$\Theta\epsilon\mu\alpha$ 1

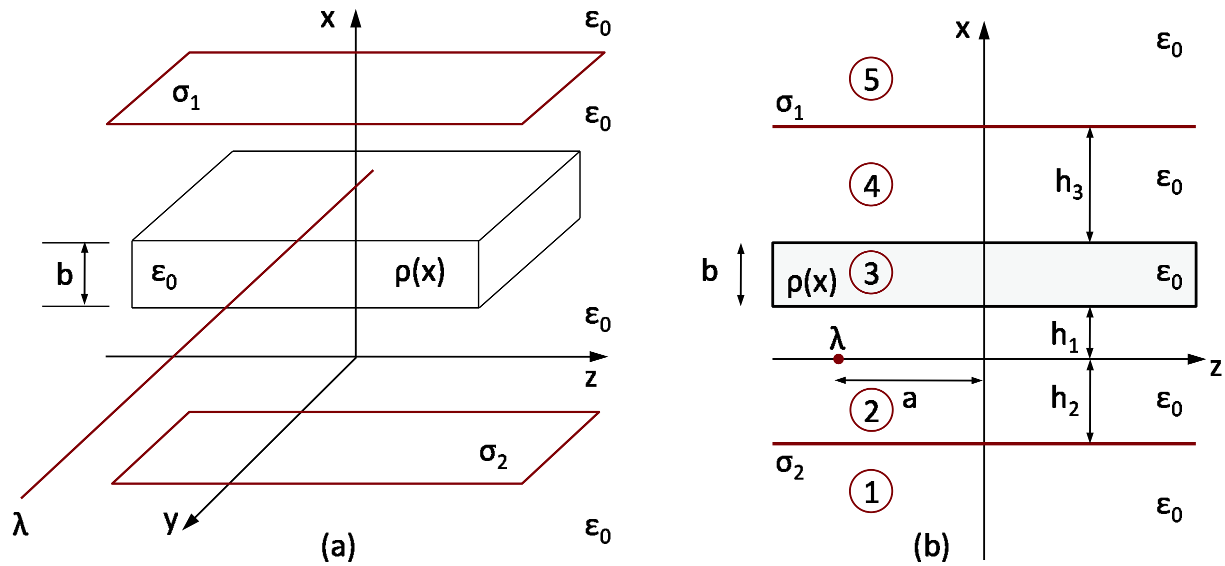

Figure 1: (Problem 1) Geometry of the structure under study with distinguished regions from 1 to 5.

(α)

$$\vec{E}_1 = -\frac{1}{2\epsilon_0}\left[\sigma_2+\rho_0\frac{2h_1+b}{2}+\sigma_1\right]\,\hat{i}_x+\frac{\lambda}{2\pi\epsilon_0}\frac{x\,\hat{i}_x+(z+\mathrm{a})\,\hat{i}_z}{x^2+(z+\mathrm{a})^2},$$

$$\vec{E}_2 = -\frac{1}{2\epsilon_0}\left[-\sigma_2+\rho_0\frac{2h_1+b}{2}+\sigma_1\right]\,\hat{i}_x+\frac{\lambda}{2\pi\epsilon_0}\frac{x\,\hat{i}_x+(z+\mathrm{a})\,\hat{i}_z}{x^2+(z+\mathrm{a})^2},$$

$$\vec{E}_3 = \frac{1}{2\epsilon_0}\left[\sigma_2+\rho_0\frac{x^2-h_1^2}{2b}-\rho_0\frac{(h_1+b)^2-x^2}{2b}-\sigma_1\right]\,\hat{i}_x+\frac{\lambda}{2\pi\epsilon_0}\frac{x\,\hat{i}_x+(z+\mathrm{a})\,\hat{i}_z}{x^2+(z+\mathrm{a})^2},$$

$$\vec{E}_4 = \frac{1}{2\epsilon_0}\left[\sigma_2+\rho_0\frac{2h_1+b}{2}-\sigma_1\right]\,\hat{i}_x+\frac{\lambda}{2\pi\epsilon_0}\frac{x\,\hat{i}_x+(z+\mathrm{a})\,\hat{i}_z}{x^2+(z+\mathrm{a})^2},$$

$$\vec{E}_5 = \frac{1}{2\epsilon_0}\left[\sigma_2+\rho_0\frac{2h_1+b}{2}+\sigma_1\right]\,\hat{i}_x+\frac{\lambda}{2\pi\epsilon_0}\frac{x\,\hat{i}_x+(z+\mathrm{a})\,\hat{i}_z}{x^2+(z+\mathrm{a})^2},$$

(β)

$$\vec{f}_\lambda = \frac{\lambda}{2\epsilon_0}\left[\sigma_2-\rho_0\frac{2h_1+b}{2}-\sigma_1\right]\,\hat{i}_x,$$

$$\vec{f}_{\sigma_1} = \sigma_1 \left[\frac{1}{2\epsilon_0} \left(\sigma_2 + \rho_0 \frac{2h_1 + b}{2} \right) \hat{i}_x + \frac{\lambda}{2\pi\epsilon_0} \frac{x\, \hat{i}_x + (z + \mathrm{a})\, \hat{i}_z}{x^2 + (z+\mathrm{a})^2} \right] \Bigg|_{x = h_1 + b + h_3} .$$

Θεμα 2

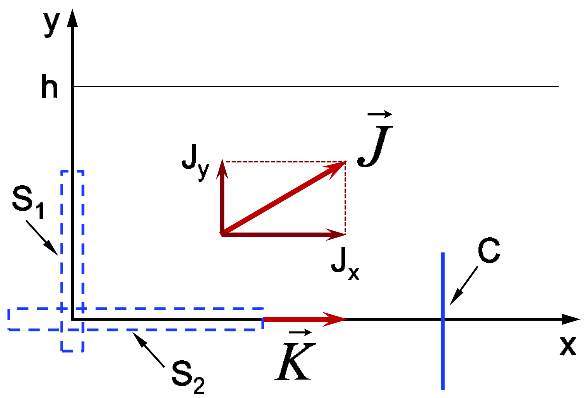

Figure 2: (Problem 2) Geometry of the structure under study which is assumed uniform along the z-axis. The closed surfaces S_1 and S_2 and the contour C are shown (actually their projections in the xy-plane.

(α)

$$\begin{aligned} J_x(x,y) &= J_0 \left[1 - \exp(-kx)\right], \\ J_y(x,y) &= kJ_0 \exp(-kx)(h-y), \\ K(x) &= -J_0 h \left[1 - \exp(-kx)\right]. \end{aligned}$$

(β)

$$\vec{H}(x,y) = \hat{i}_z \begin{cases} J_0(y-h)\left[1 - \exp(-kx)\right], & \text{for } x > 0 \text{ and } 0 < y < h, \\ 0 & \text{elsewhere} \end{cases}$$

(γ)

$$\vec{\nabla} \times \vec{E} = 0 \Longrightarrow \left[1 - e^{-kx}\right] \frac{\partial \sigma}{\partial y} = \left[\frac{\partial \sigma}{\partial x} + \sigma k\right] k e^{-kx}(h-y)$$

$$\vec{\nabla} \cdot \vec{D} = 0 \Longrightarrow \frac{\partial}{\partial x}\left(\frac{\epsilon J_x}{\sigma}\right) + \frac{\partial}{\partial y}\left(\frac{\epsilon J_y}{\sigma}\right) = 0$$

$\Theta\epsilon\mu\alpha\ \ 3$

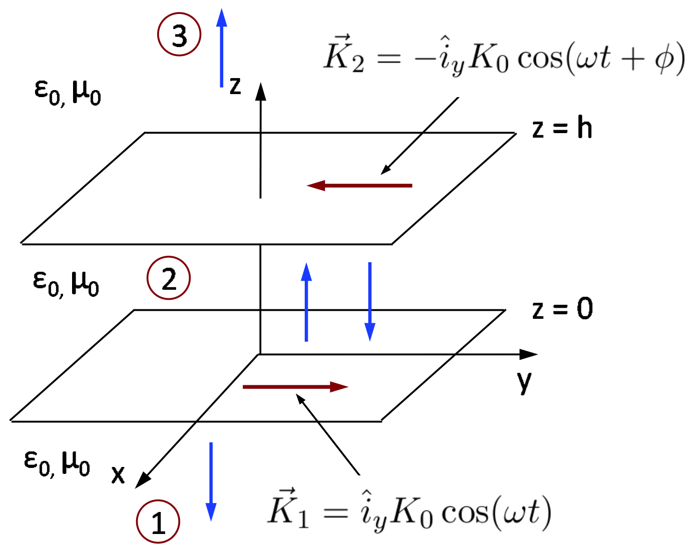

Figure 3: (Problem 3) Geometry of the structure of the two infinite surface current densities. The different regions are marked as 1, 2, and 3.

(α)

$$
\begin{aligned}
\vec{H}_1 &= \hat{i}_x H_1 \exp(+jk_0 z) \\
\vec{E}_1 &= \hat{i}_y Z_0 H_1 \exp(+jk_0 z) \\
\vec{H}_2 &= \hat{i}_x \left[H_{2+} \exp(-jk_0 z) + H_{2-} \exp(+jk_0 z)\right] \\
\vec{E}_2 &= \hat{i}_y Z_0 \left[-H_{2+} \exp(-jk_0 z) + H_{2-} \exp(+jk_0 z)\right] \\
\vec{H}_3 &= \hat{i}_x H_3 \exp(+jk_0 z) \\
\vec{E}_3 &= -\hat{i}_y Z_0 H_3 \exp(-jk_0 z)
\end{aligned}
$$

(β)

$$
\begin{bmatrix} 1 & 1 & -1 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & -1 & e^{+j2k_0h} & 1 \\ 0 & -1 & -e^{+j2k_0h} & 1 \end{bmatrix}
\begin{bmatrix} H_1 \\ H_{2+} \\ H_{2-} \\ H_3 \end{bmatrix}
=
\begin{bmatrix} 0 \\ K_0 \\ 0 \\ -K_0 e^{j\phi} e^{jk_0 h} \end{bmatrix}.
$$

(γ)

$$
\begin{aligned}
\vec{H}_1 &= -\hat{i}_x K_0 \exp(+jk_0 z), \\
\vec{H}_2 &= -\hat{i}_x j K_0 \sin(k_0 z),
\end{aligned}
$$

$$
\begin{aligned}
\vec{\mathcal{H}}_2(z,t) &= \hat{i}_x K_0 \sin(k_0 z)\sin(\omega t), \\
\vec{H}_3 &= 0.
\end{aligned}
$$

(δ)

$$
\begin{aligned}
\vec{f} &= \frac{1}{2} K_0^2 \mu_0 \sin^2(\omega t)\ \hat{i}_z, \\
\langle \vec{f} \rangle &= \hat{i}_z \frac{K_0^2 \mu_0}{4}.
\end{aligned}
$$