ΠΕΔΙΑ A 29/10/2007

Θεμα 1

(α)

$$\vec{E} = \begin{cases} -\hat{i}_x\Big[\dfrac{d\rho_0}{2\epsilon_0} + \dfrac{x}{x^2+y^2}\dfrac{\mathrm{a}^2\rho_0}{2\epsilon_0}\Big] - \hat{i}_y\Big[\dfrac{y}{x^2+y^2}\dfrac{\mathrm{a}^2\rho_0}{2\epsilon_0}\Big] & -\infty < x < -(d/2), \\ +\hat{i}_x\Big[\dfrac{2x\rho_0}{2\epsilon_0} - \dfrac{x}{x^2+y^2}\dfrac{\mathrm{a}^2\rho_0}{2\epsilon_0}\Big] - \hat{i}_y\Big[\dfrac{y}{x^2+y^2}\dfrac{\mathrm{a}^2\rho_0}{2\epsilon_0}\Big] & -(d/2) < x < +(d/2) \text{ and } r_T > \mathrm{a}, \\ +\hat{i}_x\dfrac{\rho_0}{2\epsilon_0}x - \hat{i}_y\dfrac{\rho_0}{2\epsilon_0}y, & -(d/2) < x < +(d/2) \text{ and } r_T < \mathrm{a}, \\ +\hat{i}_x\Big[\dfrac{d\rho_0}{2\epsilon_0} - \dfrac{x}{x^2+y^2}\dfrac{\mathrm{a}^2\rho_0}{2\epsilon_0}\Big] - \hat{i}_y\Big[\dfrac{y}{x^2+y^2}\dfrac{\mathrm{a}^2\rho_0}{2\epsilon_0}\Big] & +(d/2) < x < +\infty. \end{cases}$$

(β)

$$W_e/\ell = \frac{\rho_0^2\mathrm{a}^4\pi}{16\epsilon_0}$$

Θεμα 2

(α)

$$I(z) = -2\pi\mathrm{a}K_0, \qquad (z > 0)$$

$$\vec{K}_1 = \hat{i}_z K_0, \qquad (z > 0)$$

$$\vec{J}(r_T, z) = \hat{i}_z\frac{2\pi K_0}{r_T}\sin\big(\frac{2\pi r_T}{\mathrm{a}}\big) \qquad (z > 0).$$

(β)

$$\vec{H} = \hat{i}_\phi\begin{cases} -K_0\dfrac{\mathrm{a}}{r_T}\cos\big(\dfrac{2\pi r_T}{\mathrm{a}}\big), & \text{for } z > 0 \text{ and } r_T < \mathrm{a}, \\ 0, & \text{for } z < 0 \text{ or } (z > 0 \text{ and } r_T > \mathrm{a}). \end{cases}$$

Θεμα 3

(α)

$$\begin{aligned} \vec{E}_i &= +\hat{i}_z E_0\exp(-jk_0(-x\cos\theta - y\sin\theta), \\ \vec{E}_r &= +\hat{i}_z E_0\exp(-jk_0(+x\cos\theta + y\sin\theta), \\ \vec{E}_d &= -\hat{i}_z E_0\exp(-jk_0(+x\cos\theta - y\sin\theta), \\ \vec{E}_u &= -\hat{i}_z E_0\exp(-jk_0(-x\cos\theta + y\sin\theta), \end{aligned}$$

(β)

$$\vec{E} = \hat{i}_z(-4E_0)\sin(k_0 x\cos\theta)\sin(k_0 y\sin\theta)\cos(\omega t)$$

$$x = m\frac{\lambda_0}{2\cos\theta}, \qquad y = \ell\frac{\lambda_0}{2\sin\theta}, \qquad m,\ell \in \mathcal{Z}.$$

(γ)

$$\vec{f} = -\hat{i}_x\frac{8\mu_0 E_0^2}{Z_0^2}\cos^2\theta\sin^2(k_0 y\sin\theta)\sin^2(\omega t),$$

$$\langle\vec{f}\rangle = -\hat{i}_x\frac{4\mu_0 E_0^2}{Z_0^2}\cos^2\theta\sin^2(k_0 y\sin\theta).$$