Performance-Based Seismic Design: Avant-garde and code-compatible approaches

Dimitrios Vamvatsikos[1], Athanasia K. Kazantzi[2] and Mark A. Aschheim[3]

Abstract: Current force-based codes for the seismic design of structures use design spectra and system-specific behavior factors to satisfy one or two pre-defined structural limit-states. In contrast, performance-based seismic design aims to design a structure to fulfill any number of target performance objectives, defined as user-prescribed levels of structural response, loss, or casualties to be exceeded at a mean annual frequency less than a given maximum. First, a review of recent advances in probabilistic performance assessment is offered. Second, we discuss the salient characteristics of methodologies that have been proposed to solve the inverse problem of design. Finally, an alternative approach is proposed that relies on a new format for visualizing seismic performance, termed Yield Frequency Spectra (YFS). YFS offer a unique view of the entire solution space for structural performance of a surrogate single-degree-of-freedom oscillator, incorporating uncertainty and propagating it to the output response to enable rapid determination of a good preliminary design that satisfies any number of performance objectives.

CE Database subject headings: Seismic response; Earthquakes; Performance evaluation; Safety; Structural reliability

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[1] Lecturer, School of Civil Engineering, National Technical Univ. of Athens, Greece (corresponding author). E-mail: divamva@mail.ntua.gr

[2] Research Engineer, J.B. ATE, Herakleion Crete, Greece. E-mail: kazantzi@mail.ntua.gr

[3] Professor, Dept. of Civil Engineering, Santa Clara Univ., 500 El Camino Real
Santa Clara, CA 95053. E-mail: maschheim@scu.edu

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Introduction

Earthquake engineering is a fascinating scientific field where multiple disciplines come together to mitigate the threat of one of the deadliest natural hazards on Earth. Of immediate concern is the entirety of the built environment that houses the majority of human societal and economic activities. Earthquakes thus threaten immeasurable wealth, representing past investments in existing infrastructure and ever-increasing future ones as new structures are added or renovated yearly. Damage to the built infrastructure ricochets through the transportation, communication, financial, cultural, and economic dimensions of society. Recent seismic events, e.g., Northridge (1994), Kobe (1995), China (2008), Tohoku (2011), have shown that despite considerable advances in research, we still have a long way to go: while modern buildings have lower fatality rates, the staggering monetary losses and disruption of function from seismic events can cripple entire cities, or even countries.

At the core of earthquake engineering stand the dual problems of assessment and design. Assessment is the direct process of estimating structural behavior given the structure and the hazard. Design is the inverse problem, whereby a structure, its members and properties are established to achieve an acceptable behavior under the given seismic hazard. As typically befits such dualities, the direct path of assessment is by far the simpler of the two. Thus, while the earthquake engineering field has benefited enormously from recent advances in computer science and technology, these have been realized in an asymmetric way. Important solutions have mostly been achieved for the assessment problem—e.g., Fajfar and Dolsek (2010) introduced the concept of performance quantification within a probabilistic context as a viable approach for design. Therein, a structure's properties may be taken into account together with the seismic hazard obtained by probabilistic seismic hazard analysis (PSHA) to

provide relatively accurate estimates of the distributions of structural demand, repair cost, time-to-repair or even human casualties. While most of these capabilities are still only available at the academic level, some of the benefits are influencing the new generation of seismic codes dealing with the assessment of existing structures, such as EN1998-3 (CEN 2005) and ASCE/SEI 41-06 (2007), and in the establishment of design parameters for seismic force-resisting systems, e.g., FEMA P-695 (2009) and FEMA P-795 (2011).

In contrast, no such revolution has been realized for routine design. Current seismic design codes follow a classical force-based design paradigm that has been in place for decades, in which design lateral forces are based on elastic demands reduced by a system-specific "behavior" *R*- or *q*-factor to account for inelasticity. Generally, the smoothed (pseudo-acceleration) design spectra and *R*- or *q*-factors are intended to approximately achieve Life Safety performance for ground motions having a probability of exceedance of about 10% in 50 years. The *R*- or *q*-factors are prescribed by the code, are specific to each lateral load-resisting system type, and purportedly account for the ductility capacity and overstrength inherent in buildings of various configurations, in view of local construction practice. Demands at critical sections are determined by elastic analysis (static or dynamic) in a simplified process that is intuitive to engineers. The design is deemed to provide acceptable Life Safety performance simply by ensuring critical member sections have sufficient strength and conform with prescribed detailing requirements. Serviceability is also addressed, by limiting interstory drifts, either under the design lateral loads or for a reduced design spectrum that relates to a more frequent level of seismic loads.

While the simplicity and utility of the classical approach is appealing, many drawbacks exist. The emphasis on forces obscures the importance of stiffness, deformation demands, mechanism development, and deformation compatibility. Moehle (1992) noted the value of explicitly considering displacements in design, while Priestley (2000) promoted the use of an

equivalent linear approach to displacement-based design. Performance-based design approaches often aim to limit drift and ductility demands and rely on displacement-based approaches for doing so. Displacement-based approaches may be based on an estimate of the fundamental period or the yield displacement of the structure in a first-mode pushover analysis. The benefits of both force- and displacement-based approaches can be realized in hybrid approaches that combine the two, as described recently by Tzimas et al. (2013).

Besides the emphasis on forces, another drawback of the classical approach is that nonlinear effects are addressed only through the necessarily imprecise reduction/behavior factor, while uncertainties are incorporated via safety factors at the input level of material and load values instead of the output response. Under the severe demands of strong ground shaking, this formulation leads to substantial variability in performance, which either leaves some fraction of the building inventory vulnerable to deficient performance or wastes resources in needlessly improving the performance of others, attempting to uniformly raise the bar across a non-uniform population. A shift in paradigm is needed, from the present emphasis on imbuing conservatism in the design of members to meet code requirements to explicit accounting for uncertainty to achieve desired performance objectives for the specific system.

Theoretical and practical difficulties have hampered the much sought-after leap to performance-based seismic design, whereby a structure would be *directly designed* to satisfy a range of performance objectives with specific allowable exceedance rates and explicit consideration of uncertainty. The present paper responds to this challenge, providing a direct way to incorporate uncertainty into a simple seismic design process that respects user-determined limits on drift and system ductility. To lay the proper groundwork, the following discusses the state-of-art in performance-based design, focusing on uncertainties, the probabilistic basis of performance assessment, and salient characteristics of both

conventional and novel approaches to seismic design. In the end, the representation of seismic demands using Yield Frequency Spectra (YFS) is offered as a new approach to achieving performance targets with reasonable computational complexity.

Current state of art

Aleatory and epistemic uncertainty

During the last decades, the use of probabilistic tools for assessing the reliability of structural systems under seismic excitations has attracted considerable research worldwide (e.g. Bazzurro et al. 1998, Cornell et al. 2002; Wen et al. 2003; Krawinkler and Miranda 2004). Both assessment and design become more challenging when coupled within a probabilistic framework, needed to account for the various sources of uncertainty sources that are intrinsic to engineering applications. Uncertainties can be due to the inherently random nature (aleatory), e.g., of seismic loads, or can be attributed to our incomplete knowledge (epistemic), for example regarding the modeling or the actual properties of an existing structure. The fundamental difference is that epistemic uncertainty can be reduced, for example by conducting additional measurements, tests or experiments. However, aleatory randomness is naturally occurring in a process, e.g., the sequence and magnitude of earthquakes, and cannot be alleviated (at least not with any current or foreseeable technology). In general, most existing performance-based assessment formats treat these two flavors in a unified way (despite some objections, e.g., Der Kiureghian 2005) prompting us to refer to them collectively as simply “uncertainty”.

So far, several researchers have concluded that, among all uncertainties, the ground motion waveform has the most profound influence on the structural reliability, especially in the case of performance levels associated with severe structural and non-structural damage (Dymiotis et al. 1999; Kwon and Elnashai 2006; Kazantzi et al. 2008; Kazantzi et al. 2011).

A comprehensive site hazard representation that is compatible with current design norms is provided by the seismic hazard surface, a 3D plot of the mean annual frequency (MAF) of exceeding any level of spectral acceleration $S_a(T)$ for the full practical range of periods T (Fig. 1). This is the true representation of seismic demands for elastic single-degree-of-freedom (SDOF) oscillators at any given site. More familiar 2D pictures can be produced by taking cross sections (or contours) of the hazard surface. Cutting horizontally at given values of MAF will provide the corresponding uniform hazard spectra (UHS). For example, at $P_o = -\ln(1 - 0.1)/50 = 0.0021$, or a 10% in 50 yrs probability of exceedance (Fig. 2a), one obtains the spectrum typically associated with design at the ultimate limit state (or Life Safety). Taking a cross section at a given period T produces the corresponding $S_a(T)$ hazard curve (Fig. 2b).

Considerable uncertainty also may be introduced due to material properties and due to the choice of model and analysis type. Details of soil, foundation and structural modeling may heavily influence the outcome of any analysis method, as shown in Fig. 3 for the capacity curve of a simple 5-story structure. With models under cyclic loading still under development (e.g. Lignos and Krawinkler 2011) and with considerable uncertainty about behavior in the high-deformation region (Fig. 4a), the accurate evaluation of structural response within an analytical context remains a difficult task, and one that may lead to inaccurate response estimates and consequently to structural designs with unsatisfactory or non-homogeneous seismic reliability. This is particularly true in cases where simplified modeling assumptions and analysis options have been employed and the structure approaches its collapse capacity, where large deformations and complicated degradation come into play as prominently shown in Fig. 4b (see also Villaverde 2007).

Performance-based assessment

Performance-based earthquake engineering has recently emerged to quantify in probabilistic terms the performance of structures using metrics that are of immediate use to both engineers and stakeholders (Yang et al. 2009). The use of a proper probabilistic framework for propagating both aleatory and epistemic uncertainties to the final results obtained from a variety of nonlinear analysis methods (e.g. static pushover and nonlinear response history analysis) is best exemplified by the Cornell-Krawinkler framing equation adopted by the Pacific Earthquake Engineering Research (PEER) Center (Cornell and Krawinkler 2000):

$$\lambda(DV) = \iiint G(DV \mid DM) \left| \mathrm{d}G(DM \mid EDP) \right| \left| \mathrm{d}G(EDP \mid IM) \right| \left| \mathrm{d}\lambda(IM) \right| \quad (1)$$

for which *DV* is one or more decision variables, such as cost, time-to-repair or human casualties that are meant to enable decision-making by the stakeholders; *DM* represents the damage measures, typically discretized in a number of progressive damage states for structural and non-structural elements and even building contents; *EDP* contains the engineering demand parameters such as interstory drift or peak floor acceleration that the engineers are accustomed to using when determining the structural behavior; and, *IM* is the seismic intensity measure, for example represented by the 5%-damped first-mode spectral acceleration $S_a(T_1)$. The relationship of *EDP* and *IM* is obtained by structural analysis and can be established through incremental dynamic analysis (IDA, Vamvatsikos and Cornell 2002), using multiple ground motion records scaled to different intensity levels (and exceedance frequencies). An example is shown in Fig. 5, where even for an SDOF system, considerable variability is present. The function $\lambda(y)$ provides the MAF of exceedance of values of its operand y, thus making $\lambda(IM)$ the seismic hazard, while $G(x)$ is the complementary cumulative distribution function (CCDF) of its variable x. Finally, note that all differentials in Eq. (1) appear as absolute values since they concern monotonically decreasing functions, and thus they would otherwise be negative.

Stakeholders care about *DV* and perhaps even *DM*, or damage states. Engineers, on the other hand, prefer to focus on *EDP* to express performance. This may be best achieved by moving to the familiar territory of limit states and appropriately modifying the Cornell-Krawinkler framing equation (Vamvatsikos and Cornell 2004). Defining *DV* and *DM* to be simple indicator variables that become 1.0 when a given limit-state (LS) is exceeded, transforms Eq. (1) to estimate λ_{LS}, the MAF of violating LS:

$$\lambda_{\mathrm{LS}} = \int G(EDP \mid IM) \left|\mathrm{d}\lambda(IM)\right| \tag{2}$$

Both approaches represented by Eq. (1) and Eq. (2) find excellent uses but they are presently limited to assessment, i.e., the forward derivation of a given structure's performance. A proper performance-based design would mean at least inverting such equations to allow deriving the desired properties of the structure that would satisfy a given value of λ_{LS}, for example the 0.002 per annum to successfully fulfill the Life Safety requirements. Closed-form approximations of the integral in Eq. (2) can offer significant help in achieving such a design-oriented solution. The best known such expression is offered by Cornell et al. (2002), using a power-law approximation for both the hazard curve and the median *EDP* versus *IM* relationship. To improve the inherent accuracy issues (see Bradley and Dhakal 2008), Lazar and Dolsek (2014) have suggested the introduction of appropriate integration bounds, while Vamvatsikos (2014) has recommended the use of a biased fit to the hazard curve that favors lower intensities, and/or the introduction of a second-order power-law approximation for the hazard curve.

Design approach characteristics

Each approach to design has its own characteristics in terms of how the design problem is defined and how one goes about solving it. The design problem is essentially an optimization problem, where specific performance objectives are set and an approach is offered on how to

achieve them. How these objectives are defined and what approach is taken to meet them is what essentially establishes the basis of design. In the following we distinguish different methodologies that have appeared in the literature based on three essential characteristics: how one chooses to (a) express the performance targets, (b) treat uncertainty, and (c) connect performance targets to the design of members and thus solve the optimization problem.

Performance targets

Each performance objective can be thought of as a specific value of a "performance variable" paired with a (maximum allowable) target MAF of exceeding it. Eqs (1) and (2) represent two different approaches for defining the performance variable and estimating the corresponding MAF. For example, let us consider a Life Safety performance objective tied to the well-known 10% probability of exceedance in 50 years, or MAF of 0.0021. The difference in employing Eq. (1) versus Eq. (2) appears in the metrics used to define Life Safety itself. In Eq. (1) this is expressed in terms of one or more DVs, for example by requesting casualties less than *x*% of the inhabitants and property damage less than *y*% of the total investment at the given MAF. In Eq. (2), this could be a more familiar limitation in terms of one or more EDPs, e.g., having a maximum interstory drift ratio lower than 2% at the given MAF without any brittle failure of structural components.

While being able to design a structure that can satisfy performance objectives that actually make sense to stakeholders may be considered to be the ultimate goal of performance-based design, estimating such losses is not a trivial procedure (see Baker and Cornell 2008). Absent crude approximations for seismic losses, only full-scale optimization approaches can accurately solve this problem. A prime example of the challenges involved in properly accounting for seismic losses in design is offered by the conceptual approach of Krawinkler et al. (2006).

Propagation of uncertainty

The uncertainty in seismic hazard is of such magnitude that a design method that does not account for it is inconceivable. The difference among design methods is in the way that this uncertainty is taken into account. Herein, performance objectives were explicitly defined as specific DV or EDP values tied to a target MAF, the latter conveying the probabilistic nature of the (output) measure of performance of the design. In contrast, the typical approach is to consider hazard uncertainty at the input intensity level, i.e., by designating the MAF of exceeding specific IM levels of the (input) ground motion. This is incorporated in all current design codes, in the form of the (smoothed) uniform hazard design spectra (see Fig. 2a) that contain all the $S_a(T_1)$ values that correspond to a given MAF of exceedance. By subjecting the structure to forces consistent with such a spectrum one derives the output values of DV or EDP; if the limit is not violated, the design complies with code requirements and is deemed adequate (although the MAF of exceedance of the DV or EDP remains unknown).

Code design formats have their roots in the partial safety factor approach (load and resistance factor design) that is generally used in the linear (or near-linear) range of structural response. Therein, the linear equations that tie the loads and their effects essentially ensure that uncertainty applied at the input is properly propagated to the output, at least in terms of preserving the first and second moment (mean and variability), if not the distribution type. On the other hand, the nonlinearity that is inherent in extreme seismic events means that no such assumptions can be made. Further to this point, Cornell et al. (2002) showed that the shape of the hazard curve and the variability in the EDP–IM relationship mean that multiple hazard levels need to be considered to evaluate performance. Actually, since the lognormal distribution of EDP given IM has a heavy right tail (i.e., a high proportion of extreme responses) and the lower IM values appear more frequently (i.e., correspond to a higher MAF of exceedance in Fig. 2b), they contribute significantly more to the system's rate of exceeding

any limit-state. For example, the MAF of exceedance of a prescribed (output) EDP value is guaranteed to be higher than the MAF of the (input) IM that it corresponds to, on average, often by factors higher than 100%. Thus, whenever probability is controlled/applied/checked at the input intensity of a nonlinear structure, it will often result to a (mean) output of unknown magnitude.

The inconsistency of specifying probability at the input rather than the output undermines the probabilistic basis of all such approaches and becomes an important liability. There is truly no "right" value for the safety factors (typically incorporated within material safety factors and the conservativeness in choosing R, q) that can be both *safe* and *economical* when applied to an entire class of buildings. Designs complying with code requirements may be too safe (and therefore, costly) or the opposite, as blanket safety factors do not provide the same performance across the range of building configurations (or archetypes, in the terminology of FEMA P-695 (2009)) allowed within a building code. As well, since the calibration of code design parameters results in a range of performance for code compliant designs, it is not clear what (stricter) performance criteria should be imposed to obtain user-prescribed "better than code" performance. The use of importance factors to amplify the design spectrum or equivalently, reduce demands for the given design spectrum, are an imprecise substitute. Only design formats that accurately propagate the uncertainty from input IM to output DV or EDP and are theoretically consistent can allow the MAF of exceedance of the output to be controlled; this is necessary in our opinion for a design approach to truly be "performance-based".

Design solution process

Having defined the performance objectives, the grand question of how one goes about establishing a structure that can meet them must be addressed. All design approaches are essentially methods to solve this inverse problem, which is challenging because the

functional relationship between the design variables, e.g., member sizes, and the target objectives is not invertible, and may not be precisely known. Thus, iteration is required. Engineers can easily solve the forward problem of applying the "seismic loads" to determine individual member forces or deformations and invert (explicitly or implicitly through iterations) the equations that define section or member resistance to derive the minimum required member size. The question is how to determine the loads to apply, since (a) they depend on the dynamic characteristics of the yet to be determined structure and (b) they need to be consistent with the design objectives. Here one needs to choose between working with the multi-degree-of-freedom (MDOF) system using a simpler proxy, or making use of a database (or sequence) of relevant candidate MDOF designs.

Assessing the performance of candidate MDOF designs is a conceptually simple direct-search problem that is well suited to optimization. At its simplest implementation, an engineer is guided by his/her intuition to incrementally change an initial design to satisfaction. Each iteration, though, is equivalent to a cycle of re-design and re-analysis, where the latter is a full-blown performance-based assessment involving nonlinear static or dynamic runs, as discussed in Krawinkler et al. (2006). Any method built on this paradigm essentially is an iterated assessment procedure. Many researchers have also chosen to improve upon the efficiency of the re-design to achieve a fast convergence, often leading to the use of numerical optimization. An optimization algorithm provides candidate structures for performance assessment. Force and deformation checks are encoded as constraints, while the initial or lifecycle cost of the structure can be the optimization objective (Fragiadakis and Lagaros 2011). Mackie and Stojadinovic (2007) have suggested this approach for bridges while Fragiadakis and Papadrakakis (2008), Franchin and Pinto (2012) and Lazar and Dolsek (2012) have all used optimization techniques for the performance-based seismic design of reinforced-concrete structures.

While using the MDOF system is arguably more comprehensive and accurate, it is also considerably more expensive. A simpler approach entails using a surrogate system, typically an SDOF oscillator, to help translate the MAFs of exceedance of EDP limits to design loads. Thus, the complexity of guessing the MDOF system's properties is reduced to determining the strength and stiffness that define the surrogate oscillator (often assumed to have elasto-plastic behavior). Any two of the following quantities define the oscillator yield point: (a) period, T, (b) yield strength, F_y, (usually normalized by the (known) weight W to equal the base shear coefficient at yield, $C_y = F_y / W$), and (c) yield displacement, δ_y. These three quantities are functionally related:

$$T = 2\pi\sqrt{\frac{\delta_y}{C_y g}} \tag{3}$$

Assuming now that a structural configuration has been decided, one needs only to determine the minimal member sizes that satisfy the objectives. This is essentially akin to a Newton-Raphson algorithm for finding the root (or minimum) of a nonlinear "black-box" equation: a local estimate of the function tangent is employed and assumed to remain constant to linearize the problem in order to point to a potential solution. This tangent is essentially an assumed invariant term that is updated at the end of each iteration until convergence. Presently, there are two distinct flavors of the SDOF-proxy approach, based on what term is considered to be *invariant* (or approximately stable).

Force-based approaches are the basis of practically all current seismic codes, and operate on an invariant T basis. This is in turn used to determine $S_a(T)$ that is multiplied by the system mass m (hence the "force-basis") and divided by the ubiquitous R or q factor to determine the desired base shear at yield, V_y. For a given mass, the same information as V_y is conveyed by the spectral acceleration at yield, $S_{ay}(T)$, or similarly, C_y where $C_y = S_{ay}$ when S_{ay} is expressed in units of g. The R- or q-factors are approximate methods to account for the effect of

overstrength and ductility on peak response. Note that while higher mode periods can be used to determine the loads in a modal response analysis, the approach remains essentially SDOF-based as R and q always refer back to the first mode only. The MDOF structure is then designed to be safe against the required effects of loads consistent with C_y. An eigenvalue analysis of the newly designed system provides an updated period and if a significant difference is found, a new cycle of design and analysis is performed. A key feature of this approach is the use of approximate formulas to provide an initial guess for T. A good guess essentially eliminates the need for repeated cycles, something that is often the case when an experienced engineer is facing a structure and associated performance objectives he/she is comfortable with. Nevertheless, inaccurate estimates may be expected where more stringent performance objectives call for more than usual lateral stiffness.

Alternatively, one may assume instead that the yield displacement δ_y is the invariant term. This has been suggested by Priestley (2000) and Aschheim (2002) as a much more stable parameter compared to the period T, thus significantly reducing the need for iterations. Here, the design process starts by deriving an initial guess for δ_y (rather than T), and then using derived limits on ductility, μ, and appropriate R-μ-T relationships to determine the seismic coefficient C_y. The process has been encoded by Aschheim (2002) into a visual design tool termed Yield Point Spectra (Fig. 6). Having established C_y, member sizes may be determined. In the infrequent cases where iteration on δ_y is needed, a new yield displacement is derived and the process is repeated. The refinement of the yield displacement is much like the refinement of period that occurs in current force-based methods of design.

Table 1 summarizes the defining characteristics of different design approaches. Practically all code-sanctioned methods use an EDP-basis and a force-based SDOF proxy, while lacking uncertainty propagation. This description covers nearly 100% of current engineering practice. At the academic level, considerable advances have appeared in the

MDOF-based indirect approach for performance-based design, often employing proper uncertainty propagation and either an EDP (Franchin and Pinto 2012) or a DV basis (Mackie and Stojadinovic 2007). Still, there are considerable usability barriers in adopting such powerful approaches in practice. A methodology that, in the terms of Table 1, employs full uncertainty propagation, an EDP-basis and a displacement-invariant SDOF proxy to achieve "directness" may have a better chance. Only recently, Vamvatsikos and Aschheim (2015) proposed such a methodology to correctly propagate uncertainty at the SDOF level through the use of YFS.

Origin, definition, and use of Yield Frequency Spectra

The essential ingredients of the YFS approach to performance-based design are (a) the site hazard and (b) some assumption about the system's behavior (e.g. elastic, elastoplastic, etc) that is used to define the SDOF proxy. Then, for a given capacity curve shape (or system type) we are asked to estimate the yield strength and the period, T, for not exceeding a limiting displacement δ_{lim} at a rate higher than P_o. Even for an SDOF system, the introduction of yielding, ductility and the resulting record-to-record response variability fundamentally changes the nature of the problem. This is best represented in the familiar coordinates of the IM, taken here as the first mode spectral acceleration $S_a(T)$, and the EDP, i.e., the displacement response δ. The structural response then appears in the form of IDA curves as shown in Fig. 5 for a T = 1 s system with a capacity curve having positive and then negative post-yield stiffness. Formally, this relationship may be represented by the following integral (Jalayer 2003; Vamvatsikos and Cornell 2004), that is equivalent to Eq. (2):

$$\lambda(\delta) = \int_0^{+\infty} P\left[EDP > \delta \,|\, \mathrm{IM} = s\right] \left|\mathrm{d}\lambda(s)\right| \tag{4}$$

where $P[EDP > \delta \mid IM = s]$ is the probability of exceeding a certain level of response δ given a seismic intensity of s, equivalent to $G(EDP \mid IM)$, the CCDF of *EDP* given *IM* evaluated at specific values of each (e.g., Fig. 5). $\lambda(s)$ is the associated hazard rate.

Using Eq. (4) together with *R*-μ-*T* relationships that offer the probabilistic distribution of structural response given intensity $P[EDP > \delta \mid IM = s]$ (e.g. Vamvatsikos and Cornell 2006), rather than just the mean, allows the estimation of the so-called inelastic displacement (or drift) hazard curves. For a yielding system these are the direct equivalent of spectral displacement hazard curves. They have appeared at least in the work of Inoue and Cornell (1990) and subsequently further discussed by Bazzurro and Cornell (1994) and Jalayer (2003). While useful for assessment, they lack the necessary parameterization to become helpful for design. An appropriate normalization may be achieved for oscillators with yield strength and displacement F_y and δ_y, respectively, by employing ductility μ, rather than displacement δ and the seismic coefficient C_y instead of the strength.

Up to this point, what has been proposed is similar to the results presented by Ruiz-Garcia and Miranda (2007) on the derivation of maximum inelastic displacement hazard curves. What truly makes the difference is defining δ_y as a constant for a given structural system, for relevance to displacement-based design. Then C_y essentially replaces the period *T*, as expressed in Eq. (3).

For a given site hazard and oscillator properties—system damping, δ_y, value of C_y (or period), and capacity curve *shape* (e.g. as normalized in terms of $R = F/F_y$ and μ)—a unique representation of the oscillator's probabilistic response may be gained through the displacement (or ductility) hazard curves produced via Eq. (4). Damping, δ_y and the capacity curve shape are considered as stable system characteristics, as they would be in a displacement-based design process. By plotting such curves of $\lambda(\mu)$, for a range of μ_{lim} limiting values and a range of C_y, we obtain contours of the inelastic displacement hazard

surface for constant values of C_y. These contours allow the direct evaluation of system strength and period—i.e., the C_y required to satisfy any combination of performance objectives defined as $P_o = \lambda(\mu_{lim})$, where each limiting value of ductility μ_{lim} is associated with a maximum MAF of exceedance P_o, as shown in Fig. 7.

The resulting YFS are thus proposed as a design aid, being a direct visual representation of a system's performance that quantitatively links the MAF of exceeding any displacement value (or ductility μ) with the system yield strength (or seismic coefficient C_y). YFS are plotted for a specified yield displacement; thus, periods of vibration represented in YFS vary with C_y. Fig. 7 presents an example for an elastic-perfectly-plastic oscillator. In this case, three performance objectives are specified (the red "x" symbols) while curves representing the site hazard convolved with the system fragility are plotted for fixed values of C_y. Of course, increases in C_y always reduce the MAF of exceeding a given ductility value. Thus, the minimum acceptable C_y (within some tolerance) that fulfils the set of performance objectives for the site hazard can be determined for an SDOF system. This strength is used as a starting point for the performance-based design of more complex structures, potentially solving the problem in a single step having begun with a good estimate of the yield displacement.

At a certain level, YFS can be considered as a building- and user-specific extension of concepts behind the IBC (2011) risk-targeted design spectra. Whereas the latter are meant to offer a uniform measure of safety, they only do so for one limit-state (global collapse), one specific target probability (1% in 50 years) and a given assumed fragility regardless of the type of lateral-load resisting system. On the contrary, YFS can target any number of concurrent limit-states, each for a user-defined level of performance (or safety), and employ building-specific fragility functions, as implied by the supplied capacity curve shape. The practical estimation of YFS is thus based on the case-by-case solution of the integral in

Eq. (4). This involves a comprehensive evaluation for a number of SDOF oscillators with the same capacity curve shape and yield displacement but different periods and yield strengths. If a numerical approach is employed, then we can obtain the comprehensive view shown in Fig. 7 at the cost of a few minutes of computer time. Alternatively, if one seeks only the value of C_y corresponding to each performance objective, then an analytical approach can be used that offers accurate results with only a few iterations (Vamvatsikos and Aschheim 2015).

As an example of application, we consider an $H = 32.8$m high, 8-story reinforced-concrete space-frame building (from FEMA 2009) to be designed for a high-seismicity site having a (10% in 50 year) design spectrum anchored at $S_a(0.5s) = 1.5g$ and falling off with $1/T$ in the period range of interest. A yield displacement of 0.22m (0.67% yield roof drift) was determined for the MDOF system, translated to 0.13m for the equivalent SDOF (Vamvatsikos et al. 2015), while a serviceability requirement of 0.75% maximum interstory drift was adopted at the 10% in 10 years frequency (or a MAF of 0.0105), becoming the governing performance target. For a ratio of maximum interstory drift to roof drift of about 1.5, this translated to a ductility limit of $\mu = 0.75\% / (1.5 \cdot 0.67\%) = 0.76$. A simplified model of the hazard was employed (Cornell et al. 2002), assuming hazard curves of constant slope $k = 2.5$ in lognormal coordinates. As solution was sought in the *nominally* elastic region ($\mu < 1$) a low, yet non-zero, dispersion of 20% was assumed for the response distribution in Eq. (4). YFS suggested a required first-mode period (cracked section properties) of $T = 1.24$s and a yield base shear of $V_y \approx 2500$kN. Having determined member sizes for the entire structure on these two premises, performance was assessed through IDA. The MAF of violating the serviceability limit-state was found to be 0.0097, i.e., lower than the 0.0105 threshold, soundly validating the proposed approach.

Conclusions

There are many published design approaches for achieving the desired performance compliance for any structure. Current code approaches are arguably the simplest and most practical ones. Yet, they are invariably limited in accuracy due to their far-from-perfect handling of uncertainty, which can severely bias the output design to be either too conservative, or even unconservative, and with unknown MAFs of exceedance. Furthermore, their use of a force-basis means that some iterations may be needed, unless a good approximation of the desired period is available. On the other end of the spectrum lie the modern methods that employ full system optimization to achieve the required performance in terms of reducing actual seismic losses. These may truly fulfill the actual target of performance-based design, yet they are severely encumbered by their computational complexity. Most, if not all, are only applicable within an academic environment. Attempting to bridge this distance, Yield Frequency Spectra have been introduced as an intuitive and practical approach to performance-based design. They allow design to approximately satisfy an arbitrary number of performance objectives that can be connected to the global displacement of an equivalent single-degree-of-freedom oscillator. Within this relatively benign constraint, our approach incorporates uncertainty and accurately propagates it to the output structural response, where performance is checked. Thus, it can help deliver preliminary designs that are close to their performance targets, requiring few (if any) cycles of re-analysis and re-design to reach the final stage. Only time will tell if engineering practice is ready to adopt a new design paradigm.

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Figure Captions

Fig. 1. Spectral acceleration hazard surface.

Fig. 2. (a) Uniform hazard spectra and (b) S_a hazard curves corresponding to the hazard surface of Fig. 1.

Fig. 3. The effect of alternative structural modeling choices on the capacity curve of a 5-story reinforced concrete building, as estimated via first-mode static pushover (adapted from Zeris et al. 2007).

Fig. 4. Steel beam plastic hinge uncertainties and their effect: (a) cumulative distribution functions of θ_{pc}, or the difference between hinge rotation at maximum moment and at complete loss of strength (adapted from Lignos and Krawinkler 2011) and (b) the estimated dispersion of the median response in spectral acceleration versus maximum interstory drift terms for a 9-story frame (from Vamvatsikos and Fragiadakis 2010).

Fig. 5. IDA curves for a T = 1s oscillator showing the distribution of the EDP response given intensity s, and the estimation of the corresponding probability of exceeding a certain EDP value of δ, $P[EDP > \delta \mid IM = s]$.

Fig. 6. Yield Point Spectra computed for the 1940 NS El Centro record. The response of a system with yield displacement $\delta_y \approx 4$ cm, yield strength coefficient $C_y \approx 0.18$, and $T = 1$ s is shown. The yield point falls on the $\mu = 2$ curve, indicating that the peak displacement is twice the yield displacement (from Aschheim 2002).

Fig. 7. YFS contours at C_y = 0.1,…,1.0 determined for an elastoplastic system (δ_y = 0.06m) subjected to the hazard of Fig.1. The red "x" symbols represent three performance objectives (μ = 1, 2, 4 at 50%, 10% and 2% in 50yrs exceedance rates, respectively). The third objective

582 governs with $C_y \approx 0.93$. The corresponding period is $T \approx 0.51$s.

583 **Table 1.** Summary of different characteristics of design approaches.

Characteristic	Low fidelity	High fidelity
decision variable	structural response (DV=EDP)	cost, casualties, downtime, (actual DVs)
uncertainty propagation	None, probability controlled via input intensity only	Accurate, probability checked at output
design invariant term	Period or yield displacement via SDOF proxy	None, full MDOF is used

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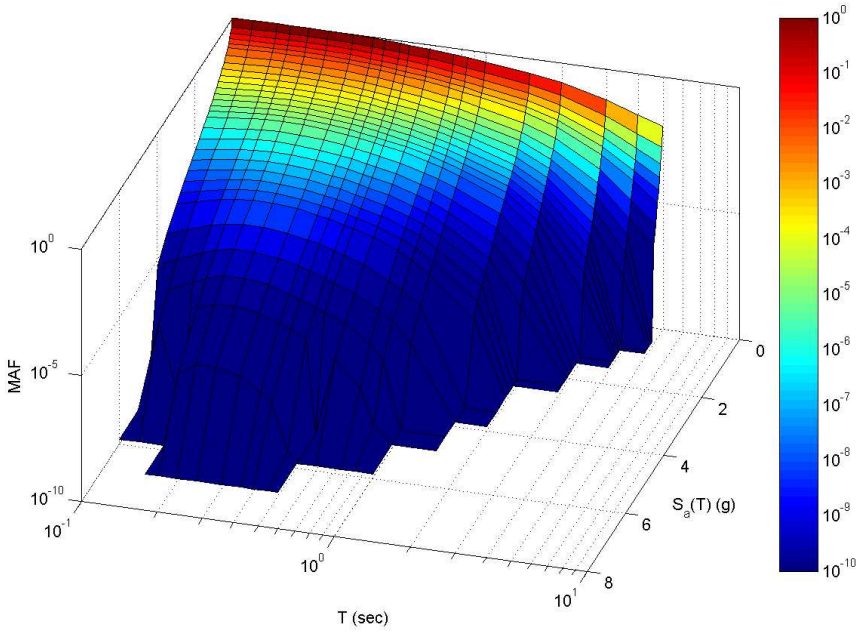

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587 **Fig. 1.** Spectral acceleration hazard surface.

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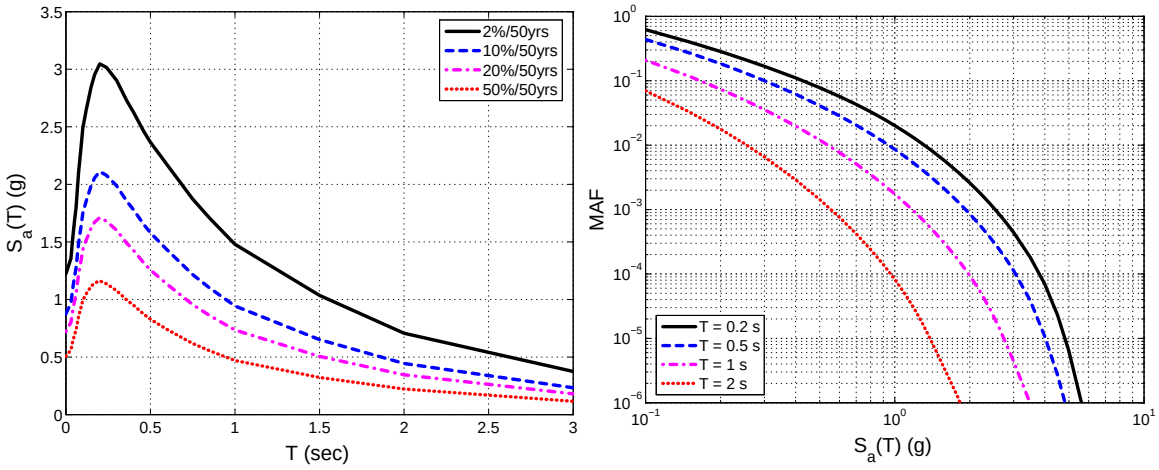

(a) (b)

589 **Fig. 2.** (a) Uniform hazard spectra and (b) S_a hazard curves corresponding to the hazard surface of

590 Fig. 1.

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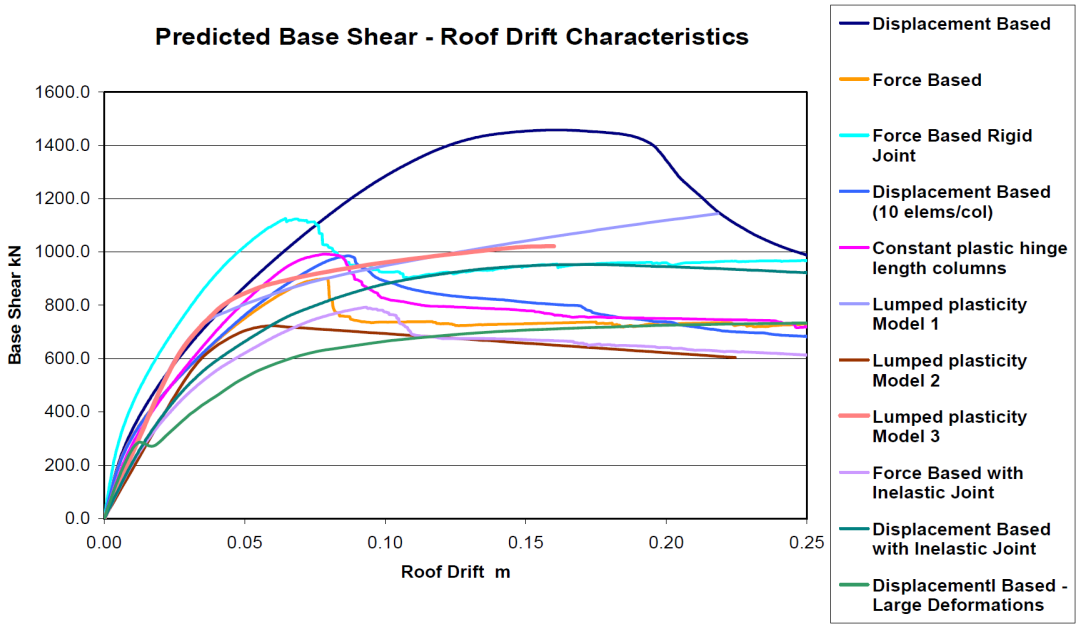

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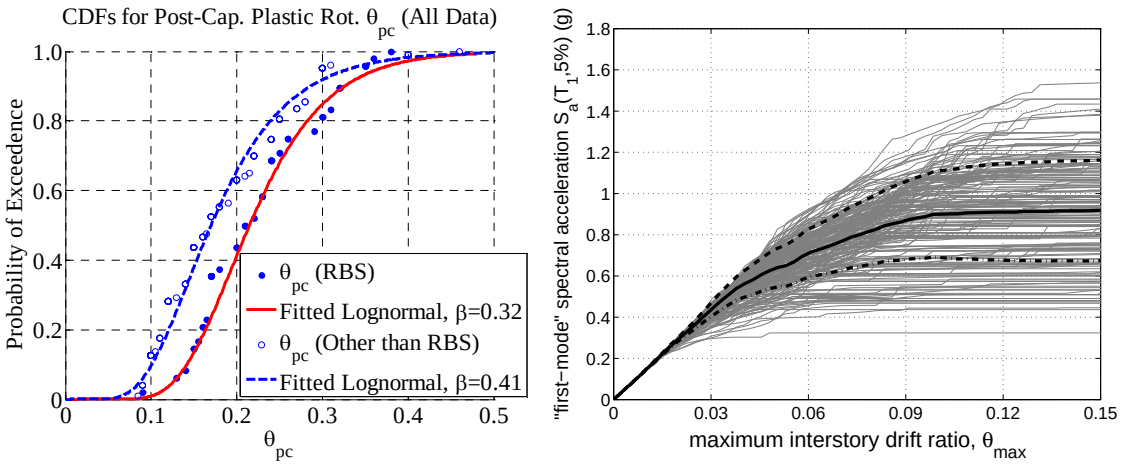

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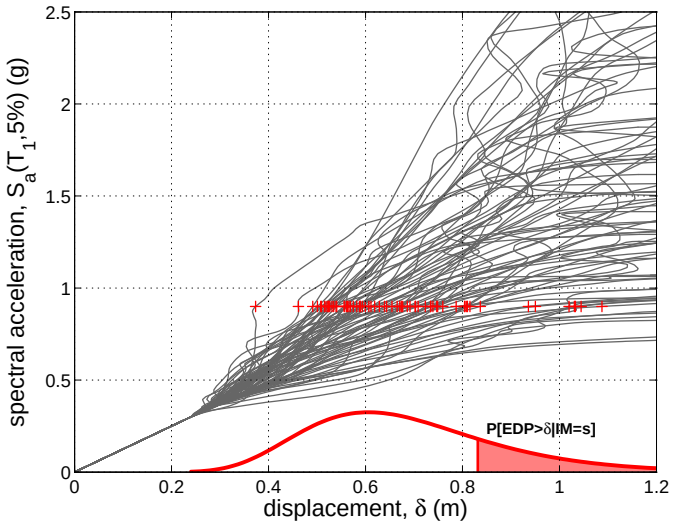

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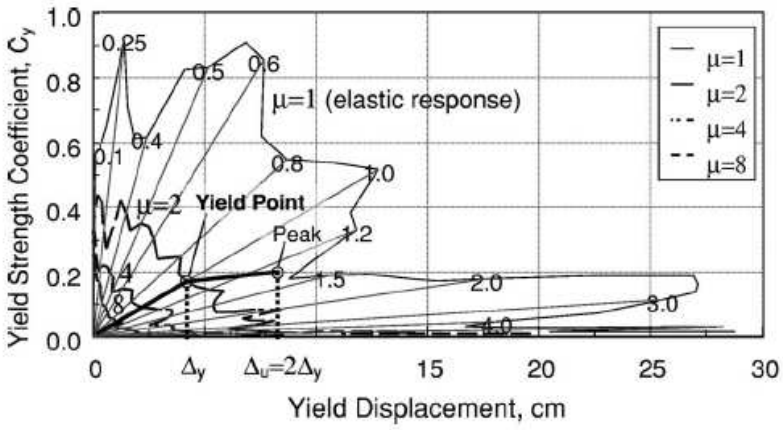

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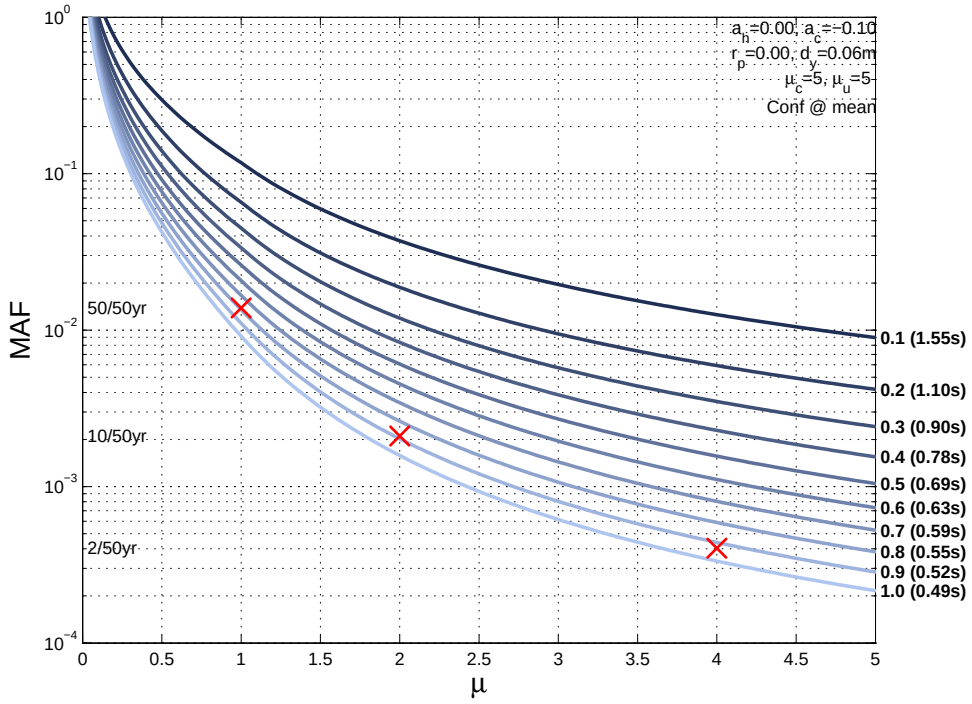

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