Seismic Performance Uncertainty Estimation via IDA with Progressive Accelerogram-wise Latin Hypercube Sampling [1]

Dimitrios Vamvatsikos[2]

Abstract: An algorithm is proposed for the rapid estimation of the influence of model parameter uncertainties on the seismic performance of structures using incremental dynamic analysis (IDA) and Monte Carlo simulation with latin hypercube sampling. It builds upon existing methods that quantify the uncertainty for structural models with non-deterministic parameters by performing IDA with multiple ground motion records on each model realization out of a predetermined sample. However, their practical application is restricted due to (a) the inability to determine *a priori* the required number of samples and (b) the disproportionate increase of the number of analyses in realistic multi-parameter models. To address these issues two fundamental changes are incorporated. First, latin hypercube sampling is applied progressively by starting with a small sample that is doubled successively until the desired accuracy is achieved. Second, parameter sampling is performed on a record-by-record basis, rather than maintaining the same model over an entire record suite, thus expanding the model sample size without increasing the number of nonlinear dynamic analyses. Using a strong-column and a weak-column model of a steel moment-resisting frame, the algorithm is shown to possess excellent scalability, extending the original methodology to be applicable to large-scale models with hundreds of random variables.

CE Database subject headings: Seismic response; Earthquakes; Performance evaluation; Safety; Structural Reliability.

Author keywords: Nonlinear Dynamic Analysis; Uncertainty.

Introduction

The estimation of the structural demand and capacity lies at the core of performance-based earthquake engineering. Being inherently probabilistic, their distribution is influenced by both aleatory randomness, e.g., due to natural ground motion variability, and epistemic uncertainty, owing to modeling assumptions, omissions or errors. Appropriately quantifying their effect is an integral point of safety assessment and needs to be explicitly accounted for in performance-based frameworks (Esteva and Ruiz 1989; Cornell et al. 2002). Recently, guidelines have emerged (SAC/FEMA 2000; FEMA 2012) that recognize the need for assessing the role of uncertainties by directly including them in seismic performance estimates. Nevertheless, computational difficulties render such assessments resource-intensive, delegating this role to ad hoc safety factors, or, at best, standardized dispersion values that often serve as placeholders.

Given a structural model, the two contributors to seismic response dispersion are (a) the ground motion record-to-record variability and (b) the model parameter uncertainty. The first is captured by analyzing the model under multiple ground motion records, for example via incremental dynamic analysis (IDA, Vamvatsikos and Cornell 2002). The second, classified either as aleatory when designing a yet-unbuilt structure, or as epistemic for assessing an existing one (Der Kiureghian and Ditlevsen 2009), remains a little-explored issue.

[1] Based on a short paper presented at the 11th ICASP Conference, Zurich, 2011

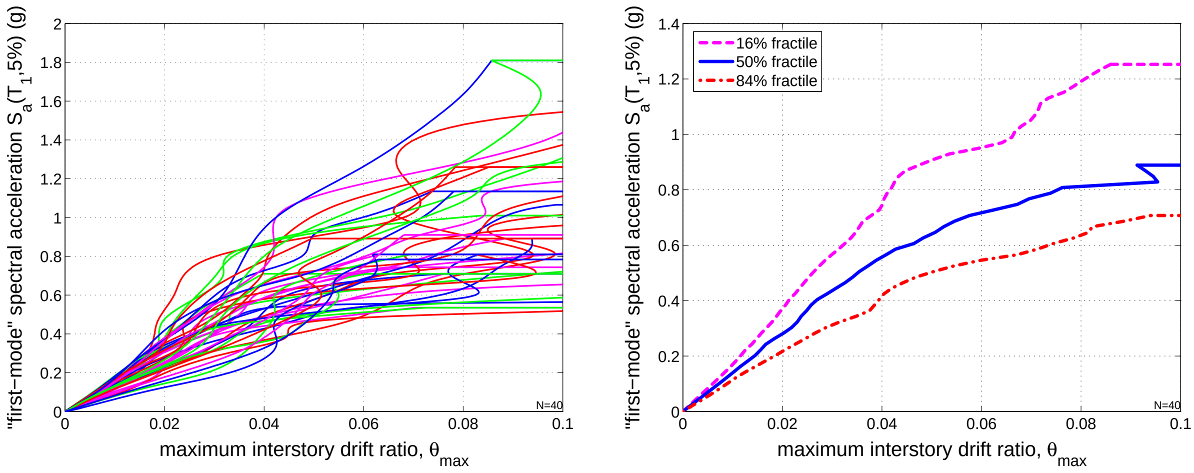
[2] School of Civil Engineering, National Technical University of Athens, Greece

Recently, the application of nonlinear dynamic analysis combined with Monte Carlo simulation has been proposed to quantify parameter uncertainty. For example, Ibarra and Krawinkler (2011) propose propagating the uncertainty from model parameters to structural behavior using first-order-second-moment (FOSM) principles (verified through Monte Carlo) to evaluate the collapse capacity uncertainty. To improve performance, Latin Hypercube Sampling (LHS, McKay et al. 1979) has also been introduced as a replacement for classic random sampling. Kazantzi et al. (2008) used Monte Carlo with LHS to incorporate uncertainty into steel frame fragility curves. Liel et al. (2009) employed IDA with Monte Carlo and FOSM coupled with a response surface approximation method to evaluate the collapse uncertainty of a reinforced-concrete building. On a similar track, Dolsek (2009) and Vamvatsikos and Fragiadakis (2010) have proposed using Monte Carlo with LHS on IDA to achieve the same goal on reinforced-concrete and steel structures, respectively. However, any large-scale application of IDA-based methodologies is severely restricted due to two important reasons.

The first is the inability to determine in advance the required number of observations for a successful Monte Carlo application. In classic random sampling, observations can be generated on the fly and the simulation can be stopped at will. Due to the nature of LHS, the entire sample has to be decided *a priori*. It is generally not possible to expand or contract a given sample to an arbitrary higher or lower size without risking a biased estimate. A change in sample size typically means starting from scratch. In other words, for LHS, the incomplete initial understanding of the model restricts our ability to select an optimal sample size.

The second issue is the disproportionate increase in the number of analyses when dealing with a plethora of random variables. Depending on their correlation structure, it may become prohibitively expensive to determine the influence of multiple random parameters, as the sample size rises disproportionately. Thus, matters of spatial uncertainty and correlation of variables at multiple locations in a structure are difficult to resolve; they necessitate a high number of observations that easily runs into the hundreds or thousands. This is what has lead all early attempts (Liel et al. 2009; Dolsek 2009; Vamvatsikos and Fragiadakis 2010; Dolsek 2012) to limit themselves to just a handful of parameters. What compounds all of the above, is that it becomes highly desirable to limit the computational cost of evaluating each sample member. It is bound to lead to a trend of reducing the size of the ground motion record suite used for IDA, making it attractive to use, e.g., 10 records rather than a healthier set of 20 or 30.

Such issues mostly reflect current concepts regarding the application of IDA. One is accustomed to using the same model over all accelerograms to determine the aleatory record-to-record variability and then add the influence of model uncertainty. On the other hand, within a performance-based framework such as SAC/FEMA (Cornell et al. 2002) or the Cornell and Krawinkler (2000) Pacific Earthquake Engineering Research Center (PEER) format, it is customary to combine aleatory and epistemic contributions in a single dispersion parameter. Furthermore, IDA is itself a record-sampling technique at its core, operating on the principle that all records in a suite have equal probability of occurrence. In statistical terms, this is a classic problem in factorial design of experiments that can be efficiently tackled by combining existing concepts from current literature. This means meshing together the model and record sampling (Nielson and DesRoches 2007) and using incremental sample sizes that have been carefully selected to allow full reuse of the earlier runs performed (Sallaberry and Helton 2005). Thus, a general algorithm is proposed that efficiently upgrades the original to be applicable to large-scale models without any need for pre-determining sample sizes.



(a) IDA curves

(b) fractile IDA curves

Fig. 1. Forty IDA curves and their summarization into 16,50,84% fractile IDA curves.

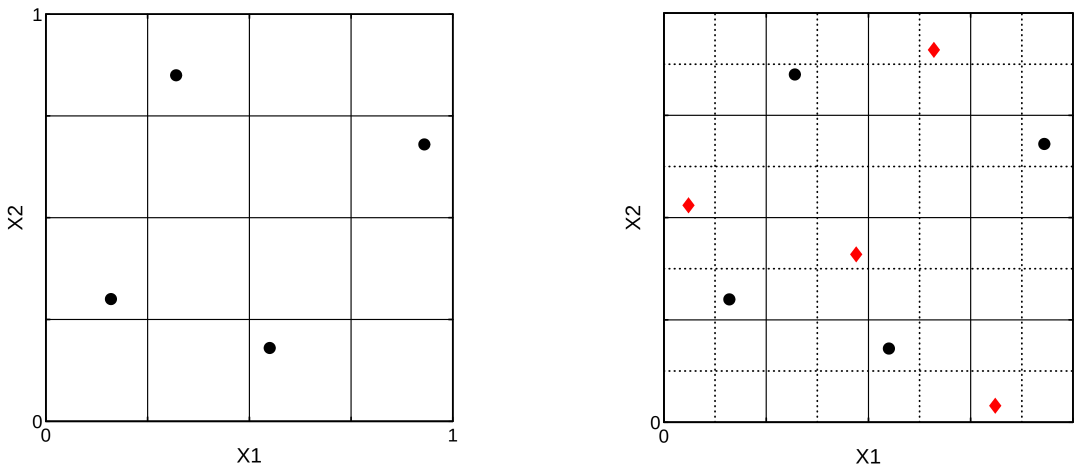
Incremental Dynamic Analysis

Incremental Dynamic Analysis (IDA) is a powerful method that offers thorough seismic demand and capacity prediction capability (Vamvatsikos and Cornell 2002). It involves performing a series of nonlinear dynamic analyses under a multiply-scaled suite of ground motion records, selecting proper Engineering Demand Parameters (EDPs) to characterize the structural response and an Intensity Measure (IM), e.g., the 5% damped first-mode spectral acceleration, $S_a(T_1, 5\%)$, to represent the seismic intensity. Whenever large scaling factors are involved, employing a *sufficient* IM with respect to seismological parameters becomes highly important to avoid bias (Luco and Cornell 2007; Luco and Bazzurro 2007). IDA results are presented as curves of EDP versus IM for each record (Fig. 1(a)) that can be further summarized into the 16,50,84% fractile IDA curves (Fig. 1(b)) by estimating the respective percentile values given a range of IM or EDP values. Appropriate limit-states can be defined by setting limits on the EDPs and estimating the probabilistic distribution of the respective capacities. For limiting values of the maximum interstory drift θ_{max}, this means reading off the median and the dispersion of the required S_a capacity from Fig. 1(b). Combining such results with probabilistic seismic hazard analysis allows the estimation of mean annual frequencies (MAFs) of exceeding the limit-states, thus offering a direct characterization of seismic performance.

Nevertheless, IDA comes at a considerable cost, even for simple structures, necessitating the use of multiple nonlinear dynamic analyses that are often beyond the abilities and the computational resources of the average practicing engineer. Wherever IDA is involved, searching for an efficient implementation is always desirable.

Incremental Recordwise LHS

To mitigate the limitations of LHS on IDA, we propose retaining the fundamental procedures while redefining their implementation in two ways. First, LHS is applied incrementally by starting with a small sample that is doubled successively until adequate accuracy has been achieved (Sallaberry and Helton 2005). By bisecting the existing equiprobable strata along each dimension, the current partitioning of the sample space is further subdivided into smaller hypercubes, half of which contain the original observations (Fig. 2). Thus, additional observations can be inserted, fully retaining the results of the previous LHS design. This is essentially the algorithm



(a) generation 1 (b) generation 2

Fig. 2. An iLHS design for two uniformly-distributed variables is doubled in size by subdividing each stratum to insert new observations.

Table 1. The format of the iLHS sample for N parameters and M records.

No.	X_1	X_2	$\dots$	X_N	X_{N+1}[1]	X_{N+2}[2]
1	$x_{1,1}$	$x_{1,2}$	$\dots$	$x_{1,N}$	ang_1	Rec_1
2	$x_{2,1}$	$x_{2,2}$	$\dots$	$x_{2,N}$	ang_2	Rec_2
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
M	$x_{M,1}$	$x_{M,2}$	$\dots$	$x_{M,N}$	ang_M	Rec_M
$M+1$	$x_{M+1,1}$	$x_{M+1,2}$	$\dots$	$x_{M+1,N}$	ang_{M+1}	Rec_1
$M+2$	$x_{M+2,1}$	$x_{M+2,2}$	$\dots$	$x_{M+2,N}$	ang_{M+2}	Rec_2
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$

[1] incident angle [2] record index

of Sallaberry et al. (2008) stripped to its core to generate only uncorrelated designs for reasons of applicability to small samples and large parameter spaces (see later discussion). More elaborate and generalized algorithms, termed hierarchical LHS, have also been presented by Tong (2006) and Vorechovsky (2010), which can be considered as supersets of the proposed implementation. The main advantage of all such incremental applications of LHS is that a rational stopping rule may be defined by comparing the convergence of the IDA estimates in successive generations. This offers an intuitive way to determine a reasonable sample size, minimizing the waste of runs over repeated tries or the (equally wasteful) tendency to overestimate the size to "get it right" in one step.

Furthermore, by taking advantage of IDA being itself a sampling process of equiprobable points (or records), LHS can be performed simultaneously on the structural properties and on the ground motion records. Instead of maintaining the same properties for a given model realization over an entire ground-motion record suite, model parameter sampling is performed on a record-by-record basis, efficiently expanding the number of observations without increasing the number of nonlinear dynamic analyses, a concept that has also been proposed by Schotanus and Franchin (2004) and applied with LHS by Nielson and DesRoches (2007). An example of such a sampling design appears in Table 1, where each row is one structural observation that also corresponds to a single ground motion record on which IDA is performed. The incident angle of the record may also be varied for higher randomization (e.g., Rigato and Medina

2007; Lagaros 2010). If more observations are needed than the M records available, the ground motions are recycled, either with the same or a different incident angle. In the customary application of LHS, each row of the table would be subject to IDA for the entire record suite, magnifying the number of analyses by a factor of 20–40.

The resulting algorithm will be termed iLHS. It can be employed with large-scale models removing all needs for guessing sample sizes or limiting the model's probabilistic complexity. It is also general enough to be applicable to any performance assessment method based on dynamic analysis under scaled ground motions (e.g., stripe or cloud scaling, Jalayer and Cornell 2009). On the other hand, iLHS comes with some minor disadvantages of its own. Perhaps the most important is that due to the small size of the first generation samples, one cannot use some simple algorithms (Iman and Conover 1982) for imposing the desired correlation structure on the sample. Instead, genetic or evolutionary algorithms need to be employed, such as the ones by Charmpis and Panteli (2004) or Vorechovsky and Novak (2009), the latter having already been used for seismic applications by Dolsek (2009). These are more resource-intensive but they offer the ability to fine-tune the correlation structure and achieve higher fidelity. Another issue is the obvious incompatibility of iLHS with any "accelerated-IDA" techniques that assume the same model over an entire record suite. Priority lists (Azarbakht and Dolsek 2007, 2011), or static-pushover-based approximations of IDA, such as SPO2IDA (Fragiadakis and Vamvatsikos 2010) and IN2 (Dolsek and Fajfar 2005) can only benefit from the hierarchical LHS part of the algorithm. Finally, with iLHS the model uncertainty effects cannot be distinguished from the record-to-record variability unless additional analyses are performed for the mean-parameter model. Nevertheless, such amendments are easy to provide, and the overall algorithm can be implemented with a minimal of programming, as shown below.

Algorithm. iLHS

```
define N random model parameters
define M ground motion records
k0 = 1; k = InitialSize
repeat
   if k0 = 1 then
      create initial LH-sample of size k
   else
      for i = 1 to M do
         divide the range of variable-i in 2k equiprobable intervals
         find the k empty intervals
         randomly select one value of variable-i in each empty interval
      end for
      optimally combine k values from M variables to form k new sample vectors with desired
      correlation
      append the k new sample vectors to the k existing ones
   end if
   for j = k0 to k0 + k − 1 do
      R = ((j − 1) mod M) + 1
      generate model for (new) sample vector j
      randomly rotate record R (optional)
      run IDA for record R on model j
   end for
   if k0 > 1 then k = 2k end if
   k0 = k0 + k;
   estimate change in total IDA results relative to previous iteration
```

25: **until** maximum change < tolerance

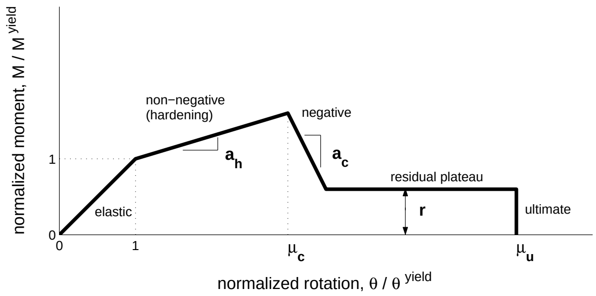


Fig. 3. The moment-rotation beam-hinge backbone to be investigated and its six controlling parameters.

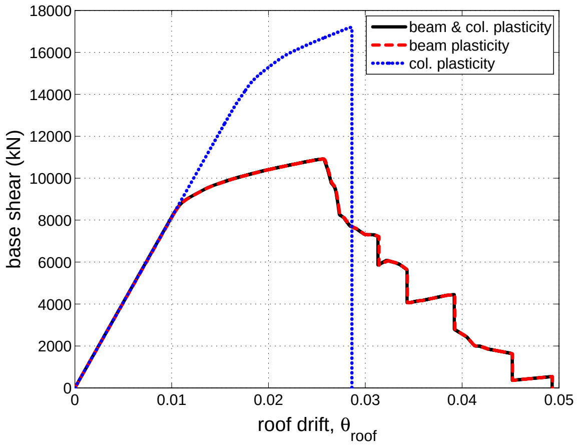


Fig. 4. First-mode pushover curves for the BCP, BP and CP mean-parameter models. Ignoring column plasticity for the BP model does not reduce its accuracy.

Table 2. The uncertain parameters in order of decreasing importance and their distribution properties. The mean and coefficient of variation (c.o.v.) are shown for the underlying normal that is truncated at the corresponding min/max limits.

	mean	c.o.v.	min	max	type
a_{My}	1.0	20%	0.70	1.30	trunc. normal
μ_c	3.0	40%	1.20	4.80	trunc. normal
μ_u	6.0	40%	2.40	9.60	trunc. normal
a_c	-0.5	40%	-0.80	-0.20	trunc. normal
a_h	0.1	40%	0.04	0.16	trunc. normal
r	0.5	40%	0.20	0.80	trunc. normal

Model and records

To perform IDA, a suite of sixty ground motion records has been selected, comprising both horizontal components from thirty recordings. They are all characterized by relatively large moment magnitudes (6.5–6.9) and moderate distances (15km–35km), while they were all recorded on firm soil and bear no marks of directivity.

The structure selected is a nine-story steel moment-resisting frame with a single-story basement that has been designed for Los Angeles, following the 1997 NEHRP (National Earthquake Hazard Reduction Program) provisions (Foutch and Yun 2002). A centerline model with non-linear beam-column connections was formed using OpenSees (McKenna et al. 2000). It allows for plastic hinge formation at beam and columns ends (Fig. 3), with the option to selectively deactivate either beam or column hinging to enable or deter the formation of story mechanisms. Thus, the same parametric model can represent a realistic capacity-designed building where plasticity can develop in both beams and columns (termed BCP), only in the beams (BP) or only in the columns (CP). As shown by static pushover analysis in Fig. 4, the BCP and BP models behave identically, at least for the mean values of the parameters. This is a direct consequence of strong-column weak-beam design that may not be generalized to all capacity-designed buildings. Even here, the uncertainty in model parameters may still change this in some cases. Still, the lower computational footprint of the BP model makes it quite attractive, and it is adopted henceforth as indicative of ductile capacity-designed structures, to be contrasted to its "equivalent" version of a brittle model that allows only column-plasticity. The true nature of these models is actually better understood as a series (CP) versus a parallel (BP) system in terms of reliability, where the appearance of damage at a single story can or cannot lead to global failure.

Rayleigh damping of 2% has been applied to the first and second modes. The structural model also includes P-Δ effects while the internal gravity frames have been directly incorporated. The fundamental mode of the reference frame is at a period of $T_1 = 2.35$s and accounts for approximately 84% of the total mass. Essentially this is a first-mode dominated structure that still allows for some sensitivity to higher modes (Vamvatsikos and Fragiadakis 2010).

The plastic hinges are modeled as rotational springs with a quadrilinear moment-rotation backbone (Fig. 3) that is symmetric for positive and negative rotations and employs a moderately pinching hysteresis without cyclic degradation (Ibarra et al. 2005; Lignos and Krawinkler 2011). The backbone hardens after a yield moment of a_{My} times the nominal, having a non-negative slope of a_h up to a normalized rotation (or rotational ductility) μ_c where the negative stiffness segment starts. The drop, at a slope of a_c, is arrested by the residual plateau appearing at normalized height r that abruptly ends at the ultimate rotational ductility μ_u.

Fragiadakis et al. (2006) have suggested that seismic performance is much more sensitive to strength, rather than stiffness or mass. Thus, in order to evaluate the effect of uncertainties, the plastic hinge backbones are varied by assigning normal distributions to their six parameters (Table 2). To remove excessive values, all distributions were truncated within ± 1.5 standard deviations and appropriately rescaled to avoid the concentration of high probabilities at the cut-off points. The truncated distributions retain the same mean due to the symmetry in the imposed boundaries, but, having lost both "tails", they shed roughly 25% of their variability. Thus, the actual coefficients of variation are 15% and 30% rather than the reported 20% and 40% values of the underlying normal distributions. While the removal of the tails of the distributions (especially the lower one) may cause some concern, this should only be an issue for, say, wind or gravity safety checking. The rarity of the seismic action dominates the problem, thus prompting the use of more probable (rather than extreme) values for the model's properties. This is reflected in all seismic guidelines that may stipulate characteristic values for checking against wind or gravity, but use averages for seismic assessment (see also Cornell et al. 2002).

Two different correlation models have been considered. In the first case, the six different types of parameters shown in Table 2 are assumed to be independent, while full spatial correlation for the same parameter type is enforced throughout the structure. Thus, only six variables govern the model strength properties, an effort level commensurable with aforementioned uncertainty studies in the literature. The second approach employs a spatially variable model that more faithfully represents the partial correlation expected in an actual structure, loosely following the suggestions of Idota et al. (2009) for high inter-member correlation within the same production lot. Thus, only the hinges at the ends of each individual element are assumed to be perfectly correlated. Within the same element, the random parameters are independent except for the ductilities μ_c and μ_u that share an 80% correlation coefficient: A hinge with a longer post-yield plateau will also have a higher ultimate ductility (Lignos and Krawinkler 2011). Among different beams or columns in any given story, a 70% correlation was employed for same-type parameters. Among elements in different stories only 50% correlation was used. If only beams (BP) or only columns (CP) are randomized, there will be 270 and 378 parameters, respectively, for a grand total of 648 for the full model (BCP).

It is important to state that the distribution properties are only based on engineering judgement. They were purposefully selected to be roughly indicative of an existing steel structure that was constructed with less-than-perfect quality control in the beam-column connection welding. Thus, the element hinges may not show the fracturing tendencies of a pre-Northridge connection, yet they may still experience reduced ductility and strength due to quality issues. As a result, the relatively broad dispersions of Table 2 were used. Laboratory-level quality-control will lead to tighter distributions as shown for example by Lignos and Krawinkler (2011). Employing higher dispersions was a conscious decision to allow more uncertainty into the model than a well-executed newly designed steel building would experience, offering a more rigorous test for the proposed iLHS procedure. Due to this obvious limitation the results shown should only be viewed qualitatively, without any quantitative connection to structural safety.

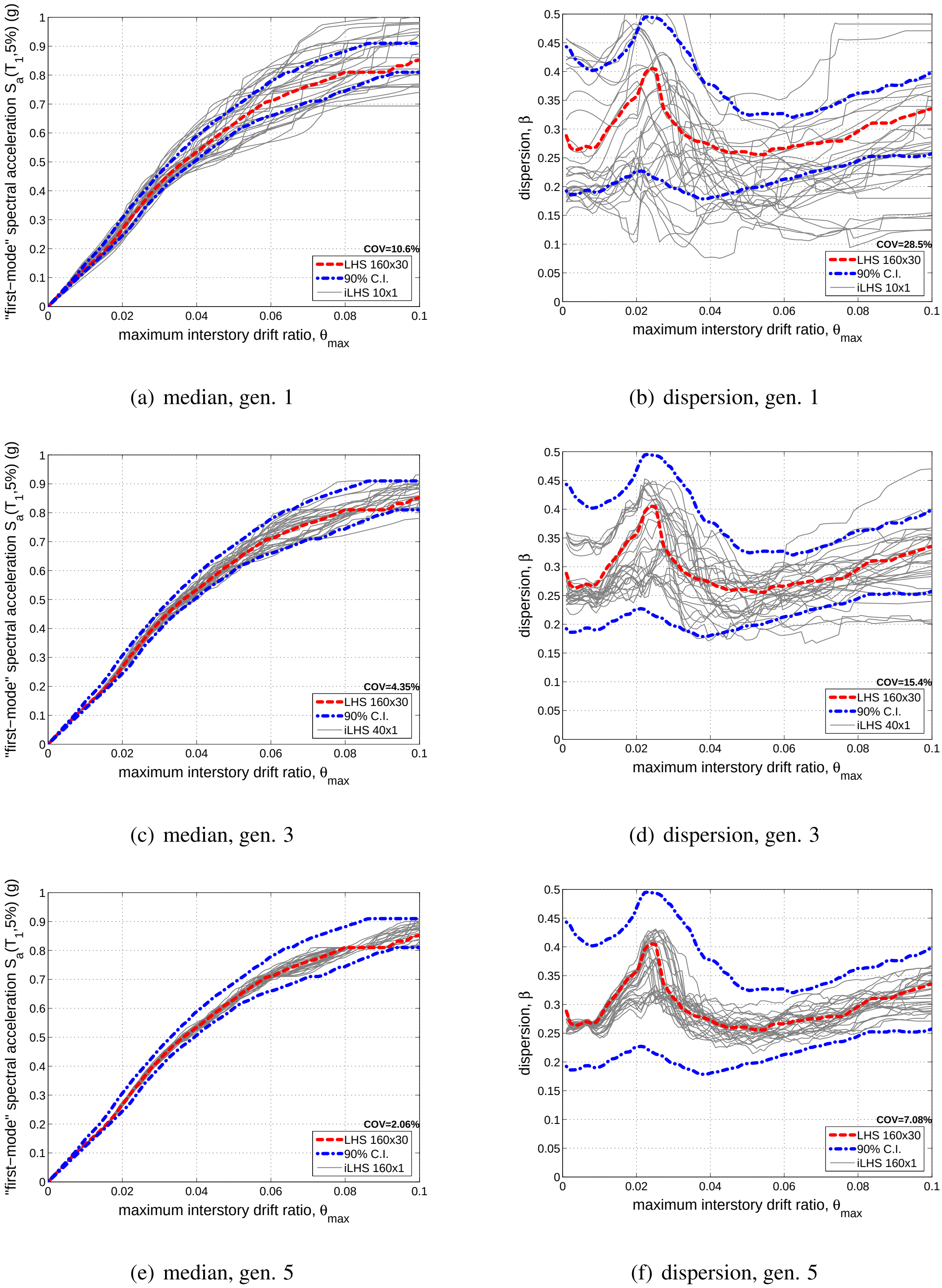
Convergence properties

The proposed iLHS algorithm, being essentially a latin hypercube design at its core, should inherit all of its useful properties. Still, given the different handling of record sampling compared to the typical application of IDA, it is important to understand its convergence properties. Two such investigations will be undertaken, contrasting small and large parameter spaces.

LHS versus iLHS

Due to the resource-intensive nature of IDA, an attempt to compare LHS versus iLHS, i.e., running 30 records for each model rather than one, has to be performed within a small parameter space. Thus, the ductile, full spatial correlation model having randomized beams is employed, having six independent parameters. In this low-dimensionality case, an initial sample of 10 models over 5 generations results to $10 \times 2^{(5-1)} = 160$ in total. To provide level ground for comparison, the same sample of 160 will be employed for both methods.

For the classic application of LHS each model is analyzed through IDA for the same 30 accelerograms. Each is one of the two horizontal components from the 30 recordings. Thus, a total of $30 \times 160 = 4800$ IDA curves are generated to comprise the LHS set. Instead of running iLHS independently, one can take advantage of these runs. The set of 4800 IDA curves can be perfectly partitioned into thirty potential realizations of iLHS. In each such realization, only one of the 30 IDA curves obtained for each structural model participates. Thus, according to the scheme of Table 1, the first iLHS sample uses record 1 from sample 1, record 2 from sample 2, cycling back to record 1 every thirty observations. Similarly the second iLHS sample uses



(a) median, gen. 1

(b) dispersion, gen. 1

(c) median, gen. 3

(d) dispersion, gen. 3

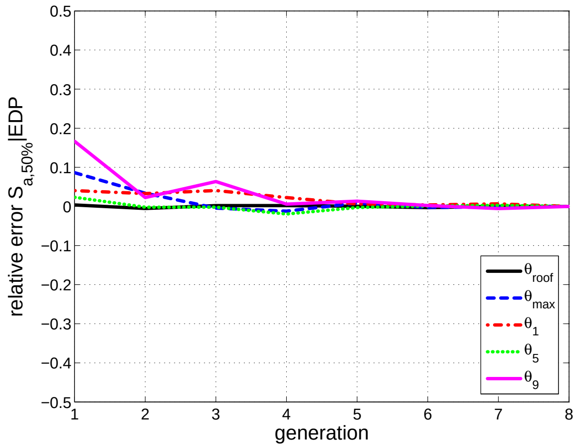
(e) median, gen. 5

(f) dispersion, gen. 5

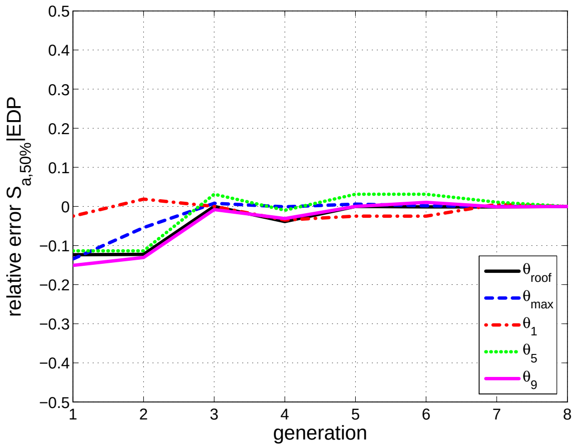
Fig. 5. Medians and dispersions of S_a capacity shown for a single LHS (160×30) application versus thirty different realizations of iLHS ($N \times 1$, $N = 10, 40, 160$) using 160 samples at most and 30 ground motion records. Bootstrap 90% confidence limits show that any iLHS achieves the same statistical significance with at least 30 times fewer runs.

record 2 from sample 1, record 3 from sample 2, and so on. As sequential selection is not a prerequisite, many more such iLHS realizations can be created if necessary.

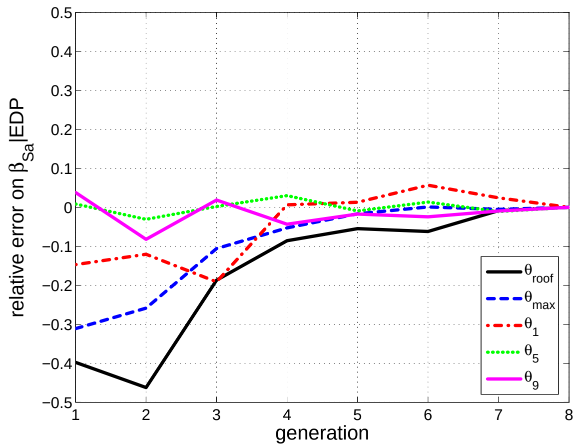
To quantify structural performance, the median and dispersion β (standard deviation of the log) results for values of S_a capacity given $\theta_{\max}$ are presented in Fig. 5 for generations 1, 3 and 5. Such values are the direct representation of structural fragility as defined by limiting values of the interstory drift and for a typical lognormal assumption. Thus, medians and dispersions corresponding to $\theta_{\max} = 1\%$ are indicative of (violating) an Immediate Occupancy or Slight Damage performance level, while values for $\theta_{\max} > 9\%$ correspond to global collapse, as the IDA curves flatline (Fig. 1). Appropriate 90% confidence intervals are also computed for the LHS dataset, by "bootstrapping" (Efron and Tibshirani 1993) the accelerograms. Any iLHS result falling within their limits cannot be said to differ significantly from the LHS result. It is obvious that successive generations quickly tighten the dispersion of iLHS; by the fifth iteration *any* iLHS realization is, statistically speaking, a perfect match for the thirty times costlier LHS. The averaged standard deviation of the errors across the entire $\theta_{\max}$ range is practically halved every two generations. Overall, from about 10% for the median and 30% for the dispersion, it drops to 2% and 7% respectively. If slightly higher errors are tolerable, the analysis can be stopped at a sample size of only 40 models.



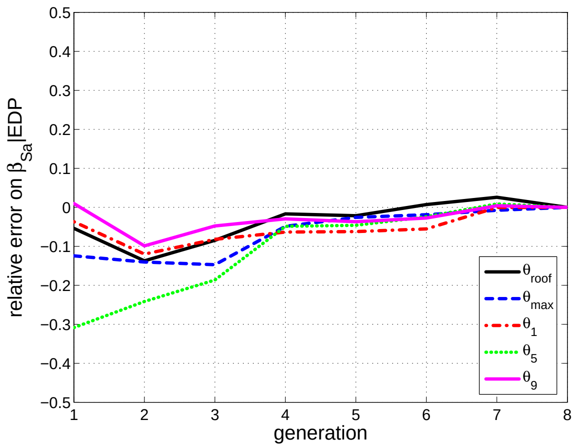
(a) $S_{a,50\%}$, near yield



(b) $S_{a,50\%}$, near collapse



(c) β_{Sa} near yield



(d) β_{Sa} near collapse

Fig. 6. The relative errors in the median and dispersion in S_a terms as estimated for two limit-states and multiple EDPs.

Large parameter spaces

A more rigorous test of iLHS can be realized for the partially correlated models. Using random beam properties and elastic columns, the ductile BP version of the 9-story is characterized by 270 random variables. Setting a starting size of 10 over 8 generations, the maximum sample size is 1280. Obviously there are too many correlation coefficients to control: $270(270-1)/2 = 36315$. It is not possible to get them all right. Even with 1280 observations the *maximum* absolute error is in the order of 0.25, i.e., at worst, correlation coefficients are matched within a ± 0.25 margin: a requested value of 0.5 could become 0.25 or 0.75. Fortunately, such errors are concentrated only in few random variables and one can choose which those will be. The Charmpis and Panteli (2004) algorithm starts with a single random variable and sequentially adds each subsequent one, using optimization techniques to rearrange the sample and best capture the correlation coefficients added in each cycle. Initially, there is only one to match, then two more, three, four and so on. Thus, random parameters that appear early in the correlation matrix have their correlation nearly perfectly captured, while most of the error is concentrated at the ones that are processed last. Therefore, it becomes advantageous to rearrange the sample, placing significant variables first. According to results supplied by Ibarra and Krawinkler (2011), Vamvatsikos and Cornell (2006) and Vamvatsikos and Fragiadakis (2010), the order of significance shown in Table 2 has been established. The most important parameter for the entire post-yield region is the hinge yield moment, as it directly scales the entire moment-rotation curve. Then come the ductilities at maximum strength and at fracture, followed by the other three parameters. Although not applied herein, the Vorechovsky and Novak (2009) algorithm can also make use of such information. As it performs optimization of the entire correlation matrix, appropriate weights can be introduced in the error calculation to promote higher accuracy for the most significant variables.

The simulation was run in parallel using algorithms developed by Vamvatsikos (2011) on five single-core processors within 10hrs. As shown in Fig. 6, the 1280 observations are excessive. The median and the dispersion β of the S_a capacity for a given response value become fairly stable for practically all EDPs after only 4–5 generations or 160–320 samples, respectively. The convergence rate only mildly differs among the global or local EDPs considered: roof drift θ_{roof}, maximum interstory drift $\theta_{\max}$ and the individual i-story drifts θ_i. Consequently, a simple rule may be devised using the relative change in the estimated IM values given EDP to determine when to stop sampling. First, let $\Delta S_{a,50\%}(EDP)$ and $\Delta\beta_{Sa}(EDP)$ represent the relative change (from generation to generation) in the median and the dispersion of S_a capacity for a given value of the EDP. Then a robust stopping criterion is formed by requesting that the maximum absolute value of all such changes, sampled at several limit-states (i.e., EDP-values) of interest, be less than a certain tolerance. Essentially, this assures that the change in the median and dispersion that characterize drift-based fragility are not altered appreciably after one stops sampling. A precise definition of limit-states and appropriate EDPs is not necessary, as long as all characteristic levels of structural response are sampled (e.g., elastic, yield, post-yield, collapse), preferably at each story: Convergence rates are similar for all EDPs. Extensive tests have shown that a tolerance of 5% on $\Delta S_{a,50\%}$ and $\Delta\beta_{Sa}$ results to an overall error of 10% at most. As shown in Table 3, sampling stops after the sixth generation using either a $\theta_{\max}$ or a more local θ_i basis for evaluating the relative error. The latter, though, is considered more robust, especially close to global collapse where severe localization of deformation may appear.

Table 3. The progression of the maximum absolute value of the relative error in the maximum story drift versus all peak story and roof drifts.

generation	2	3	4	5	6	7	8
$\max\{\lvert\Delta S_{a,50\%}(\theta_{\max})\rvert, \lvert\Delta\beta_{Sa}(\theta_{\max})\rvert\}$	8.5%	17.1%	10.3%	5.9%	2.4%	4.5%	1.6%
$\max\{\lvert\Delta S_{a,50\%}(\theta_i)\rvert, \lvert\Delta\beta_{Sa}(\theta_i)\rvert\}$	22.7%	33.9%	19.6%	7.9%	4.6%	5.7%	2.6%

Examples and applications

Having shown the effectiveness of the proposed iLHS method, it is now possible to use it for investigating the influence of parameter uncertainties on the nine-story's behavior in a variety of settings. Two illustrative examples are going to be presented, discussing (a) the effect of varying the incident angle of ground motion records and (b) the importance of spatial correlation for a ductile model (BP) without story-mechanisms versus a brittle model (CP) susceptible to localized story damage.

Incident angle effects

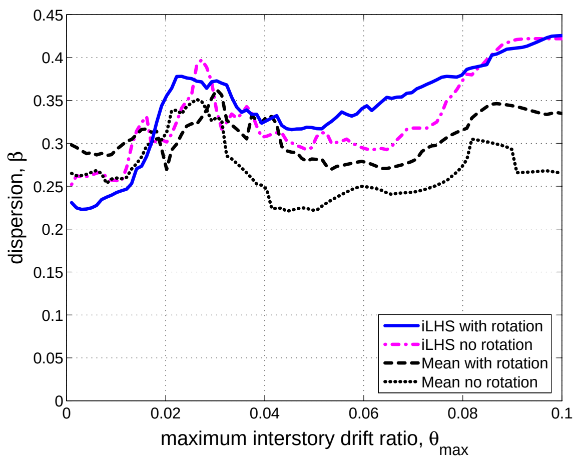


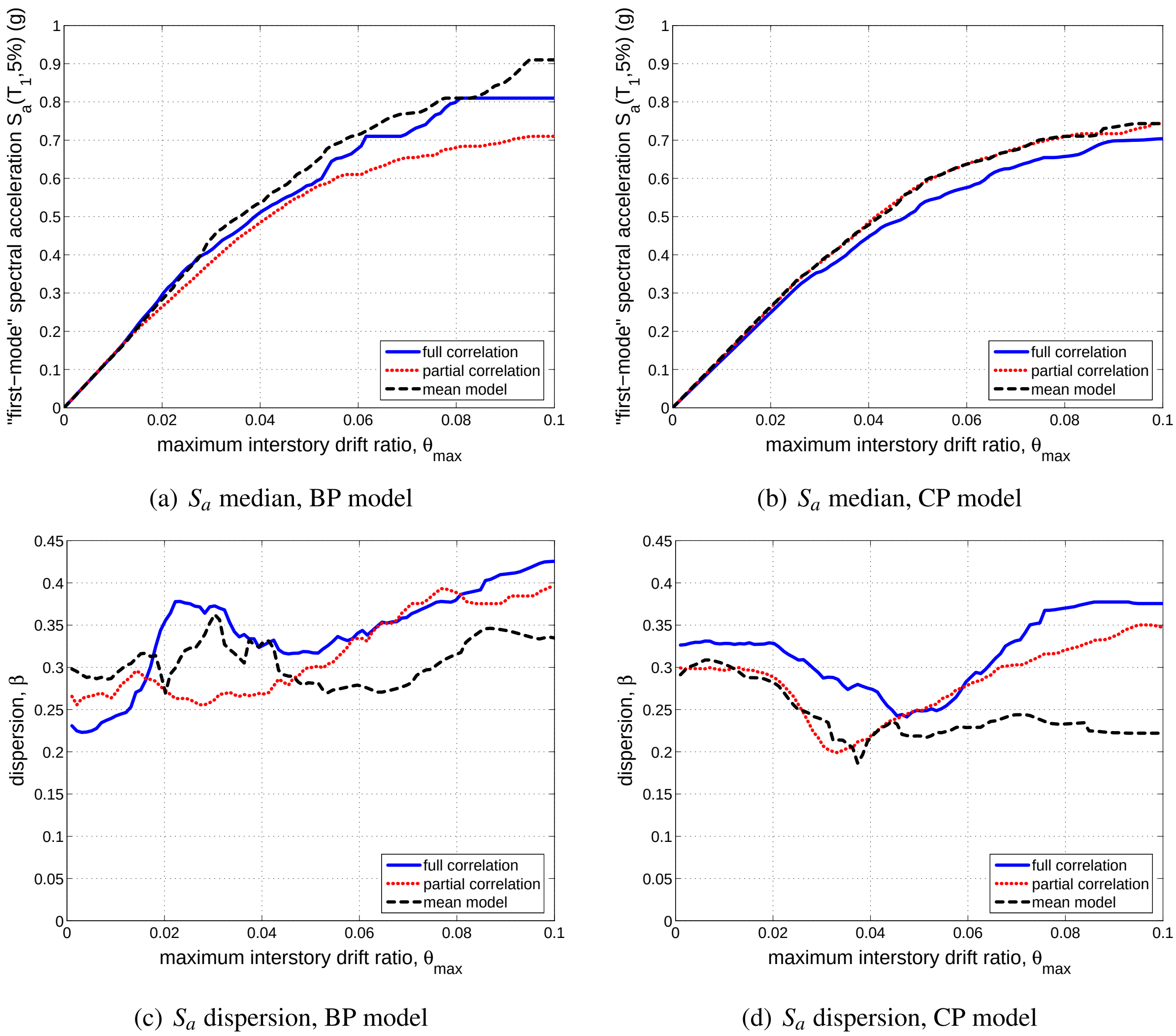
Fig. 7. S_a dispersion given values of $\theta_{\max}$ for the mean model with and without record incidence rotation compared against the respective iLHS results. Record rotation increases the aleatory dispersion, but does not dominate the total uncertainty.

iLHS can incorporate the randomization of the record incident angle. Whenever feasible, employing a larger suite of ground motions is always desirable. Still, when the size of such a set has been established, it will generally be lower than the number of models that iLHS may need to examine, so incident angle rotation may introduce further variability (Rigato and Medina 2007; Lagaros 2010). Incidence effects may be accommodated for a 2D model by rotating the two horizontal components in X-Y space and utilizing only one for the analysis.

Using the 6 parameter full spatial correlation BP model, two versions of iLHS will be shown. First, only the X-component of each recording is employed to analyze 160 models ("iLHS no rotation"). Then, both components with random rotations are used to analyze the same model sample ("iLHS with rotation"). As a basis for comparison, the mean model is subjected to the original 30 X-components ("mean no rotation") or to all 60 X/Y-components ("mean with rotation"), thus incorporating a 90^{o} incident angle change.

As shown in Fig. 7, including the rotation increases the aleatory variability of the mean model by 20%, approximately going from a 0.25 average dispersion to 0.30. A similar, albeit highly dampened, effect appears also with the iLHS results when random component rotations are introduced. Apparently, this additional randomization appears to have a small effect on the overall dispersion in comparison to its effect on the mean model. The addition of the parameter uncertainty decreases the importance of the higher record-to-record variability, mostly rendering its effect indistinguishable from the statistical fluctuations inherent in any probabilistic estimation procedure. Thus, it seems that including incident angle randomization may not be as important when a sufficiently large record suite or an uncertain model are employed. Still, to ensure a fair assessment of the influence of parameter uncertainty, incident angle rotations will be included both for the mean model and for iLHS for all comparisons that follow.

Spatial correlation, dispersion and bias



(a) S_a median, BP model (b) S_a median, CP model

(c) S_a dispersion, BP model (d) S_a dispersion, CP model

Fig. 8. The median and dispersion in S_a terms of a model with only beam-plasticity (BP) versus only column-plasticity (CP) shown for different spatial correlation models.

As iLHS is able to accommodate large parameter spaces with intricate correlation structures, it is of interest to show how spatial correlation among different elements influences the effects of uncertainty. Since uncertainty is typically neglected, it is important to verify whether this omission biases the median or reduces the dispersion of structural response. The results appear in Fig. 8 for both the BP and the CP version.

For the median response the ductile BP structure shows significant unconservative bias in Fig. 8(a). Partial correlation causes a 20% drop in S_a values beyond $\theta_{\max} = 0.05$. The same effect also appears at a much reduced scale for the CP brittle structure of Fig. 8(b). Vamvatsikos and Fragiadakis (2010) have shown that μ_u introduces most of this bias. Although its distribution is symmetric, its effect is not. Increasing μ_u relative to the backbone shape of Fig. 3 only confers a mild benefit. Decreasing it, though, especially when in correlation with μ_c, takes out a large part of the hinge ductility, severely reducing seismic performance.

Regarding dispersion, when including both contributions from record-to-record and parameter uncertainty, the total variability is not appreciably different for the two structures shown in Figs 8(c),(d). Still, the relative contribution of the two factors changes, especially in the post-yield region for the brittle structure. The record-to-record variability steadily decreases with $\theta_{\max}$. The existence of a well-defined story mechanism makes the fate of the brittle structure less uncertain with higher $\theta_{\max}$. Incorporating the model parameter uncertainty introduces a wider range of mechanisms, as any weakened story may cause global collapse. Thus, given the same brittle model, changing records will not make a large difference. Given the same record, though, different models may shift the balance one way or another. A similar effect may also appear for the ductile structure, although considerably reduced. The inherent redundancy prevents local beam failures from becoming runaway global collapses. In essence, the beam-versus column-plasticity (roughly interpreted as capacity- versus non-capacity-designed) comparison is a recasting of the classic stable versus unstable system.

The effect of full or partial spatial correlation is not as important for dispersion in general, although full correlation seems to introduce a somewhat higher overall uncertainty. Orchestrated changes, introducing weakening or strengthening throughout the structure should cause larger variation compared to uncorrelated changes that may cancel each other out within a single story. Naturally, having studied only one structure and set of random parameters, the final word has not yet been cast for the importance of parameter uncertainty. Still, there is enough evidence to claim that it should not be discounted easily.

Conclusions

The incremental accelerogram-wise Latin Hypercube Sampling procedure iLHS has been developed for efficiently estimating the effect of model parameter uncertainties on the seismic performance of structures. It builds upon the existing paradigm of incremental dynamic analysis with latin hypercube sampling and further improves it by resolving the problem of sample size determination and by partially mitigating its slow convergence, offering an improvement by a factor of 20 at least. The end result is a robust algorithm that is amenable to parallelization and automated application. It allows excellent scalability and extends the original methodology to be applicable to models with hundreds of random variables. It can be used to evaluate the actual distribution of seismic response, revealing the bias in estimates of median and dispersion that may substantially differ between capacity (beam-plasticity) and non-capacity (column-plasticity) designed structures.

Acknowledgements

Financial support was provided by the EU Research Executive Agency via the Marie Curie Continuing Integration Grant No. PCIG09-GA-2011-293855.

References

Azarbakht, A., and Dolsek, M. (2007). "Prediction of the median IDA curve by employing a limited number of ground motion records." *Earthq. Eng. Struct. Dyn.*, 36(15), 2401–2421.

Azarbakht, A., and Dolsek, M. (2011). "Progressive incremental dynamic analysis for first-mode dominated structures." *J. Struct. Eng.*, 137(3), 445–455.

Charmpis, D. C., and Panteli, P. P. (2004). "A heuristic approach for the generation of multivariate random samples with specified marginal distributions and correlation matrix." *Computational Statistics*, 19, 283–300.

Cornell, C. A., Jalayer, F., Hamburger, R. O., and Foutch, D. A. (2002). "The probabilistic basis for the 2000 SAC/FEMA steel moment frame guidelines." *J. Struct. Eng.*, 128(4), 526–533.

Cornell, C. A., and Krawinkler, H. (2000). "Progress and challenges in seismic performance assessment." *PEER Center News*, 3(2), URL `http://peer.berkeley.edu/news/2000spring/index.html`, [May 2012].

Der Kiureghian, A., and Ditlevsen, O. (2009). "Aleatory or epistemic? does it matter?" *Struct. Saf.*, 31, 105–112.

Dolsek, M. (2009). "Incremental dynamic analysis with consideration of modelling uncertainties." *Earthq. Eng. Struct. Dyn.*, 38(6), 805–825.

Dolsek, M. (2012). "Simplified method for seismic risk assessment of buildings with consideration of aleatory and epistemic uncertainty." *Struct. Infrastruct. Eng.*, 8(10), 939–953.

Dolsek, M., and Fajfar, P. (2005). "Simplified non-linear seismic analysis of infilled reinforced concrete frames." *Earthq. Eng. Struct. Dyn.*, 34(1), 49–66.

Efron, B., and Tibshirani, R. J. (1993). *An Introduction to the Bootstrap*. Chapman & Hall/CRC, New York.

Esteva, L., and Ruiz, S. E. (1989). "Seismic failure rates of multistory frames." *J. Struct. Eng.*, 115(2), 268–284.

FEMA (2012). "Seismic performance assessment of buildings, Volume 1 - Methodology." *Report No. FEMA P-58-1*, Applied Technology Council, Federal Emergency Management Agency, Washington, DC.

Foutch, D. A., and Yun, S.-Y. (2002). "Modeling of steel moment frames for seismic loads." *Journal of Constructional Steel Research*, 58, 529–564.

Fragiadakis, M., and Vamvatsikos, D. (2010). "Fast performance uncertainty estimation via pushover and approximate IDA." *Earthq. Eng. Struct. Dyn.*, 39(6), 683–703.

Fragiadakis, M., Vamvatsikos, D., and Papadrakakis, M. (2006). "Evaluation of the influence of vertical irregularities on the seismic performance of a 9-storey steel frame." *Earthq. Eng. Struct. Dyn.*, 35(12), 1489–1509.

Ibarra, L., and Krawinkler, H. (2011). "Variance of collapse capacity of SDOF systems under earthquake excitations." *Earthq. Eng. Struct. Dyn.*, 40, 1299–1314.

Ibarra, L., Medina, R., and Krawinkler, H. (2005). "Hysteretic models that incorporate strength and stiffness deterioration." *Earthq. Eng. Struct. Dyn.*, 34, 1489–1511.

Idota, H., Guan, L., and Yamazaki, K. (2009). "Statistical correlation of steel members for system reliability analysis." *Proceedings of the 9th International Conference on Structural Safety and Reliability (ICOSSAR)*, Osaka, Japan.

Iman, R. L., and Conover, W. J. (1982). "A distribution-free approach to inducing rank correlation among input variables." *Communication in Statistics Part B: Simulation and Computation*, 11(3), 311–334.

Jalayer, F., and Cornell, C. A. (2009). "Alternative non-linear demand estimation methods for probability-based seismic assessments." *Earthq. Eng. Struct. Dyn.*, 38(8), 951–1052.

Kazantzi, A. K., Righiniotis, T. D., and Chryssanthopoulos, M. K. (2008). "Fragility and hazard analysis of a welded steel moment resisting frame." *J. Earthq. Eng.*, 12(4), 596–615.

Lagaros, N. D. (2010). “Multicomponent incremental dynamic analysis considering variable incident angle.” *Struct. Infrastruct. Eng.*, 6(1-2), 77–94.

Liel, A. B., Haselton, C. B., Deierlein, G. G., and Baker, J. W. (2009). “Incorporating modeling uncertainties in the assessment of seismic collapse risk of buildings.” *Struct. Saf.*, 31(2), 197–211.

Lignos, D., and Krawinkler, H. (2011). “Deterioration modeling of steel components in support of collapse prediction of steel moment frames under earthquake loading.” *J. Struct. Eng.*, 137(11), 1291–1302.

Luco, N., and Bazzurro, P. (2007). “Does amplitude scaling of ground motion records result in biased nonlinear structural drift responses?” *Earthq. Eng. Struct. Dyn.*, 36(13), 1813–1835.

Luco, N., and Cornell, C. A. (2007). “Structure-specific, scalar intensity measures for near-source and ordinary earthquake ground motions.” *Earthq. Spectra*, 23(2), 357–392.

McKay, M. D., Conover, W. J., and Beckman, R. (1979). “A comparison of three methods for selecting values of input variables in the analysis of output from a computer code.” *Technometrics*, 21(2), 239–245.

McKenna, F., Fenves, G., Jeremic, B., and Scott, M. (2000). “Open system for earthquake engineering simulation.” URL `http://opensees.berkeley.edu`, [May 2008].

Nielson, B. G., and DesRoches, R. (2007). “Analytical seismic fragility curves for typical bridges in the central and southeastern United States.” *Earthq. Spectra*, 23(3), 615–633.

Rigato, A., and Medina, R. (2007). “Influence of angle of incidence on the seismic demands for inelastic single-storey structures subjected to bi-directional ground motions.” *Eng. Struct.*, 29(10), 2593–2601.

SAC/FEMA (2000). “Recommended seismic design criteria for new steel moment-frame buildings.” *Report No. FEMA-350*, SAC Joint Venture, Federal Emergency Management Agency, Washington, DC.

Sallaberry, C., and Helton, J. C. (2005). “A method for extending the size of a latin hypercube sample.” *Proceedings of the IMAC XXIII Conference and Exposition on Structural Dynamics - Structural Health Monitoring*, Orlando, FL.

Sallaberry, C. J., Helton, J. C., and Hora, S. C. (2008). “Extension of latin hypercube samples with correlated variables.” *Reliab. Eng. Syst. Saf.*, 93(7), 1047–1059.

Schotanus, M., and Franchin, P. (2004). “Seismic reliability analysis using response surface: a simplification.” *Proceedings of the 2nd ASRANet Colloquium*, Barcelona, Spain.

Tong, C. (2006). “Refinement strategies for stratified sampling methods.” *Reliab. Eng. Syst. Saf.*, 91(10-11), 1257–1265.

Vamvatsikos, D. (2011). “Performing incremental dynamic analysis in parallel.” *Computers and Structures*, 89(1-2), 170–180.

Vamvatsikos, D., and Cornell, C. A. (2002). “Incremental dynamic analysis.” *Earthq. Eng. Struct. Dyn.*, 31(3), 491–514.

Vamvatsikos, D., and Cornell, C. A. (2006). “Direct estimation of the seismic demand and capacity of oscillators with multi-linear static pushovers through incremental dynamic analysis.” *Earthq. Eng. Struct. Dyn.*, 35(9), 1097–1117.

Vamvatsikos, D., and Fragiadakis, M. (2010). “Incremental dynamic analysis for estimating seismic performance sensitivity and uncertainty.” *Earthq. Eng. Struct. Dyn.*, 39(2), 141–163.

Vorechovsky, M. (2010). “Extension of sample size in latin hypercube sampling with correlated variables.” *Proceedings of the 4th International Workshop on Reliable Engineering Computing (REC2010)*, 353–368, National University of Singapore, Singapore.

Vorechovsky, M., and Novak, D. (2009). “Correlation control in small-sample Monte Carlo type simulations. I. A simulated annealing approach.” *Probabilistic Engineering Mechanics*, 24(3), 452–462.