Incremental Dynamic Analysis

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SUMMARY

Incremental Dynamic Analysis (IDA) is a parametric analysis method that has recently emerged in several different forms to estimate more thoroughly structural performance under seismic loads. It involves subjecting a structural model to one (or more) ground motion record(s), each scaled to multiple levels of intensity, thus producing one (or more) curve(s) of response parameterized versus intensity level. To establish a common frame of reference, the fundamental concepts are analyzed, a unified terminology is proposed, suitable algorithms are presented, and properties of the IDA curve are looked into for both single-degree-of-freedom (SDOF) and multi-degree-of-freedom (MDOF) structures. In addition, summarization techniques for multi-record IDA studies and the association of the IDA study with the conventional Static Pushover Analysis and the yield reduction R-factor are discussed. Finally in the framework of Performance-Based Earthquake Engineering (PBEE), the assessment of demand and capacity is viewed through the lens of an IDA study.

KEY WORDS: performance-based earthquake engineering; incremental dynamic analysis; demand; collapse capacity; limit-state; nonlinear dynamic analysis

1   INTRODUCTION

The growth in computer processing power has made possible a continuous drive towards increasingly accurate but at the same time more complex analysis methods. Thus the state of the art has progressively moved from elastic static analysis to dynamic elastic, nonlinear static and finally nonlinear dynamic analysis. In the last case the convention has been to run one to several different records, each once, producing one to several "single-point" analyses, mostly used for checking the designed structure. On the other hand methods like the nonlinear static pushover (SPO) [1] or the capacity spectrum method [1] offer, by suitable scaling of the static force pattern, a "continuous" picture as the complete range of structural behavior is investigated, from elasticity to yielding and finally collapse, thus greatly facilitating our understanding.

By analogy with passing from a single static analysis to the incremental static pushover, one arrives at the extension of a single time-history analysis into an incremental one, where the seismic "loading" is scaled. The concept has been mentioned as early as 1977 by Bertero [2], and has been cast in several forms in the work of many researchers, including Luco and Cornell [3, 4], Bazurro and Cornell [5, 6], Yun and Foutch [7], Mehanny and Deierlein [8], Dubina *et al.* [9], De Matteis *et al.* [10], Nassar and

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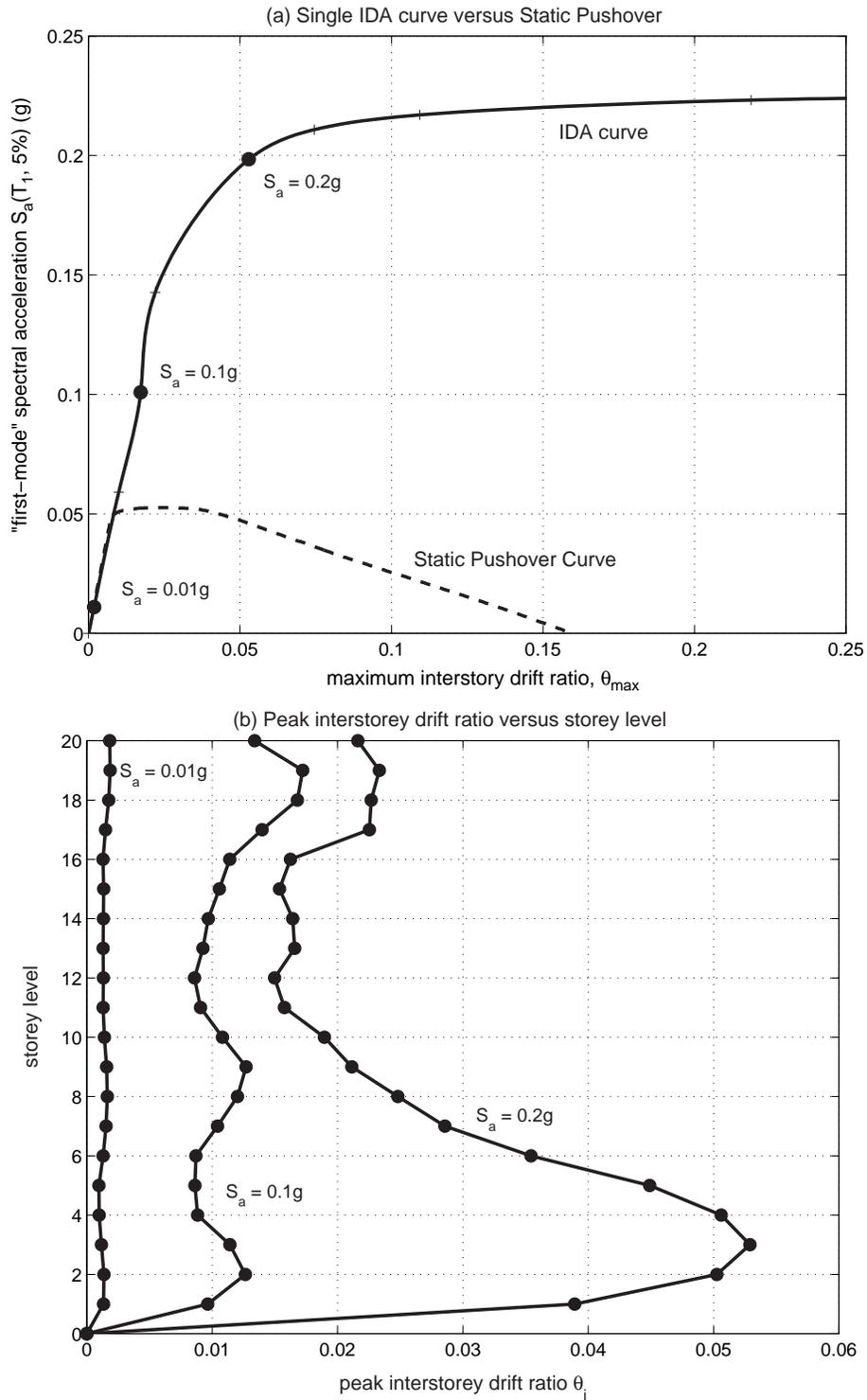
Figure 1. An example of information extracted from a single-record IDA study of a $T_1 = 4$ sec, 20-storey steel moment-resisting frame with ductile members and connections, including global geometric nonlinearities (P–Δ) subjected to the El Centro, 1940 record (fault parallel component).

Krawinkler [11, pg.62–155] and Psycharis *et al.* [12]. Recently, it has also been adopted by the U.S. Federal Emergency Management Agency (FEMA) guidelines [13, 14] as the Incremental Dynamic Analysis (IDA) and established as the state-of-the-art method to determine global collapse capacity. The IDA study is now a multi-purpose and widely applicable method and its objectives, only some of which are evident in Figure 1(a,b), include

1. thorough understanding of the range of response or "demands" versus the range of potential levels of a ground motion record,

2. better understanding of the structural implications of rarer / more severe ground motion levels,

3. better understanding of the changes in the nature of the structural response as the intensity of ground motion increases (e.g., changes in peak deformation patterns with height, onset of stiffness and strength degradation and their patterns and magnitudes),

4. producing estimates of the dynamic capacity of the global structural system and

5. finally, given a multi-record IDA study, how stable (or variable) all these items are from one ground motion record to another.

Our goal is to provide a basis and terminology to unify the existing formats of the IDA study and set up the essential background to achieve the above-mentioned objectives.

2 FUNDAMENTALS OF SINGLE-RECORD IDAS

As a first step, let us clearly define all the terms that we need, and start building our methodology using as a fundamental block the concept of scaling an acceleration time history.

Assume we are given a single acceleration time-history, selected from a ground motion database, which will be referred to as the base, "as-recorded" (although it may have been pre-processed by seismologists, e.g., baseline corrected, filtered and rotated), unscaled accelerogram a_1, a vector with elements $a_1(t_i)$, $t_i = 0, t_1, \ldots, t_{n-1}$. To account for more severe or milder ground motions, a simple transformation is introduced by uniformly scaling up or down the amplitudes by a scalar $\lambda \in [0, +\infty)$: $a_\lambda = \lambda \cdot a_1$. Such an operation can also be conveniently thought of as scaling the elastic acceleration spectrum by λ or equivalently, in the Fourier domain, as scaling by λ the amplitudes across all frequencies while keeping phase information intact.

Definition 1. *The* SCALE FACTOR *(SF) of a scaled accelerogram, a_λ, is the non-negative scalar $\lambda \in [0, +\infty)$ that produces a_λ when multiplicatively applied to the unscaled (natural) acceleration time-history a_1.*

Note how the *SF* constitutes a one-to-one mapping from the original accelerogram to all its scaled images. A value of $\lambda = 1$ signifies the natural accelerogram, $\lambda < 1$ is a scaled-down accelerogram, while $\lambda > 1$ corresponds to a scaled-up one.

Although the *SF* is the simplest way to characterize the scaled images of an accelerogram it is by no means convenient for engineering purposes as it offers no information of the real "power" of the scaled record and its effect on a given structure. Of more practical use would be a measure that would map to the *SF* one-to-one, yet would be more informative, in the sense of better relating to its damaging potential.

Definition 2. *A* MONOTONIC SCALABLE GROUND MOTION INTENSITY MEASURE *(or simply intensity measure, IM) of a scaled accelerogram,* a_λ*, is a non-negative scalar* $IM \in [0, +\infty)$ *that constitutes a function,* $IM = f_{a_1}(\lambda)$*, that depends on the unscaled accelerogram,* a_1*, and is* <u>*monotonically increasing with the scale factor,*</u> λ*.*

While many quantities have been proposed to characterize the "intensity" of a ground motion record, it may not always be apparent how to scale them, e.g., Moment Magnitude, Duration, or Modified Mercalli Intensity; they must be designated as non-scalable. Common examples of scalable *IM*s are the Peak Ground Acceleration (*PGA*), Peak Ground Velocity, the $\xi = 5\%$ damped Spectral Acceleration at the structure's first-mode period ($S_a(T_1, 5\%)$), and the normalized factor $R = \lambda / \lambda_{\text{yield}}$ (where λ_{yield} signifies, for a given record and structural model, the lowest scaling needed to cause yielding) which is numerically equivalent to the yield reduction R-factor (e.g., [15]) for, for example, bilinear SDOF systems (see later section). These *IM*s also have the property of being proportional to the *SF* as they satisfy the relation $IM_{\text{prop}} = \lambda \cdot f_{a_1}$. On the other hand the quantity $S_{am}(T_1, \xi, b, c, d) = [S_a(T_1, \xi)]^b \cdot [S_a(cT_1, \xi)]^d$ proposed by Shome and Cornell [16] and Mehanny [8] is scalable and monotonic but non-proportional, unless $b + d = 1$. Some non-monotonic *IM*s have been proposed, such as the inelastic displacement of a nonlinear oscillator by Luco and Cornell [17], but will not be focused upon, so *IM* will implicitly mean monotonic and scalable hereafter unless otherwise stated.

Now that we have the desired input to subject a structure to, we also need some way to monitor its *state*, its response to the seismic load.

Definition 3. DAMAGE MEASURE *(DM) or* STRUCTURAL STATE VARIABLE *is a non-negative scalar* $DM \in [0, +\infty]$ *that characterizes the additional response of the structural model due to a prescribed seismic loading.*

In other words a *DM* is an observable quantity that is part of, or can be deduced from, the output of the corresponding nonlinear dynamic analysis. Possible choices could be maximum base shear, node rotations, peak storey ductilities, various proposed damage indices (e.g., a global cumulative hysteretic energy, a global Park–Ang index [18] or the stability index proposed by Mehanny [8]), peak roof drift, the floor peak interstorey drift angles $\theta_1, \ldots, \theta_n$ of an *n*-storey structure, or their maximum, the maximum peak interstorey drift angle $\theta_{\max} = \max(\theta_1, \ldots, \theta_n)$. Selecting a suitable *DM* depends on the application and the structure itself; it may be desirable to use two or more *DM*s (all resulting from the same nonlinear analyses) to assess different response characteristics, limit-states or modes of failure of interest in a PBEE assessment. If the damage to non-structural contents in a multi-storey frame needs to be assessed, the peak floor accelerations are the obvious choice. On the other hand, for structural damage of frame buildings, $\theta_{\max}$ relates well to joint rotations and both global and local storey collapse, thus becoming a strong *DM* candidate. The latter, expressed in terms of the total drift, instead of the effective drift which would take into account the building tilt, (see [19, pg.88]) will be our choice of *DM* for most illustrative cases here, where foundation rotation and column shortening are not severe.

The structural response is often a signed scalar; usually, either the absolute value is used or the magnitudes of the negative and the positive parts are separately considered. Now we are able to define the IDA.

Definition 4. *A* SINGLE-RECORD IDA STUDY *is a dynamic analysis study of a given structural model parameterized by the scale factor of the given ground motion time history.*

Also known simply as Incremental Dynamic Analysis (IDA) or Dynamic Pushover (DPO), it involves a series of dynamic nonlinear runs performed under scaled images of an accelerogram, whose *IM*s are, ideally, selected to cover the whole range from elastic to nonlinear and finally to collapse of the structure. The purpose is to record *DM*s of the structural model at each level *IM* of the scaled ground motion, the resulting response values often being plotted versus the intensity level as continuous curves.

Definition 5. *An* IDA CURVE *is a plot of a state variable (DM) recorded in an IDA study versus one or more IMs that characterize the applied scaled accelerogram.*

An IDA curve can be realized in two or more dimensions depending on the number of the *IM*s. Obviously at least one must be scalable and it is such an *IM* that is used in the conventional two-dimensional (2D) plots that we will focus on hereafter. As per standard engineering practice such plots often appear "upside-down" as the independent variable is the *IM* which is considered analogous to "force" and plotted on the vertical axis (Figure 1(a)) as in stress-strain, force-deformation or SPO graphs. As is evident, the results of an IDA study can be presented in a multitude of different IDA curves, depending on the choices of *IM*s and *DM*.

To illustrate the IDA concept we will use several MDOF and SDOF models as examples in the following sections. In particular the MDOFs used are a $T_1 = 4$ sec 20-storey steel-moment resisting frame [3] with ductile members and connections, including a first-order treatment of global geometric nonlinearities (P–Δ effects), a $T_1 = 2.2$ sec 9-storey and a $T_1 = 1.3$ sec 3-storey steel-moment resisting frame [3] with ductile members, fracturing connections and P–Δ effects, and a $T_1 = 1.8$ sec 5-storey steel chevron-braced frame with ductile members and connections and realistically buckling braces including P–Δ effects [6].

3 LOOKING AT AN IDA CURVE: SOME GENERAL PROPERTIES

The IDA study is *accelerogram* and *structural model* specific; when subjected to different ground motions a model will often produce quite dissimilar responses that are difficult to predict a priori. Notice, for example, Figure 2(a–d) where a 5-storey braced frame exhibits responses ranging from a gradual degradation towards collapse to a rapid, non-monotonic, back-and-forth twisting behavior. Each graph illustrates the *demands* imposed upon the structure by each ground motion record at different intensities, and they are quite intriguing in both their similarities and dissimilarities.

All curves exhibit a distinct elastic linear region that ends at $S_a^{\text{yield}}(T_1, 5\%) \approx 0.2g$ and $\theta_{\max}^{\text{yield}} \approx$ 0.2% when the first brace-buckling occurs. Actually, any structural model with initially linearly elastic elements will display such a behavior, which terminates when the first nonlinearity comes into play, i.e., when any element reaches the end of its elasticity. The slope *IM*/*DM* of this segment on each IDA curve will be called its elastic "stiffness" for the given *DM*, *IM*. It typically varies to some degree from record to record but it will be the same across records for SDOF systems and even for MDOF systems if the *IM* takes into account the higher mode effects (i.e., Luco and Cornell [17]).

Focusing on the other end of the curves in Figure 2, notice how they terminate at different levels of *IM*. Curve (a) sharply "softens" after the initial buckling and accelerates towards large drifts and eventual collapse. On the other hand, curves (c) and (d) seem to weave around the elastic slope; they follow closely the familiar *equal displacement* rule, i.e., the empirical observation that for moderate period structures, inelastic global displacements are generally approximately equal to the displacements of the corresponding elastic model (e.g., [20]). The twisting patterns that curves (c) and (d) display in doing so are successive segments of "softening" and "hardening", regions where the local slope or
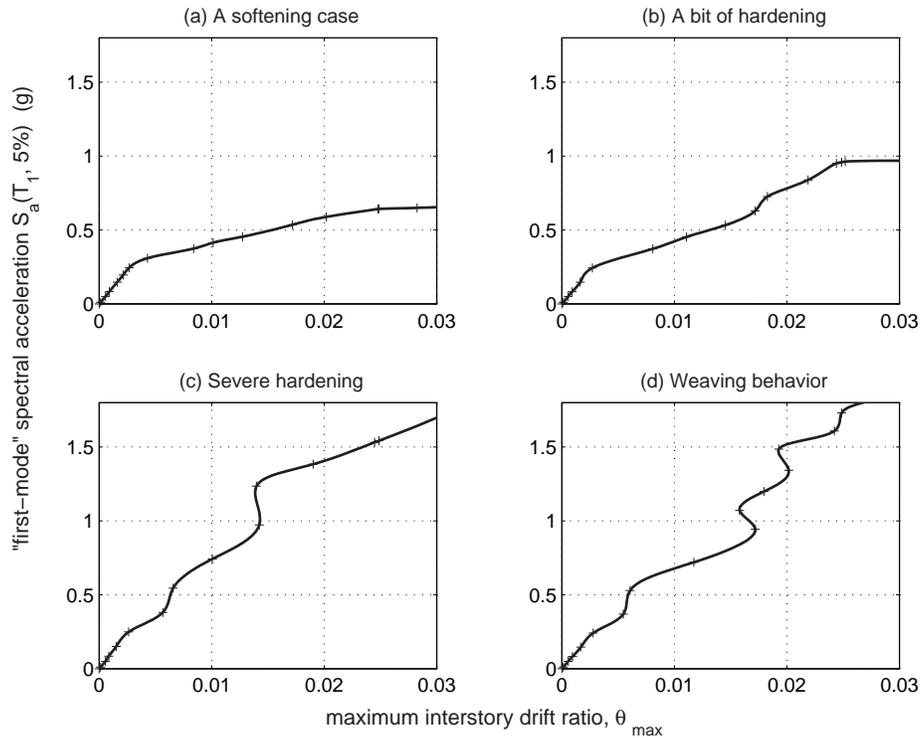
Figure 2. IDA curves of a $T_1 = 1.8$ sec, 5-storey steel braced frame subjected to 4 different records.

"stiffness" decreases with higher *IM* and others where it increases. In engineering terms this means that at times the structure experiences acceleration of the rate of *DM* accumulation and at other times a deceleration occurs that can be powerful enough to momentarily stop the *DM* accumulation or even reverse it, thus locally pulling the IDA curve to relatively lower *DM*s and making it a non-monotonic function of the *IM* (Figure 2(d)). Eventually, assuming the model allows for some collapse mechanism and the *DM* used can track it, a final softening segment occurs when the structure accumulates *DM* at increasingly higher rates, signaling the onset of *dynamic instability*. This is defined analogously to static instability, as the point where deformations increase in an unlimited manner for vanishingly small increments in the *IM*. The curve then flattens out in a plateau of the maximum value in *IM* as it reaches the *flatline* and *DM* moves towards "infinity" (Figure 2(a,b)). Although the examples shown are based on $S_a(T_1, 5\%)$ and θ_{max}, these modes of behavior are observable for a wide choice of *DM*s and *IM*s.

Hardening in IDA curves is not a novel observation, having been reported before even for simple bilinear elastic-perfectly-plastic systems, e.g., by Chopra [15, pg.257-259]. Still it remains counter-intuitive that a system that showed high response at a given intensity level, may exhibit the same or even lower response when subjected to higher seismic intensities due to excessive hardening. But it is the *pattern* and the *timing* rather than just the intensity that make the difference. As the accelerogram is scaled up, weak response cycles in the early part of the response time-history become strong enough to inflict damage (yielding) thus altering the properties of the structure for the subsequent, stronger cycles. For multi-storey buildings, a stronger ground motion may lead to earlier yielding of one floor which in turn acts as a fuse to relieve another (usually higher) one, as in Figure 3. Even simple oscillators when caused to yield in an earlier cycle, may be proven less responsive in later cycles that
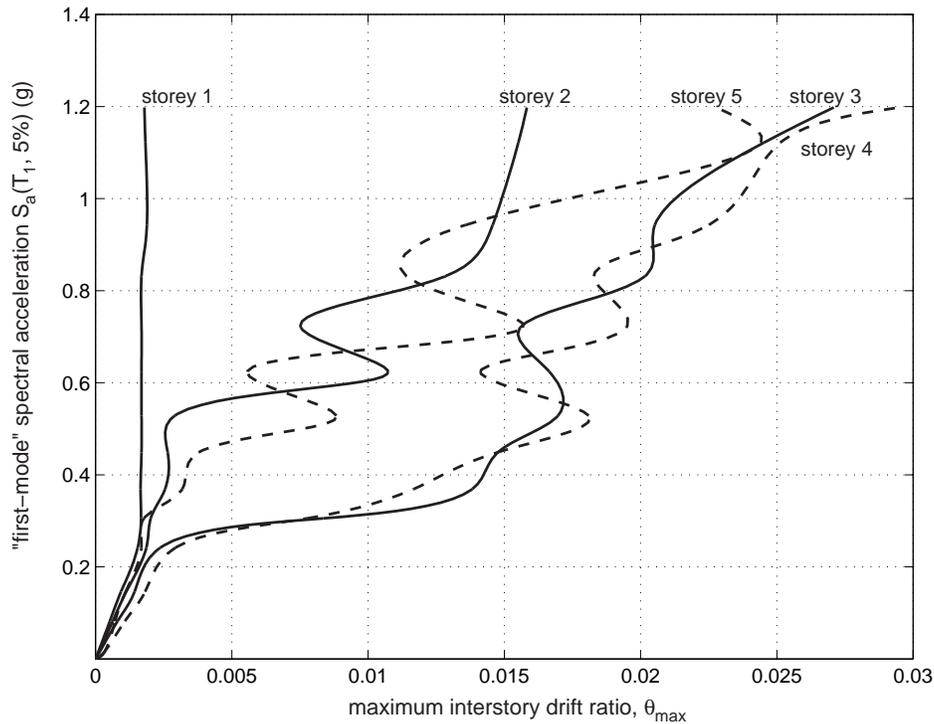
Figure 3. IDA curves of peak interstorey drifts for each floor of a $T_1 = 1.8$ sec 5-storey steel braced frame. Notice the complex "weaving" interaction where extreme softening of floor 2 acts as a fuse to relieve those above (3,4,5).

had previously caused higher *DM* values (Figure 4), perhaps due to "period elongation". The same phenomena account for the *structural resurrection*, an extreme case of hardening, where a system is pushed all the way to global collapse (i.e., the analysis code cannot converge, producing "numerically infinite" *DM*s) at some *IM*, only to reappear as non-collapsing at a higher intensity level, displaying high response but still standing (e.g., Figure 5).

As the complexity of even the 2D IDA curve becomes apparent, it is only natural to examine the properties of the curve as a mathematical entity. Assuming a monotonic *IM* the IDA curve becomes a *function* ($[0, +\infty) \rightarrow [0, +\infty]$), i.e., any value of *IM* produces a single value *DM*, while for any given *DM* value there is at least one or more (in non-monotonic IDA curves) *IM*s that generate it, since the mapping is not necessarily one-to-one. Also, the IDA curve is not necessarily smooth as the *DM* is often defined as a maximum or contains absolute values of responses, making it non-differentiable by definition. Even more, it may contain a (hopefully finite) number of discontinuities, due to multiple excursions to collapse and subsequent resurrections.

4   CAPACITY AND LIMIT-STATES ON SINGLE IDA CURVES

Performance levels or limit-states are important ingredients of Performance Based Earthquake Engineering (PBEE), and the IDA curve contains the necessary information to assess them. But we need to define them in a less abstract way that makes sense on an IDA curve, i.e., by a statement or a *rule*
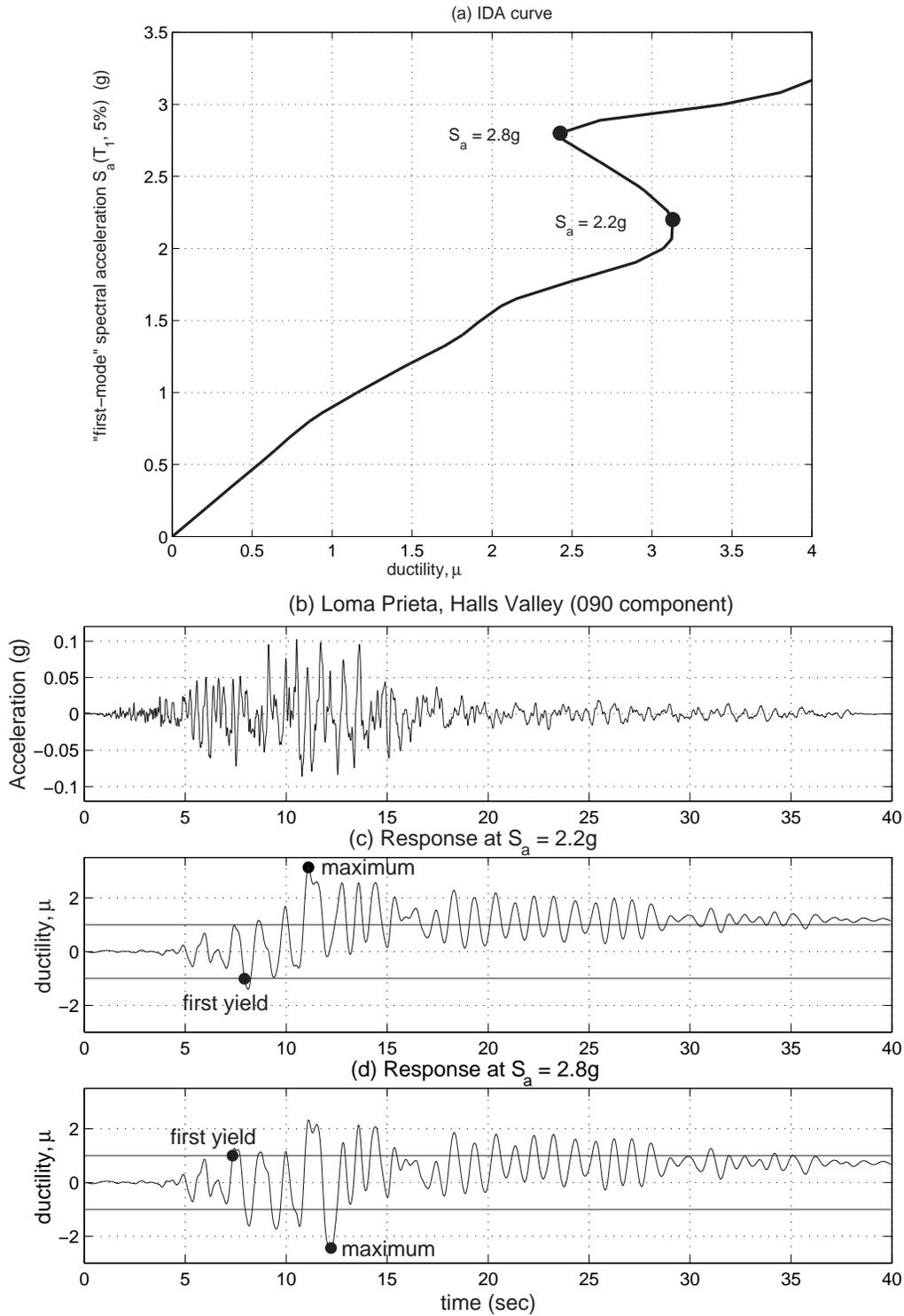
Figure 4. Ductility response of a $T = 1$ sec, elasto-plastic oscillator at multiple levels of shaking. Earlier yielding in the stronger ground motion leads to a lower absolute peak response.
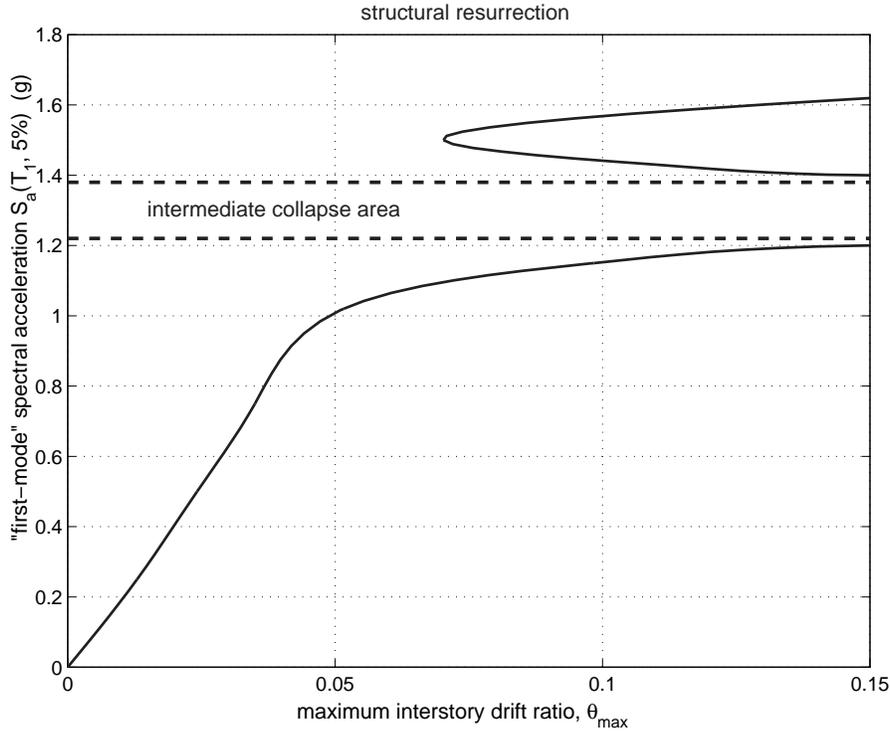
Figure 5. Structural resurrection on the IDA curve of a $T_1 = 1.3$ sec, 3-storey steel moment-resisting frame with fracturing connections.

that when satisfied, signals reaching a limit-state. For example, Immediate Occupancy [13, 14] is a structural performance level that has been associated with reaching a given *DM* value, usually in θ_{max} terms, while (in FEMA 350 [13], at least) Global Collapse is related to the *IM* or *DM* value where dynamic instability is observed. A relevant issue that then appears is what to do when multiple points (Figure 6(a,b)) satisfy such a rule? Which one is to be selected?

The cause of multiple points that can satisfy a limit-state rule is mainly the hardening issue and, in its extreme form, structural resurrection. In general, one would want to be conservative and consider the lowest, in *IM* terms, point that will signal the limit-state. Generalizing this concept to the whole IDA curve means that we will discard its portion "above" the first (in *IM* terms) flatline and just consider only points up to this first sign of dynamic instability.

Note also that for most of the discussion we will be equating dynamic instability to numerical instability in the prediction of collapse. Clearly the non-convergence of the time-integration scheme is perhaps the safest and maybe the only numerical equivalent of the actual phenomenon of dynamic collapse. But, as in all models, this one can suffer from the quality of the numerical code, the stepping of the integration and even the round-off error. Therefore, we will assume that such matters are taken care of as well as possible to allow for accurate enough predictions. That being said, let us set forth the most basic rules used to define a limit-state.

First comes the *DM-based rule*, which is generated from a statement of the format: "If $DM \geq C_{DM}$ then the limit-state is exceeded" (Figure 6(a)). The underlying concept is usually that *DM* is a damage indicator, hence, when it increases beyond a certain value the structural model is assumed to be in the limit-state. Such values of C_{DM} can be obtained through experiments, theory or engineering
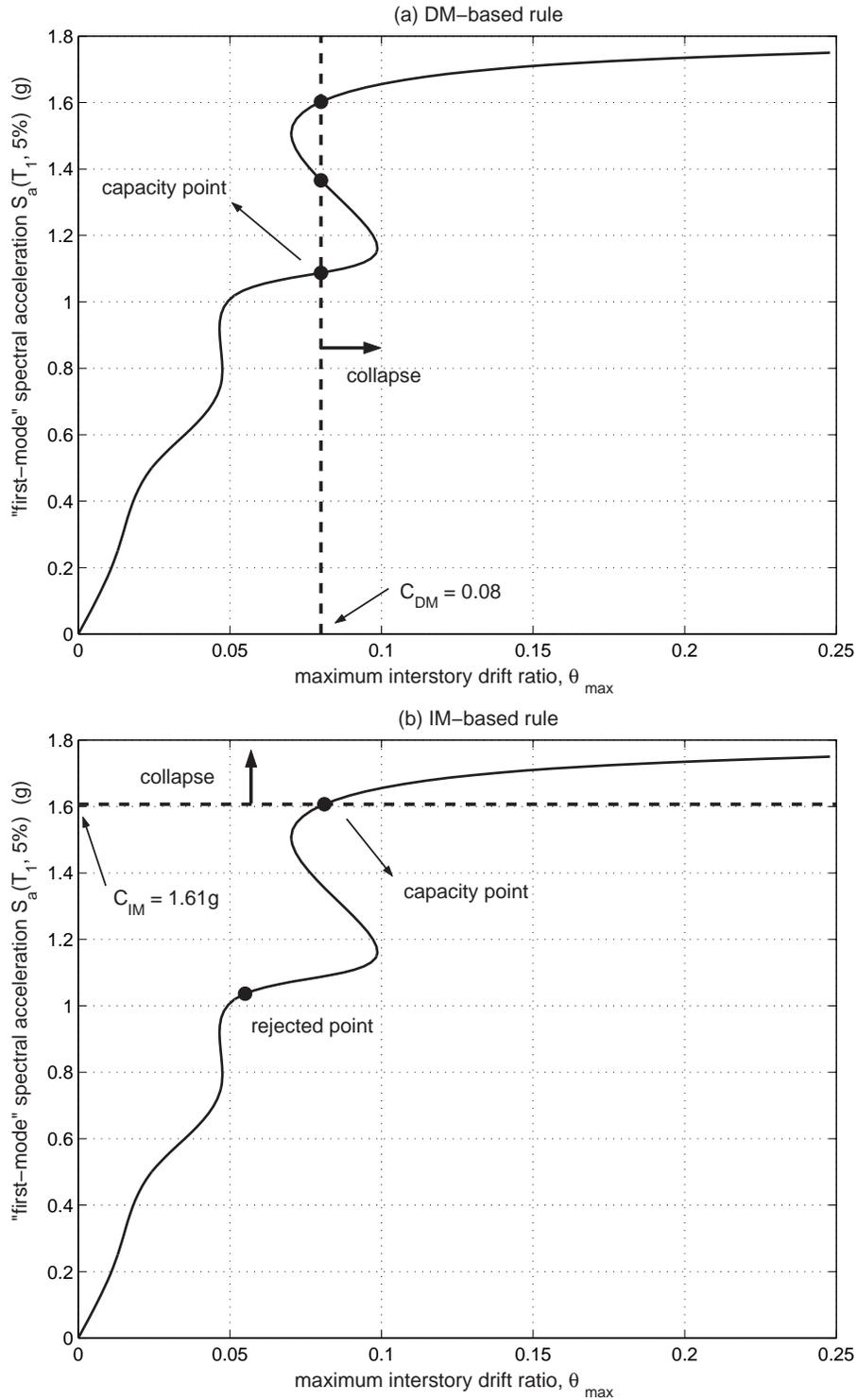
Figure 6. Two different rules producing multiple capacity points for a $T_1 = 1.3$ sec, 3-storey steel moment-resisting frame with fracturing connections. The *DM* rule, where the *DM* is $\theta_{\max}$, is set at $C_{DM} = 0.08$ and the *IM* rule uses the 20% slope criterion.

experience, and they may not be deterministic but have a probability distribution. An example is the $\theta_{\max} = 2\%$ limit that signifies the Immediate Occupancy structural performance level for steel moment-resisting frames (SMRFs) with type-1 connections in the FEMA guidelines [14]. Also the approach used by Mehanny and Deierlein [8] is another case where a structure-specific damage index is used as *DM* and when its reciprocal is greater than unity, collapse is presumed to have occurred. Such limits may have randomness incorporated, for example, FEMA 350 [13] defines a local collapse limit-state by the value of $\theta_{\max}$ that induces a connection rotation sufficient to destroy the gravity load carrying capacity of the connection. This is defined as a random variable based on tests, analysis and judgment for each connection type. Even a unique C_{DM} value may imply multiple limit-state points on an IDA curve (e.g., Figure 6(a)). This ambiguity can be handled by an ad hoc, specified procedure (e.g., by conservatively defining the limit-state point as the lowest *IM*), or by explicitly recognizing the multiple regions conforming and non-conforming with the performance level. The *DM*-based rules have the advantage of simplicity and ease of implementation, especially for performance levels other than collapse. In the case of collapse capacity though, they may actually be a sign of model deficiency. If the model is realistic enough it ought to explicitly contain such information, i.e., show a collapse by non-convergence instead of by a finite *DM* output. Still, one has to recognize that such models can be quite complicated and resource-intensive, while numerics can often be unstable. Hence *DM*-based collapse limit-state rules can be quite useful. They also have the advantage of being consistent with other less severe limit-states which are more naturally identified in *DM* terms, e.g., $\theta_{\max}$.

The alternative *IM-based rule*, is primarily generated from the need to better assess collapse capacity, by having a single point on the IDA curve that clearly divides it to two regions, one of non-collapse (lower *IM*) and one of collapse (higher *IM*). For monotonic *IM*s, such a rule is generated by a statement of the form: "If $IM \geq C_{IM}$ then the limit-state is exceeded" (Figure 6(b)). A major difference with the previous category is the difficulty in prescribing a C_{IM} value that signals collapse for all IDA curves, so it has to be done individually, curve by curve. Still, the advantage is that it clearly generates a single collapse region, and the disadvantage is the difficulty of defining such a point for each curve in a consistent fashion. In general, such a rule results in both *IM* and *DM* descriptions of capacity. A special (extreme) case is taking the "final" point of the curve as the capacity, i.e., by using the (lowest) flatline to define capacity (in *IM* terms), where all of the IDA curve up to the first appearance of dynamic instability is considered as non-collapse.

The FEMA [13] 20% tangent slope approach is, in effect, an *IM*-based rule; the *last* point on the curve with a tangent slope equal to 20% of the elastic slope is defined to be the capacity point. The idea is that the flattening of the curve is an indicator of dynamic instability (i.e., the *DM* increasing at ever higher rates and accelerating towards "infinity"). Since "infinity" is not a possible numerical result, we content ourselves with pulling back to a rate of $\theta_{\max}$ increase equal to five times the initial or elastic rate, as the place where we mark the capacity point. Care needs to be exercised, as the possible "weaving" behavior of an IDA curve can provide several such points where the structure seems to head towards collapse, only to recover at a somewhat higher *IM* level, as in Figure 6(b); in principle, these lower points should thus be discarded as capacity candidates. Also the non-smoothness of the actual curve may prove to be a problem. As mentioned above, the IDA curve is at best piecewise smooth, but even so, approximate tangent slopes can be assigned to every point along it by employing a smooth interpolation. For sceptics this may also be thought of as a discrete derivative on a grid of points that is a good "engineering" approximation to the "rate-of-change".

The above mentioned simple rules are the building blocks to construct composite rules, i.e., composite logical clauses of the above types, most often joined by logical OR operators. For example, when a structure has several collapse modes, not detectable by a single *DM*, it is advantageous to

detect global collapse with an OR clause for each individual mode. An example is offshore platforms where pile or soil failure modes are evident in deck drift while failures of braces are more evident in maximum peak inter-tier drift. The first — in *IM* terms — event that occurs is the one that governs collapse capacity. Another case is Global Collapse capacity, which as defined by FEMA in [13, 14] is in fact an OR conjunction of the 20% slope *IM*-based rule and a $C_{DM} = 10\%$ *DM*-based rule, where $S_a(T_1, 5\%)$ and $\theta_{\max}$ are the *IM* and *DM* of choice. If either of the two rules obtains, it defines capacity. This means that the 20% stiffness detects impending collapse, while the 10% cap guards against excessive values of $\theta_{\max}$, indicative of regions where the model may not be trustworthy. As a *DM* description of capacity is proposed, this definition may suffer from inaccuracies, since close to the flatline a wide range of *DM* values may correspond to only a small range of *IM*s, thus making the actual value of *DM* selected sensitive to the quality of IDA curve tracing and to the (ad hoc) 20% value. If, on the other hand, an *IM* description is used, the rule becomes more robust. This is a general observation for collapse capacity; it appears that it can be best expressed in *IM* terms.

5 MULTI-RECORD IDAS AND THEIR SUMMARY

As should be evident by now, a single-record IDA study cannot fully capture the behavior a building may display in a future event. The IDA can be highly dependent on the record chosen, so a sufficient number of records will be needed to cover the full range of responses. Hence, we have to resort to subjecting the structural model to a suite of ground motion records.

Definition 6. *A* MULTI-RECORD IDA STUDY *is a collection of single-record IDA studies of the same structural model, under different accelerograms.*

Such a study, correspondingly produces sets of IDA curves, which by sharing a common selection of *IM*s and the same *DM*, can be plotted on the same graph, as in Figure 7(a) for a 5-storey steel braced frame.

Definition 7. *An* IDA CURVE SET *is a collection of IDA curves of the same structural model under different accelerograms, that are all parameterized on the same IMs and DM.*

While each curve, given the structural model and the ground motion record, is a completely defined deterministic entity, if we wish to take into account the inherent randomness with respect to what record the building might experience, we have to bring a probabilistic characterization into play. The IDA given the structural model and a statistical population of records is no longer deterministic; it is a *random line*, or a random function $DM = f(IM)$ (for a single, monotonic *IM*). Then, just as we are able to summarize a suite of records by having, for example, mean, median, and 16%, 84% response spectra, so we can define mean, median and 16%, 84% IDA curves (e.g., Figure 7(b)) to (marginally) summarize an IDA curve set. We, therefore, need methods for estimating statistics of a sample of 2D random lines (assuming a single *IM*), a topic of Functional Data Analysis [21]. They conveniently fall in two main categories.

First are the parametric methods. In this case a parametric model of the *DM* given the *IM* is assumed, each line is separately fit, providing a sample of parameter values, and then statistics of the parameters are obtained. Alternatively a parametric model of the median *DM* given the *IM* can be fit to all the lines simultaneously. As an example, consider the 2-parameter, power-law model $\theta_{\max} = \alpha \cdot [S_a(T_1, 5\%)]^\beta$ introduced by Shome and Cornell [16], which under the well-documented assumption of lognormality of the conditional distribution of $\theta_{\max}$ given $S_a(T_1, 5\%)$, often provides a
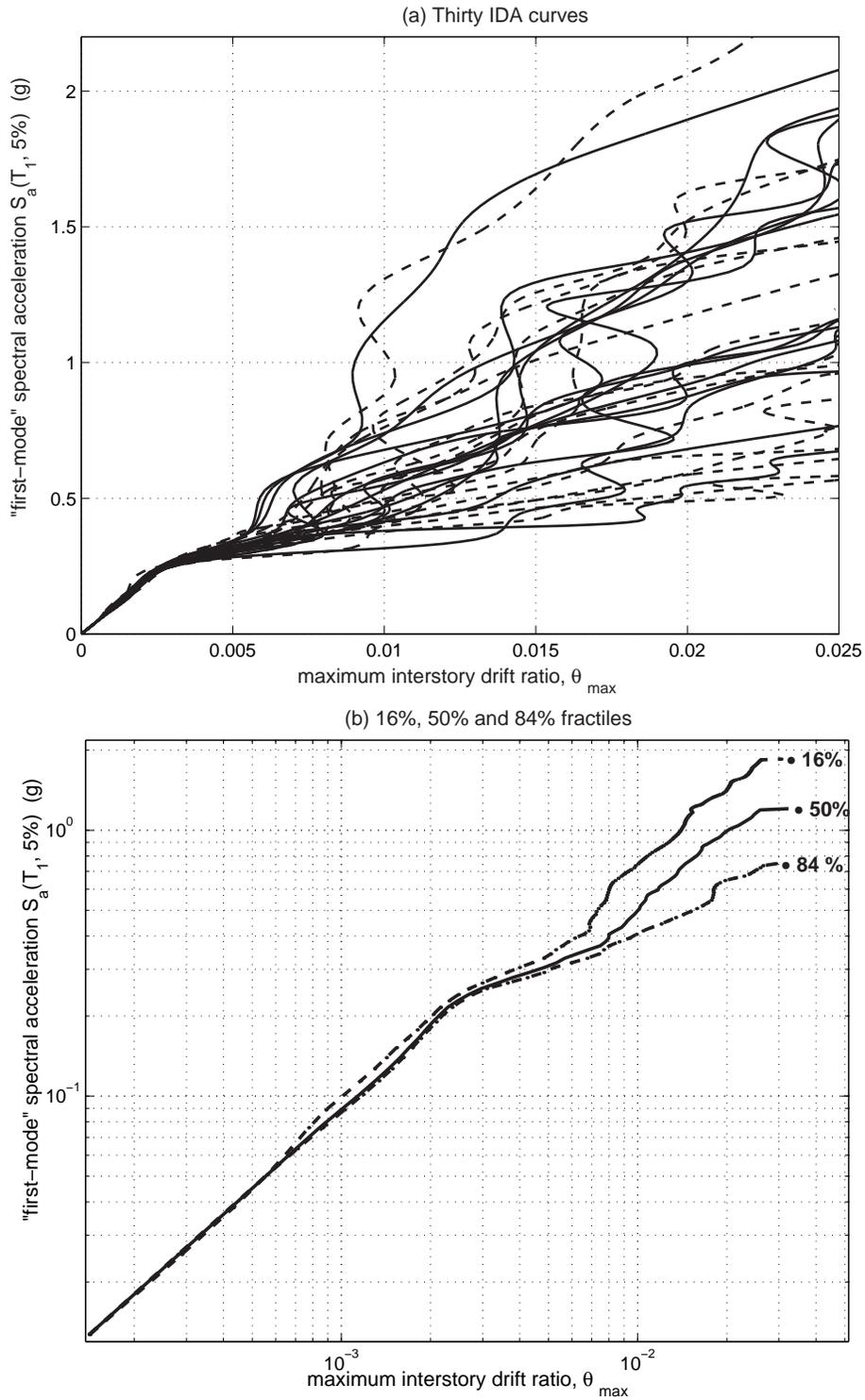
Figure 7. An IDA study for thirty records on a $T_1 = 1.8$ sec, 5-storey steel braced frame, showing (a) the thirty individual curves and (b) their summary (16%, 50% and 84%) fractile curves (in log-log scale).

13

simple yet powerful description of the curves, allowing some important analytic results to be obtained [22, 23]. This is a general property of parametric methods; while they lack the flexibility to accurately capture each curve, they make up by allowing simple descriptions to be extracted.

On the other end of the spectrum are the non-parametric methods, which mainly involve the use of "scatterplot smoothers" like the running mean, running median, LOESS or the smoothing spline [24]. Perhaps the simplest of them all, the running mean with a zero-length window (or cross-sectional mean), involves simply calculating values of the *DM* at each level of *IM* and then finding the average and standard deviation of *DM* given the *IM* level. This works well up to the point where the first IDA curve reaches capacity, when *DM* becomes infinite, and so does the mean IDA curve. Unfortunately most smoothers suffer from the same problem, but the cross-sectional median, or cross-sectional fractile is, in general, more robust. Instead of calculating means at each *IM* level, we now calculate sample medians, 16% and 84% fractiles, which become infinite only when collapse occurs in 50%, 84% and 16% of the records respectively. Another advantage is that under suitable assumptions (e.g., continuity and monotonicity of the curves), the line connecting the *x*% fractiles of *DM* given *IM* is the same as the one connecting the $(100-x)$% fractiles of *IM* given *DM*. Furthermore, this scheme fits nicely with the well-supported assumption of lognormal distribution of $\theta_{\max}$ given $S_a(T_1, 5\%)$, where the median is the natural "central value" and the 16%, 84% fractiles correspond to the median times $e^{\mp \mathrm{dispersion}}$, where "dispersion" is the standard deviation of the logarithms of the values [22].

Finally, a variant for treating collapses is proposed by Shome and Cornell [25], where the conventional moments are used to characterize non-collapses, thus removing the problem of infinities, while the probability of collapse given the *IM* is summarized separately by a logistic regression.

A simpler, yet important problem is the summarizing of the capacities of a sample of *N* curves, expressed either in *DM* (e.g., $\{C^i_{\theta_{\max}}\}$, $i = 1 \ldots N$) or *IM* (e.g., $\{C^i_{S_a(T_1, 5\%)}\}$, $i = 1 \ldots N$) terms. Since there are neither random lines nor infinities involved, the problem reduces to conventional sample statistics, so we can get means, standard deviations or fractiles as usual. Still, the observed lognormality in the capacity data, often suggests the use of the median (e.g., $\widehat{C}_{S_a(T_1, 5\%)}$ or $\widehat{C}_{\theta_{\max}}$), estimated either as the 50% fractile or as the antilog of the mean of the logarithms, and the standard deviation of the logarithms as dispersion. Finally, when considering limit-state probability computations (see section below), one needs to address potential dependence (e.g., correlation) between capacity and demand. Limited investigation to date has revealed little if any systematic correlation between *DM* capacity and *DM* demand (given *IM*).

6   THE IDA IN A PBEE FRAMEWORK

The power of the IDA as an analysis method is put to use well in a probabilistic framework, where we are concerned with the estimation of the annual likelihood of the event that the demand exceeds the limit-state or capacity *C*. This is the likelihood of exceeding a certain limit-state, or of failing a performance level (e.g., Immediate Occupancy or Collapse Prevention in [13]), within a given period of time. The calculation can be summarized in the framing equation adopted by the Pacific Earthquake Engineering Center [26]

$$\lambda(\boldsymbol{DV}) = \iint G(\boldsymbol{DV}|\boldsymbol{DM}) \; |\mathrm{d}G(\boldsymbol{DM}|\boldsymbol{IM})| \; |\mathrm{d}\lambda(\boldsymbol{IM})| \qquad (1)$$

in which *IM*, *DM* and *DV* are vectors of intensity measures, damage measures and "decision variables" respectively. In this paper we have generally used scalar *IM* (e.g., $S_a(T_1, 5\%)$) and *DM*

(e.g., $\theta_{\max}$) for the limit-state case of interest. The decision variable here is simply a scalar "indicator variable": $DV = 1$ if the limit-state is exceeded (and zero otherwise). $\lambda(IM)$ is the conventional hazard curve, i.e., the mean annual frequency of IM exceeding, say, x. The quantity $|\mathrm{d}\lambda(x)| = |\mathrm{d}\lambda(x)/\mathrm{d}x|\,\mathrm{d}x$ is its differential (i.e., $|\mathrm{d}\lambda(x)/\mathrm{d}x|$ is the mean rate density). $|\mathrm{d}G(DM|IM)|$ is the differential of the (conditional) complementary cumulative distribution function (CCDF) of DM given IM, or $f_{DM|IM}(y|x)\,\mathrm{d}y$. In the previous sections we discussed the statistical characterization of the random IDA lines. These distributions are precisely this characterization of $|\mathrm{d}G(DM|IM)|$. Finally in the limit-state case, when on the left-hand side of Equation (1) we seek $\lambda(DV{=}1) = \lambda(0)$, $G(0|DM)$ becomes simply the probability that the capacity C is less than some level of the DM, say, y; so $G(0|DM) = F_C(y)$, where $F_C(y)$ is the cumulative distribution function of C, i.e., the statistical characterization of capacity, also discussed at the end of the previous section. In the global collapse case, capacity estimates also come from IDA analyses. In short, save for the seismicity characterization, $\lambda(IM)$, given an intelligent selection of IM, DM and structural model, the IDA produces, in arguably the most comprehensive way, precisely the information needed both for PBEE demand characterization and for global collapse capacity characterization.

7 SCALING LEGITIMACY AND IM SELECTION

As discussed above, we believe there is useful engineering insight to be gained by conducting individual and sets of IDA studies. However, concern is often expressed about the "validity" of DM results obtained from records that have been scaled (up or down), an operation that is not uncommon both in research and in practice. While not always well expressed, the concern usually has something to do with "weaker" records not being "representative" of "stronger" ones. The issue can be more precisely stated in the context of the last two sections as: will the median (or any other statistic of) DM obtained from records that have been scaled to some level of IM estimate accurately the median DM of a population of unscaled records all with that same level of IM. Because of current record catalog limitations, where few records of any single given IM level can be found, and because we have interest usually in a range of IM levels (e.g., in integrations such as Equation (1)), it is both more practical and more complete to ask: will the (regression-like) function median DM versus IM obtained from scaled records (whether via IDAs or otherwise) estimate well that same function obtained from unscaled records? There is a growing body of literature related to such questions that is too long to summarize here (e.g., Shome and Cornell [16, 27]). An example of such a comparison is given in Figure 8 from Bazzuro *et al.* [28], where the two regressions are so close to one another that only one was plotted by the authors. Suffice it to say that, in general, the answer to the question depends on the structure, the DM, the IM and the population in mind. For example, the answer is "yes" for the case pictured in Figure 8, i.e., for a moderate period (1 sec) steel frame, for which DM is maximum interstorey drift and IM is first-mode-period spectral acceleration, and for a fairly general class of records (moderate to large magnitudes, M, all but directivity-influenced distances, R, etc.). On the other hand, for all else equal except IM defined now as PGA, the answer would be "no" for this same case. Why? Because such a (first-mode dominated) structure is sensitive to the strength of the frequency content near its first-mode frequency, which is well characterized by the $S_a(1\,\text{sec},5\%)$ but not by PGA, and as magnitude changes, spectral shape changes implying that the average ratio of $S_a(1\,\text{sec},5\%)$ to PGA changes with magnitude. Therefore the scaled-record median drift versus PGA curve will depend on the fractions of magnitudes of different sizes present in the sample, and may or may not represent well such a curve for any (other) specified population of magnitudes. On the other hand, the IM first-mode spectral acceleration will no longer work well for a tall, long-period building

15
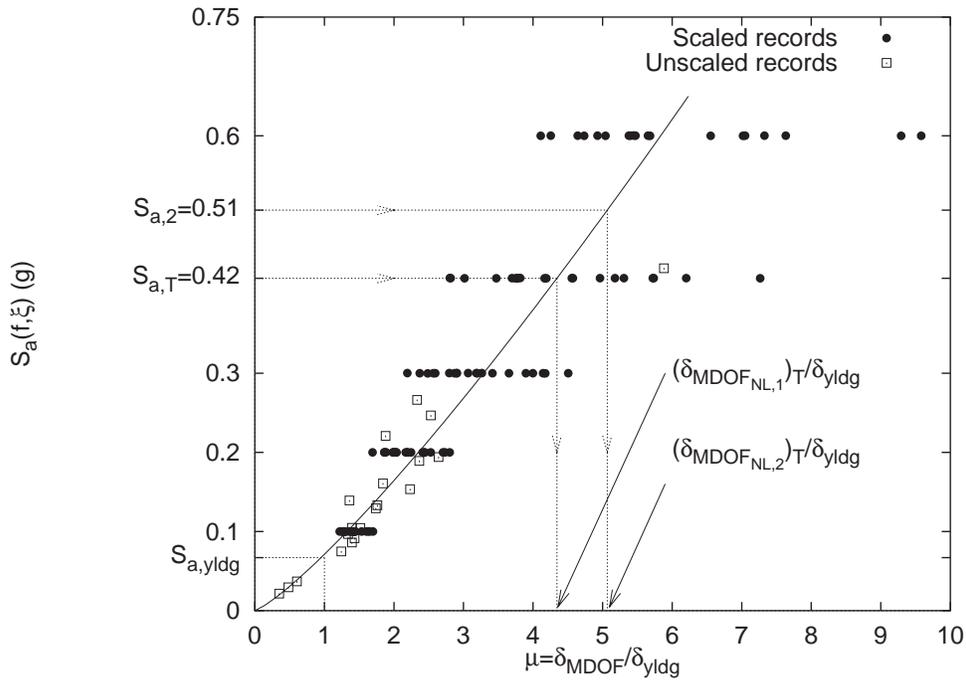
Figure 8. Roof ductility response of a $T_1 = 1$ sec, MDOF steel frame subjected to 20 records, scaled to 5 levels of $S_a(T_1, 5\%)$. The unscaled record response and the power-law fit are also shown for comparison (from Bazzurro *et al.* [28]).
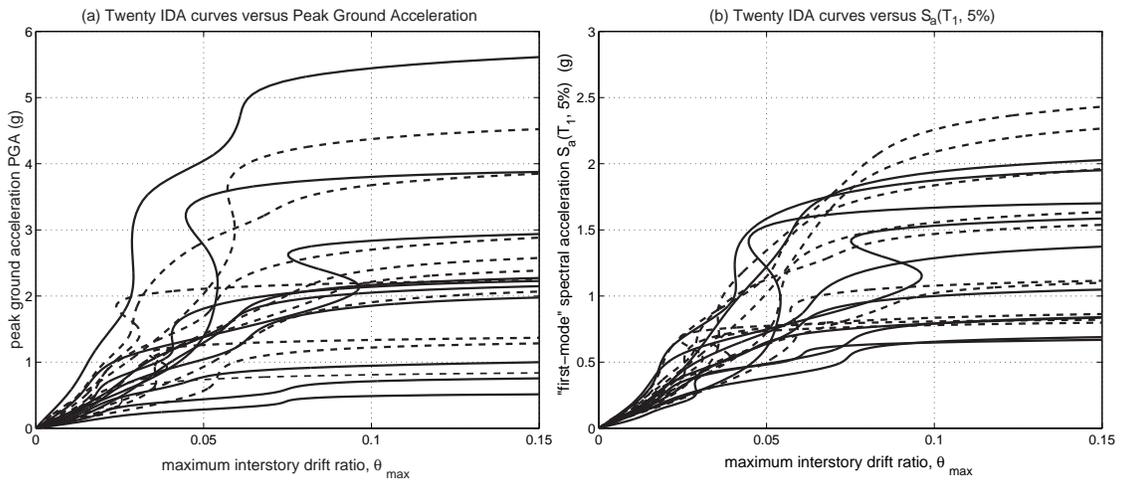
Figure 9. IDA curves for a $T_1 = 2.2$ sec, 9-storey steel moment-resisting frame with fracturing connections plotted against (a) *PGA* and (b) $S_a(T_1, 5\%)$.

that is sensitive to shorter periods, again because of spectral shape dependence on magnitude.

There are a variety of questions of efficiency, accuracy and practicality associated with the wise choice of the *IM* for any particular application (e.g., Luco and Cornell [17]), but it can generally be said here that if the *IM* has been chosen such that the regression of *DM* jointly on *IM*, *M* and *R* is found to be effectively independent of *M* and *R* (in the range of interest), then, yes, scaling of records will provide good estimates of the distribution of *DM* given *IM*. Hence we can conclude that scaling is indeed (in this sense) "legitimate", and finally that IDAs provide accurate estimates of *DM* given *IM* statistics (as required, for example, for PBEE use; see [16, 28]).

IDA studies may also bring a fresh perspective to the larger question of the effective *IM* choice. For example, smaller dispersion of *DM* given *IM* implies that a smaller sample of records and fewer nonlinear runs are necessary to estimate median *DM* versus *IM*. Therefore, a desirable property of a candidate *IM* is small dispersion. Figure 9 shows IDAs from a 9-storey steel moment-resisting frame in which the *DM* is θ_{max} and the *IM* is either (a) *PGA* or (b) $S_a(T_1, 5\%)$. The latter produces a lower dispersion over the full range of *DM* values, as the IDA-based results clearly display. Furthermore, the IDA can be used to study how well (with what dispersion) particular *IM*s predict collapse capacity; again $S_a(T_1, 5\%)$ appears preferable to *PGA* for this structure as the dispersion of *IM* values associated with the "flatlines" is less in the former case.

8   THE IDA VERSUS THE *R*-FACTOR

A popular form of incremental seismic analysis, especially for SDOF oscillators, has been that leading to the yield reduction *R*-factor (e.g., Chopra [15]). In this case the record is left unscaled, avoiding record scaling concerns; instead, the yield force (or yield deformation, or, in the multi-member MDOF case, the yield stress) of the model is scaled down from that level that coincides with the onset of inelastic behavior. If both are similarly normalized (e.g., ductility = deformation/yield-deformation and $R = S_a(T_1, 5\%)/S_a^{yield}(T_1, 5\%)$), the results of this scaling and those of an IDA will be identical for those classes of systems for which such simple structural scaling is appropriate, e.g., most SDOF models, and certain MDOF models without axial-force–moment interaction, without second-or-higher-order geometric nonlinearities, etc. One might argue that these cases of common results are another justification for the legitimacy of scaling records in the IDA. It can be said that the difference between the *R*-factor and IDA perspectives is one of design versus assessment. For design one has an allowable ductility in mind and seeks the design yield force that will achieve this; for assessment one has a fixed design (or existing structure) in hand and seeks to understand its behavior under a range of potential future ground motion intensities.

9   THE IDA VERSUS THE NONLINEAR STATIC PUSHOVER

The common incremental loading nature of the IDA study and the SPO suggests an investigation of the connection between their results. As they are both intended to describe the same structure, we should expect some correlation between the SPO curve and any IDA curve of the building (Figure 1), and even more so between the SPO and the summarized (median) IDA curve, as the latter is less variable and less record dependent. Still, to plot both on the same graph, we should preferably express the SPO in the *IM*, *DM* coordinates chosen for the summarized IDA. While some *DM*s (e.g., θ_{max}) can easily be obtained from both the static and the dynamic analysis, it may not be so natural to convert the *IM*s, e.g., base shear to $S_a(T_1, 5\%)$. The proposed approach is to adjust the "elastic stiffness" of the SPO to
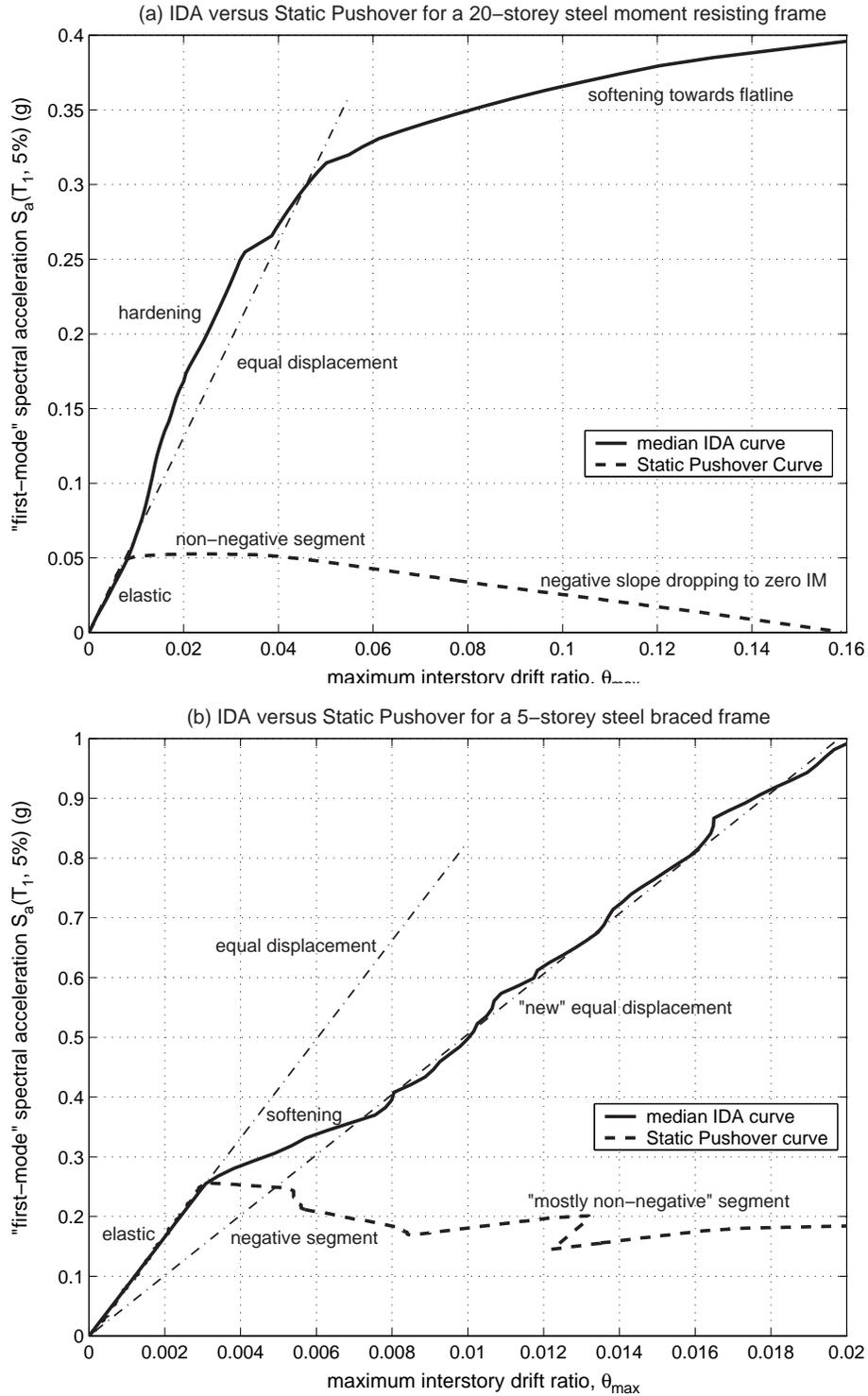
Figure 10. The median IDA versus the Static Pushover curve for (a) a $T_1 = 4$ sec, 20-storey steel moment-resisting frame with ductile connections and (b) a $T_1 = 1.8$ sec, 5-storey steel braced frame.

be the same as that of the IDA, i.e., by matching their elastic segments. This can be achieved in the aforementioned example by dividing the base shear by the building mass, which is all that is needed for SDOF systems, times an appropriate factor for MDOF systems.

The results of such a procedure are shown in Figures 10(a,b) where we plot the SPO curve, obtained using a first-mode force pattern, versus the median IDA for a 20-storey steel moment-resisting frame with ductile connections and for a 5-storey steel braced frame using $S_a(T_1, 5\%)$ and θ_{max} coordinates. Clearly both the IDA and the SPO curves display similar ranges of *DM* values. The IDA always rises much higher than the SPO in *IM* terms, however. While a quantitative relation between the two curves may be difficult, deserving further study (e.g., [29]), qualitatively we can make some, apparently, quite general observations that permit inference of the approximate *shape* of the median IDA simply by looking at the SPO.

1. By construction, the elastic region of the SPO matches well the IDA, including the first sign of nonlinearity appearing at the same values of *IM* and *DM* for both.

2. A subsequent reduced, but still non-negative stiffness region of the SPO correlates on the IDA with the approximate "equal-displacement" rule (for moderate-period structures) [20], i.e., a near continuation of the elastic regime slope; in fact this near-elastic part of the IDA is often preceded by a *hardening* portion (Figure 10(a)). Shorter-period structures will instead display some softening.

3. A negative slope on the SPO translates to a (softening) region of the IDA, which can lead to collapse, i.e., IDA flat-lining (Figure 10(a)), unless it is arrested by a non-negative segment of the SPO before it reaches zero in *IM* terms (Figure 10(b)).

4. A non-negative region of the SPO that follows after a negative slope that has caused a significant *IM* drop, apparently presents itself in the IDA as a new, modified "equal-displacement" rule (i.e., an near-linear segment that lies on a secant) that has lower "stiffness" than the elastic (Figure 10(b)).

10   IDA ALGORITHMS

Despite the theoretical simplicity of an IDA study, actually performing one can potentially be resource intensive. Although we would like to have an almost continuous representation of IDA curves, for most structural models the sheer cost of each dynamic nonlinear run forces us to think of algorithms designed to select an optimal grid of discrete *IM* values that will provide the desired coverage. The density of a grid on the curve is best quantified in terms of the *IM* values used, the objectives being: a high *demand resolution*, achieved by evenly *spreading* the points and thus having no gap in our *IM* values larger than some tolerance, and a high *capacity resolution*, which calls for a *concentration* of points around the flatline to bracket it appropriately, e.g., by having a distance between the highest (in terms of *IM*) "non-collapsing" run and the lowest "collapsing" run less than some tolerance. Here, by collapsing run we mean a dynamic analysis performed at some *IM* level that is determined to have caused collapse, either by satisfying some collapse-rule (*IM* or *DM* based, or more complex) or simply by failing to converge to a solution. Obviously, if we allow only a fixed number of runs for a given record, these two objectives compete with one another.

In a multi-record IDA study, there are some advantages to be gained by using information from the results of one record to adapt the grid of points to be used on the next. Even without exploiting these

opportunities, we can still design efficient methods to tackle each record separately that are simpler and more amenable to parallelization on multiple processors. Therefore we will focus on the *tracing* of single IDA curves.

Probably the simplest solution is a *stepping* algorithm, where the *IM* is increased by a constant step from zero to collapse, a version of which is also described in Yun *et al.* [7]. The end result is a uniformly-spaced (in *IM*) grid of points on the curve. The algorithm needs only a pre-defined step value and a rule to determine when to stop, i.e., when a run is collapsing.

> **repeat**
>   increase *IM* by the step
>   scale record, run analysis and extract *DM*(s)
> **until** collapse is reached

Although it is an easily programmable routine it may not be cost-efficient as its quality is largely dependent on the choice of the *IM* step. Even if information from previously processed ground motion records is used, the step size may easily be too large or too small for this record. Even then, the variability in the "height" (in *IM* terms) of the flatline, which is both accelerogram and *IM* dependent, tends to unbalance the distribution of runs; IDA curves that reach the flatline at a low *IM* level receive fewer runs, while those that collapse at higher *IM* levels get more points. The effect can be reduced by selecting a good *IM*, e.g., $S_a(T_1, 5\%)$ instead of *PGA*, as *IM*s with higher *DM* variability tend to produce more widely dispersed flatlines (Figure 9). Another disadvantage is the implicit coupling of the capacity and demand estimation, as the demand and the capacity resolutions are effectively the same and equal to the step size.

Trying to improve upon the basis of the stepping algorithm, one can use the ideas on searching techniques available in the literature (e.g., [30]). A simple enhancement that increases the speed of convergence to the flatline is to allow the steps to increase, for example by a factor, resulting in a geometric series of *IM*s, or by a constant, which produces a quadratic series. This is the *hunting phase* of the code where the flatline is bracketed without expending more than a few runs.

> **repeat**
>   increase *IM* by the step
>   scale record, run analysis and extract *DM*(s)
>   increase the step
> **until** collapse is reached

Furthermore, to improve upon the capacity resolution, a simple enhancement is to add a step-reducing routine, for example bisection, when collapse (e.g., non-convergence) is detected, so as to tighten the bracketing of the flatline. This will enable a prescribed accuracy for the capacity to be reached regardless of the demand resolution.

> **repeat**
>   select an *IM* in the gap between the highest non-collapsing and lowest non-collapsing *IM*s
>   scale record, run analysis and extract *DM*(s)
> **until** highest collapsing and lowest non-collapsing *IM*-gap < tolerance

Even up to this point, this method is a logical replacement for the algorithm proposed in Yun *et al.* [7] and in the FEMA guidelines [13] as this algorithm is focused on optimally locating the capacity,

20

which is the only use made of the IDA in those two references. If we also wish to use the algorithm for demand estimation, coming back to fill in the gaps created by the enlarged steps is desirable to improve upon the demand resolution there.

> **repeat**
>   select an *IM* that halves the largest gap between the *IM* levels run
>   scale record, run analysis and extract *DM*(s)
> **until** largest non-collapsing *IM*-gap < tolerance

When all three pieces are run sequentially, they make for a more efficient procedure, a *hunt & fill* tracing algorithm, described in detail by Vamvatsikos and Cornell [31], that performs increasingly larger leaps, attempting to bound the *IM* parameter space, and then fills in the gaps, both capacity and demand-wise. It needs an initial step and a stopping rule, just like the stepping algorithm, plus a step increasing function, a capacity and a demand resolution. The latter two can be selected so that a prescribed number of runs is performed on each record, thus tracing each curve with the same load of resources.

A subtle issue here is the summarization of the IDA curves produced by the algorithm. Obviously, if the same step is used for all records and if there is no need to use another *IM*, the stepping algorithm immediately provides us with stripes of *DM* at given values of *IM* (e.g., Figure 8). So a "cross-sectional median" scheme could be implemented immediately on the output, without any post-processing. On the other hand, a hunt & fill algorithm would produce *DM* values at non-matching levels of *IM* across the set of records, which necessitates the interpolation of the resulting IDA curve to calculate intermediate values. Naturally, the same idea can be applied to the output of any IDA tracing algorithm, even a stepping one, to increase the density of discrete points without the need for additional dynamic analyses. Ideally, a flexible, highly non-parametric scheme should be used. Coordinate-transformed natural splines, as presented in [31], are a good candidate. Actually all the IDA curves in the figures of the paper are results of such an interpolation of discrete points. However, before implementing any interpolation scheme, one should provide a dense enough grid of *IM* values to obtain a high confidence of detecting any structural resurrections that might occur before the final flatline.

11   CONCLUSIONS

The results of Incremental Dynamic Analyses of structures suggest that the method can become a valuable additional tool of seismic engineering. IDA addresses both demand and capacity of structures. This paper has presented a number of examples of such analyses (from simple oscillators to 20-storey frames), and it has used these examples to call attention to various interesting properties of individual IDAs and sets of IDAs. In addition to the peculiarities of non-monotonic behavior, discontinuities, "flatlining" and even "resurrection" behavior within individual IDAs, one predominate impression left is that of the extraordinary variability from record to record of the forms and amplitudes of the IDA curves for a single building (e.g., Figure 7(a)). The (deterministic) vagaries of a nonlinear structural system under irregular input present a challenge to researchers to understand, categorize and possibly predict. This variability also leads to the need for statistical treatment of multi-record IDA output in order to summarize the results and in order to use them effectively in a predictive mode, as for example in a PBEE context. The paper has proposed some definitions and examples of a variety of issues such as these IDA properties, the scaling variables (*IM*s), limit-state forms, and collapse definitions.

Further we have addressed the question of "legitimacy" of scaling records and the relationships between IDAs and R-factors as well as between IDAs and the Static Pushover Analysis. Finally, while the computational resources necessary to conduct IDAs may appear to limit them currently to the research domain, computation is an ever-cheaper resource, the operations lend themselves naturally to parallel computation, IDAs have already been used to develop information for practical guidelines [13, 14], and algorithms presented here can reduce the number of nonlinear runs per record to a handful, especially when the results of interest are not the curious details of an individual IDA curve, but smooth statistical summaries of demands and capacities.

ACKNOWLEDGEMENTS

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