

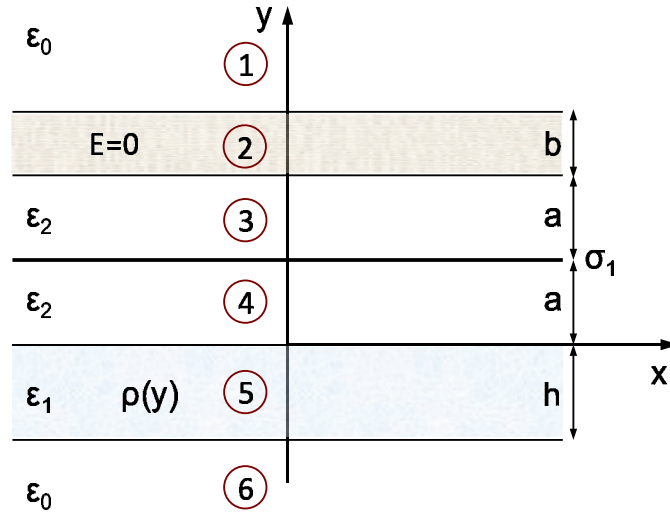
Θεμα 1

Figure 1: (Problem 1) Given structure geometry with specified (numbered) regions of interest.

(α)

$$\begin{aligned}\sigma(y = 2a) &= - \left[ \frac{\sigma_1}{2} - \frac{\rho_0 h}{\pi} \right], \\ \sigma(y = 2a + b) &= + \left[ \frac{\sigma_1}{2} - \frac{\rho_0 h}{\pi} \right],\end{aligned}$$

(β)

$$\vec{E}_1 = \hat{i}_y \frac{1}{\epsilon_0} \left[ \frac{\sigma_1}{2} - \frac{\rho_0 h}{\pi} \right],$$

$$\vec{E}_2 = 0,$$

$$\vec{E}_3 = \hat{i}_y \frac{1}{\epsilon_2} \left[ \frac{\sigma_1}{2} - \frac{\rho_0 h}{\pi} \right],$$

$$\vec{E}_4 = -\hat{i}_y \frac{1}{\epsilon_2} \left[ \frac{\sigma_1}{2} + \frac{\rho_0 h}{\pi} \right],$$

$$\vec{E}_5 = -\hat{i}_y \frac{1}{\epsilon_1} \left[ \frac{\sigma_1}{2} + \frac{\rho_0 h}{\pi} \cos(\pi y/h) \right],$$

$$\vec{E}_6 = -\hat{i}_y \frac{1}{\epsilon_0} \left[ \frac{\sigma_1}{2} - \frac{\rho_0 h}{\pi} \right],$$

(\gamma)

$$\begin{aligned} \vec{f}_1 &= -\hat{i}_y \sigma_1 \frac{1}{\epsilon_2} \frac{\rho_0 h}{\pi}, \\ \vec{f}(y=2a) &= -\hat{i}_y \frac{1}{2\epsilon_2} \left[ \frac{\sigma_1}{2} - \frac{\rho_0 h}{\pi} \right]^2, \\ \vec{f}(y=2a+b) &= +\hat{i}_y \frac{1}{2\epsilon_0} \left[ \frac{\sigma_1}{2} - \frac{\rho_0 h}{\pi} \right]^2. \end{aligned}$$

### \Theta\epsilon\mu\alpha 2

(\alpha)

$$\vec{H}_1 = -\frac{K_1}{2} \left[ \hat{i}_z \cos \Phi - \hat{i}_x \sin \Phi \right] + aK_0 \cos \vartheta \left[ \frac{(y-y_0)\hat{i}_z - (z-z_0)\hat{i}_y}{(y-y_0)^2 + (z-z_0)^2} \right],$$

$$\vec{H}_2 = +\frac{K_1}{2} \left[ \hat{i}_z \cos \Phi - \hat{i}_x \sin \Phi \right] + aK_0 \cos \vartheta \left[ \frac{(y-y_0)\hat{i}_z - (z-z_0)\hat{i}_y}{(y-y_0)^2 + (z-z_0)^2} \right],$$

$$\vec{H}_3 = +\frac{K_1}{2} \left[ \hat{i}_z \cos \Phi - \hat{i}_x \sin \Phi \right] + \hat{i}_x K_0 \sin \vartheta.$$

(\beta)

$$W_m = \frac{1}{2} \mu_0 \left[ \left( \frac{K_1}{2} \right)^2 + K_0^2 \sin^2 \vartheta - K_0 K_1 \sin \vartheta \sin \Phi \right] \pi a^2 L$$

and the maxima or minima are given for

$$\begin{aligned} \vartheta &= \frac{\pi}{2}, \\ \vartheta &= \sin^{-1} \left\{ \frac{K_1 \sin \Phi}{2K_0} \right\} \quad \text{if } K_1 \sin \Phi < 2K_0. \end{aligned}$$

(\gamma)

$$\vec{f}_1 = \frac{a\mu_0 K_1 K_0 \cos \vartheta}{(y_0+d)^2 + (z-z_0)^2} \left[ \hat{i}_x (z-z_0) \sin \Phi + \hat{i}_y (y_0+d) \cos \Phi - \hat{i}_z (z-z_0) \cos \Phi \right],$$

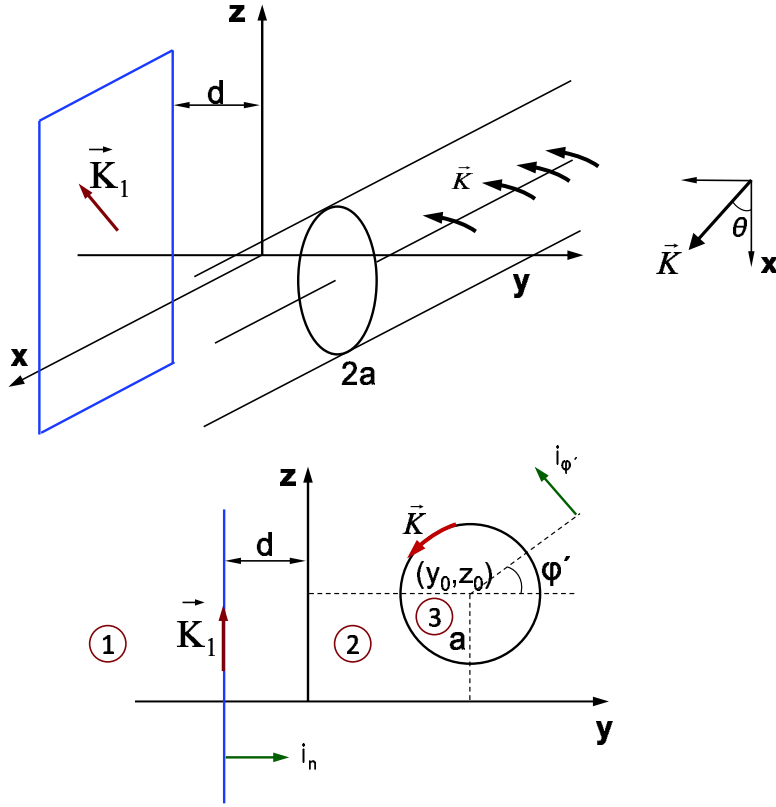


Figure 2: (Problem 2) Given structure geometry with specified (numbered) regions of interest.

$$\vec{f} = \hat{i}_y K_0 \mu_0 \left[ \frac{K_0}{2} \cos(2\vartheta) - \frac{K_1}{2} \cos(\vartheta + \Phi) \right].$$

$\Theta\epsilon\mu\alpha$  3

( $\alpha$ )

$$\vec{E}_1 = \hat{i}_x [E_0 \exp(-jk_0z) + E_r \exp(+jk_0z)],$$

$$\vec{E}_2 = \hat{i}_x [E_{2+} \exp(-\alpha_2z) \exp(-j\beta_2z) + E_{2-} \exp(+\alpha_2z) \exp(+j\beta_2z)],$$

$$\alpha_2 = \omega \sqrt{\mu_0 \epsilon_2} \left\{ \frac{1}{2} \left[ -1 + \left( 1 + \left( \frac{\sigma_2}{\omega \epsilon_2} \right)^2 \right)^{1/2} \right] \right\}^{1/2},$$

$$\beta_2 = \omega \sqrt{\mu_0 \epsilon_2} \left\{ \frac{1}{2} \left[ +1 + \left( 1 + \left( \frac{\sigma_2}{\omega \epsilon_2} \right)^2 \right)^{1/2} \right] \right\}^{1/2},$$

$$\vec{E}_3 = 0.$$

( $\beta$ )

$$\vec{H}_1 = \hat{i}_y \frac{1}{Z_0} [E_0 \exp(-jk_0z) - E_r \exp(+jk_0z)],$$

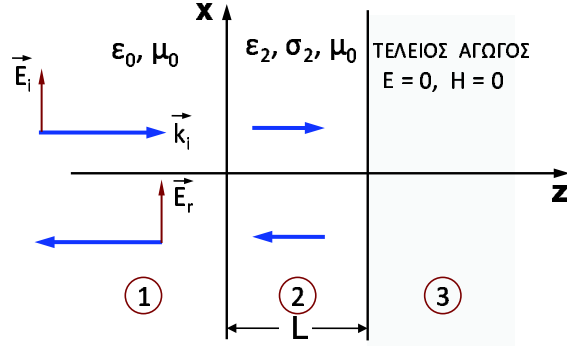


Figure 3: (Problem 3) Given structure geometry with specified (numbered) regions of interest.

$$\begin{aligned}
 Z_0 &= \sqrt{\frac{\mu_0}{\epsilon_0}}, \\
 \vec{H}_2 &= \hat{i}_y \frac{1}{Z_2} [E_{2+} \exp(-\alpha_2 z) \exp(-j\beta_2 z) - E_{2-} \exp(+\alpha_2 z) \exp(+j\beta_2 z)], \\
 Z_2 &= \sqrt{\frac{j\omega\mu_0}{\sigma_2 + j\omega\epsilon_0}}, \\
 \vec{H}_3 &= 0.
 \end{aligned}$$

(γ)

$$\begin{aligned}
 E_0 + E_r &= E_{2+} + E_{2-}, \\
 \frac{1}{Z_0} [E_0 - E_r] &= \frac{1}{Z_2} [E_{2+} - E_{2-}], \\
 E_{2+} \exp(-j\beta_2 L) \exp(-\alpha_2 L) + E_{2-} \exp(+j\beta_2 L) \exp(+\alpha_2 L) &= 0.
 \end{aligned}$$

$$E_r = E_0 \frac{-(1+A) + (1-A)(Z_2/Z_0)}{(1+A) + (1-A)(Z_2/Z_0)},$$

$$\begin{aligned}
E_{2+} &= \frac{2E_0}{(1-A) + (1+A)(Z_0/Z_2)}, \\
E_{2-} &= \frac{-2AE_0}{(1-A) + (1+A)(Z_0/Z_2)}, \\
A &= \exp(-j\beta_2 L) \exp(-2\alpha_2 L).
\end{aligned}$$

( $\delta$ )

$$\vec{K} = \hat{i}_x \frac{2 \exp(-j\beta_2 L) \exp(-\alpha_2 L)}{Z_2} \frac{2E_0}{(1-A) + (1+A)(Z_0/Z_2)}$$

( $\epsilon$ )

$$\begin{aligned}
\vec{f} &= \hat{i}_z \frac{2E_0^2}{Z_0^2} \mu_0 \cos^2(\omega t), \\
\langle \vec{f} \rangle &= \hat{i}_z \frac{E_0^2}{Z_0^2} \mu_0
\end{aligned}$$