Accurate application and second-order improvement of the SAC/FEMA probabilistic formats for seismic performance assessment [1]

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Abstract: The SAC/FEMA probabilistic framework is based on a closed-form expression to analytically estimate the value of the risk integral convolving seismic hazard and structural response. Despite its practicality, implementation has been hindered by reduced accuracy due to a number of approximations needed to achieve a desirable form, the most significant being the power-law fitting of the seismic hazard curve. To mitigate this problem, two approaches are hereby offered, namely (a) selecting an appropriately-biased power-law fit and (b) offering a novel closed-form expression involving a second-order approximation. Where blind application of the original format could involve error in excess of 100% for the predicted mean annual frequency of limit-state exceedance, biased fitting reduces it to less than 50% for many practical cases, whereas the new closed-form consistently lies below 10%. While other sources of error still remain, the robustness achieved opens new avenues of application for this popular format.

CE Database keywords: Seismic response; Earthquakes; Performance evaluation; Safety; Approximation methods.

Introduction

The SAC/FEMA project was conceived in the wake of the Northridge 1994 earthquake to improve the performance of steel moment-resisting frame buildings. Its results have been summarized in a series of documents and guidelines, the most prominent being FEMA-350/351 (SAC/FEMA 2000a,b). One of the enduring legacies of the work generated by this project is the popularization by Cornell et al. (2002) of the concept of assessing the seismic performance of a structure in terms of the mean annual frequency (MAF) of limit-state exceedance. Equally important is the introduction by the same authors of the only closed-form solution for evaluating the probabilistic integral and of a safety checking format similar to the familiar Load and Resistance Factored Design (LRFD) (AISC 2003).

The SAC/FEMA MAF format offers a simple expression to convolve the seismic hazard with the structural response and derive estimates of the mean annual rate (or the mean return period) of exceeding any limit-state that can be defined in terms of structural response. It incorporates the effects of aleatory variability, attributed to natural randomness, and epistemic uncertainty, as caused by incomplete knowledge, offering performance assessment with a user-selected level of confidence. Thanks to the simplicity of this formulation, it has found widespread recognition and it has been used in numerous contexts as a basis for performance-based earthquake engineering calculations. An non-exhaustive list includes the seismic performance assessment of steel (e.g., Yun et al. 2002; Kazantzi et al. 2011) and reinforced concrete buildings (e.g., Dolsek and Fajfar 2008; Lupoi et al. 2002; Vamvatsikos et al. 2011a; Fajfar and Dolsek 2012), the time-dependent evaluation of corroded or aging structures (Torres and

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Ruiz 2007; Vamvatsikos and Dolsek 2011b) and the design of new structures (e.g., Franchin and Pinto 2012). At its basis, it can also be thought to have formed the core of the highly influential Pacific Earthquake Engineering Research (PEER) Center probabilistic framework via the Cornell-Krawinkler framing equation (Cornell and Krawinkler 2000), where performance is characterized by decision variables such as cost, casualties and downtime.

Nevertheless, the closed-form SAC/FEMA expressions have also been criticized for their lack of accuracy (Aslani and Miranda 2005; Bradley and Dhakal 2008). The main issue is the adequacy of the approximations used in deriving the analytic expressions. The most controversial among them is the power-law fit of the seismic hazard curve that is only locally accurate and can potentially introduce massive errors. It can be argued that, despite such issues, the SAC/FEMA format retains its primary use as a tool for developing intuition by *qualitatively* understanding the parameters influencing seismic performance. Still, having an accurate closed-form expression can be highly useful, both for design and assessment, allowing *quantitative* evaluations to be performed nearly instantaneously. Thus, following in the steps of the original derivation, we aim to address its main shortcomings by exploring the possibilities offered by two recent improvements, namely using the original expression with a first-order biased hazard fit (Dolsek and Fajfar 2008) and employing a second-order hazard fit in the log-log domain to derive a novel closed-form solution (Vamvatsikos 2013).

The SAC/FEMA format

Estimating the probability of violating a certain performance level or limit-state starts with the evaluation of a site's seismic hazard. By adopting a Poisson model for earthquake occurrence, probabilistic seismic hazard analysis (PSHA, Cornell 1968) offers a quantification of site hazard via the hazard curve $H(s)$. This is the function of the mean annual frequency (MAF) of exceeding levels (values) of s, i.e., of the adopted seismic intensity measure (IM). Typical scalar IM choices include the peak ground acceleration or velocity, the spectral acceleration S_a at a given period T and combinations of elastic (Cordova et al. 2000; Vamvatsikos and Cornell 2005; Mehanny 2009) and/or inelastic spectral values (Luco and Cornell 2007). Let then C, D be scalar capacity and demand characteristics, respectively, of the structure. They can be expressed either in IM or engineering demand parameter (EDP) terms to check for violating the limit-state LS. Thus, in the absence of uncertainty, failure is simply stated as $C < D$, or the capacity being less than the demand. For example this could be cast as the maximum interstory drift demand of the structure being more than a limiting value of, say, 1% (EDP basis), or the 5%-damped first-mode spectral acceleration $S_a(T_1, 5\%)$ of the ground motion excitation being higher than, e.g., 0.4g (IM basis). In the presence of uncertainty, the conditional failure probability $P(C < D|s)$, also known as fragility, is used instead. By convolving with the seismic hazard, the MAF of limit-state (LS) exceedance λ_{LS} can be estimated via any of the following three integrals (Jalayer 2003; Vamvatsikos 2013):

$$\begin{aligned} \lambda_{\mathrm{LS}} &= \int_0^{+\infty} P(C < D\,|s)\,|\mathrm{d}H(s)| \\ &= \int_0^{+\infty} P(C < D\,|s) \left|\frac{\mathrm{d}H(s)}{\mathrm{d}s}\right| \mathrm{d}s \\ &= \int_0^{+\infty} \frac{\mathrm{d}P(C < D\,|s)}{\mathrm{d}s} H(s)\mathrm{d}s \end{aligned} \tag{1}$$

Cornell et al. (2002) have shown that a closed-form solution may be derived by making a series of rational assumptions and approximations. First, a power-law fit is adopted for the hazard curve (see also Kennedy and Short 1994):

$$H(s) = k_0(s)^{-k_1} = k_0 \exp(-k_1 \ln s)\,, \tag{2}$$

where k_0 and k_1 are positive real numbers. If the capacity C and demand D of the structure in Eq. (1) are expressed in terms of the IM, then we have the IM-based format. Assuming that the IM-capacity s_c is lognormally distributed with median $\hat{s}_c$ and dispersion (standard deviation of the log of the data) β_{Sc}, then the MAF of the limit-state can be approximated as:

$$\lambda_{\mathrm{LS}} = H(\hat{s}_c) \exp\left(\frac{1}{2} k_1^2 \beta_{Sc}^2\right) \tag{3}$$

If, instead, the capacity C in Eq. (1) is represented as an EDP value that when exceeded by the seismic EDP demand D signals violation, further approximations are needed. Accordingly, it is assumed that the EDP capacity θ_c follows a lognormal distribution with median $\hat{\theta}_c$ and dispersion $\beta_{\theta c}$. If, additionally, the EDP demand θ given the IM is also lognormal with a constant dispersion of $\beta_{\theta d}$ and a conditional median demand provided by a power law

$$\hat{\theta}(s) \approx a s^b, \tag{4}$$

where a, b are positive real numbers, the closed-form approximation becomes:

$$\lambda_{\mathrm{LS}} = H\left[\left(\frac{\hat{\theta}_c}{a}\right)^{\frac{1}{b}}\right] \exp\left[\frac{k_1^2}{2b^2}(\beta_{\theta d}^2 + \beta_{\theta c}^2)\right]. \tag{5}$$

In cases where instead of estimating the MAF one is interested in simply checking whether the structure violates a certain limit-state, the Demand-Capacity Factor Design Format (DCFD) can be used. This was introduced by Cornell et al. (2002) for safety checking in a manner resembling the popular Load and Resistance Factor Design (LRFD) format. Let's say that we wish to verify whether the structure complies with a limit-state performance objective P_o, e.g., the typical 10% in 50yrs for Life Safety corresponding to a $P_o = -\ln(1-0.10)/50 = 0.00211$. We need to estimate the median demand $\hat{\theta}_{po}$ and its dispersion $\beta_{\theta d}$ that are representative of P_o. This entails statistically summarizing the EDP results from several nonlinear dynamic analyses using ground motion records having (or scaled to) an intensity level $s_{po} = H^{-1}(P_o)$, where $H^{-1}(\cdot)$ is the inverse of the seismic hazard function. Given our earlier assumptions about the lognormality of the EDP capacity and the conditional EDP demand, safety can be verified as

$$\hat{\theta}_c \exp\left(-\frac{k_1}{2b}\beta_{\theta c}^2\right) \geq \hat{\theta}_{po} \exp\left(\frac{k_1}{2b}\beta_{\theta d}^2\right). \tag{6}$$

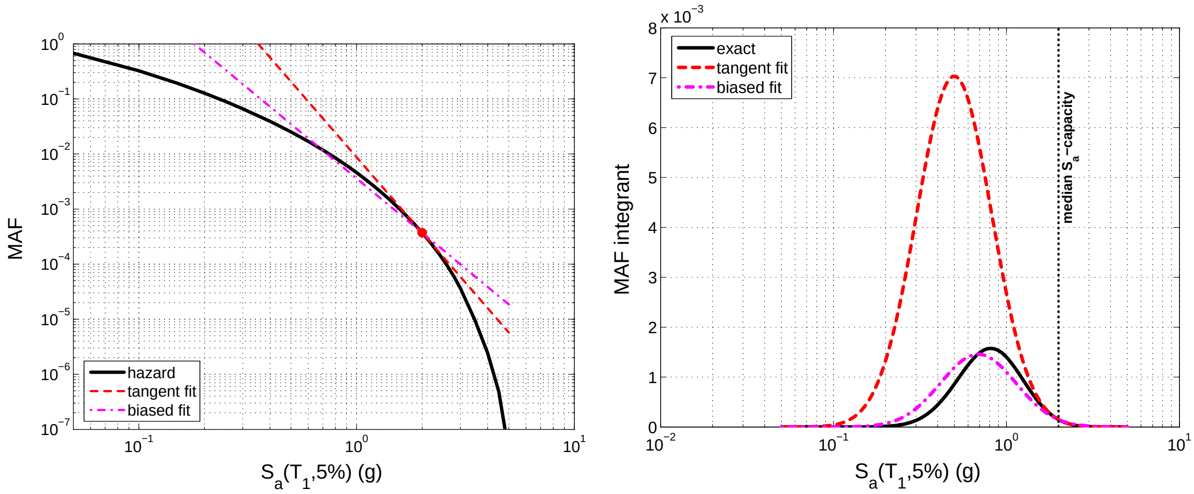
Epistemic uncertainty is typically handled under the first-order assumption, i.e., by taking it to affect only the variability and not the central value of D and C. This is achieved by inflating the dispersion of demand and capacity through the incorporation of dispersions $\beta_{U\theta d}$ and $\beta_{U\theta c}$, respectively, in a square-root-sum-of-squares manner. To offer a choice of treating the effect of epistemic uncertainty with the desired confidence, an additional exponential factor can be appended to the right side of Eq. (6):

$$\hat{\theta}_c \exp\left(-\frac{k_1}{2b}\beta_{\theta c}^2\right) \geq \hat{\theta}_{po} \exp\left(\frac{k_1}{2b}\beta_{\theta d}^2 + K_x \beta_{U\theta}\right), \tag{7}$$

where $\beta_{U\theta}^2 = \beta_{U\theta d}^2 + \beta_{U\theta c}^2$ is the total uncertainty in demand and capacity. K_x is the standard normal variate corresponding to the desired confidence level. Formally, $K_x = \Phi^{-1}(x)$, where

$\Phi^{-1}(\cdot)$ is the inverse cumulative distribution function (CDF) of a standard normal variable (Benjamin and Cornell 1970), readily available in any probability textbook or spreadsheet program (for example, $K_x = 1.28$ for a 90% confidence level estimate).

Summing up, beyond the various lognormality assumptions, the IM-based format requires only the hazard curve approximation of Eq. (2), while the EDP-based formats need the additional approximation of structural response via Eq. (4). In both cases, though, it is the common local hazard fit that creates most accuracy problems due to the rapid monotonically decreasing nature of the hazard function $H(s)$. The effect can be overwhelming, resulting to MAF estimates that can be off target by orders of magnitude (Bradley and Dhakal 2008). To resolve this situation, we intend to offer two complementary fitting solutions.



(a) hazard curve fits (b) MAF integrands

Fig. 1. The tangent and the biased power-law fits versus the corresponding MAF integrands for $\hat{s}_c = 2$g, $\beta_{Sc} = 0.5$ at the Van Nuys site ($T_1 = 0.7$s).

First-order biased hazard fitting

Approximating the curved seismic hazard function by a straight line in log-log space (Fig. 1(a)) can be a tricky endeavour. Jalayer (2003) proposed locally fitting Eq. (2) as a tangent at the median IM-capacity. By construction this will always assure a conservative fit due to the concave shape of the hazard curve (Fig. 1(a)). Unfortunately, as the hazard curvature and the capacity dispersion increase, this approach results in excessive overestimation of the MAF integrand (Fig. 1(b)). If we adopt an IM-basis and assume the third form of Eq. (1), the MAF integrand is simply the probability density function (PDF) of the IM capacity (essentially symmetric around the median in log-log) multiplied by the geometrically decreasing hazard value. Thus, most of the contribution to the MAF integral of Eq. (1) comes from the higher frequency earthquakes to the left of the median capacity $\hat{s}_c$ (Bradley and Dhakal 2008; Eads et al. 2012) as shown in the high-curvature high-dispersion example of Fig. 1(b).

Thus, it makes sense to introduce a biased fit of Eq. (2), a fact originally recognized by Dolsek and Fajfar (2008). They suggested performing a linear regression in the region of $[0.25\hat{s}_c, 1.25\hat{s}_c]$ to derive the hazard fit. Since this requires a regression and does not take into account the dispersion of capacity, a simpler solution is proposed that involves left-weighted, right-biased fitting, to be termed the first-order biased fit. The seismic hazard is approximated by a secant line that passes through the median capacity but adopts a k_1-slope determined at points 0.5 and 1.5 standard deviations away. For the IM-based format of Eq. (3):

$$k_1 = -\frac{\ln H(s_2) - \ln H(s_1)}{\ln s_2 - \ln s_1} \tag{8}$$

$$k_0 = H(\hat{s}_c) \cdot \hat{s}_c^{k_1} \tag{9}$$

$$s_i = \hat{s}_c \exp(c_i \beta_{Sc}), \qquad i = 1, 2 \tag{10}$$

where, $c_{1,2} = -0.5, -1.5$. An example of its efficiency appears in Fig. 1(b) where the biased fit offers a close match of the MAF integrand in a high-curvature high-dispersion situation. In this case, the exact MAF is 0.0015, with the tangent fit producing 0.0050 and the biased fit 0.0014.

For application with the EDP-based format of Eq. (5), the same idea is used, only now the anchor points are determined by employing the total dispersion of EDP demand and capacity, divided by the slope (in log-log) of the IM-EDP relationship. Thus, Eq. (10) is replaced by:

$$s_i = \left(\frac{\hat{\theta}_c}{a}\right)^{1/b} \exp\left(c_i \frac{\sqrt{\beta_{\theta c}^2 + \beta_{\theta d}^2}}{b}\right). \tag{11}$$

When employing the DCFD format of Eq. (6), another modification is needed. Hazard fitting has to be performed at s_{po}, this being the intensity of the ground motion records used for the nonlinear analyses: It is the only intensity for which demand is estimated. Ideally, the fit should remain close to the IM value corresponding to the median capacity, as in Eq. (11). When the difference between s_{po} and $\hat{s}_c$ is substantially in favour of the latter, i.e., the unfactored capacity (the opposite guarantees failure), the point is mute as any reasonable fit will correctly suggest a safe result. When they are close, though, the accuracy of Eq. (5) becomes critical. Then, it helps to shift the fit closer to $\hat{s}_c$ (rather than s_{po}), or equivalently shift the c_i values closer to zero, say by 0.5 standard deviations. Thus, $c_{1,2} = 0.0, -1.0$ is used together with

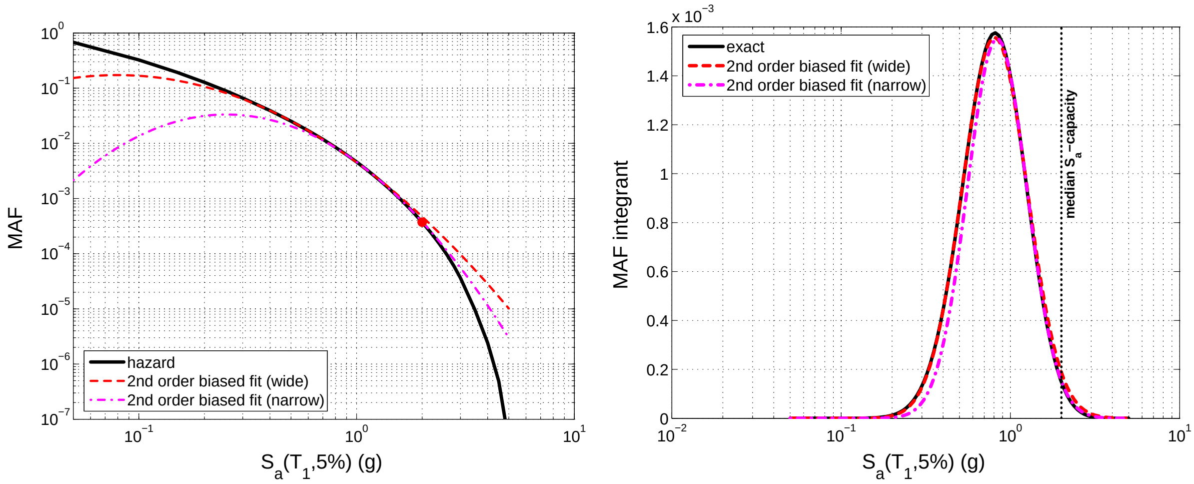
$$s_i = s_{po} \exp\left(c_i \frac{\sqrt{\beta_{\theta c}^2 + \beta_{\theta d}^2}}{b}\right). \tag{12}$$

Second-order hazard fitting solutions

While the biased fit of the power-law approximation can be a definite improvement over the tangent fit, it does not take into account the curvature of the seismic hazard function. Predictably, its performance will degrade when higher curvatures are present. Employing a second-order polynomial fit in log-space can go a long way in resolving this problem. Thus, letting

$$H(s) = k_0 \exp\left(-k_2 \ln^2 s - k_1 \ln s\right) \tag{13}$$

new closed-form solutions have been derived by Vamvatsikos (2013) following the path laid out by Cornell et al. (2002).



(a) hazard curve fits

(b) MAF integrands

Fig. 2. Two biased 2nd order power-law fits, on a narrow or a wide interval, and the corresponding MAF integrands for $\hat{s}_c = 2\text{g}$, $\beta_{Sc} = 0.5$ at the Van Nuys site ($T_1 = 0.7\text{s}$).

MAF format on IM-basis

Let us first assume that demand and capacity are expressed in an IM-basis. Then, the third form of Eq. (1) can be integrated analytically (Vamvatsikos 2013) to become

$$\begin{aligned}\lambda_{\text{LS}} &= \sqrt{p}\; k_0^{1-p}\; [H(\hat{s}_c)]^p \,\exp\left(\frac{1}{2} p k_1^2 \beta_{Sc}^2\right) \\ &= \sqrt{p}\; k_0^{1-p}\; [H(\hat{s}_c)]^p \,\exp\left[\frac{k_1^2}{4k_2}(1-p)\right] ,\end{aligned} \tag{14}$$

where the positive real $p \in (0,1]$ is defined as:

$$p = \frac{1}{1 + 2k_2\beta_{Sc}^2} . \tag{15}$$

The first form of the solution in Eq. (14) shows that for zero curvature ($k_2 = 0$), i.e., for the classic power-law fit of Eq. (2), $p = 1$ and the expression reverts back to the SAC/FEMA original of Eq. (3). The second form is valid only for $k_2 \neq 0$, this always being the case anywhere on a realistic hazard curve.

Introducing the effect of epistemic uncertainty is equally simple. Uncertainty in the hazard curve is approximately included by using the mean hazard function $\overline{H}(\hat{s}_c)$. IM capacity uncertainty is taken into account by having the dispersion incorporate both epistemic and aleatory contributions. Hence, employing the square-root-sum-of-squares rule as in SAC/FEMA to combine dispersions we only need to replace p with its respective counterpart p':

$$p' = \frac{1}{1 + 2k_2\left(\beta_{Sc}^2 + \beta_{USc}^2\right)} \tag{16}$$

with β_{USc} being the dispersion due to uncertainty in IM capacity. This would in turn produce the mean (vis-à-vis epistemic uncertainty) estimate of the MAF. If instead a certain percentile MAF value is required, reflecting, e.g., the $x = 90\%$ confidence level, the associated dispersion in the MAF due to epistemic uncertainty is defined as:

$$\beta_{TUSc} = \beta_{USc}\, p\left(k_1 + 2k_2 \ln \hat{s}_c\right) . \tag{17}$$

Then, letting K_x be the standard normal variate corresponding to the desired x confidence level and using (for example) the second form of Eq. (14) we reach

$$\lambda_{\mathrm{LS}}^{x} = \sqrt{p}\ k_0^{1-p}\ \left[\overline{H}(\hat{s}_c)\right]^p \exp\left[\frac{k_1^2}{4k_2}(1-p) + K_x\beta_{TUSc} - \gamma_{Sx}\right], \tag{18}$$

where a skewness correction factor is employed for added accuracy (Vamvatsikos 2013):

$$\gamma_{Sx} = k_2\beta_{USc}^2 p \cdot \frac{(1-2x)^2}{(1-x)^{0.4}}. \tag{19}$$

Had we used the equivalent first form of Eq. (14) to derive the above formula, it would be obvious that for $k_2 = 0$, Eq. (18) reverts again back to the SAC/FEMA original form.

When coupled with a biased fit concept, the new format promises excellent results. Using Eq. (10) with $c_{1,2,3} = -0.5, -1.5, -3.0$ offers a useful set of interpolation points for Eq. (13). Still, the selection is now quite more flexible. An example appears in Fig. 2(a) where the second-order power-law has been bias-fitted over a narrow or a much wider interval than the one suggested. While the fits themselves seem to differ markedly, they both perform equally well in the region that matters, as seen in Fig. 2(b). Therein, the MAF integrand from Eq. (1) is practically identical, either using the actual hazard curve or any of the two second-order approximations. The resulting MAF estimates match the exact value of 0.0015 within 1%.

MAF format on EDP-basis

To utilize the second-order fit within an EDP-based format, we let

$$q = \frac{1}{1 + 2k_2\beta_{\theta d}^2/b^2}, \tag{20}$$

$$\phi = \frac{1}{1 + 2k_2\left(\beta_{\theta d}^2 + \beta_{\theta c}^2\right)/b^2}, \tag{21}$$

and define the IM-level corresponding to the median EDP-capacity according to Eq. (4)

$$s_{\hat{\theta}c} = \left(\frac{\hat{\theta}_c}{a}\right)^{1/b}. \tag{22}$$

Then Eq. (1) becomes (Vamvatsikos 2013):

$$\begin{aligned}\lambda_{\mathrm{LS}} &= \sqrt{\phi}\ k_0^{1-\phi}\ \left[H(s_{\hat{\theta}c})\right]^\phi \exp\left[\frac{1}{2b^2}qk_1^2(\beta_{\theta d}^2 + \phi\beta_{\theta c}^2)\right] \\ &= \sqrt{\phi}\ k_0^{1-\phi}\ \left[H(s_{\hat{\theta}c})\right]^\phi \exp\left[\frac{k_1^2}{4k_2}(1-\phi)\right],\end{aligned} \tag{23}$$

The first form of Eq. (23) turns into the SAC/FEMA original when $k_2 = 0$, since $q = \phi = 1$. Introducing epistemic uncertainty again involves using the mean hazard value $\overline{H}(s_{\hat{\theta}c})$ and the updated value of ϕ' for estimating the overall mean estimate of the MAF via Eq. (23):

$$\phi' = \frac{1}{1 + 2k_2\left(\beta_{\theta d}^2 + \beta_{\theta c}^2 + \beta_{U\theta d}^2 + \beta_{U\theta c}^2\right)/b^2}. \tag{24}$$

$\beta_{U\theta d}$, $\beta_{U\theta c}$ are the demand and capacity dispersions, respectively, due to epistemic uncertainty.

If instead we are interested in a specific x-confidence value for the MAF, the standard normal variate $K_x = \Phi^{-1}(x)$ is needed. The corresponding MAF becomes:

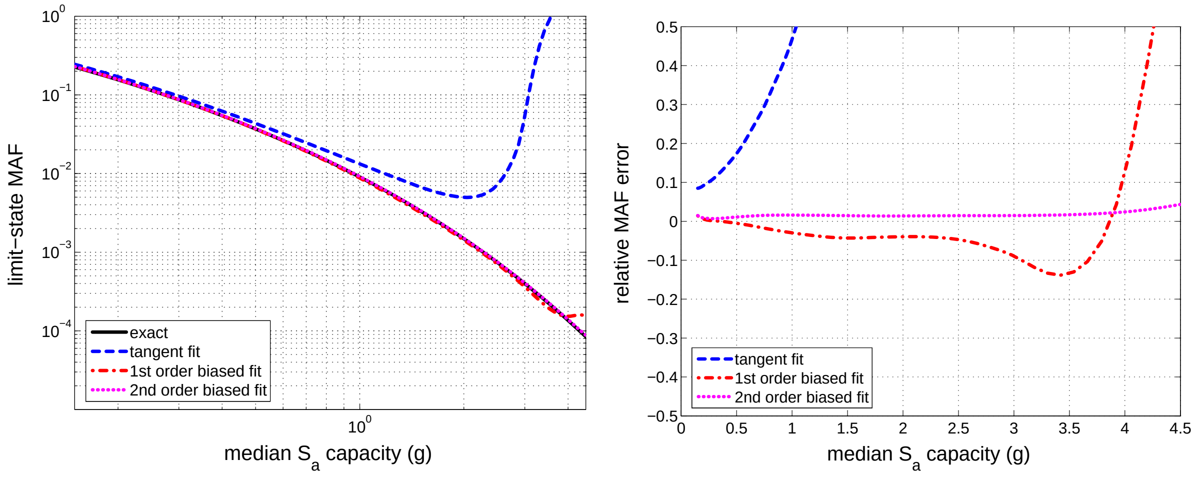
$$\lambda_{\mathrm{LS}}^{x} = \sqrt{\phi}\ k_0^{1-\phi}\ \left[\overline{H}(s_{\hat{\theta}c})\right]^{\phi} \cdot \exp\left[\frac{k_1^2}{4k_2}(1-\phi) + K_x \beta_{TU\theta} - \gamma_{\theta x}\right] . \tag{25}$$

The total MAF dispersion due to uncertainty and the respective skewness factor are:

$$\beta_{TU\theta} = \beta_{U\theta} \frac{\phi}{b} \left(k_1 + 2k_2 \ln s_{\hat{\theta}c}\right) , \tag{26}$$

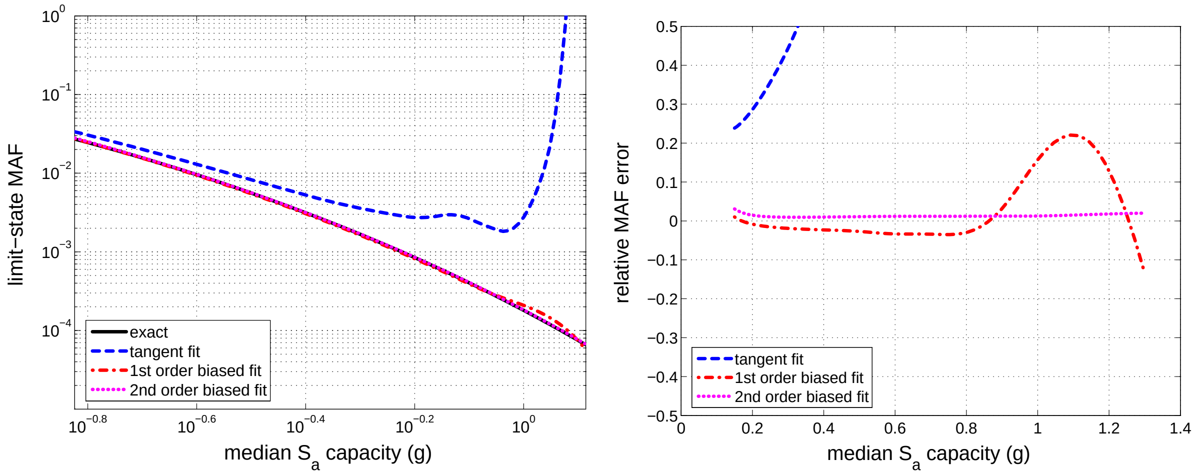
$$\gamma_{\theta x} = k_2 \beta_{U\theta}^2 \frac{\phi}{b} \cdot \frac{(1-2x)^2}{(1-x)^{0.4}} , \tag{27}$$

where $\beta_{U\theta}^2 = \beta_{U\theta d}^2 + \beta_{U\theta c}^2$, assuming uncorrelated demand and capacity. In all cases, biased fitting is suggested with interpolation points defined via Eq. (11) for $c_{1,2,3} = -0.5, -1.5, -3.0$.



(a) estimated MAFs (b) MAF relative error

Fig. 3. Estimates of limit-state MAFs and corresponding relative errors for varying $\hat{s}_c$ and $\beta_{Sc} = 0.5$ at the Van Nuys site ($T_1 = 0.7$s).



(a) estimated MAFs (b) MAF relative error

Fig. 4. Estimates of limit-state MAFs and corresponding relative errors for varying $\hat{s}_c$ and $\beta_{Sc} = 0.5$ at the Van Nuys site ($T_1 = 2.4$s).

Illustrative MAF application

To showcase the improvements brought by the biased fitting and the second-order approximation, a level playing field is needed. To avoid any situation that would favour one over the other, we will adopt full compliance with all but one of the assumptions required by the SAC/FEMA approach. Thus, the median EDP demand is defined as a power-law function of $S_a(T_1, 5\%)$ and it is assumed to have constant dispersion due to epistemic and aleatory sources, regardless of the intensity. The marked exception from adherence to theory will be the use of the highly-curved seismic hazard of the Van Nuys site in Los Angeles CA that is expected to pose a severe test for the proposed approximations. Obviously, the accuracy achieved in all subsequent analyses may further degrade in real-structure situations, wherever the EDP versus IM relationship does not adhere well to the above two assumptions.

For MAF estimation, the above setup makes the IM-basis and the EDP-basis exactly equivalent; therefore results will be shown only for the simpler IM-format. A typical example of the estimates obtained by numerical integration and the closed-form solutions for the first-order and second-order hazard fits appears in Figs 3 and 4 for periods of 0.7s and 2.4s, respectively. The IM-dispersion is a constant $\beta_{Sc} = 0.5$ while a continuum of limit-states is used, having median S_a-capacities ranging from 0.15g up to 1.3g. Due to the nature of the seismic hazard (see for example Fig. 1(a)), as the median capacity increases, so does the local curvature of the hazard curve. Thus, it is not surprising that the MAF estimates of the first-order tangent fit widely miss the exact value (sometimes by several orders of magnitude) for practically any capacity beyond 0.2g for this severe case. Even for low capacities, the error observed is more than 20% of the numerical integration result (Figs 3(b),4(b)). On the other hand, the first-order biased fit achieves errors less than 20%, at least for $\hat{s}_c < 1$g in Fig. 4(b). Predictably, its accuracy degrades as curvature increases. This is in contrast to the second-order fit that manages excellent predictions with less than 5% error regardless of curvature. As this is only a single example, further tests will be conducted in the following sections to better establish the expected errors.

DCFD format

If one is interested in simply checking whether the structure violates a certain limit-state, the Demand-Capacity Factor Design (DCFD) format becomes convenient. To derive the improved format for the seismic hazard fit of Eq. (13) we set λ_{LS} less than or equal to the performance objective P_o. A second-order expression is formed whose solution can be conservatively approximated as (Vamvatsikos 2013):

$$\hat{\theta}_c \geq \left(\hat{\theta}_{po}\right)^{1/\sqrt{\phi}} \exp\left[\left(\frac{bk_1}{2k_2} - \ln a\right)\left(\frac{1}{\sqrt{\phi}} - 1\right)\right]. \quad (28)$$

Using the above expression with ϕ' instead of ϕ and deriving k_1 and k_2 for the mean hazard curve also includes the effect of epistemic uncertainties and it incorporates a confidence level consistent with the mean estimate of the MAF, i.e., somewhere above 50%. If checking at an arbitrary confidence level $x\%$ is desired, the following inequality can be employed:

$$\hat{\theta}_c \geq \left(\hat{\theta}_{po}\right)^{1/\sqrt{\phi}} \exp\left[\left(\frac{bk_1}{2k_2} - \ln a\right)\left(\frac{1}{\sqrt{\phi}} - 1\right) + K_x \beta_{U\theta}\right]. \quad (29)$$

For improved accuracy, shifted bias fitting is suggested by using the interpolation points of Eq. (12) for $c_{1,2,3} = 0.0, -1.0, -2.5$.

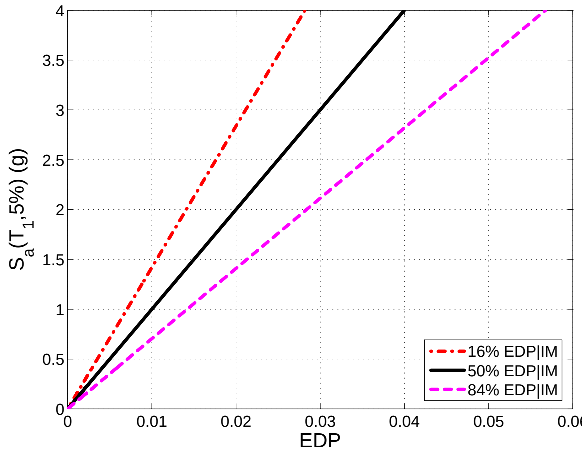
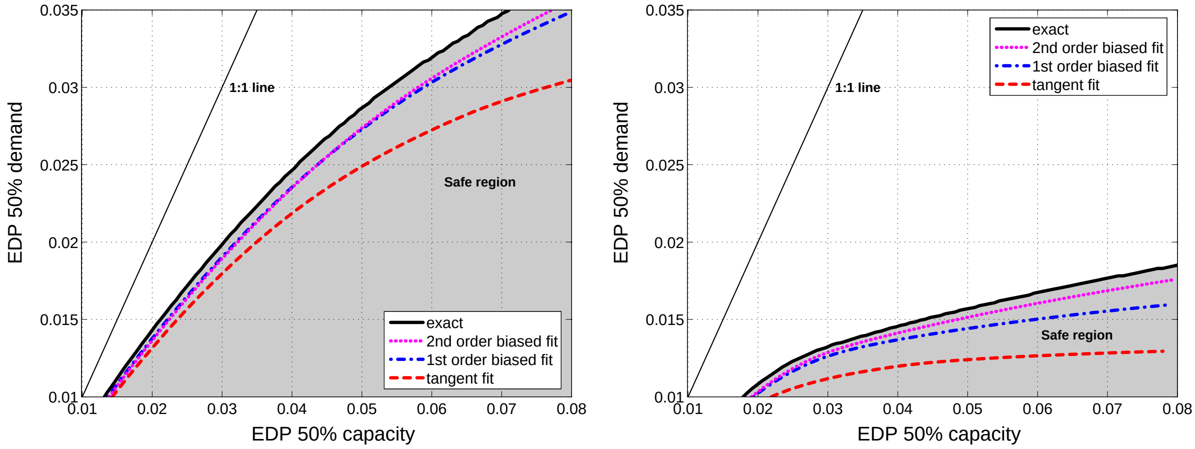


Fig. 5. The fractile IDA curves used for the DCFD comparisons ($\hat{\theta} = 0.01S_a^{1.0}$, $\beta_{\theta d} = \beta_{\theta c} = 0.35$).



(a) Case A: $T_1 = 0.7$s (b) Case B: $T_1 = 2.4$s

Fig. 6. The allowable EDP demand versus capacity via DCFD comparison by Eqs (6), (28) showing a low and a high curvature case.

Illustrative DCFD application

To illustrate the performance of the new concepts on the DCFD format, we will employ the same assumptions as for the MAF. Testing will be performed on two fictitious structures. Building A and B have first-mode periods of $T_1 = 0.7$s and 2.4s, respectively, while their median structural EDP response follows $\hat{\theta} = 0.01S_a^{1.0}$ (Fig. 5). The longer period of the second structure engages a seismic hazard curve with higher local curvatures, resulting in a more severe test. In both cases the constant demand and capacity dispersions are $\beta_{\theta d} = \beta_{\theta c} = 0.35$. These values could be attributed either to aleatory randomness only or both aleatory and epistemic uncertainty sources, a choice that has no impact on the mean MAF results. To properly visualize the discriminatory ability of the DCFD format, we will display the edge of the safe region in terms of the median demand $\hat{\theta}_{po}$ versus the median capacity $\hat{\theta}_c$, i.e., as determined by assuming equality in Eqs (6) and (28). To serve as the basis of comparison, we will also plot the edge corresponding to the exact result of P_o being equal to the numerically estimated MAF. In all cases, safety checking will be performed at the level of the mean estimates consistent with Eqs (6) and (28), corresponding to a confidence level somewhere above 50%.

Fig. 6(a) shows the boundary of the safety region between demand and capacity for building A. The tangent fit is clearly off mark with its error predictably increasing for higher capacities

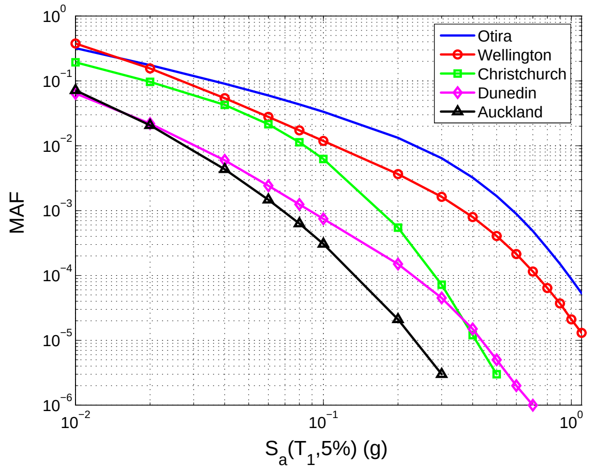
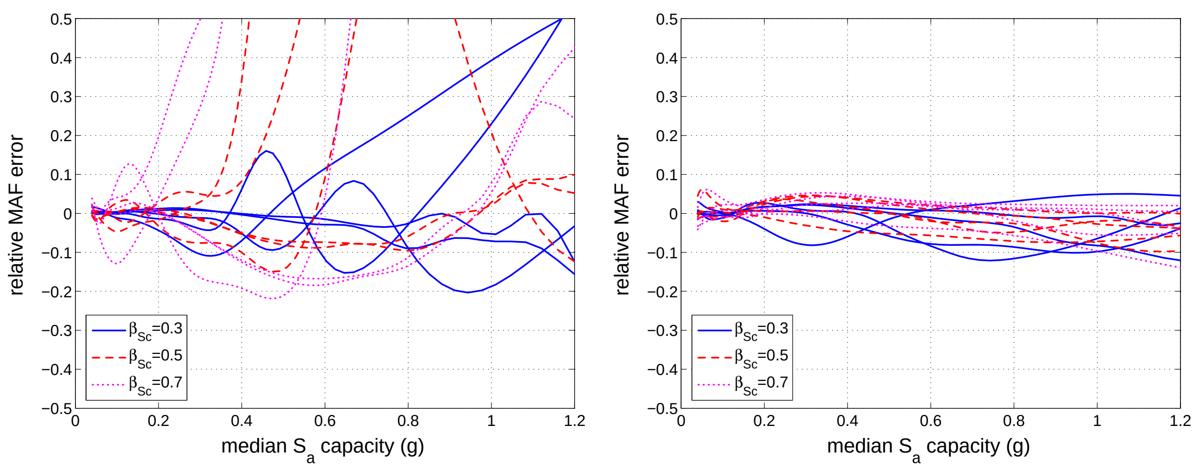


Fig. 7. Seismic hazard curves for New Zealand for $T_1 = 1.5$s (adapted from Bradley and Dhakal 2008).

(i.e., higher hazard curvature). Conversely, the biased first- and second-order fits offer a relatively good, slightly conservative approximation. For example, a median capacity of 0.050 should be able to resist an EDP demand of exactly 0.028, but this is estimated as 0.027 by the two improved methods. The original tangent fit instead would only accept a median demand lower than 0.025, obviously restricting the estimated structure's ability to absorb damage. The differences in Fig. 6(b) tell a worse story, especially if "horizontal" (i.e., given demand) rather than "vertical" (i.e., given capacity) statistics are employed. For a median demand of 0.015 the structure should in reality have an EDP capacity of 0.043 to remain safe. If approximated by a tangent fit DCFD format, then the needed capacity is off the charts, practically infinite. A biased first-order fit manages a better approximation at 0.059, while the second-order fit achieves a much improved estimate of 0.048 (at an error of 12%). In general, as the safety boundary veers away from the 1:1 demand/capacity ratio, fitting accuracy matters more.

Blind testing results



(a) 1st order biased fit

(b) 2nd order biased fit

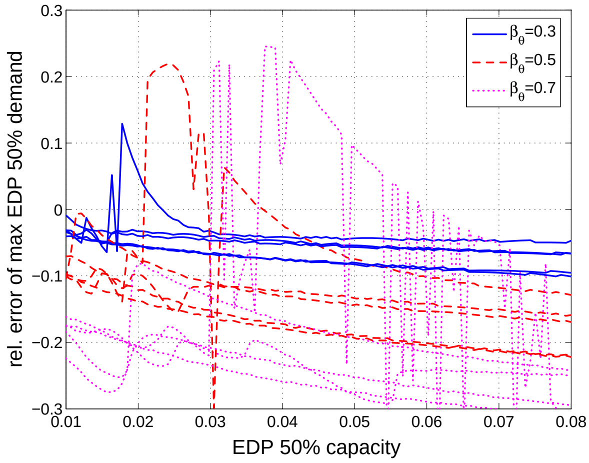
Fig. 8. Estimates of MAF relative errors for varying $\hat{s}_c$ and β_{Sc} at the New Zealand sites ($T_1 = 1.5$s).

The results shown so far indicate a remarkable improvement brought by the first- and second-order biased fits. It would be interesting to investigate whether their performance holds for a more varied set of hazard curves that were not available during development. For such a blind test we will employ five spectral acceleration hazard curves for $T_1 = 1.5$s by Bradley and Dhakal (2008), shown in Fig. 7.

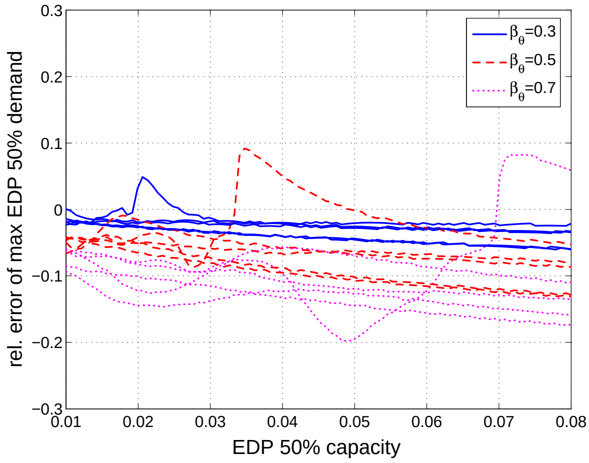
Fig. 8 presents the MAF-format relative errors for the two biased fits. The tangent fit is not shown as it is clearly inferior for MAF estimation purposes. Three different values of the dispersion are employed: $\beta_{Sc} \in \{0.3, 0.5, 0.7\}$. Obviously, as the dispersion increases a larger part of the hazard curve is involved in the determination of the MAF, putting more emphasis on the ability of the fit to accurately represent the hazard. Thus, it is no wonder that the first-order fit can provide relatively good results for $\beta_{Sc} = 0.3$ nearly for any value of the median IM capacity $\hat{s}_c$, as shown in Fig. 8(a). On the other hand, it sustains errors in excess of $+50\%$ whenever a higher dispersion is combined with a highly-curved segment of the hazard curve. Overall, the first-order biased fit provides generally conservative values at least for the values $c_{1,2} = -0.5, -1.5$ selected for Eq. (10). A more balanced overall result can be achieved by pushing the second point s_2 to the left, e.g., at $c_2 = -1.75$ or even -2.0, at the loss of the general conservatism of our initial result. There can be no doubt that the first-order biased fit remains a better alternative than the original tangent approach. Still, it cannot be recommended for highly curved seismic hazard functions when the overall (capacity and demand) dispersion is above, say, 0.4. On the other hand, the efficiency of the second-order fit is beyond doubt. The results in Fig. 8(b) show that the error rarely exceeds 10%, and even then only by a slim margin. Actually, this value is even less than the MAF error (reported to be in the order of 50% by Eads et al. 2012) due to the use of a limited number of ground motion records, typically 20–40, to estimate capacity. Thus, for all practical purposes, the second-order fit can be declared to be a near perfect match to the numerical results at a fraction of the effort.

To investigate the efficiency of the DCFD format approximations, we select again three different levels of total dispersion β_θ equal to 0.3, 0.5 and 0.7, split evenly between demand and capacity, e.g., $\beta_{\theta d} = \beta_{\theta c} \in \{0.212, 0.354, 0.495\}$ if all variability is allocated to aleatory sources. Instead of showing the safety regions of Fig. 6, the corresponding "vertical" error estimates are used: For given values of the median EDP capacity $\hat{\theta}_c$ we calculate the relative error between the actual and estimated maximum allowable value of the median EDP demand $\hat{\theta}_{po}$ according to Eqs (6) and (28). This corresponds to what a user would face when using the DCFD format in an assessment situation, where for a given structure (with a given $\hat{\theta}_c$) we are interested in estimating the maximum median demand that the proposed formats would deem acceptable. As seen earlier, the alternative representation via "horizontal" estimates, where for a given demand we seek the estimated minimum capacity required, would result to larger error estimates for high demand values. We chose not to focus on these as they are confined to a disproportionately small performance range and may give the wrong impression.

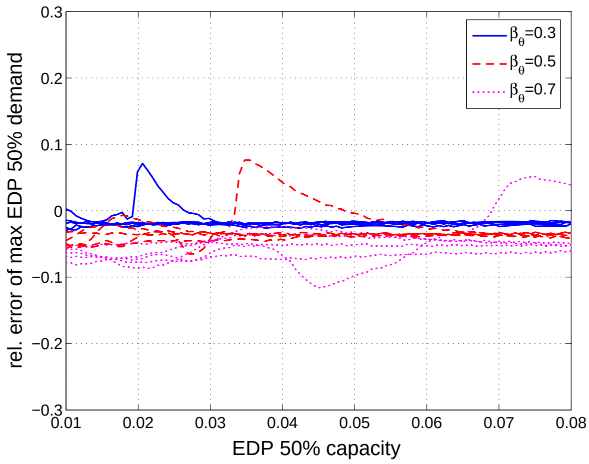
Fig. 9 presents the results for the New Zealand hazard curves of Fig. 7 and a wide range of EDP capacities. Negative errors imply conservatism; they correspond to an estimate of a lower acceptable demand than what is actually possible. Thus, a primary observation is that all formats are conservative (to different degrees) with the exception of some isolated cases computed for the Wellington site. As Fig. 9(a) shows, the tangent fit fares the worst, showing errors in excess of 30%. Still, these are at least an order of magnitude less than the discrepancies estimated at the MAF level, a clear effect of the hazard function introducing an almost exponential difference between the MAF space and the EDP-IM space. Nevertheless, such errors remain unacceptable for practical applications. The first-order biased fit in Fig. 9(b) performs considerably better, offering errors at most equal to 20%. In both of the above cases the absolute error



(a) tangent fit



(b) 1st order biased fit



(c) 2nd order biased fit

Fig. 9. Estimates of the maximum allowable $\hat{\theta}_{po}$ demand relative error for varying EDP capacity $\hat{\theta}_c$ and dispersion β_θ at the New Zealand sites ($T_1 = 1.5$s).

increases with capacity, indicating the inability of the original SAC/FEMA format to capture high curvatures in the hazard function. On the contrary, the second-order biased fits in Fig. 9(c) restrict the error to less than 10% practically for any hazard curve, dispersion or capacity value, without any sign of degradation with curvature. All points considered, the consistently low conservative error of the second-order fit makes it the clear winner of this test as well.

Conclusions

An investigation of methods to mitigate the hazard approximation errors in the SAC/FEMA closed-form solution has been presented. Two hazard fitting approaches have been proposed, offering increased accuracy at a negligible cost. The first is a biased fit of the first-order power-law function. This allows retaining the original SAC/FEMA expressions and achieve estimates with a reasonable error, at least as long as the hazard function is not highly curved and the overall dispersion of demand and capacity is less than 40%. The second approach involves the biased fitting of a second-order power-law function together with novel closed-form expressions for estimating the mean annual frequency of limit-state exceedance or for safety checking in an LRFD-like format. This innovative approach is shown to be remarkably efficient, nearly zeroing-out the excessive errors due to the curvature of the seismic hazard regardless of the dispersion. The improvement is so overwhelming that we can emphatically say that tangentially fitting a power-law function to the hazard curve in log-space should be avoided.

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