



National Technical University of Athens  
School of Civil Engineering

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# **Accurate application and higher-order solutions of the SAC/FEMA probabilistic format for performance assessment**

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The banner features a background image of a historic stone building with a tower and a blue sky with white clouds. Overlaid text includes "15<sup>TH</sup> WORLD CONFERENCE ON EARTHQUAKE ENGINEERING", "24 TO 28 SEPTEMBER 2012", and "LISBON - PORTUGAL". To the right is the WCEE logo, which consists of three overlapping circles in blue, red, green, and yellow, with the text "15 WCEE LISBOA 2012".

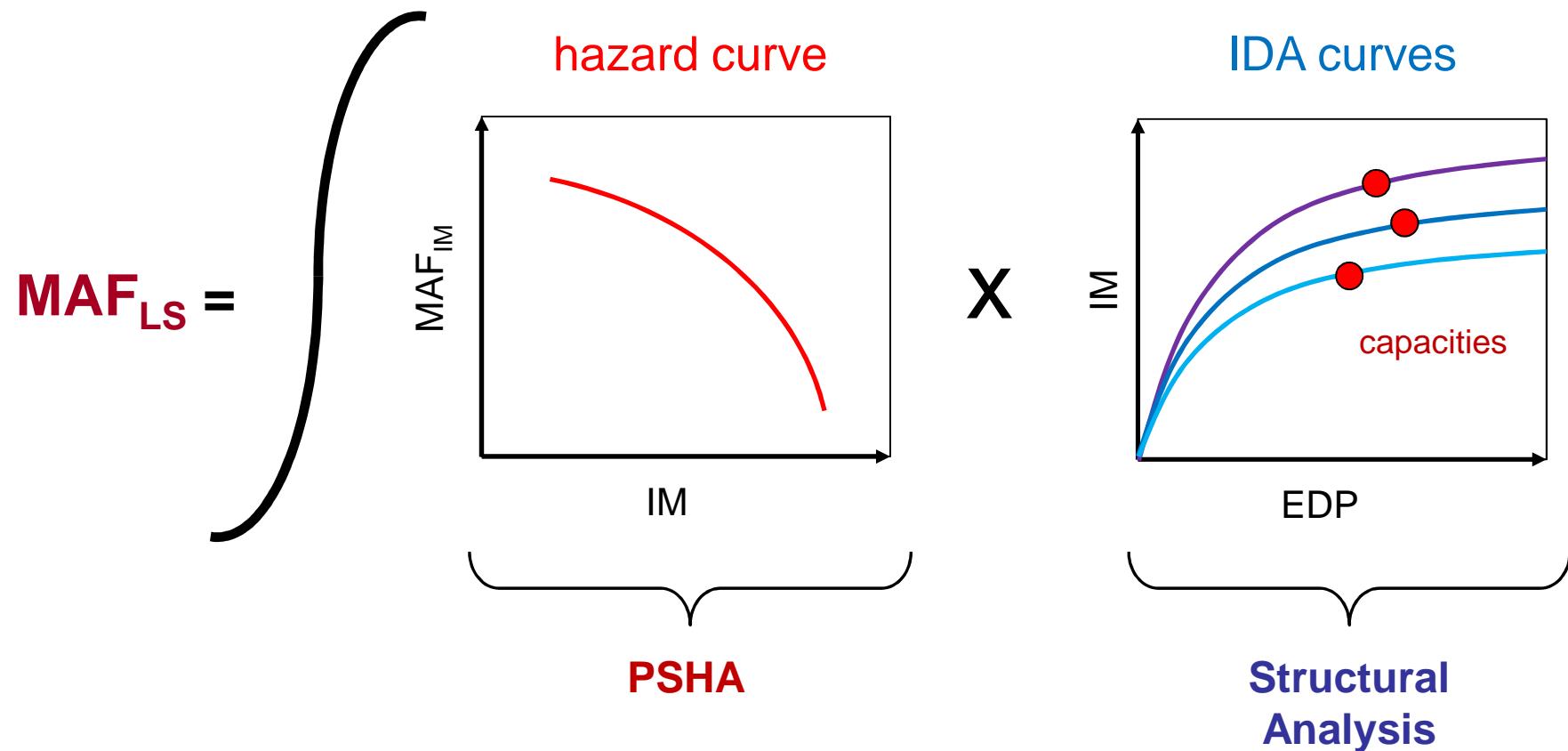
15<sup>TH</sup> WORLD CONFERENCE ON  
EARTHQUAKE ENGINEERING  
24 TO 28 SEPTEMBER 2012  
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15 WCEE  
LISBOA 2012

# Limit-State MAF Estimation

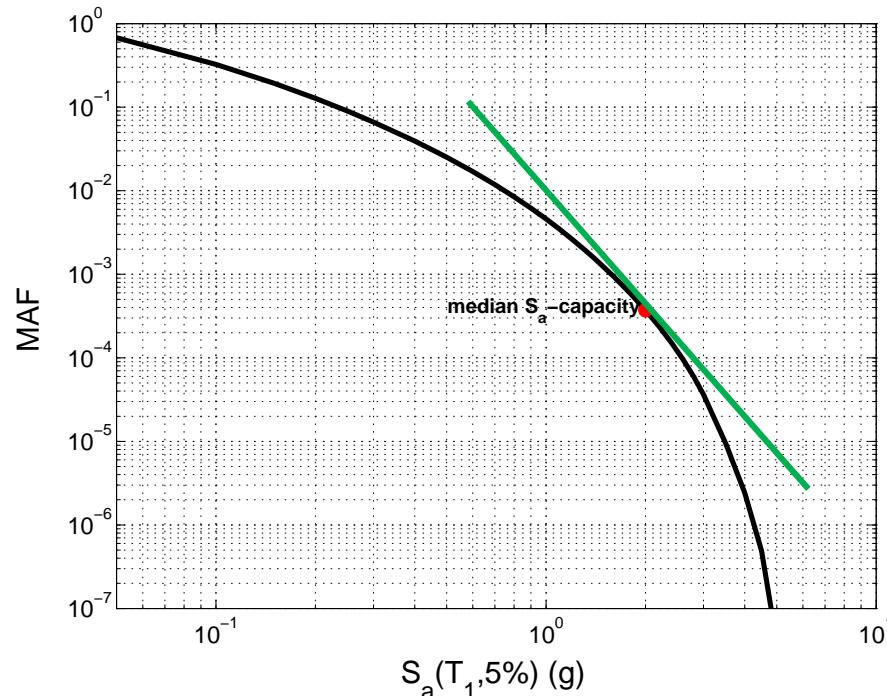
- SAC/FEMA: Probabilistic basis becomes “mainstream”
- Uses Mean Annual Frequency of limit-state exceedance
  - Or “mean return period” or “X% in 50yrs”
  - Use to characterize performance
- How to compute for arbitrary limit-state?
  - Seismic hazard curve (PSHA)
  - Structural analysis (e.g., IDA)
- Numerically integrate!

# Numerical Integration

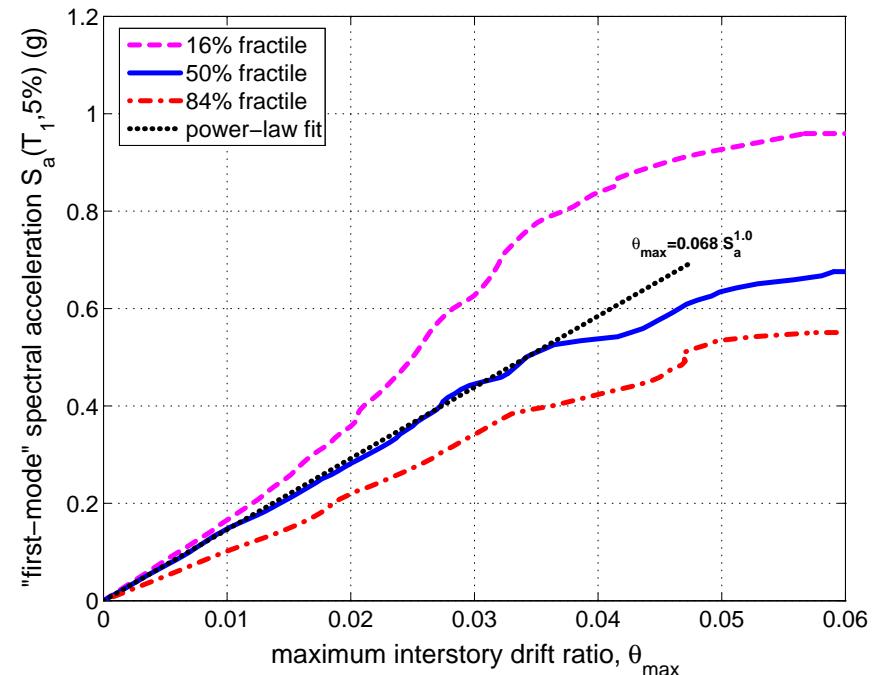


# Cornell et al: Analytical solution

Approximate hazard



Approximate IDA

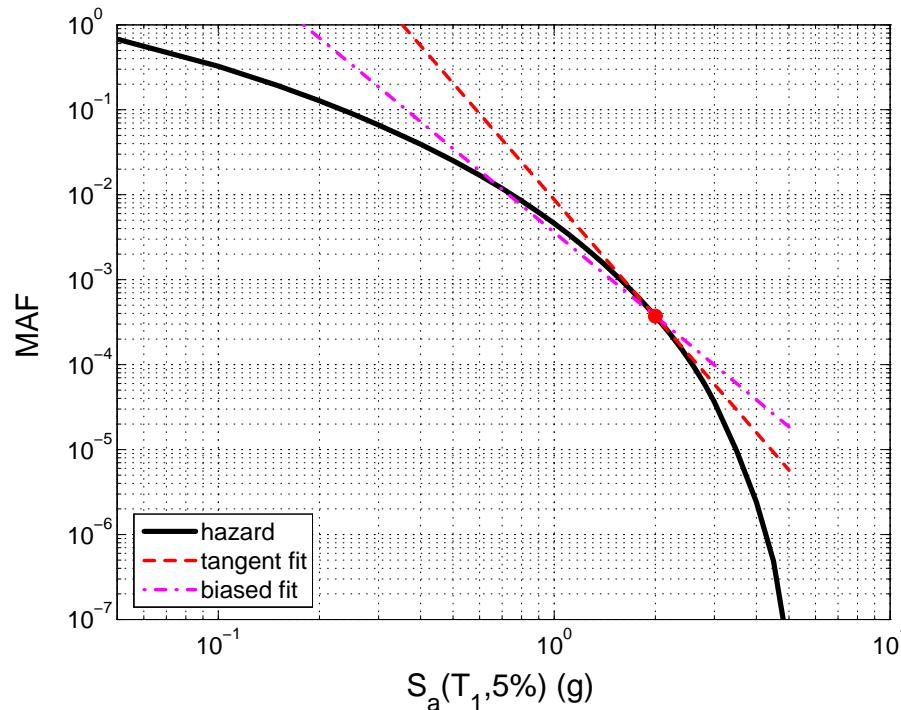


- Tangent fit in log-log
- Obviously conservative
- **Problems with curvature**

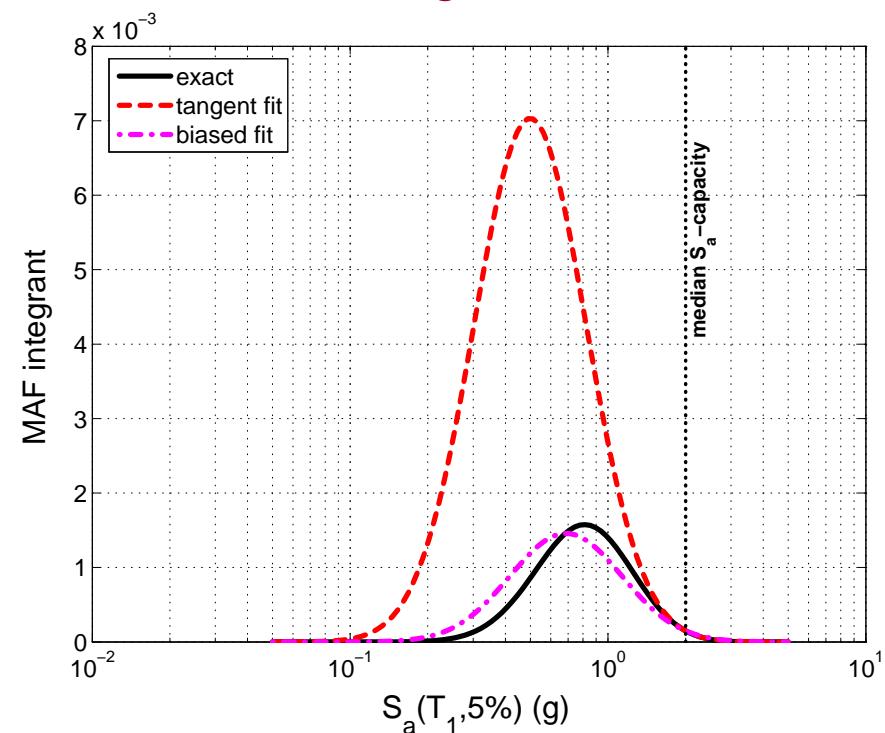
- Fit median by power-law
- Assume constant dispersion
- Locally ok!

# Improving the hazard fit (1)

Approximate hazard



Integrants

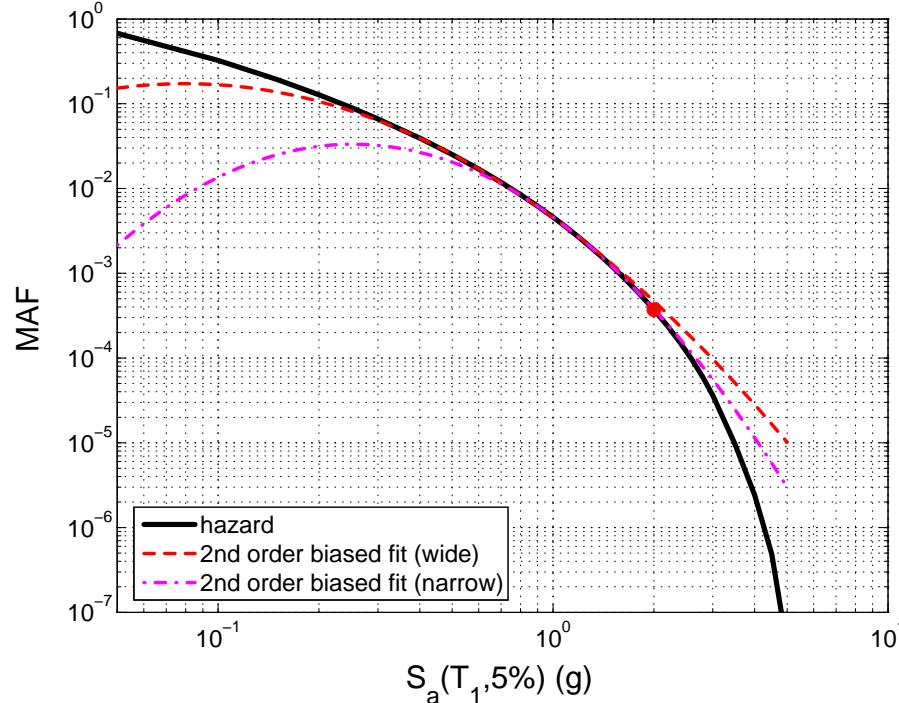


- Bias the fit!
- Higher weight on frequent events

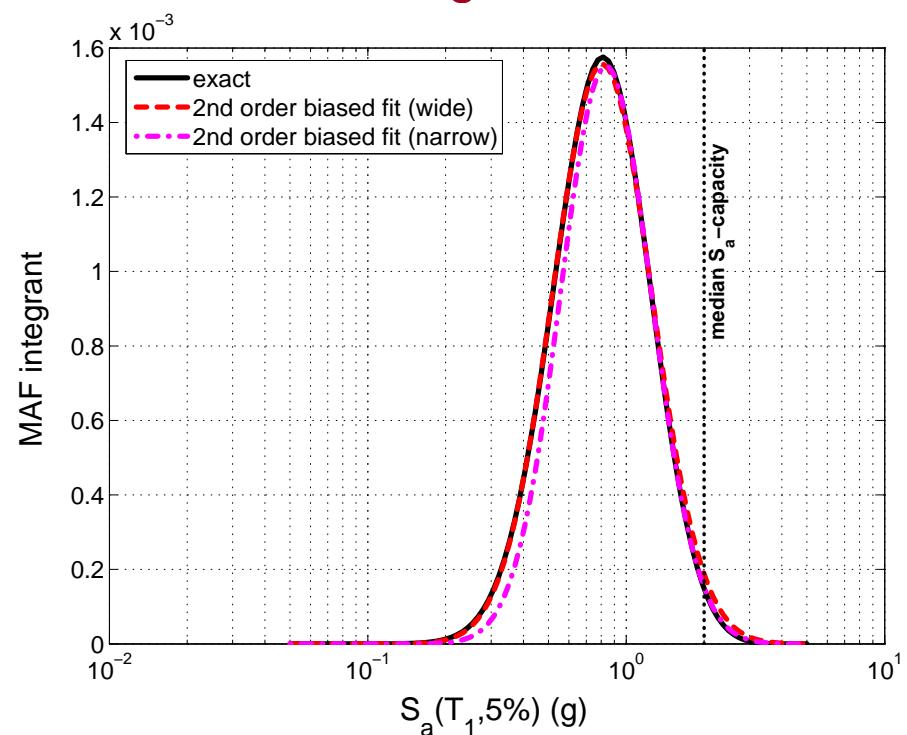
- Tangent fit is too conservative

# Improving the hazard fit (2)

Approximate hazard



Integrants



- Biased 2<sup>nd</sup> order fit
- Less local
- Not sensitive to fitting details

# No free lunch...

- 2<sup>nd</sup> order fitting means new closed-form solutions
- MAF Estimation
  - Original
  - New
- Safety Checking
  - Original
  - New

$$\lambda_{LS} = H(\hat{s}_c) \exp\left(\frac{1}{2} k_1^2 \beta_{Sc}^2\right)$$

$p < 1$

$$\lambda_{LS} = \sqrt{p} k_0^{1-p} [H(\hat{s}_c)]^p \exp\left(\frac{1}{2} p k_1^2 \beta_{Sc}^2\right)$$

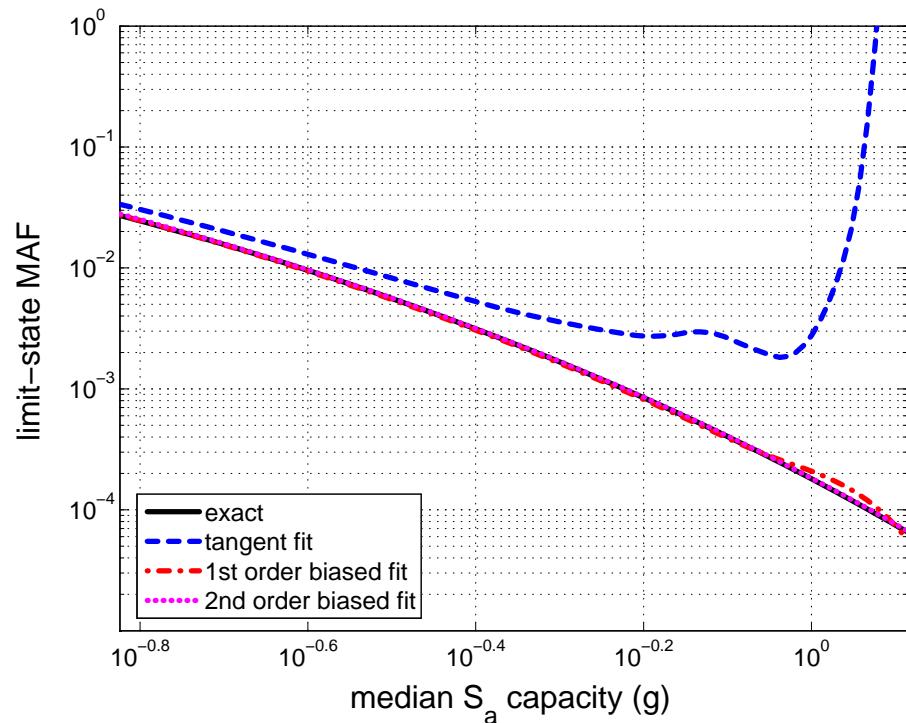
$$\hat{\theta}_c \exp\left(-\frac{k_1}{2b} \beta_{\theta c}^2\right) \geq \hat{\theta}_{Po} \exp\left(\frac{k_1}{2b} \beta_{\theta d}^2\right)$$

$\phi < 1$

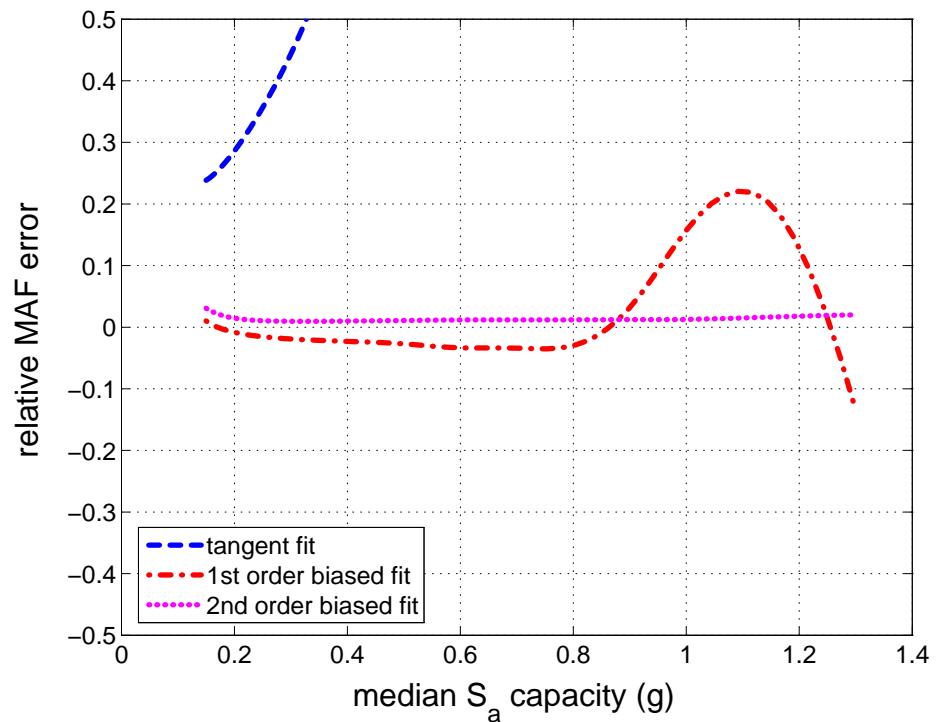
$$\frac{\hat{\theta}_c}{a} \exp\left(\frac{bk_1}{2k_2}\right) \geq \left[ \frac{\hat{\theta}_{po}}{a} \exp\left(\frac{bk_1}{2k_2}\right) \right]^{1/\sqrt{\phi}}$$

# MAF Estimation Errors

Displacement hazard



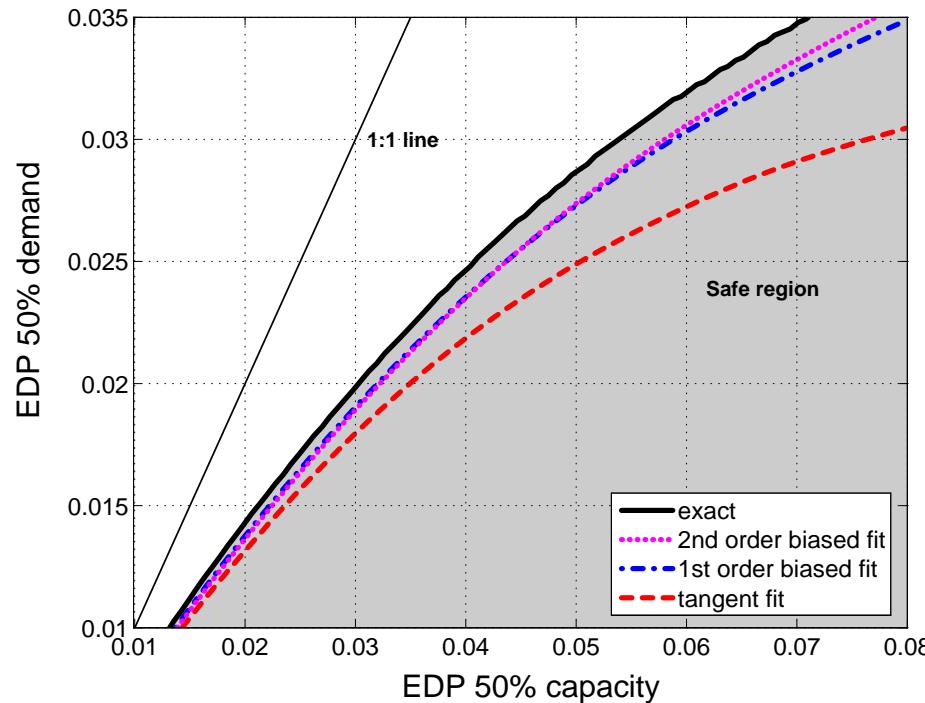
MAF error



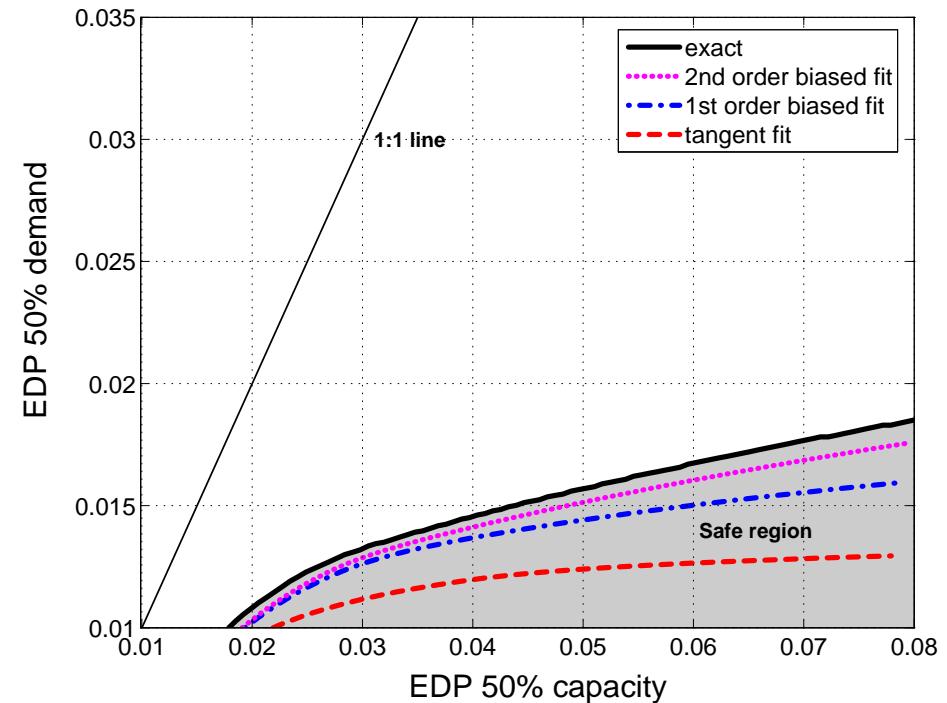
- Tangent fit is off-scale
- Biased fits look ok
- Biased 1<sup>st</sup> order: 20-70% error
- Biased 2<sup>nd</sup> order: 5% !

# Safety Checking Errors

Benign case



Tough case



- Same observations
- Errors at least non-astronomic

- Errors increase away from 1:1
- Demand of 1.5% may “need” infinite capacity!

## Final remarks

- ✓ Novel analytical solutions
- ✓ Fully resolve hazard-fitting errors
- ✓ Other, smaller error sources remain
- ✓ Cornell et al. solutions still usable
  - They just need a biased fit to improve
  - .... only avoid if hazard function is strongly curved

**Please never use a tangent fit!**