - \frac{\pi}{3} - \frac{\pi}{2} - 941.03 \cdot 0.5 \right] \\ &= \hat{i}_x 6.044 \cdot 10^{-20} \cos \left[10^{10} \pi t - \frac{\pi}{3} - \frac{\pi}{2} - 74.884 (2\pi) \right] \\ &= \hat{i}_x 6.044 \cdot 10^{-20} \cos \left[10^{10} \pi t - \frac{\pi}{3} - \frac{\pi}{2} - 0.884 \cdot 2\pi \right] \\ &= \hat{i}_x 6.044 \cdot 10^{-20} \cos \left[10^{10} \pi t - 2.6029\pi \right] \\ &= \hat{i}_x 6.044 \cdot 10^{-20} \cos \left[10^{10} \pi t - 0.6029\pi \right] \end{aligned}$$

Είναι ευκολότερο να βρούμε τον φασιδέτη του \vec{E} και μετά το στιγμιαίο πεδίο. Ο φασιδέτης του μαγνητικού πεδίου είναι :

$$\vec{H} = \hat{i}_x 0.1 e^{-\alpha y} e^{-j\beta y} e^{-j5\pi/6}$$

Από τις εξισώσεις του Maxwell: (για επίπεδο κύμα):

$$-j(\vec{k} \times \vec{H}) = (\sigma + j\omega\epsilon) \vec{E} \Rightarrow \vec{E} = \frac{-j}{\sigma + j\omega\epsilon} k_c (\hat{k} \times \vec{H}) \Rightarrow$$

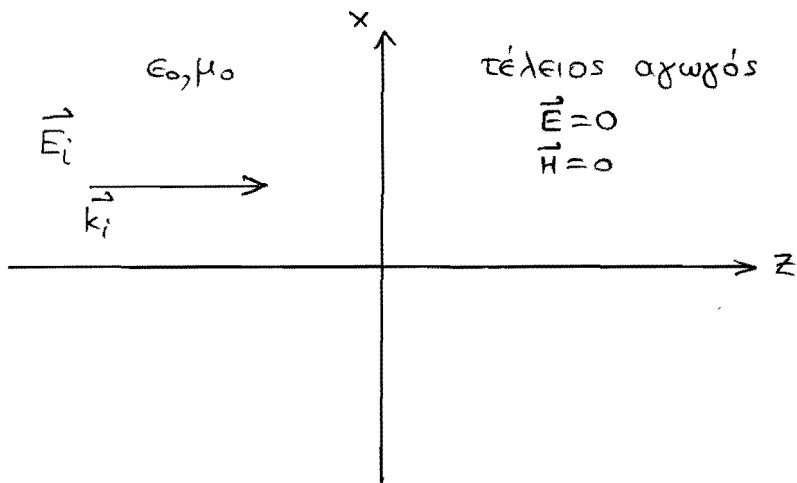
$$\vec{E} = -\frac{\sqrt{j\omega\mu(\sigma + j\omega\epsilon)}}{(\sigma + j\omega\epsilon)} (\hat{k} \times \vec{H}) = -\left(\frac{j\omega\mu}{\sigma + j\omega\epsilon}\right)^{1/2} \hat{k} \times \vec{H} =$$

$$= -Z (\hat{k} \times \vec{H}) = -Z [\hat{i}_y \times \hat{i}_x H] =$$

$$= -ZH(-\hat{i}_z) = \hat{i}_z 41.786 e^{j5.0952^\circ} 0.1 e^{-\alpha y} e^{-j\beta y} e^{-j5\pi/6}$$

$$\begin{aligned} \Rightarrow \vec{E}(y=0.5 \text{ m}, t) &= \hat{i}_z 41.786 \cdot 0.1 \cdot e^{-\alpha \cdot 0.5} \cos \left[10^{10} \pi t - \frac{5\pi}{6} - 941.03 \cdot 0.5 \right] \\ &= \hat{i}_z 2.526 \cdot 10^{-18} \cos \left[10^{10} \pi t - 0.6029\pi + \frac{5.0952 \times \pi}{180} \right] \end{aligned}$$

ΑΣΚΗΣΗ 4:



$$\vec{E}_i = E_0 [2\hat{i}_x - j\hat{i}_y] e^{-j\beta z}$$

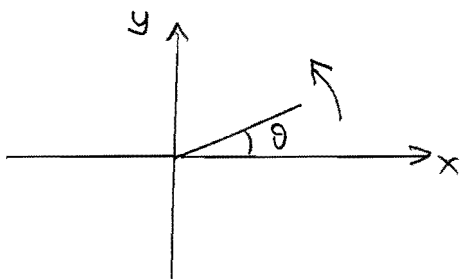
(α) $\vec{k}_i = k_0 \hat{i}_z \quad k_0 = \omega \sqrt{\mu_0 \epsilon_0}$

$$\vec{k}_i \cdot \vec{r} = k_0 z = \beta z \quad \sim \quad \beta = \omega \sqrt{\mu_0 \epsilon_0}$$

$$E_x = 2E_0 e^{-j\beta z} \quad \sim \quad \mathcal{E}_x = 2E_0 \cos(\omega t - \beta z)$$

$$E_y = -jE_0 e^{-j\beta z} \quad \sim \quad \mathcal{E}_y = E_0 \cos(\omega t - \beta z - \frac{\pi}{2}) = E_0 \sin(\omega t - \beta z)$$

$$\left(\frac{\mathcal{E}_x}{2E_0}\right)^2 + \left(\frac{\mathcal{E}_y}{E_0}\right)^2 = \cos^2(\quad) + \sin^2(\quad) = 1 \quad \sim \quad \text{ελλαπττική πόρωση}$$



$$\phi = \phi_y - \phi_x = -\pi/2$$

$$\tan \vartheta = \frac{\mathcal{E}_y}{\mathcal{E}_x} = \frac{1}{2} \tan(\omega t - \beta z)$$

RH ελλαπττική πόρωση

(β) $\vec{E}_1 = \vec{E}_i + \vec{E}_r$

$$\vec{E}_r = (E_{rx} \hat{i}_x + E_{ry} \hat{i}_y) e^{+jk_0 z}$$

Εφαρμοσμένη συνιστώσα του ηλεκτρικού πεδίου συνεχώς:

$$E_{x1} = E_{x2} = 0 \quad \sim \quad 2E_0 + E_{rx} = 0 \quad \sim \quad E_{rx} = -2E_0$$

$$E_{y1} = E_{y2} = 0 \quad \sim \quad -jE_0 + E_{ry} = 0 \quad \sim \quad E_{ry} = +jE_0$$

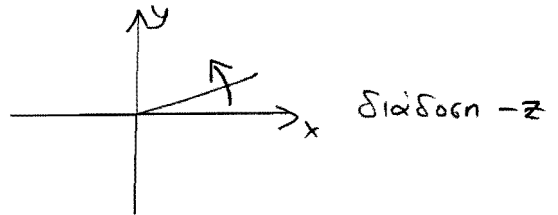
$$\vec{E}_r = (-2E_0 \hat{x} + jE_0 \hat{y}) e^{jk_0 z}$$

$$E_{rx} = 2E_0 \cos(\omega t + kz + \pi) = -2E_0 \cos(\omega t + k_0 z)$$

$$E_{ry} = E_0 \cos(\omega t + kz + \pi/2) = -E_0 \sin(\omega t + k_0 z)$$

$$\left(\frac{E_{rx}}{2E_0}\right)^2 + \left(\frac{E_{ry}}{E_0}\right)^2 = 1 \sim \text{ελλειπτική πόαση}$$

$$\tan \vartheta = \frac{E_{ry}}{E_{rx}} = \frac{1}{2} \tan(\omega t + k_0 z)$$



οπότε είναι LH ελλειπτική πόαση

$$\vec{H}_r = \frac{1}{Z_0} (\hat{L}_z \times \vec{E}_r) = \frac{-1}{Z_0} \hat{L}_z \times (-2E_0 \hat{x} + jE_0 \hat{y}) e^{jk_0 z}$$

$$= \left(\frac{2E_0}{Z_0} \hat{y} + j \frac{E_0}{Z_0} \hat{x} \right) e^{jk_0 z}$$

$$\vec{H}_i = \frac{1}{Z_0} (+\hat{L}_z \times \vec{E}_i) = \frac{1}{Z_0} \hat{L}_z \times (2E_0 \hat{x} - jE_0 \hat{y}) e^{-jk_0 z} =$$

$$= \left(\frac{2E_0}{Z_0} \hat{y} + j \frac{E_0}{Z_0} \hat{x} \right) e^{-jk_0 z}$$

$$(\gamma) \vec{E}_1 = \vec{E}_i + \vec{E}_r = E_0 [2\hat{x} - j\hat{y}] e^{-jk_0 z} + E_0 [-2\hat{x} + j\hat{y}] e^{jk_0 z}$$

$$= 2E_0 (e^{-jk_0 z} - e^{jk_0 z}) \hat{x} + E_0 j [e^{jk_0 z} - e^{-jk_0 z}] \hat{y} =$$

$$= 2E_0 (-2j \sin(k_0 z)) \hat{x} + E_0 j 2j \sin(k_0 z) \hat{y}$$

$$\vec{E}_1 = \hat{x} 4E_0 \sin(k_0 z) \cos(\omega t - \frac{\pi}{2}) \hat{x} + \hat{y} 2E_0 \sin(k_0 z) \cos(\omega t)$$

$$= \hat{x} 4E_0 \sin(k_0 z) \sin(\omega t) + \hat{y} 2E_0 \sin(k_0 z) \cos(\omega t)$$

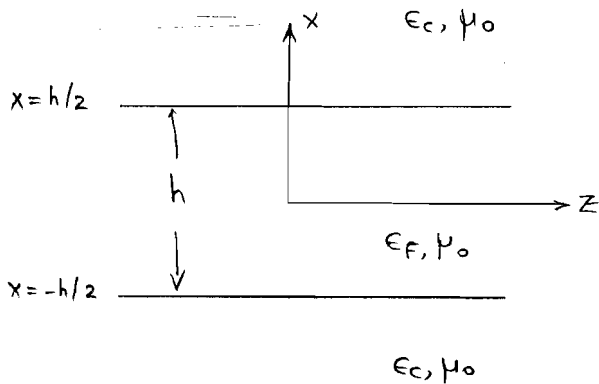
$$= E_0 \sin(k_0 z) [4\hat{x} \sin(\omega t) - 2\hat{y} \cos(\omega t)]$$

$$\begin{aligned}
 (8) \quad \sigma &= \hat{l}_n \cdot (\vec{D}^+ - \vec{D}^-) = \\
 &= (-\hat{l}_z) \cdot (\epsilon_0 \vec{E}_1) = 0 \quad \sim \sigma = 0
 \end{aligned}$$

$$\begin{aligned}
 \vec{K} &= \hat{l}_n \times (\vec{H}^+ - \vec{H}^-) = (-\hat{l}_z) \times \{ \vec{H}_i + \vec{H}_r \} \Big|_{z=0} = \\
 &= (-\hat{l}_z) \times \left[\frac{4E_0}{Z_0} \hat{l}_y + j \frac{2E_0}{Z_0} \hat{l}_x \right] = \\
 &= \hat{l}_x \frac{4E_0}{Z_0} - \hat{l}_y \frac{2E_0}{Z_0} j
 \end{aligned}$$

$$\begin{aligned}
 \vec{K} &= \text{Re} \{ \vec{K} e^{j\omega t} \} = \frac{4E_0}{Z_0} \cos(\omega t) \hat{l}_x + \frac{2E_0}{Z_0} \cos(\omega t - \frac{\pi}{2}) \hat{l}_y \\
 &= \frac{2E_0}{Z_0} [2\cos(\omega t) \hat{l}_x + \sin(\omega t) \hat{l}_y]
 \end{aligned}$$

ΑΣΚΗΣΗ 5:



$$\vec{E} = \hat{y} \begin{cases} E_c e^{-\gamma_c(x-h/2)} & h/2 < x < \infty \\ E_f \cos(k_f x) & -h/2 < x < h/2 \\ E_c e^{\gamma_c(x+h/2)} & -\infty < x < -h/2 \end{cases} e^{-j\beta z}$$

(α) Από τις εξισώσεις Maxwell στην μόνιμη ημιτονοειδή κατάσταση έχουμε:

$$\vec{\nabla} \times \vec{E} = -j\omega\mu_0 \vec{H} \Rightarrow \vec{H} = -\frac{1}{j\omega\mu_0} \vec{\nabla} \times \vec{E} \Rightarrow$$

$$\vec{H} = -\frac{1}{j\omega\mu_0} \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} = -\frac{1}{j\omega\mu_0} \left[\hat{i}_x \left(-\frac{\partial E_y}{\partial z} \right) + \hat{i}_z \left(\frac{\partial E_y}{\partial x} \right) \right]$$

Επομένως

$$H_x = \frac{1}{j\omega\mu_0} \frac{\partial E_y}{\partial z} = \frac{1}{j\omega\mu_0} (-j\beta) \begin{cases} E_c e^{-\gamma_c(x-h/2)} \\ E_f \cos(k_f x) \\ E_c e^{+\gamma_c(x+h/2)} \end{cases} e^{-j\beta z}$$

$$= -\frac{\beta}{\omega\mu_0} e^{-j\beta z} \begin{cases} E_c e^{-\gamma_c(x-h/2)} & h/2 < x < \infty \\ E_f \cos(k_f x) & -h/2 < x < h/2 \\ E_c e^{+\gamma_c(x+h/2)} & -\infty < x < -h/2 \end{cases}$$

$$H_z = -\frac{1}{j\omega\mu_0} \frac{\partial E_y}{\partial x} = -\frac{1}{j\omega\mu_0} e^{-j\beta z} \begin{cases} E_c (-\gamma_c) e^{-\gamma_c(x-h/2)} \\ E_f (-k_f) \sin(k_f x) \\ E_c (+\gamma_c) e^{\gamma_c(x+h/2)} \end{cases}$$

$$= \frac{1}{j\omega\mu_0} e^{-j\beta z} \begin{cases} \gamma_c E_c e^{-\gamma_c(x-h/2)} & h/2 < x < \infty \\ k_f E_f \sin(k_f x) & -h/2 < x < h/2 \\ -\gamma_c E_c e^{\gamma_c(x+h/2)} & -\infty < x < -h/2 \end{cases}$$

Επομένως $\vec{H} = H_x \hat{i}_x + H_z \hat{i}_z$

AMPAD 22-142 100 SHEETS 22-144 200 SHEETS

(β) Εφόσον τα πεδία είναι άθροισμα επιπέδων υφάρων ικανοποιούν τις εξισώσεις Maxwell. Αρκεί λοιπόν να ικανοποιούν και τις οριακές συνθήκες.

$$\begin{aligned} \underline{x = h/2}: \quad E_y(x = h/2^+) - E_y(x = h/2^-) &= 0 \\ H_z(x = h/2^+) - H_z(x = h/2^-) &= 0 \end{aligned} \quad \left. \vphantom{\begin{aligned} E_y(x = h/2^+) - E_y(x = h/2^-) &= 0 \\ H_z(x = h/2^+) - H_z(x = h/2^-) &= 0 \end{aligned}} \right\} \Rightarrow$$

$$\left. \vphantom{\begin{aligned} E_c &= E_f \cos(k_f h/2) \\ \gamma_c E_c &= k_f E_f \sin(k_f h/2) \end{aligned}} \right\} \Rightarrow$$

$$E_c = E_f \cos(k_f h/2)$$

$$\gamma_c E_c = k_f E_f \sin(k_f h/2)$$

$$\underline{x = -h/2}: \quad E_y(x = -h/2^+) - E_y(x = -h/2^-) = 0$$

$$H_z(x = -h/2^+) - H_z(x = -h/2^-) = 0 \quad \left. \vphantom{\begin{aligned} E_y(x = -h/2^+) - E_y(x = -h/2^-) &= 0 \\ H_z(x = -h/2^+) - H_z(x = -h/2^-) &= 0 \end{aligned}} \right\} \Rightarrow$$

$$E_f \cos(k_f (-h/2)) = E_c$$

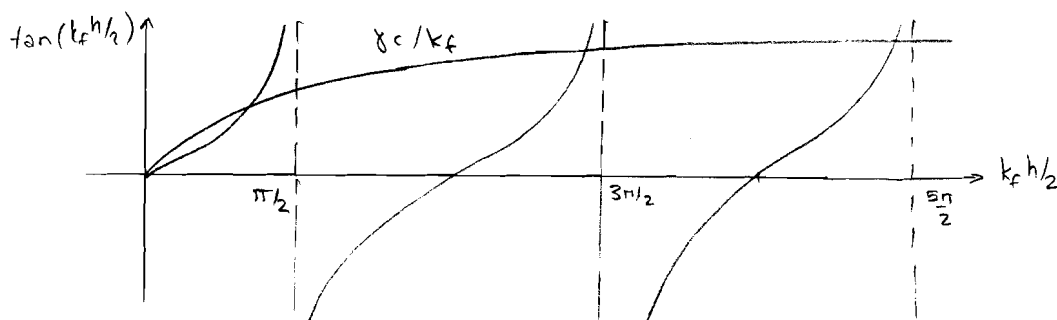
$$k_f E_f \sin(k_f (-h/2)) = -\gamma_c E_c$$

Οπότε για να ικανοποιούνται και οι οριακές συνθήκες πρέπει να ισχύουν οι σχέσεις:

$$\left. \begin{aligned} E_c &= E_f \cos(k_f h/2) \\ \gamma_c E_c &= k_f E_f \sin(k_f h/2) \end{aligned} \right\} \Rightarrow$$

$$\gamma_c = k_f \tan(k_f h/2) \Rightarrow \tan(k_f h/2) = \frac{\gamma_c}{k_f}$$

Όμως $\gamma_c = (\beta^2 - k_0^2 n_c^2)^{1/2}$, $k_f = (k_0^2 n_f^2 - \beta^2)^{1/2}$, είναι πιθανόν να πάρει λύσεις.



$$(γ) \quad \vec{\mathcal{E}} = \text{Re} \{ \vec{E} e^{j\omega t} \} \Rightarrow$$

$$\vec{\mathcal{E}}(x, z, t) = \hat{y} \begin{cases} E_c e^{-\gamma_c(x-h/2)} \cos(\omega t - \beta z) & h/2 < x < \infty \\ E_f \cos(k_f x) \cos(\omega t - \beta z) & -h/2 < x < h/2 \\ E_c e^{\gamma_c(x+h/2)} \cos(\omega t - \beta z) & -h/2 < x < \infty \end{cases}$$

$$\text{ουα} \quad E_c = E_f \cos(k_f h/2)$$

(δ) Το μικρότερο διάνυσμα Poynting δίδεται από την σχέση :

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* \Rightarrow$$

$$\begin{aligned} \vec{S} &= \frac{1}{2} E_y \hat{y} \times (H_x^* \hat{x} + H_z^* \hat{z}) = \\ &= \frac{1}{2} E_y H_x^* (-\hat{z}) + \frac{1}{2} E_y H_z^* \hat{x} = S_x \hat{x} + S_z \hat{z} \end{aligned}$$

$$\frac{h}{2} < x < \infty:$$

$$\begin{aligned} S_z &= -\frac{1}{2} E_c e^{-\gamma_c(x-h/2)} e^{-j\beta z} \left(-\frac{\beta}{\omega\mu_0}\right) E_c^* e^{-\gamma_c(x-h/2)} e^{+j\beta z} \\ &= \frac{1}{2} |E_c|^2 e^{-2\gamma_c(x-h/2)} \left(\frac{\beta}{\omega\mu_0}\right) \end{aligned}$$

$$\begin{aligned} S_x &= \frac{1}{2} E_c e^{-\gamma_c(x-h/2)} e^{-j\beta z} \frac{1}{-j\omega\mu_0} e^{+j\beta z} \gamma_c E_c^* e^{-\gamma_c(x-h/2)} \\ &= \frac{1}{2} |E_c|^2 \frac{j\gamma_c}{\omega\mu_0} e^{-2\gamma_c(x-h/2)} \end{aligned}$$

$$\text{Επομένως, } \langle N_z \rangle = \text{Re} \{ S_z \} = \frac{1}{2} |E_c|^2 e^{-2\gamma_c(x-h/2)} \left(\frac{\beta}{\omega\mu_0}\right)$$

$$\langle N_x \rangle = 0$$

$$-h/2 < x < h/2:$$

$$\begin{aligned} S_z &= -\frac{1}{2} E_f \cos(k_f x) e^{-j\beta z} \left(-\frac{\beta}{\omega\mu_0}\right) e^{+j\beta z} E_f^* \cos(k_f x) = \\ &= \frac{1}{2} |E_f|^2 \cos^2(k_f x) \left(\frac{\beta}{\omega\mu_0}\right) \end{aligned}$$

$$\begin{aligned} S_x &= \frac{1}{2} E_f \cos(k_f x) e^{-j\beta z} \frac{1}{-j\omega\mu_0} e^{+j\beta z} E_f^* k_f \sin(k_f x) = \\ &= \frac{1}{2} |E_f|^2 \frac{j k_f}{\omega\mu_0} \sin(k_f x) \cos(k_f x) \end{aligned}$$

$$\langle N_z \rangle = \text{Re} \{ S_z \} = \frac{1}{2} |E_f|^2 \cos^2(k_f x) \left(\frac{\beta}{\omega\mu_0}\right), \quad \langle N_x \rangle = 0$$

$$\underline{-\infty < x < -h/2:}$$

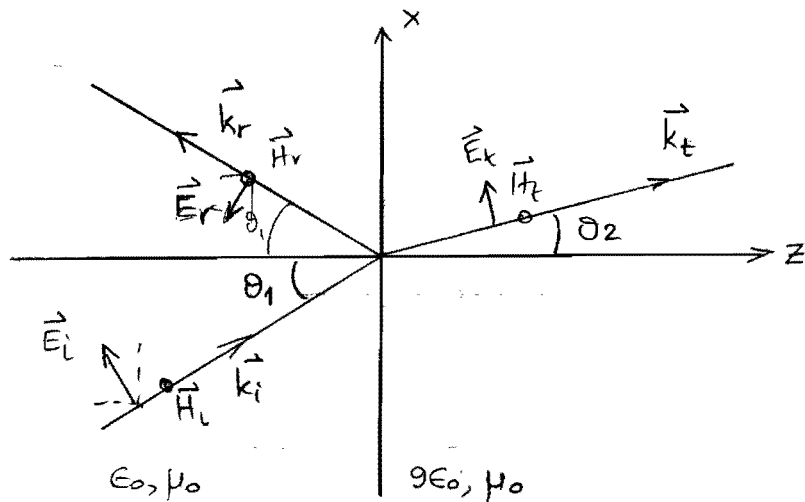
$$\begin{aligned} S_z &= -\frac{1}{2} E_c e^{\gamma_c(x+h/2)} e^{-j\beta z} \left(-\frac{\beta}{\omega\mu_0}\right) E_c^* e^{+\gamma_c(x+h/2)} e^{+j\beta z} \\ &= \frac{1}{2} |E_c|^2 \left(\frac{\beta}{\omega\mu_0}\right) e^{2\gamma_c(x+h/2)} \end{aligned}$$

$$\begin{aligned} S_x &= \frac{1}{2} E_c e^{\gamma_c(x+h/2)} e^{-j\beta z} \left(\frac{1}{-j\omega\mu_0}\right) e^{+j\beta z} (-\gamma_c E_c^*) e^{\gamma_c(x+h/2)} \\ &= \frac{1}{2} |E_c|^2 \left(\frac{-j\gamma_c}{\omega\mu_0}\right) e^{2\gamma_c(x+h/2)} \end{aligned}$$

$$\langle N_z \rangle = \text{Re}\{S_z\} = \frac{1}{2} |E_c|^2 \left(\frac{\beta}{\omega\mu_0}\right) e^{2\gamma_c(x+h/2)}$$

$$\langle N_x \rangle = \text{Re}\{S_x\} = 0$$

ΑΣΚΗΣΗ 6:



$$(α) \vec{H}_i = 100 \hat{y} e^{-j(50x + 50\sqrt{3}z)}$$

$$k_x = 50 \text{ m}^{-1} \quad k_z = 50\sqrt{3} \text{ m}^{-1} \quad k_x = k_0 \sin\theta_1 \quad k_z = k_0 \cos\theta_1$$

$$k_i = (k_x^2 + k_z^2)^{1/2} = [50^2 + (50\sqrt{3})^2]^{1/2} = 50 \cdot 2 = 100 \text{ m}^{-1}$$

$$k_i = k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda_0} \quad \lambda_0 = \frac{2\pi}{k_i} = 0.0628 \text{ m}^{-1} = 6.28 \text{ cm}$$

$$\omega = k_0 c = 100 \frac{1}{\text{m}} \cdot 300\,000 \cdot 10^3 \frac{\text{m}}{\text{s}} = 3 \cdot 10^{10} \text{ rad/s} \Rightarrow$$

$$f = \frac{\omega}{2\pi} = 4.7746 \cdot 10^9 \text{ Hz} = 4.7746 \text{ GHz}$$

$$\theta_1 = \tan^{-1}(k_x/k_z) = \tan^{-1}(1/\sqrt{3}) = 30^\circ$$

Η πόλωση είναι γραμμική αφού $\vec{H} \parallel \hat{y}$ και συγκευρισμένα

TM (//).

$$(β) \vec{E}_i = -z_1 (\vec{k}_i \times \vec{H}_i) = -z_1 \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \sin\theta_1 & 0 & \cos\theta_1 \\ 0 & H_i & 0 \end{vmatrix} e^{-j\vec{k}_i \cdot \vec{r}}$$

$$= -z_1 e^{-j\vec{k}_i \cdot \vec{r}} [\hat{x} (-H_i \cos\theta_1) - \hat{y} 0 + \hat{z} H_i \sin\theta_1]$$

$$= (z_1 H_i) e^{-j\vec{k}_i \cdot \vec{r}} [\cos\theta_1 \hat{x} - \hat{z} \sin\theta_1] =$$

$$= E_i [\cos\theta_1 \hat{x} - \hat{z} \sin\theta_1] e^{-j(50x + 50\sqrt{3}z)}$$

$$E_r = r_{TM} E_i$$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2 \sim$$

$$\theta_2 = \sin^{-1} \left(\frac{n_1 \sin \theta_1}{n_2} \right)$$

$$n_1 = 1.0, n_2 = \sqrt{9} = 3$$

$$\theta_2 = 9.59^\circ$$

$$r_{TM} = \frac{Z_1 \cos \theta_1 - Z_2 \cos \theta_2}{Z_1 \cos \theta_1 + Z_2 \cos \theta_2}$$

$$r_{TM} = 0.44978$$

$$t_{TM} = \frac{2 Z_2 \cos \theta_1}{Z_1 \cos \theta_1 + Z_2 \cos \theta_2} = 0.48326$$

Επομένως,

$$\vec{E}_r = 0.44978 \cdot 377 \cdot 100 e^{-j \vec{k}_r \cdot \vec{r}} [-\cos \theta_1 \hat{1}_x - \sin \theta_1 \hat{1}_z]$$

$$\vec{k}_r = k_0 [-\hat{1}_z \cos \theta_1 + \sin \theta_1 \hat{1}_x]$$

$$\vec{E}_r = 16956.7 \left(\frac{V}{m} \right) \left[-\frac{\sqrt{3}}{2} \hat{1}_x - \frac{1}{2} \hat{1}_z \right] e^{-j(50x - 50\sqrt{3}z)}$$

Η πόλωση του ανακλωμένου κύματος παραμένει γραμμική (TM).

$$(\gamma) E_t = t_{TM} E_i = 0.48326 \cdot 377 \cdot 100 = 18218.9 \text{ (V/m)}$$

$$\vec{E}_t = 18218.9 [-\sin \theta_2 \hat{1}_z + \cos \theta_2 \hat{1}_x] e^{-j \vec{k}_t \cdot \vec{r}}$$

$$\vec{k}_t = k_0 n_2 [\cos \theta_2 \hat{1}_z + \sin \theta_2 \hat{1}_x] =$$

$$= 300 [0.986 \hat{1}_z + 0.16667 \hat{1}_x]$$

$$= 295.8 \hat{1}_z + 50 \hat{1}_x$$

$$\vec{E}_t = 18218.9 [-\sin \theta_2 \hat{1}_z + \cos \theta_2 \hat{1}_x] e^{-j(295.8z + 50x)}$$

$$(\delta) \vec{H}_t = \frac{1}{Z_2} \hat{k}_t \times \vec{E}_t = \frac{E_t}{Z_2} \begin{vmatrix} \hat{1}_x & \hat{1}_y & \hat{1}_z \\ \sin \theta_2 & 0 & \cos \theta_2 \\ \cos \theta_2 & 0 & -\sin \theta_2 \end{vmatrix} e^{-j \vec{k}_t \cdot \vec{r}}$$

$$Z_2 = \frac{377}{3} = 125.67 \Omega$$

$$\vec{H}_t = 144,98 \left[\hat{L}_x \emptyset - \hat{L}_y (-\sin^2 \theta_2 - \cos^2 \theta_2) + \hat{L}_z \emptyset \right] e^{-j \vec{k}_t \cdot \vec{r}}$$

$$= 144,98 \hat{L}_y e^{-j(295,8z + 50x)}$$

$$\vec{H}_t = 144,98 \hat{L}_y \cos [2\pi ft - (295,8z + 50x)]$$

$$f = 4,7746 \cdot 10^9 \text{ Hz}$$

$$(E) \vec{P}_{avg_2} = \frac{1}{2} \text{Re} \{ \vec{E}_t \times \vec{H}_t^* \} =$$

$$= \frac{1}{2} \text{Re} \{ -z_2 (\hat{k}_t \times \vec{H}_t) \times \vec{H}_t^* \} =$$

$$= \frac{z_2}{2} \text{Re} \{ \vec{H}_t^* \times (\hat{k}_t \times \vec{H}_t) \} =$$

$$= \frac{z_2}{2} \text{Re} \{ \hat{k}_t |\vec{H}_t|^2 - \vec{H}_t (\hat{k}_t \cdot \vec{H}_t^*) \} =$$

$$= \frac{1}{2} z_2 |\vec{H}_t|^2 \hat{k}_t$$

$$\vec{P}_{avg_2} = \frac{1}{2} \frac{377}{3} |144,98|^2 [\sin \theta_2 \hat{L}_x + \cos \theta_2 \hat{L}_z]$$

$$= 1,3207 \cdot 10^6 \left(\frac{W}{m^2} \right) [0,1666 \hat{L}_x + 0,986 \hat{L}_z]$$

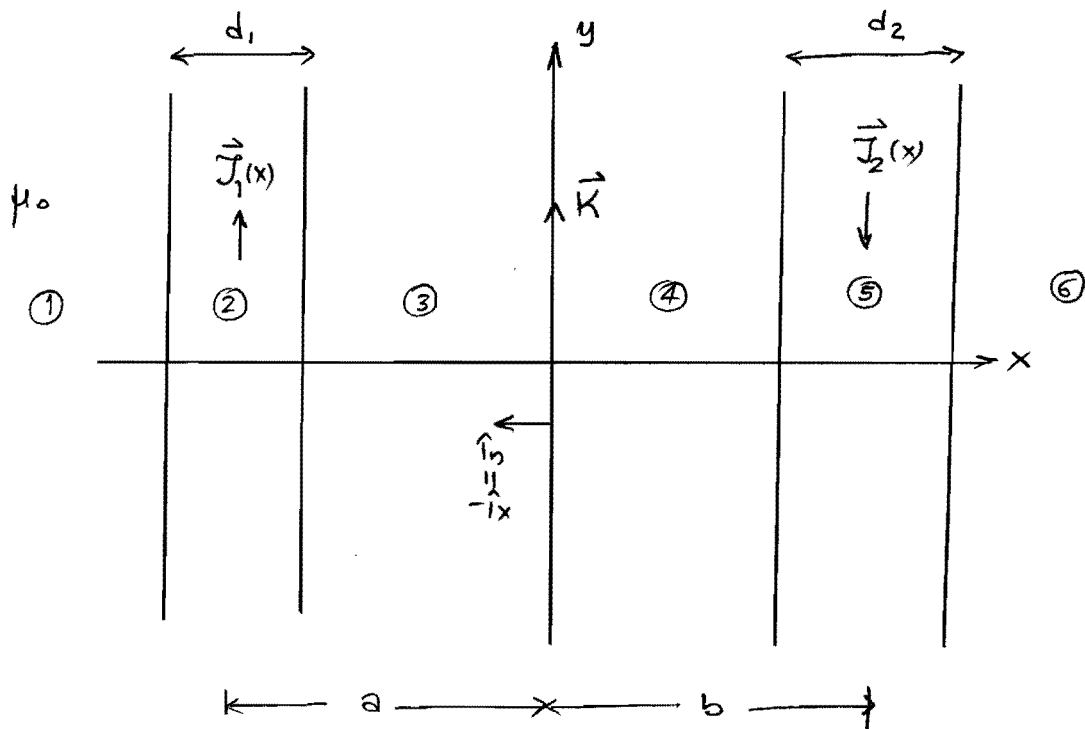
$$(OC) P_r = |r_{TM}|^2 = 0,2023$$

$$P_t = \frac{z_1 \cos \theta_2}{z_2 \cos \theta_1} |t_{TM}|^2 = \frac{3 \cos \theta_2}{1 \cos \theta_1} |t_{TM}|^2 =$$

$$= 0,7977$$

'Αρα το 20,23% ανακλάται και το 79,77% διαθλάται.

ΑΣΚΗΣΗ 7:



$$\vec{J}_1(x) = J_{01} \cos(\pi(x+a)/d_1) \hat{y}$$

$$\vec{J}_2(x) = -J_{02} \cos(\pi(x-b)/d_2) \hat{y} \quad \vec{K} = K_0 \hat{y}$$

$$(a) \quad \vec{H}_1 = \frac{\vec{K} \times \hat{n}}{2} (+1) + \int_{-a-d_1/2}^{-a+d_1/2} \frac{J_1(x) dx}{2} \hat{y} \times \hat{n} + \int_{b-d_2/2}^{b+d_2/2} \frac{J_2(x) dx}{2} (-\hat{y}) \times \hat{n} (+1)$$

$$\hat{y} \times \hat{n} = \hat{y} \times (-\hat{x}) = -\hat{y} \times \hat{x} = \hat{z}$$

$$\text{Άρα } \vec{H}_1 = \hat{z} \left\{ \frac{K_0}{2} + \frac{1}{2} \int_{-a-d_1/2}^{-a+d_1/2} J_{01} \cos\left(\pi \frac{x+a}{d_1}\right) dx - \frac{1}{2} \int_{b-d_2/2}^{b+d_2/2} J_{02} \cos\left(\pi \frac{x-b}{d_2}\right) dx \right\}$$

$$\int_{-a-d_1/2}^{-a+d_1/2} J_{01} \cos\left(\pi \frac{x+a}{d_1}\right) dx = J_{01} \left(\frac{\sin\left(\pi \frac{x+a}{d_1}\right)}{\pi/d_1} \right) \Big|_{-a-d_1/2}^{-a+d_1/2} = J_{01} \frac{2d_1}{\pi}$$

$$\int_{b-d_2/2}^{b+d_2/2} J_{02} \cos\left(\pi \frac{x-b}{d_2}\right) dx = J_{02} \frac{2d_2}{\pi}$$

$$\vec{H}_1 = \hat{z} \left\{ \frac{K_0}{2} + J_{01} \frac{2d_1}{\pi} \frac{1}{2} - J_{02} \frac{2d_2}{\pi} \frac{1}{2} \right\}$$

Όμοια για την ⑥ περιοχή:

$$\vec{H}_6 = \frac{\vec{K} \times \hat{i}_n}{2} (-1) + \frac{1}{2} \int (\quad) (-1) dx + \frac{1}{2} \int (\quad) (-1) dx =$$

$$= -\vec{H}_1 = -\hat{i}_z \left\{ \frac{K_0}{2} + J_{01} \frac{2d_1}{\pi} - J_{02} \frac{2d_2}{\pi} \right\}$$

$$H_1 = 0 = H_6 \Rightarrow \frac{K_0}{2} + \frac{J_{01} d_1}{\pi} - \frac{J_{02} d_2}{\pi} = 0$$

(β) $-a - \frac{d_1}{2} < x < b + \frac{d_2}{2}$: περιόχεται ②, ③, ④, ⑤

$$\vec{H}_2 = \frac{K_0}{2} \hat{i}_z - \hat{i}_z \int_{b+\frac{d_1}{2}}^x J_{01} \cos\left(\pi \frac{x+a}{d_1}\right) dx + \hat{i}_z \int_{-a+d_1/2}^x \frac{1}{2} J_{01} \cos\left(\pi \frac{x+a}{d_1}\right) dx$$

$$- \hat{i}_z \int_{b-\frac{d_2}{2}}^{\frac{1}{2} J_{02} \cos\left(\pi \frac{x-b}{d_2}\right) dx} =$$

$$= \hat{i}_z \left\{ \frac{K_0}{2} - \frac{1}{2} J_{01} \frac{d_1}{\pi} \left\{ \sin\left(\pi \frac{x+a}{d_1}\right) + 1 \right\} + J_{01} \frac{1}{2} \frac{d_1}{\pi} \left\{ 1 - \sin\left(\pi \frac{x+a}{d_1}\right) \right\} \right.$$

$$\left. - \frac{1}{2} J_{02} \frac{2d_2}{\pi} \right\}$$

$$= \hat{i}_z \left\{ \frac{K_0}{2} - \frac{J_{01} d_1}{\pi} \sin\left(\pi \frac{x+a}{d_1}\right) - J_{02} \frac{d_2}{\pi} \right\}$$

$$\vec{H}_3 = \hat{i}_z \left\{ \frac{K_0}{2} - J_{01} \frac{d_1}{\pi} - J_{02} \frac{d_2}{\pi} \right\} = \hat{i}_z \left(-\frac{2J_{01} d_1}{\pi} \right)$$

$$\vec{H}_4 = \hat{i}_z \left\{ -\frac{K_0}{2} - J_{01} \frac{d_1}{\pi} - J_{02} \frac{d_2}{\pi} \right\} = \hat{i}_z \left(-\frac{2J_{02} d_2}{\pi} \right)$$

$$\vec{H}_5 = \hat{i}_z \left\{ -\frac{K_0}{2} - J_{01} \frac{1}{2} \frac{2d_1}{\pi} + \frac{1}{2} \int_{b-\frac{d_2}{2}}^x J_{02} \cos\left(\pi \frac{x-b}{d_2}\right) dx + \frac{1}{2} \int_x J_{02} \cos\left(\pi \frac{x-b}{d_2}\right) dx \right\}$$

$$= \hat{i}_z \left\{ -\frac{K_0}{2} - J_{01} \frac{d_1}{\pi} + J_{02} \frac{d_2}{2\pi} \left(\sin\left(\pi \frac{x-b}{d_2}\right) + 1 \right) \right.$$

$$\left. - J_{02} \frac{d_2}{2\pi} \left(1 - \sin\left(\pi \frac{x-b}{d_2}\right) \right) \right\}$$

$$= \hat{i}_z \left\{ -\frac{K_0}{2} - J_{01} \frac{d_1}{\pi} + J_{02} \frac{d_2}{\pi} \sin\left(\pi \frac{x-b}{d_2}\right) \right\}$$