$\Theta\epsilon\mu\alpha$ 1

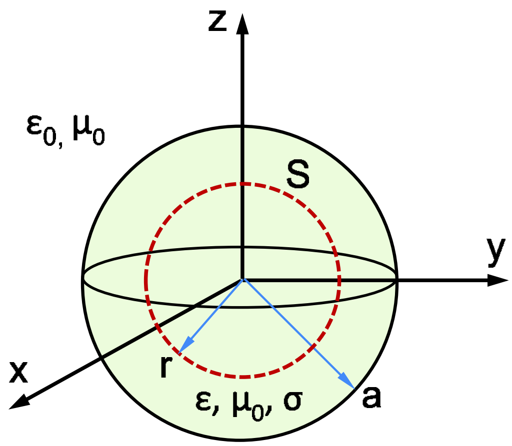

Figure 1: (Problem 1) Charged conducting sphere.

(α)

$$\vec{E}(r,t) = \hat{i}_r \begin{cases} \dfrac{\rho_0 r}{3\epsilon} e^{-t/\tau}, \qquad \rho_0 = \dfrac{3Q_0}{4\pi \mathrm{a}^3}, \quad \tau = \dfrac{\epsilon}{\sigma}, & \text{for } r < \mathrm{a}, \\ \dfrac{Q_0}{4\pi\epsilon_0 r^2} & \text{for } r > \mathrm{a} \end{cases}$$

$$\vec{J}(r,t) = \hat{i}_r \begin{cases} \sigma \dfrac{\rho_0 r}{3\epsilon} e^{-t/\tau}, & \text{for } r < \mathrm{a}, \\ 0 & \text{for } r > \mathrm{a} \end{cases}$$

$$\sigma(t) = \frac{Q_0}{4\pi \mathrm{a}^2}\left(1 - e^{-t/\tau}\right) = \frac{\rho_0 \mathrm{a}}{3}\left(1 - e^{-t/\tau}\right).$$

(β)

$$\begin{aligned} W_e(t=0) &= \frac{Q_0^2}{8\pi\epsilon_0 \mathrm{a}} + \frac{2\pi\rho_0^2 \mathrm{a}^5}{45\epsilon} = \frac{Q_0^2}{8\pi\epsilon_0 \mathrm{a}} + \frac{Q_0^2}{40\pi\epsilon \mathrm{a}} \\ W_e(t=\infty) &= \frac{Q_0^2}{8\pi\epsilon_0 \mathrm{a}} \\ W_{Loss} &= W_e(t=0) - W_e(t=\infty) = \frac{2\pi\rho_0^2 \mathrm{a}^5}{45\epsilon} = \frac{Q_0^2}{40\pi\epsilon \mathrm{a}} \end{aligned}$$

(γ)

$$\vec{\nabla} \times \vec{H} = 0, \quad \vec{\nabla} \cdot \vec{H} = 0, \quad \text{B.C. for H} = 0 \Longrightarrow \vec{H} = 0.$$

(δ)

$$\vec{F}_e(t) = \hat{i}_z \frac{\rho_0^2 \pi \mathrm{a}^4}{36} \left(\frac{e^{-2t/\tau}}{\epsilon} + \frac{2}{\epsilon_0} \right)$$

$\Theta\epsilon\mu\alpha$ 2

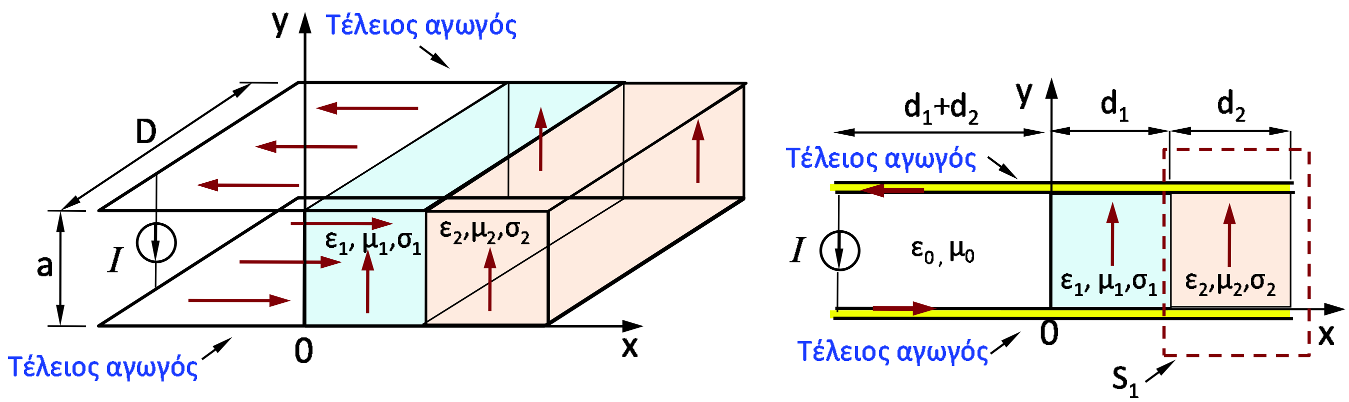

Figure 2: (Problem 2) Geometry of the leaky capacitor.

(α)

$$\vec{H}(x,y) = \hat{i}_z \begin{cases} \dfrac{I}{D} & \text{for } -(d_1+d_2) < x < 0 \text{ and } 0 < y < \mathrm{a}, \\ 0 & \text{for } x < -(d_1+d_2), \quad x > (d_1+d_2) \text{ and } \forall y, \end{cases}$$

(β)

$$\begin{aligned} \vec{H}_1 &= \vec{i}_z \left(A_1 x + B_1\right) \qquad \text{for } 0 < x < d, \\ \vec{H}_2 &= \vec{i}_z \left(A_2 x + B_2\right) \qquad \text{for } d < x < 2d, \\ A_1 &= \frac{-I/D}{\left(1 + \dfrac{\sigma_2}{\sigma_1}\right) d}, \\ A_2 &= \frac{-I/D}{\left(1 + \dfrac{\sigma_1}{\sigma_2}\right) d}, \\ B_1 &= \frac{I}{D}, \\ B_2 &= \frac{2I/D}{\left(1 + \dfrac{\sigma_1}{\sigma_2}\right) d}, \end{aligned}$$

$$
\begin{aligned}
\vec{J}_1 &= \hat{i}_y \frac{I/D}{\left(1+\dfrac{\sigma_2}{\sigma_1}\right) d}, \qquad \text{for } 0 < x < d, \\
\vec{J}_2 &= \hat{i}_y \frac{I/D}{\left(1+\dfrac{\sigma_1}{\sigma_2}\right) d}, \qquad \text{for } d < x < 2d, \\
\vec{E}_1 &= \hat{i}_y \frac{I/D}{(\sigma_1+\sigma_2)\, d} = \vec{E}_2, \qquad \text{for } 0 < x < 2d.
\end{aligned}
$$

(γ)

$$
\vec{F}_m = \hat{i}_x \frac{\mu_1}{2} \frac{\mathrm{a} I^2}{D} \left(\frac{\sigma_2}{\sigma_1+\sigma_2} \right)^2 .
$$

Θεμα 3

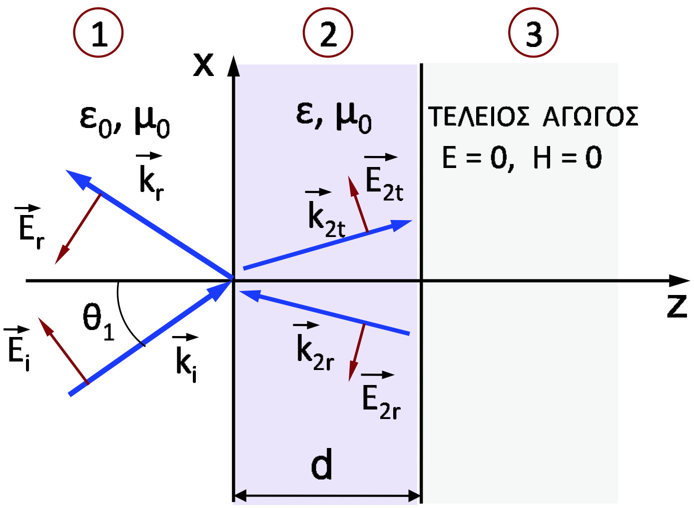

Figure 3: (Problem 3) Geometry of the a dielectric layer on top of a perfect conductor.

(α)

$$
\begin{aligned}
\vec{E}_i &= E_0 \left(\hat{i}_x \cos\theta_1 - \hat{i}_z \sin\theta_1\right) \exp(-j\vec{k}_i \cdot \vec{r}), \\
\vec{k}_i &= \omega\sqrt{\epsilon_0\mu_0} \left(\hat{i}_z \cos\theta_1 + \hat{i}_x \sin\theta_1\right), \\
\vec{E}_r &= -E_r \left(\hat{i}_x \cos\theta_1 + \hat{i}_z \sin\theta_1\right) \exp(-j\vec{k}_r \cdot \vec{r}), \\
\vec{k}_r &= \omega\sqrt{\epsilon_0\mu_0} \left(-\hat{i}_z \cos\theta_1 + \hat{i}_x \sin\theta_1\right), \\
\vec{E}_1 &= \vec{E}_i + \vec{E}_r, \\
\vec{E}_{2t} &= E_{2t} \left(\hat{i}_x \cos\theta_2 - \hat{i}_z \sin\theta_2\right) \exp(-j\vec{k}_{2t} \cdot \vec{r}), \\
\vec{k}_{2t} &= \omega\sqrt{\epsilon\mu_0} \left(\hat{i}_z \cos\theta_2 + \hat{i}_x \sin\theta_2\right), \\
\vec{E}_{2r} &= -E_{2r} \left(\hat{i}_x \cos\theta_2 + \hat{i}_z \sin\theta_2\right) \exp(-j\vec{k}_{2r} \cdot \vec{r}), \\
\vec{k}_{2r} &= \omega\sqrt{\epsilon\mu_0} \left(-\hat{i}_z \cos\theta_2 + \hat{i}_x \sin\theta_2\right), \\
\vec{E}_2 &= \vec{E}_{2t} + \vec{E}_{2r}, \\
\vec{E}_3 &= 0.
\end{aligned}
$$

(β)

$$
\begin{aligned}
\vec{H}_1 &= \hat{i}_y \frac{1}{Z_0}\left[E_0 \exp(-j\vec{k}_i \cdot \vec{r}) + E_r \exp(-j\vec{k}_r \cdot \vec{r})\right], \\
Z_0 &= \sqrt{\frac{\mu_0}{\epsilon_0}}, \\
\vec{H}_2 &= \hat{i}_y \frac{1}{Z_2}\left[E_{2t} \exp(-j\vec{k}_{2t} \cdot \vec{r}) + E_{2r} \exp(-j\vec{k}_{2r} \cdot \vec{r})\right], \\
Z_2 &= \sqrt{\frac{\mu_0}{\epsilon}}, \\
\vec{H}_3 &= 0.
\end{aligned}
$$

(γ)

$$
\begin{aligned}
\frac{E_r}{E_0} &= 1 - \frac{2(e^{j2k_{2z}d} - 1)\dfrac{\cos\theta_2}{\cos\theta_1}}{\left(\dfrac{Z_0}{Z_2} - \dfrac{\cos\theta_2}{\cos\theta_1}\right) + \left(\dfrac{Z_0}{Z_2} + \dfrac{\cos\theta_2}{\cos\theta_1}\right)e^{j2k_{2z}d}}, \\
\frac{E_{2r}}{E_0} &= \frac{2}{\left(\dfrac{Z_0}{Z_2} - \dfrac{\cos\theta_2}{\cos\theta_1}\right) + \left(\dfrac{Z_0}{Z_2} + \dfrac{\cos\theta_2}{\cos\theta_1}\right)e^{j2k_{2z}d}}, \\
\frac{E_{2t}}{E_0} &= \frac{2e^{j2k_{2z}d}}{\left(\dfrac{Z_0}{Z_2} - \dfrac{\cos\theta_2}{\cos\theta_1}\right) + \left(\dfrac{Z_0}{Z_2} + \dfrac{\cos\theta_2}{\cos\theta_1}\right)e^{j2k_{2z}d}}, \\
k_{2z} &= k_2 \cos\theta_2, \qquad \sin\theta_1 = \sqrt{\epsilon/\epsilon_0}\sin\theta_2.
\end{aligned}
$$

(δ)

$$
\begin{aligned}
P_{2x} &= \frac{E_{02}^2 \sin\theta_2}{Z_2}\left[1 + \cos(2k_{2z}z)\right], \\
E_{02} &= |E_{2r}| = |E_{2t}|, \\
P_{2z} &= 0.
\end{aligned}
$$

(ϵ)

$$
\begin{aligned}
\vec{\mathcal{K}} &= \hat{i}_x \frac{2E_{02}}{Z_2}\cos\left(\omega t - k_{2x}x + k_{2z}d + \phi_{2r}\right), \\
E_{2r} &= E_{02}\exp(j\phi_{2r}), \\
k_{2x} &= k_2 \sin\theta_2 = k_0 \sin\theta_1.
\end{aligned}
$$

($\sigma\tau$)

$$
\vec{f} = \hat{i}_z \frac{1}{4}(2E_0)^2 \epsilon \cos^2\theta_2
$$