

Nonlinear dynamics and recurrence analysis of extreme precipitation for observed and general circulation model generated climates

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Abstract:

A statistical framework based on nonlinear dynamics theory and recurrence quantification analysis of dynamical systems is proposed to quantitatively identify the temporal characteristics of extreme (maximum) daily precipitation series. The methodology focuses on both observed and general circulation model (GCM) generated climates for present (1961–2000) and future (2061–2100) periods which correspond to 1xCO₂ and 2xCO₂ simulations. The daily precipitation has been modelled as a stochastic process coupled with atmospheric circulation. An automated and objective classification of daily circulation patterns (CPs) based on optimized fuzzy rules was used to classify both observed CPs and ECHAM4 GCM-generated CPs for 1xCO₂ and 2xCO₂ climate simulations (scenarios). The coupled model ‘CP-precipitation’ was suitable for precipitation downscaling. The overall methodology was applied to the medium-sized mountainous Mesochora catchment in Central-Western Greece. Results reveal substantial differences between the observed maximum daily precipitation statistical patterns and those produced by the two climate scenarios. A variable nonlinear deterministic behaviour characterizes all climate scenarios examined. Transitions’ patterns differ in terms of duration and intensity. The 2xCO₂ scenario contains the strongest transitions highlighting an unusual shift between floods and droughts. The implications of the results to the predictability of the phenomenon are also discussed. Copyright © 2013 John Wiley & Sons, Ltd.

KEY WORDS extreme precipitation patterns; GCM-downscaled precipitation conditioned on atmospheric circulation; nonlinear dynamics; recurrence quantification analysis; chaotic time series

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INTRODUCTION/MOTIVATION

Advanced time series analysis techniques, such as generalized regression models (El-Aldouni *et al.*, 2005; Underwood 2009; van Ogtrop *et al.*, 2011), trend analysis (Pujol *et al.*, 2007; Beguería *et al.*, 2010; Tabari and Talaei, 2011), and univariate and multivariate autoregressive modelling (Langousis and Koutsoyiannis, 2006; Lins *et al.*, 2011; Koirala *et al.*, 2011) have been extensively considered in hydrometeorology. Also, shifting mean models (Sveinsson *et al.*, 2003; Rea *et al.*, 2011), extreme values theory, and other frequency analysis methods, such as spectral analysis, wavelets, and detrended fluctuation analysis (Khaliq *et al.*, 2006; Kantelhardt *et al.*, 2006; Zhang *et al.*, 2008; Koirala *et al.*, 2011) have a history of more than two decades. Most recently, the dynamical systems approach (e.g. Jayawardena and Gurung, 2000) and Bayesian techniques (Coles and Tawn, 1996; Coles *et al.*, 2003; Renard *et al.*, 2006; Kottogoda *et al.*, 2011) have gained the interest of researchers due to

their flexibility in modelling of complex and temporally oscillating behaviour of variables, as well as the modelling of uncertainty in the temporal evolution of variables.

A large part of literature on hydro-meteorological processes has reported to the modelling of extreme values of time series such as daily, monthly, or annually extremes of precipitation data. Extreme values most likely reflect changes in the magnitude and spatiotemporal variation of the phenomenon under study. Acquiring knowledge on such behaviour could lead to effective proactive management of extreme hydrological phenomena with significant social and economic implications (Fowler *et al.*, 2005). Several modelling approaches of extreme values can be found in literature. These include fitting distributions to data with or without spatial pooling (Frei and Schär, 2001; Yue and Wang, 2002; Stuart *et al.*, 2003), extreme multivariate analysis (Renard and Lang, 2007), extreme value models with time-dependent covariates (Tramblay *et al.*, 2011), Bayesian change point modelling (Perreault *et al.*, 2000; Rasmussen, 2001; Coles *et al.*, 2003; Xiong and Guo, 2004; Renard *et al.*, 2006; Seidou *et al.*, 2007), and multi-fractal analysis (Sun and Barros, 2010). A comparison of several alternative distributions for extreme value modelling using an extensive global dataset can be found in Papalexiou *et al.* (2013).

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A critical issue in modelling the complex temporal behaviour of hydro-meteorological time series is the treatment of non-stationarity and the temporal persistence, either being short or long, in the hydrological time series. Hydrological time series have been found to incorporate long term dependence and scaling behaviour (Douglas and Barros, 2003; Langousis and Koutsoyiannis, 2006; Koutsoyiannis and Montanari, 2007; Koutsoyiannis *et al.*, 2007; Koutsoyiannis, 2011) as well as multi-fractal behaviour (Cortis *et al.*, 2008). Classical time series incorporate several constraints regarding stationarity and independence, while it has been proven that most real-world time series are most likely to be non-stationary and nonlinear in nature (Schreiber, 1999). Coles *et al.* (2003) stated that most classical statistical approaches, which do not take into account the modelling and prediction of uncertainties in the temporal evolution of hydro-meteorological time series, are bounded to produce an over-optimistic appraisal of anticipated extreme conditions that is often contradicted by real measured data. This false assumption of a stationary model may also lead to a considerable underestimation of the probability of an extreme event occurrence.

Klemes (1974) argued that long memory effects may be revealed in hydrological time series due to the implementation of stationary constrained models to complex non-stationary time series. However, Klemes (1974) recognized the non-stationary means in the Hurst phenomenon. Khaliq *et al.* (2006), in a thorough review of existing frequency approaches to treating hydro – meteorological data, underlined the need to apply methodologies that could account for the complex non-stationary behaviour of hydro-meteorological observations. A similar comment on the inadequacy of most stochastic approaches to model environmental processes is made by Cortis *et al.* (2008).

A general remark on the existing literature on hydro-meteorological time series applications is that, in most cases, the time series are usually erroneously modelled as having a homogeneous behaviour, for example being stochastic or nonlinear deterministic and so on. In time series modelling, there exist two paradigms to model irregular time series, the deterministic nonlinear and the linear stochastic behaviour (Schreiber, 1999). Between these two boundaries of modelling exists a wide range of different models and behaviour defined by different degrees of nonlinearity and stochasticity that cannot be addressed by a single model.

The knowledge of whether the series behave deterministically or stochastically, or even, whether the series oscillate or shift from periodic to chaotic behaviour is of great significance in the data-driven prediction process. Hence, a key question arises concerning the simulated hydro-meteorological time series for 1xCO₂ and 2xCO₂ climate scenarios: The most up-to-date approaches are related to dynamical or statistical downscaling of general circulation model (GCM) outputs such as, precipitation, temperature, airflow, and others which are conditioned or not on atmospheric circulation patterns (CPs) (Giorgi and Mearns, 1991; Leavseley, 1994; Matyasovsky *et al.*, 1994; Wilby *et al.*, 1998; Bárdossy and Mierlo, 2002; Wilby and Wingley, 2000; Panagoulia *et al.*, 2008). However, what actually

happens with the GCM generated climates series themselves? Can these series keep the same degree of nonlinearity and stochasticity or the same oscillations and shifts from periodic to chaotic behaviour with the observed (historical) ones? If not, what differences could be anticipated?

In the remainder of this paper, we provide an alternative framework based on the nonlinear dynamics theory and the recurrence analysis of dynamical systems in order to quantitatively identify the temporal characteristics of extreme (maximum) daily precipitation series. The available daily precipitation time series are conditioned on atmospheric circulation. More specifically, an automated objective classification of daily CPs based on optimized fuzzy rules was used to classify the observed CPs (Panagoulia *et al.*, 2006a). The ECHAM4 GCM-generated scenarios of daily CPs were used to classify the 1xCO₂ and 2xCO₂ climate scenarios (Panagoulia *et al.*, 2008). The daily precipitation was modelled as a stochastic process coupled with atmospheric circulation (Stehlik and Bárdossy, 2002; Panagoulia *et al.*, 2006b). This type of classification defines circulation (CPs) that can explain the variability of precipitation in a locally specific form ensuring so the dependence between large-scale atmospheric circulation and precipitation which is necessary for precipitation downscaling. The overall methodology was applied to the medium-sized mountainous Mesochora catchment in Central-Western Greece using observed climate time series for 1972–1992, and also using GCM generated climates for present period (1961–2000) and for future period (2061–2100), which correspond to 1xCO₂ and 2xCO₂ simulations (scenarios).

METHODOLOGICAL FRAMEWORK

Phase-space reconstruction

Let W_i be the sliding window of time where $i = 1, 2, \dots, n$, then, a pattern of a variable X in the i -th window can be defined by: $\mathbf{X}_i(t) = (x_t, x_{t-\tau}, \dots, x_{(m-1)t-\tau})$, where τ is the time delay and m is the dimension. These two parameters define the depth of information that a pattern carries. The process of constructing the previous vector is named phase-space reconstruction and aims at converting a scalar time data into a multivariable system. It has been shown by Takens (1981) that the reconstruction of the dynamics of a system by one variable is possible if m is chosen such as: $2m + 1 > d$, where d is the actual system's dimension. The optimum values of τ and m can be estimated using the mutual information that measures the information flow between sequential time delays (Fraser and Swinney, 1986) and the false nearest neighbors. This last feature examines the behaviour of near neighbors under changes in the dimension from m to $m+1$ (Kennel *et al.* 1992) equally.

Visualizing and quantifying dynamics in the phase-space

The basic characteristic of dynamical systems is that they exhibit a recurrent behaviour in terms of their temporal

characteristics in the phase-space. The recurrence of a system's state x at time i in a different time j is given by (Marwan *et al.*, 2007):

$$R_{i,j}^{m,\varepsilon_i} = \theta(\varepsilon_i - \|\vec{x}_i - \vec{x}_j\|), \rightarrow x_i \in \mathbb{R}^m, i, j = 1 \dots N \quad (1)$$

where N is the number of states x_i in the time window of study, ε_i is the threshold (Euclidean distance) for the distances $R_{i,j}$, $\|\cdot\|$ a norm (Euclidean norm), and $\theta(\cdot)$ the Heaviside function: $(\theta < 0) = 0, (\theta \geq 0) = 1$. The plot of all distances observed provides the recurrence plot (RP) (Eckmann *et al.*, 1987). Given constant value of ε value, the RP is symmetric. Moreover, the RP has a diagonal line of $\pi/4$ angle which is called *Line of Identity* (LOI) and corresponds to states recurrent state of $R_{i,j}^{m,\varepsilon_i} = 1$ (Marwan *et al.*, 2007).

The RP provides a visual manner in inspecting the recurrent behaviour of variables and various complex patterns (Marwan *et al.*, 2007). For example, in a time window of study, states can be recurrent or isolated in time. From the recurrent ones, those that may form diagonal lines parallel to the LOI reveal a deterministic behaviour. These lines may be short expressing nonlinear or chaotic behaviour or substantially long that signifies a periodic behaviour. Moreover, vertical lines and the patterns they form correspond to the tangential motion in the phase-space (laminar states in intermittent regimes) (Gao and Cai, 2000, Marwan *et al.*, 2007). The laminar states detection is important in order to identify the slow varying states. Table I depicts the statistical analogies of observed patterns on RPs (Marwan *et al.*, 2007).

Zbilut and Webber (1992) and, later, Webber and Zbilut (1994) provided a statistical manner of quantifying the RPs named as Recurrence Qualitative Analysis (RQA). RQA is established on a set of statistical measures that quantify the previously mentioned RP patterns. These measures are seen in Table II. In brief, RQA is based on the percentage of recurrent points observed in a time window of study (%REC), which is an indication of the correlation observed in the data (Casdagli, 1997). This hints that the near past holds information on the manner the system evolves in time.

Based on this, the percentage of recurrent points that form lines parallel to the main diagonal can be defined as %DET. %DET is a sign of the deterministic evolution in which high values of %DET hint a deterministic structure in the phase-space. Next, the complexity of the observed deterministic structure is quantified using the ENT

variable which corresponds to the frequency distribution of the diagonal line lengths and can be related to the Shannon entropy (Marwan *et al.*, 2007). The more complex the determinism observed in the system under study, the higher the ENT value, the higher the complexity in the time series structure, the more difficult to predict its short-term evolution. Moreover, the maximum length of the diagonal line provided by the L_{max} is related to the inverse of the largest positive Lyapunov Exponent (Eckmann *et al.*, 1987; Zbilut *et al.*, 1998). Low values of L_{max} are an indication of strong nonlinearity that is characterized by chaotic behaviour. The joint consideration of such statistics is essential in analyzing systems with complex dynamics.

From a methodological point of view, the proposed data analysis approach has been found to be particularly effective for highlighting hidden structures of the dynamics of the system under consideration without imposing any pre-defined constraint to the data. Some applications include nonlinear analysis of Electroencephalogram signals and heart rate data (Thomasson *et al.*, 2001, Wessel *et al.*, 2001), financial time series analysis and stock market indices (McKenzie, 2001; Belaire-Franch, 2004; Fabretti and Ausloos, 2005), IP-network traffic (Masugi, 2006), analysis of seismic processes (Matcharashvili *et al.*, 2008), and road traffic analysis (Vlahogianni *et al.*, 2006; Vlahogianni *et al.*, 2007; Vlahogianni *et al.*, 2008).

RQA incorporates two essential modelling features. The first is that quantification does not depend upon mathematical transformations such as the Fourier transform which is linear, wavelets, and so on. The second is that quantification can be carried out in multiple dimensions through the method of time delays. This last is important when treating chaotic-like systems that are multidimensional and nonlinear (Zbilut, 2000).

IMPLEMENTATION AND RESULTS

The data sets

The proposed methodological framework was applied to mountainous Mesochora catchment that is the most upstream sub-catchment of the Acheloos R. catchment. At the outlet of the catchment, a reservoir has been constructed (useful capacity of 228 hm³) with a hydropower plant with installed capacity of 160 MW. Mean annual discharge is equal to 23.2 m³/s. The

Table I. Statistical analogies of patterns on Recurrence Plots (Marwan *et al.* 2007)

RP patterns	Statistical analogies
Homogeneity	Stationary process
Fading	Non-stationarity (trend)
Disruptions	Non-stationarity; transitional behaviour
Periodic Patterns	Cyclicities in the process
Single Isolated Points	Heavy fluctuation in the process; random process
Diagonal Lines	Similar evolution of states at different times. Sign of deterministic process
Vertical and Horizontal lines/clusters	Some states do not change or change slowly for some time (laminarity)

Table II. Definition of RQA variables

Variable	Definition	
REC	Percent of recurrent points in the recurrence matrix	$\frac{1}{N^2} \sum_{l=1}^N R_{i,j}^{m,\varepsilon}$
DET	Percent of recurrent points in the recurrence matrix that form diagonal lines	$\frac{\sum_{l=1}^N IP(l)}{\sum_{l=1}^N IP(l)}$
ENT	Reflects the complexity of the deterministic structure in the system	$-\sum_{l=l_{\min}}^N P(l) \ln P(l)$
L_{\max}	Maximum length of the diagonal lines	$\max(\{l_i ; i = 1, \dots, N\})$

$P(l)$: the frequency distribution of the lengths l of the diagonal lines
 N : the length of the time window W considered
 m : the embedding dimension
 ε : the distance threshold (Euclidean distance)

catchment with an area of about 633 km² lies in the central-western mountain region of Greece (Figure 1) and extends nearly 32 km from north (39° 50') to south (39° 42') with an average width of about 20 km. The mean catchment elevation is 1390 m, while the elevation from the highest point to the catchment outlet ranges from 2200 m to 780 m.

The climate of the area is dominated by cold and wet winters, as well as by warm and dry summers, and the soils have been formed from decay of hard limestones and flysch (clayish, psammitic and mixed). The precipitation stations are installed within and around the catchment. The most of the stations are located at the lower half of the catchment over a range of elevations from 780 to 1160 m. The daily values of precipitation were available at 12 stations for the period of 1972–1992. The precipitation variability at the stations was determined by conditioning on atmospheric CPs. The CPs were classified via the fuzzy-rules-based approach combined with the simulated annealing algorithm (Bárdossy *et al.*, 2002; Panagoulia *et al.*, 2006a). The pressure data used were obtained from the National Meteorological Centre (NMC) grid-point data set for different windows over Europe with a grid resolution of 5° × 5°. It was found that the 700 hPa data in the window 20°–65° N, 20°W–50° E provided the best results, while the optimal number of CPs defined to 12 based on the automated objective optimization procedure (Panagoulia *et al.*, 2006a).

The space-time intermittence, the occurrence probability of dry days, the rainfall amounts on wet days, as well as the clustering of wet and dry day occurrence that has great impacts on the CPs persistence have been taken into account in mathematical modelling of daily precipitation adopting the methodology of Stehlik and Bárdossy (2002). The observed daily precipitation series for all the available periods were used to estimate the precipi-

tation coupling parameters, which describe the stochastic links between CPs and point precipitation data. Using these attained parameters, the precipitation time series were simulated (generated) (Panagoulia *et al.*, 2006b). A spatial covariance function for observed and simulated time series was assessed. The interpolation of point precipitation data to a regular grid and subsequently to nine elevation zones of the whole catchment was carried out using external drift kriging (Ahmed and de Marsily, 1987). The weighted mean precipitation from all zones was treated as the mean areal precipitation (the weighting was proportional to the zone area). Beyond the CP-dependent observed daily precipitation over the Mesochora catchment for the period 1972–1992, down-scaling was carried out for ECHAM4 GCM-generated CPs. The analysis was based on daily values at the aforesaid sector 20°–65° N, 20°W–50° E over the 700 hPa pressure field for 1xCO₂ and 2xCO₂ climate scenarios in the corresponding periods 1961–2000 and 2061–2100. The geo-potential pressure heights (the 700 hPa pressure) for both scenarios were classified by applying the same method as described above for the observed data. With the estimated parameters of the stochastic precipitation model for the observed data, the classified GCM-CPs point precipitation time series were generated representing the two climate scenarios (Panagoulia *et al.* 2008). Subsequently, the zone and mean areal precipitation for the two climate scenarios were calculated as described above for the observed data.

From the resulting daily precipitation, we calculated and further analysed three extreme time series: The first was the CP-dependent historical maximum daily mean areal precipitation series calculated in each month for each year for the period 1972–1992. This series is depicted in Figure 2. The other two were the maximum daily mean areal precipitation series in each month for each year with regards to the two ECHAM4 GCM-

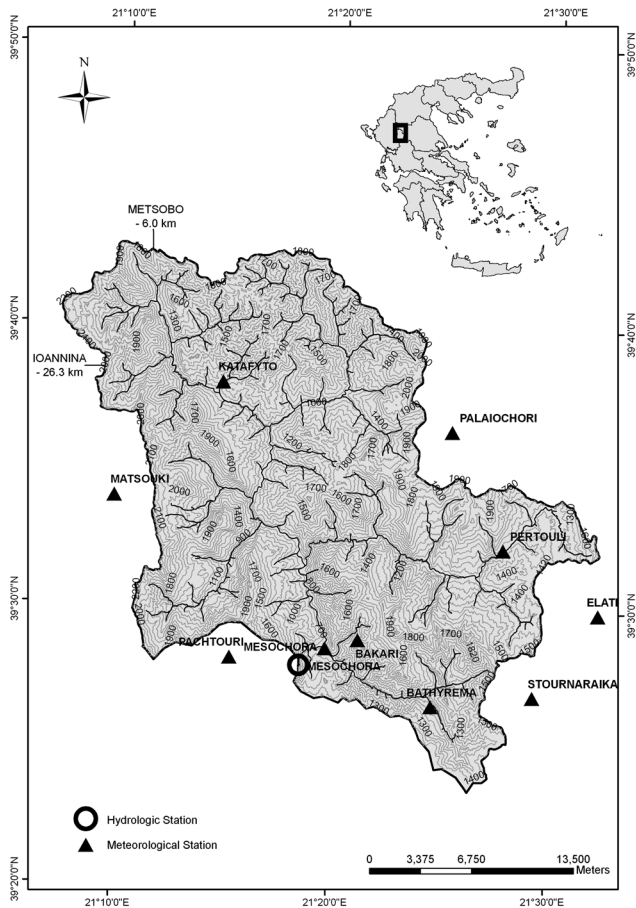


Figure 1. The Mesochora catchment, Greece: Topography and hydro-meteorological stations

generated scenarios of daily CPs for 1xCO₂ and 2xCO₂ climate scenarios corresponding to the periods 1961–2000 and 2061–2100 as these are depicted in Figure 3.

Visual inspection of the precipitation patterns

In order to apply the recurrence quantification analysis (RQA), the available (calculated) series are embedded in the phase-space. Embedding is accomplished using the mutual information and the false nearest neighbor algorithm in order to estimate the optimum values of the time delay τ and the dimension m , respectively.

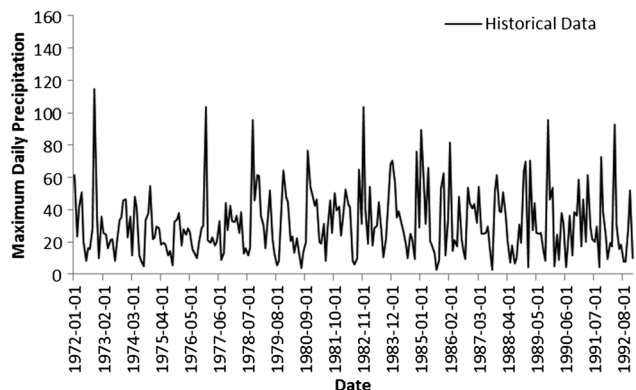


Figure 2. Time series of the historical maximum daily mean areal precipitation (in mm) in each month for each year for the period 1972–1992

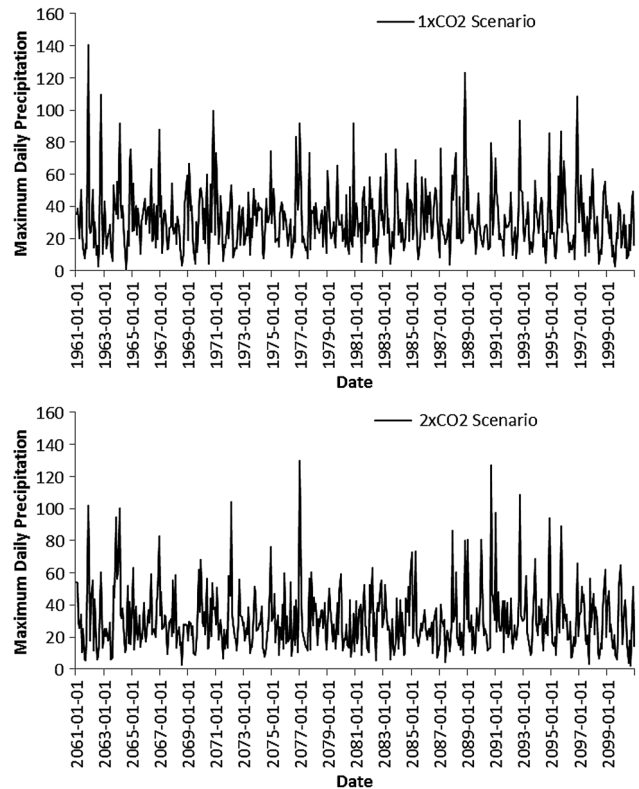


Figure 3. Time series of the maximum daily mean areal precipitation (in mm) in each month for each year for the 1xCO₂ and 2xCO₂ climate scenarios corresponding to the periods 1961–2000 and 2061–2100

Concerning the observed (historical) maximum daily precipitation, from Figure 4(a) (left), we can observe that the optimum value for τ equals to 1, a value that corresponds to the first local minimum. Moreover, Figure 4(a) (right) also shows that the value of m that corresponds to the lowest percent of false neighbors in the reconstructed phase-space is 12. Consequently, the temporal evolution of the maximum daily precipitation is studied through the vector $\mathbf{P}_i(t) = (P_t, P_{t-1}, \dots, P_{t-12})$. The same analysis is conducted for the 1xCO₂ and 2xCO₂ climate scenarios and the results for the τ and m values reflected from Figure 4(b) and (c) are similar to those of the observed (historical) maximum daily precipitation.

Figure 5 shows the RPs of the reconstructed series for the entire time period under study. From the patterns observed in these graphs, various interesting remarks arise. First, each month's historical maximum daily precipitation series exhibits a nonlinear deterministic (chaotic-like) behaviour because short lines parallel to the LOI are reflected in the pattern of the RP (Figure 5a). The length of these lines varies with time indicating a changing degree of nonlinearity in historical maximum daily precipitation series. A different behaviour is observed in maximum daily precipitation series of the 1xCO₂ (Figure 5b) and the 2xCO₂ (Figure 5c) climate scenarios. Square-like patterns and disruptions are evident in both scenarios. Such features characterize a process that exhibits strong variability and non-stationary behaviour that, when compared to those in the RP of the historical

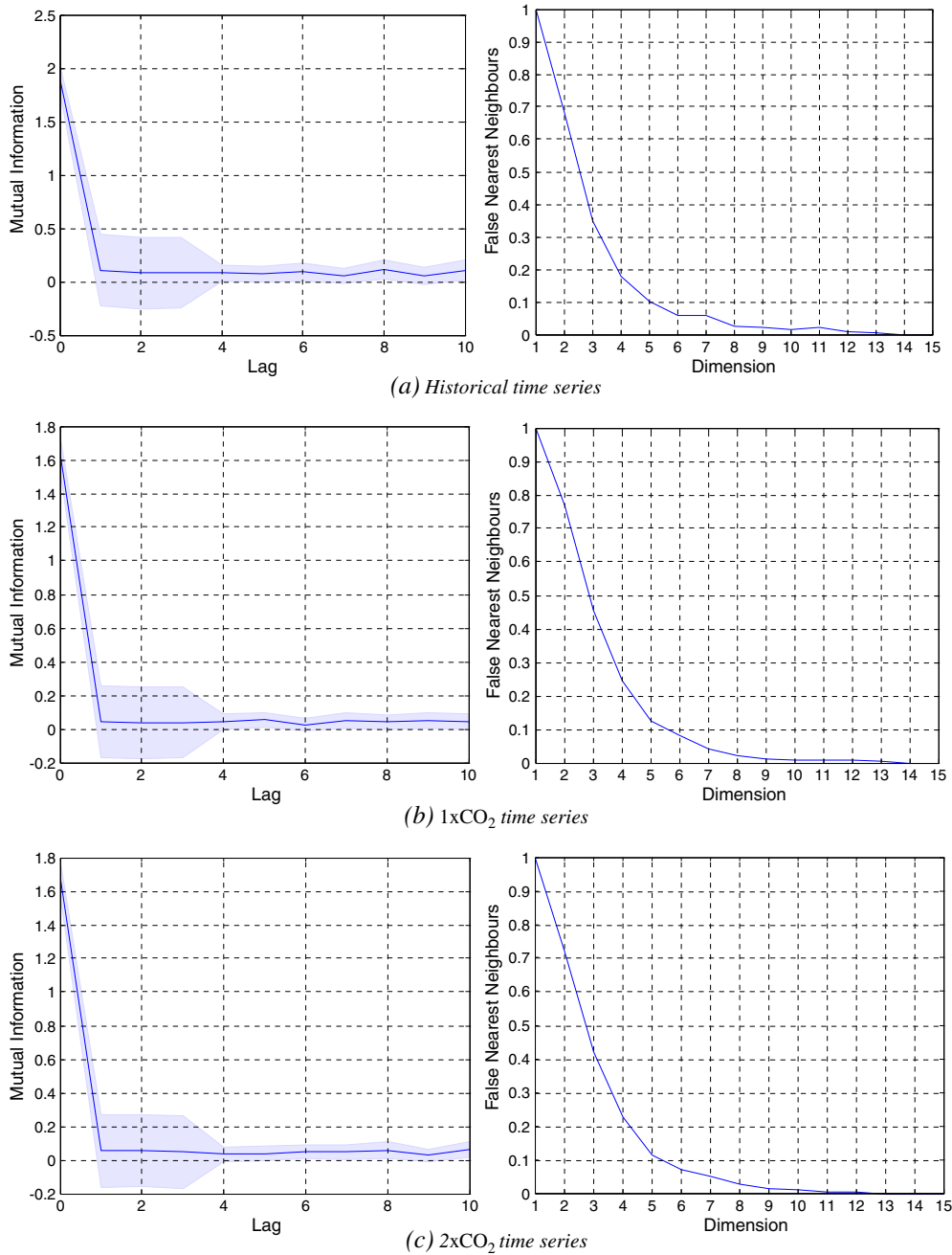


Figure 4. Mutual information with respect to the time delay τ and false nearest neighbors algorithm with respect to the dimension m of maximum daily mean areal precipitation in each month for each year for (a) historical, (b) 1xCO₂, and (c) 2xCO₂ climate scenarios

maximum daily precipitation, a stronger nonlinearity (shorter diagonal lines) is evident.

Table III reports the values of the REC statistic for the three available time series which are the historical (observed) series and the series of 1xCO₂ and 2xCO₂. RQA analysis is conducted using stable distance threshold ε_i equal to 30% of the distances calculated via the Equation (1). In Table III, the percent change between the REC values for the 1xCO₂ and 2xCO₂ and the historical series is also demonstrated. As can be observed, there is a downward change in the values of the REC statistic for the 1xCO₂ and 2xCO₂ series. This implies that the recurrence of the maximum precipitation time series for 1xCO₂ and 2xCO₂ climate scenarios is decreased when compared to that of the historical series.

The reduced recurrence highlights a weaker memory or stochasticity in the downscaled extreme precipitation series implying uncorrelated abrupt hydro-meteorological events such as heavy rainfall episodes and flash floods.

In order to directly compare the three datasets in terms of their deterministic and nonlinear features, the DET, ENT, and L_{\max} statistics are calculated while holding the recurrence stable (REC = 1%). As seen in Table IV, the values of DET and ENT statistics reflect weaker but less complex deterministic behaviour of precipitation in the 1xCO₂ and 2xCO₂ series than that of the historical series. Moreover, the lower values of L_{\max} statistic for the precipitation in the 1xCO₂ and 2xCO₂ series show stronger nonlinearity than this of historical series. Under constant recurrent conditions, the combined values of the

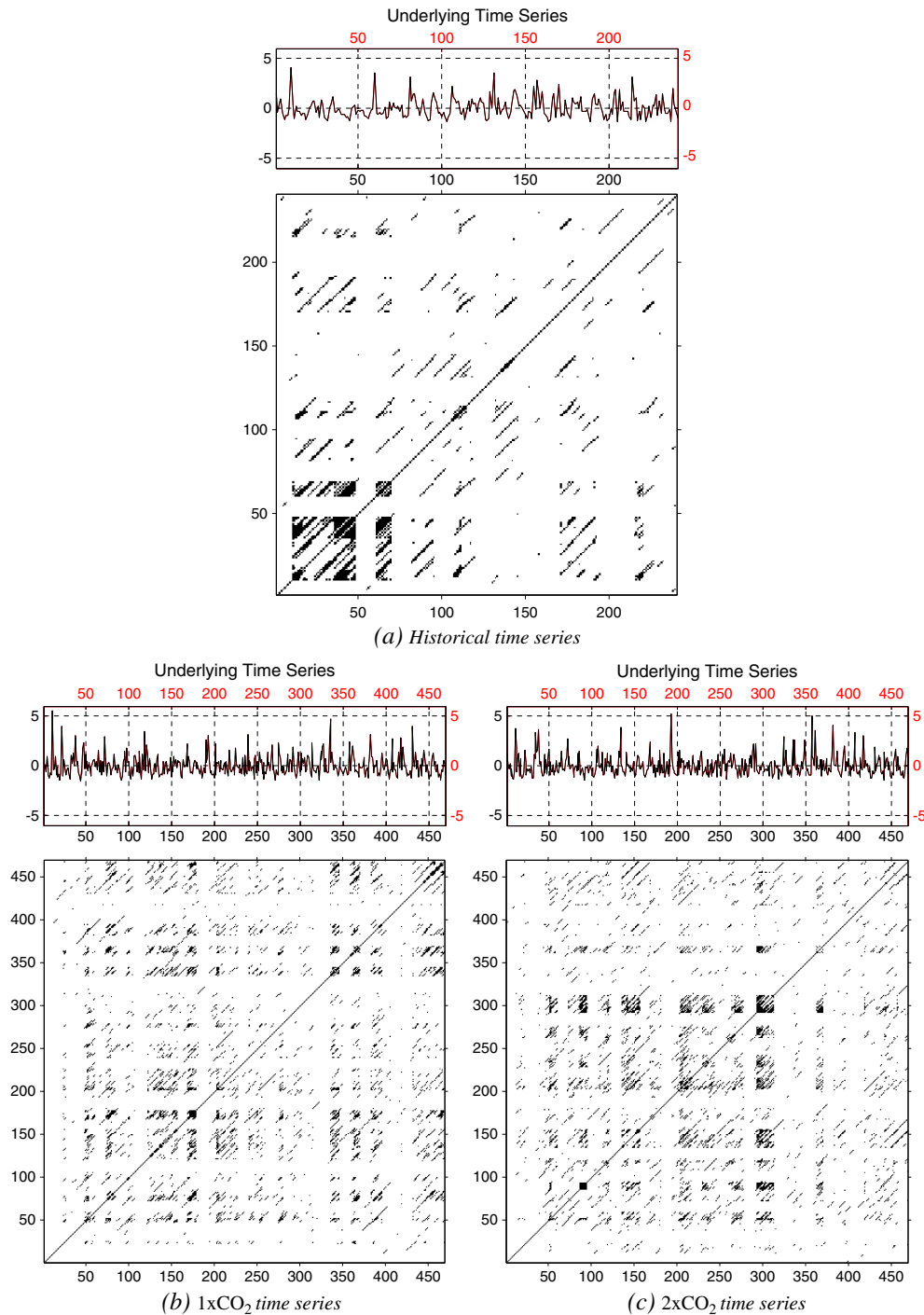


Figure 5. Recurrence plots of maximum daily mean areal precipitation in each month for each year for (a) historical, (b) 1xCO₂, and (c) 2xCO₂ climate scenarios

three statistics support more stochastic, nonlinear and unpredictable structure in future precipitation (2xCO₂ scenario). The hydrological effect of such a precipitation may be a chaotic occurrence of flood events.

Variability of the statistical characteristics of precipitation patterns

In order to examine more thoroughly the temporal behaviour of the precipitation patterns, as well as the patterns for 1xCO₂ and 2xCO₂ climate scenarios, an RQA is conducted in 10-year time window that slides through

time. The choice of the extent of the time window of study is empirically selected so as to have a significant amount of patterns to study the evolution of the average daily maximum precipitation. The results are demonstrated in Figure 6. The time series for all RQA variables start 10 years after the original time series (Figures 2 and 3) in order to construct the first precipitation pattern.

The graph of 10-year average of the historical maximum daily mean areal precipitation sliding over the study period depicts an increasing trend (Figure 6a). In contrast, the corresponding graph for the 1xCO₂ and

Table III. Values of RQA variables for the three available time series

	Historical data (Hist)	1xCO ₂	2xCO ₂	(1xCO ₂ - Hist.)/Hist %	(2xCO ₂ - Hist)/Hist%	(2xCO ₂ -1xCO ₂)/1xCO ₂ %
<i>Mean</i>	31.64	31.86	30.78	0.70	-2.72	-3.39
<i>St. Deviation</i>	20.38	19.58	19.11	-3.93	-6.23	-2.40
<i>REC</i>	0.72	0.56	0.29	-22.22	-59.72	-48.21

Table IV. Values of RQA variables for the three available time series with stable recurrence (REC = 1%)

	Historical data (Hist)	1xCO ₂	2xCO ₂	(1xCO ₂ - Hist.)/Hist%	(2xCO ₂ - Hist)/Hist %	(2xCO ₂ -1xCO ₂)/1xCO ₂ %
<i>DET</i>	87.92	82.95	83.87	-5.2	-4.2	1.1
<i>ENT</i>	2.94	2.50	2.52	-15.0	-14.3	0.8
<i>L_{max}</i>	14.00	12.00	10.00	-14.3	-28.6	-16.7

2xCO₂ climate scenarios (Figure 6b and c) shows a more variable evolution with intensely upward and downward trends. From the REC statistic, it is evident that historical precipitation patterns exhibit a decreasing recurrence structure. From January 1982 and forward, the recurrence of each month's maximum daily mean areal precipitation per year systematically decreases. This is not the case with 1xCO₂ and 2xCO₂ climate scenarios whereas the recurrence is cyclic. In the scenario of 1xCO₂, three time periods are observed. The first period is until 1981 in which the recurrence increases. The second period is until January 1989 in which the recurrence significantly decreases, and in the third (last) period the recurrence starts to increase. For the 2xCO₂ scenario (Figure 6c), the recurrence is increasing until January 2088 where a sudden drop is observed. Observing the recurrence in the three time series, it seems that there is a time period where meteorological characteristics lose their strong dependence from their past states; for the historical data, this coincides with the period after 1986 (Figure 6a), whereas for the 1xCO₂ and 2xCO₂ scenarios with the periods after 1989 and 2091, respectively. This behaviour is associated to an acute increase in the values of all studied maximum daily precipitation time series. This feature is crucial to the predictability of extreme precipitation, as it denotes the time period in which the near past information losses its criticality on the manner by which the system evolves in time.

A more thorough investigation is attempted through an examination of determinism evolution in the three precipitation series (resulting from the historical, and 1xCO₂ and 2xCO₂ climate scenarios). They all exhibit a variable determinism evolution which is stronger to the series of 2xCO₂ climate scenarios. If this evolution is combined with the evolution of the L_{max} , that is related to the inverse of the largest positive Lyapunov Exponent, then all three time series and especially the 1xCO₂ and 2xCO₂ reflect a piecewise nonlinear deterministic structure. In all time series substantial drop to DET value is observed, pointing to a shift from deterministic to stochastic structure. This is illustrated by the drop in 1991–1992 for the observed precipitation series, and by a

similar drop in 1987–1990 for the 1xCO₂ series. For the 2xCO₂ series, a shift to stochasticity is observed around 2096 and 2099. Apart from these examples, there are numerous transitions between determinism and stochasticity observed in 1xCO₂ and 2xCO₂ series. The corresponding behaviour of the historical precipitation patterns seems to be more straightforward.

The shifts between determinism and stochasticity are also evident in the series for the ENT pattern (Figure 6a, b, and c). It is observed that drops in DET are related to drops in ENT. This argument is somehow expected because transitions between nonlinear determinism and stochasticity are more likely to be associated with a decrease in the entropy of the time series.

Another significant element that can be extracted from the analysed time series concerns the intensity and duration of several transitions' patterns. The oscillating behaviour in the observed data is associated with peaks in the RQA statistics which reveal periodic-to-periodic or periodic-to-chaotic transitions that differ in duration and intensity. A characteristic example is the peaks in the 2xCO₂ series observed in 1/2079 and 3/2086 which are reflected in more acute drops in DET statistic than those related to the peak observed in 1/2074 (Figure 6c). These peaks in DET have different characteristics with respect to the nonlinearity and entropy. Moreover, between the 1/2079 and 3/2086 drops on the determinism in 2xCO₂ series, there is a chaotic-to-periodic transition (low to high L_{max} values) which is shifting from low to high complexity, as indicated by the ENT pattern (Figure 6c). In this case, the system of the 2xCO₂ series behaves as a nonlinear deterministic one until 3/2086. Then, a change takes place and the system returns to a state of low periodicity (L_{max}) but with more complex structure than before (higher ENT value).

It is suggested that the proposed approach may be used to examine the variability in the statistical characteristics of hydrological time series. However, this may not straightforwardly reveal the significance in the differences of the three time series (historical, 1xCO₂, and 2xCO₂) which are explored through their statistics patterns evolution. Resampling strategies could be implemented

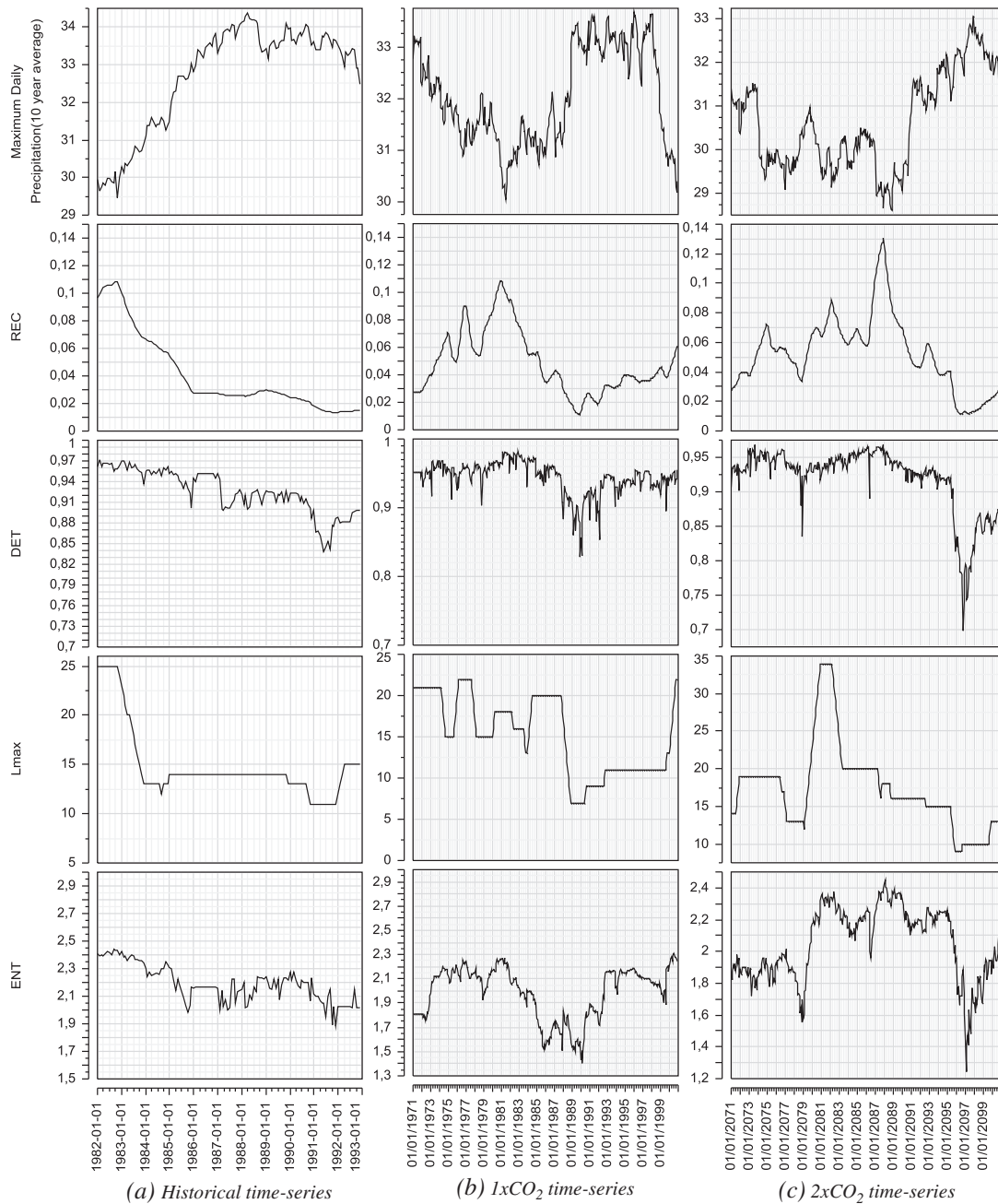


Figure 6. Time series of the maximum daily mean areal precipitation (in mm) in each month for each year for (a) historical, (b) 1xCO₂, and (c) 2xCO₂ climate scenarios for 10-year time window, as well as the REC, DET Lmax, and ENT statistics patterns

in order to assess the significance of RQA statistics or to compare the three univariate time series.

CONCLUSIONS

In the literature, hydro-meteorological time series applications have systematically supported the complex and non-stationary nature of precipitation patterns. However, these have usually failed to provide a comprehensive statistical characterization of both temporal evolution and observed transitions, which is necessary for the process adopted for model selection and prediction. In the present paper, a statistical framework based on nonlinear dynamics theory and recurrence analysis of dynamical

systems has been proposed in order to quantitatively identify the temporal characteristics of extreme (maximum) daily precipitation series in each month for each year for present and 1xCO₂ and 2xCO₂ climate scenarios.

Recognizing the peculiarities that are induced in the obtained maximum daily mean areal precipitation in each month for each year (historical and two climate scenarios) by the employed GCM-generated CPs and approaches for estimation and modelling of areal precipitation, the summarized results of the paper are as follows:

1. A substantial difference exists between the extreme (maximum) daily mean areal precipitation series in each month for each year patterns for historical and two climate scenarios evaluated.

2. The historical patterns exhibit decreasing recurrence with time that is not the case for the two climate scenarios examined where recurrence is cyclic.
3. A variable nonlinear determinism exists in all datasets and especially for 1xCO₂ and 2xCO₂ climate scenarios.
4. Periodic-to-chaotic and chaotic-to-chaotic transitional patterns are evident for all climates presenting though differences in terms of duration and intensity. The strongest transitions are noted to the 2xCO₂ scenario.

From the methodological standpoint, the proposed approach was based on methods that are able to quantify the statistical characteristics of the time series by a multidimensional and nonlinear system without depending on mathematical linear or nonlinear transformations, such as the Fourier transform, wavelets, and so. Moreover, the above mentioned characteristics of time series evolution may have significant implications to future water resources systems design and planning underling an extraordinary situation of shifts between droughts and floods conditions. The analysis presented helps to statistically discern the temporal patterns of the precipitation phenomena under investigation and characterize the transitions observed. These aspects of time series analysis are essential in the process of prediction and could be used to detect basic statistical characteristics of the time series, such as on which time period the near past information loses its criticality on the manner the system evolves in time, and, consequently help towards proper selection of a prediction model. To this end, we could say that the characteristics of the GCM-downscaled time series for future time horizon may point to the need to develop different prediction models than those of historical ones for predicting maximum daily mean areal precipitation.

It should be noted, that the developed methodology is novel for the hydro-meteorological domain and the results presented herein should be considered preliminary. As such, several improvements could be implemented in the methodology, such as resampling strategies in order to judge the significance of RQA statistics or to compare two univariate time series under the developed or other relative (classical) methods, e.g. spectral analysis, wavelet analysis, cascade analysis, etc. Future research in RQA focused on GCM-downscaled outputs could offer further insights in hydro-meteorological predictions for an evolving climate. Finally, ongoing research is focused on developing a pattern-based prediction scheme that would incorporate spatiotemporal patterns and would select the appropriate modelling strategy based on the short-term statistical characteristics of precipitation series.

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APPENDIX

The ECHAM4 atmospheric general circulation model has been developed at the Max Planck Institute for Meteorology (MPI), Hamburg, Germany. The model is a spectral transform model with 19 atmospheric layers, and the results used here derived from experiments performed with spatial resolution T42. The model structure of ECHAM4, including

dynamics and numerics, is documented in detail in Roeckner *et al.* (1996). The experiment with CO₂ concentrations for the period 1961–2000 is 346 ppmv corresponding to 1xCO₂ and for the period 2061–2100 is 748 ppmv corresponding to 2xCO₂ (Monika Esch, Max Planck Institute for Meteorology, Atmosphere in the Earth System).