

Θεμα 1

(α)

$$\vec{E} = \frac{1}{2\epsilon_0} \left\{ \begin{array}{ll} [(\sigma_1 + \sigma_2) \sin \phi \hat{i}_x + [(\sigma_1 - \sigma_2) \cos \phi + \sigma_3] \hat{i}_z] & \text{for region 1,} \\ [(\sigma_1 - \sigma_2) \sin \phi \hat{i}_x + [(\sigma_1 + \sigma_2) \cos \phi + \sigma_3] \hat{i}_z] & \text{for region 2,} \\ [-(\sigma_1 + \sigma_2) \sin \phi \hat{i}_x + [(-\sigma_1 + \sigma_2) \cos \phi + \sigma_3] \hat{i}_z] & \text{for region 3,} \\ [-(\sigma_1 + \sigma_2) \sin \phi \hat{i}_x + [(-\sigma_1 + \sigma_2) \cos \phi - \sigma_3] \hat{i}_z] & \text{for region 4,} \\ [(-\sigma_1 + \sigma_2) \sin \phi \hat{i}_x - [(\sigma_1 + \sigma_2) \cos \phi + \sigma_3] \hat{i}_z] & \text{for region 5,} \\ [(\sigma_1 + \sigma_2) \sin \phi \hat{i}_x + [(\sigma_1 - \sigma_2) \cos \phi - \sigma_3] \hat{i}_z] & \text{for region 6.} \end{array} \right.$$

(β)

$$\sigma_1 = -\sigma_2$$

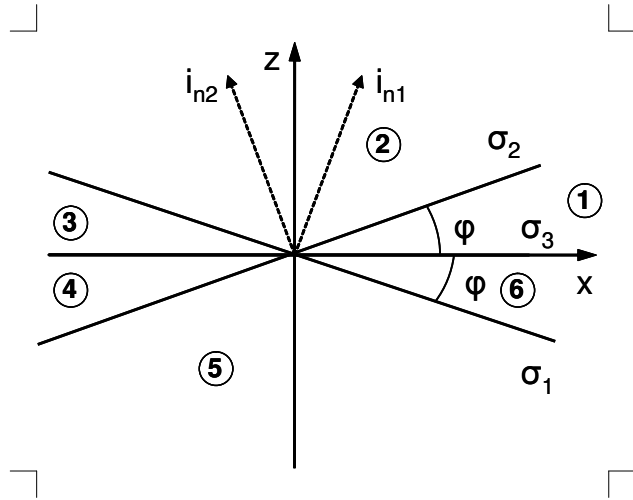
$$\sigma_3 = -2\sigma_1 \cos \phi$$

(γ)

$$W_e = \left\{ \begin{array}{ll} 0 & \text{for region 1,} \\ \frac{1}{2} \frac{\sigma_1^2}{\epsilon_0} & \text{for region 2,} \\ \frac{2\sigma_1^2 \cos^2 \phi}{\epsilon_0} & \text{for region 3,} \\ 0 & \text{for region 4,} \\ \frac{1}{2} \frac{\sigma_1^2}{\epsilon_0} & \text{for region 5,} \\ \frac{2\sigma_1^2 \cos^2 \phi}{\epsilon_0} & \text{for region 6,} \end{array} \right.$$

If $0 \leq \phi \leq \pi/3$ then $(W_e)_{max} = 2\sigma_1^2 \cos^2 \phi / \epsilon_0$

If $\pi/3 < \phi < \pi/2$ then $(W_e)_{max} = \sigma_1^2 / 2\epsilon_0$



Θεµα 2

(α)

$$\frac{K_0}{2} + \frac{J_{01}d_1}{\pi} - \frac{J_{02}d_2}{\pi} = 0$$

(β)

$$\vec{H} = \hat{i}_z \left\{ \begin{array}{ll} 0 & -\infty < x \leq -a - d_1/2 \\ \frac{K_0}{2} - \frac{J_{01}d_1}{\pi} \sin\left(\pi \frac{x+a}{d_1}\right) - \frac{J_{02}d_2}{\pi} & -a - d_1/2 \leq x \leq -a + d_1/2, \\ \frac{K_0}{2} - \frac{J_{01}d_1}{\pi} - \frac{J_{02}d_2}{\pi} & -a + d_1/2 \leq x < 0, \\ -\frac{K_0}{2} - \frac{J_{01}d_1}{\pi} - \frac{J_{02}d_2}{\pi} & 0 < x \leq b - d_2/2, \\ -\frac{K_0}{2} - \frac{J_{01}d_1}{\pi} + \frac{J_{02}d_2}{\pi} \sin\left(\pi \frac{x-b}{d_2}\right) & b - d_2/2 \leq x \leq b + d_2/2, \\ 0 & b + d_2/2 \leq x < +\infty \end{array} \right.$$

(γ)

$$\frac{W_m}{S} = \frac{3}{4} \mu_0 \frac{J_{01}^2 d_1^3}{\pi^2}$$

(δ)

$$\vec{f} = -\hat{i}_x \mu_0 K_0 \left(\frac{K_0}{2} + \frac{2J_{01}d_1}{\pi} \right) = -\hat{i}_x \mu_0 K_0 \left(\frac{J_{01}d_1}{\pi} + \frac{J_{02}d_2}{\pi} \right)$$

Θεµα 3

(α)

$$\nu = 4.775 \times 10^9 \text{ Hz}$$

$$\lambda_0 = 0.628 \text{ m}$$

$$\theta = 36.87 \text{ deg} = 0.6435 \text{ rad}$$

Polarization = Linear (parallel or TM)

(β)

$$\vec{E}_i = -Z_0 H_{i0} [\hat{i}_x \cos \theta - \hat{i}_y \sin \theta] \exp[-j(k_{ix}x + k_{iy}y)]$$

$$\vec{E}_i = [-3013.84 \hat{i}_x + 2260.38 \hat{i}_y] \exp[-j(6x + 8y)] \text{ (V/m)}$$

$$\begin{aligned}\vec{\mathcal{E}}_i &= -Z_0 H_{i0} [\hat{i}_x \cos \theta - \hat{i}_y \sin \theta] \cos[\omega t - k_{ix}x - k_{iy}y] \\ \vec{\mathcal{E}}_i &= [-3013.84\hat{i}_x + 2260.38\hat{i}_y] \cos[3 \times 10^9 t - 6x - 8y] (V/m)\end{aligned}$$

(γ)

$$\begin{aligned}\vec{E}_r &= Z_0 H_{i0} [\hat{i}_x \cos \theta + \hat{i}_y \sin \theta] \exp[-j(k_{ix}x - k_{iy}y)] \\ \vec{E}_r &= [3013.84\hat{i}_x + 2260.38\hat{i}_y] \exp[-j(6x - 8y)] (V/m) \\ \vec{H}_r &= H_{i0} \hat{i}_z \exp[-j(k_{ix}x - k_{iy}y)] \\ \vec{H}_r &= 10\hat{i}_z \exp[-j(6x - 8y)] (A/m)\end{aligned}$$

(δ)

$$\begin{aligned}\vec{H}_1 &= H_{i0} \hat{i}_z \exp(-jk_{ix}x) 2 \cos(k_{iy}y) \\ \vec{\mathcal{H}}_1 &= 20\hat{i}_z \cos(8y) \cos[3 \times 10^9 t - 6x] (A/m) \\ \vec{E}_1 &= -Z_0 H_{i0} [\hat{i}_x \cos \theta - \hat{i}_y \sin \theta] \exp[-j(k_{ix}x + k_{iy}y)] + \\ &\quad Z_0 H_{i0} [\hat{i}_x \cos \theta + \hat{i}_y \sin \theta] \exp[-j(k_{ix}x - k_{iy}y)] + \\ \vec{\mathcal{E}}_1 &= 3013.84 \times 2 \sin(8y) \cos[3 \times 10^9 t - 6x + \pi/2] \hat{i}_x + \\ &\quad 2260.38 \times 2 \cos(8y) \cos[3 \times 10^9 t - 6x] \hat{i}_y (V/m)\end{aligned}$$

(ϵ)

$$\begin{aligned}\vec{K} &= -2H_{i0} \hat{i}_x \exp(-jk_{ix}x) \\ \vec{\mathcal{K}} &= -20\hat{i}_x \cos[3 \times 10^9 t - 6x] (A/m) \\ \sigma &= -2\epsilon_0 Z_0 H_{i0} \sin \theta \exp(-jk_{ix}x) \\ \sigma &= -4520.76\epsilon_0 \cos[3 \times 10^9 t - 6x] (C/m^2)\end{aligned}$$

($\sigma\tau$)

$$\begin{aligned}\vec{f} &= \vec{f}_e + \vec{f}_m \\ \vec{f}_e &= -\hat{i}_y 2\epsilon_0 (Z_0 H_{i0})^2 \sin^2 \theta \cos^2(\omega t - k_{ix}x) (Nt/m^2) \\ \vec{f}_e &= -\hat{i}_y 9.0475 \times 10^{-5} \cos^2(3 \times 10^9 t - 6x) (Nt/m^2)\end{aligned}$$

$$\langle \vec{f}_e \rangle = -\hat{i}_y 4.523 \times 10^{-5} (Nt/m^2)$$

$$\vec{f}_m = \hat{i}_y 2\mu_0 (H_{i0})^2 \cos^2(\omega t - k_{ix}x) (Nt/m^2)$$

$$\vec{f}_m = \hat{i}_y 2.5133 \times 10^{-4} \cos^2(3 \times 10^9 t - 6x) (Nt/m^2)$$

$$\langle \vec{f}_m \rangle = \hat{i}_y 1.2566 \times 10^{-4} (Nt/m^2)$$

$$\langle \vec{f} \rangle = \langle \vec{f}_e \rangle + \langle \vec{f}_m \rangle$$