Vector and Scalar IMs in Structural Response Estimation: Part II – Building Demand Assessment

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The advantages and disadvantages of using scalar and vector ground motion Intensity Measures (IMs) are discussed for local, story-level seismic response assessment of 3D buildings. Candidate IMs are spectral accelerations, at a single period (*Sa*) or averaged over a period range (Sa_{avg}). Consistent scalar and vector probabilistic seismic hazard analysis results were derived for each IM as described in the companion paper (Kohrangi et al., 2015b). The response hazard curves were computed for three buildings with reinforced concrete infilled frames using the different IMs as predictors. Among the scalar IMs, Sa_{avg} tends to be the best predictor of both floor accelerations and inter story drift ratios at practically any floor. However, there is an improvement in response estimation efficiency when employing vector IMs, specifically for 3D buildings subjected to both horizontal components of ground motion. This improvement is shown to be most significant for a tall plan-asymmetric building.

INTRODUCTION

Performance Based Earthquake Engineering (PBEE) has been the focus of research on seismic assessment of buildings and bridges for more than a decade. The main scope of PBEE is to support decision-making regarding the seismic performance of structures within a probabilistic framework. This methodology consists of four steps that require a

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broad knowledge of: 1) Seismic Hazard Analysis; 2) Demand Analysis; 3) Damage Analysis and 4) Loss Estimation (Cornell and Krawinkler, 2000). The first step uses an IM of the ground motion to predict the distribution of the structural response in terms of different Engineering Demand Parameters (EDPs), such as Inter Story Drift Ratio (IDR) and absolute Peak Floor Acceleration (PFA). EDP distributions are then used to measure the structural/nonstructural damages using discrete damage states for each building component (FEMA-P-58 2012). Finally, the likelihood of occurrence or exceedance of monetary losses, injuries and estimated downtime is computed using the consequence functions that link damage states and their repair strategies to repair costs, repair time, and physical consequences to inhabitants. This paper focuses on the second step of this procedure namely: Probabilistic Seismic Demand Analysis (PSDA). The companion paper (Kohrangi et al., 2015b) addresses the first step that is Probabilistic Seismic Hazard Analysis (PSHA).

PSHA evaluates the Mean Annual Rate (MAR) of exceeding certain levels of IM at the building site. Some classical IMs are the ground motion peak values expressed in terms of the peak ground acceleration, velocity or displacement (PGA, PGV or PGD, respectively), and the Spectral acceleration at the first mode period of the structure, $Sa(T_1)$. Of interest are the efficiency and sufficiency of such IMs (See Luco and Cornell, 2007). An efficient IM provides low dispersion of the predicted response given IM and a sufficient IM offers statistical independence of the response given IM from ground motion characteristics, such as magnitude, distance, etc. Efficiency helps reduce the number of time history analysis for reliable assessment of response while sufficiency is a sine qua non requirement for combining PSHA with structural analysis results. For example, $Sa(T_1)$ is shown to be efficient and sufficient for assessing the response of some structures (e.g., first-mode dominated, low-rise buildings) and types of ground motion (e.g., ordinary far field motions) and not as effective in other cases, such as tall buildings (Shome and Cornell, 1999), and any building that are likely to be subject mainly to near source ground motions (Luco and Cornell, 2007). In addition, due to the 3D nature of structures and the multi-directionality of ground shaking excitations, a single scalar IM at a single ordinate of the spectrum does not seem to be a good predictor for the structural response (Faggella et al., 2013; Lucchini et al., 2011). In general, the response of the structure is correlated to the spectral acceleration at higher modes and, when beyond the elastic range, also to the spectral acceleration at elongated periods. In addition, the response in one direction might be correlated to the excitation of the orthogonal direction, especially for torsionally unrestrained buildings and when some local failure modes are triggered (e.g., out-of-plane collapse of walls being facilitated by loss of in-plane strength).

Some more complex scalar IMs have been proposed by researchers to improve the predictive performance of traditional scalar IMs. For example, Tothong and Cornell (2007) showed that the inelastic displacement of the building could be effectively estimated by the Inelastic Spectral Displacement, $S_{di}(T_1)$, of a nonlinear SDOF with vibration period consistent with the first modal period of the structure. Alternatively, for tall buildings, a combination of $S_{di}(T_1)$ with elastic spectral displacement at the second mode, $S_{de}(T_2)$, and the elastic participation factors was shown to be an effective predictor of building deformation response. Cordova et al. (2000) followed by Vamvatsikos and Cornell (2005) and Mehanny (2009) introduced the power-law form, scalar-based seismic IM that was shown to reduce the dispersion in structural inelastic displacement response. Bianchini et al. (2010) proposed the similarly defined average spectral acceleration (Sa_{avg}), which is the geometric mean of the logarithmic spectral acceleration at multiple periods, in a relevant period range. At the other end, a number of record selection schemes have been introduced that rely on simpler IMs. First, Baker and Cornell (2005) employed epsilon, i.e. a measure of the difference between ground motion's $Sa(T_1)$ and the median estimate of a ground motion prediction Equation (GMPE) for the given earthquake scenario, while Bradley (2010) proposed the Generalized Conditional Intensity Measure (GCIM) to select records using one or more simple IMs and record parameters to allow accounting for the conditional distribution of ground motion characteristics to remove issues of insufficiency.

There is no theoretical limitation forcing us to considering a scalar IM for response prediction. If advantageous, one could consider a vector of several relevant IMs for structural response estimations. The use of a set of different scalar IMs was introduced by Bazzurro (1998) as a vector that included two spectral accelerations at different periods. Vamvatsikos and Cornell (2005) studied the efficiency of a vector of $Sa(T_1)$ and a ratio of spectral accelerations at T_1 and a secondary period, while Baker and Cornell (2005, 2008a, 2008b) also investigated the further addition of epsilon. Although, it is intuitive to expect that the use of a vector IM would provide more efficiency and sufficiency in response estimation, it has not caught on in the scientific community due, in part, to the complexity of linking the response assessment with the joint hazard estimation at the site via Vector Probabilistic Seismic Hazard Analysis (VPSHA). In addition, some researchers, such as Rajeev et al. (2008) using a 2D model, did not find the gain in response prediction accuracy worth the extra effort.

This paper and its companion (Kohrangi et al., 2015b), intend to offer a fresh look into the issue of using scalar and vector IMs for probabilistic response estimation of 3D buildings under two horizontal components of ground motion. Three building examples of common Mediterranean construction practice in reinforced concrete (RC) were examined. The companion paper (Part I) presents the approaches followed to carry out VPSHA. This paper (Part II) covers the record selection approach, the structural modeling and analysis, as well as the response estimation. Finally, the results obtained are discussed, with emphasis on the effectiveness of several scalar and vector IMs as response predictors to obtain the localized-response hazard curves for these buildings.

BUILDING EXAMPLES AND MODELING DESCRIPTIONS

This study considers three examples of 3-, 5- and 8-story buildings representative of typical Southern Europe old constructions designed without specific provisions for earthquake resistance (Figure 1). The 3-story RC frame building (SPEAR) is non-symmetric in X and Y directions with 3.0 m story height. The full-scale structure was built and tested within a European research project at JRC-ELSA (Fardis, 2002). The 5- and 8-story buildings are models of real RC buildings in Turkey. The 5-story is regular in plan and height with 2.85 m story height, whereas the 8-story is irregular in plan (i.e., stiffer in Y than X direction) and height (i.e., first floor story height is 5.0m and other stories 2.7m). Detailed information about the 5- and 8- story structures is available in

Bal et al. (2007). The outer frames of all three buildings are filled with masonry walls except for the 8-story building in which no walls are present at the ground floor in the X direction.

Figure 1. Plan view of the three considered structures (dimensions in cm)

3-D nonlinear models of the building structures were created in OpenSees (McKenna, 2000). Beams and columns are modeled using force-based distributed plasticity elements and the actual properties of the floor diaphragms are considered by means of equivalent X-diagonal braces that represent the in-plan stiffness of the slab. The masonry infill panels were considered based on the model proposed by Kadysiewski and Mosalam (2009). All material and structural properties are taken at their best deterministic (typically mean) estimates. The results of the modal analysis after application of gravity loads are listed in Table 1.

The fundamental translational modes of the buildings have mass participation factors that are generally lower than 80%. The sole exception is the X-direction response of the 8-story building in which 99.04% of the modal participation is reached in the first mode due to the presence of a soft first story in this direction only. In addition, the translational response of all three buildings is coupled with torsion at least in one of the two horizontal directions, even for the 8-story structure: Due to the alignment of all walls with the Y axis, the 8-story building is flexible along X with a vibrational period of 1.30s and stiff along Y with a period of 0.44s. The pushover curves of all three buildings with and without masonry infill walls in the two orthogonal directions are shown in

Figure **2**. The difference between the stiffness and the base shear capacity in the two orthogonal directions of the 8-story building could be noticed in

Figure **2**(c). Note also that, because of the absence of infills at the ground floor, the values of the base shear capacity of the 8-story building in the X axis computed with and without infill walls are basically the same. More discussion about the building properties and modelling can be found in Kohrangi (Kohrangi, 2015a).

Table 1. Periods and participating mass ratios (PMR) of buildings' eigen modes for the translational (X, Y) and rotational (RZ) degrees of freedom after the application of gravity loads. The prefix Σ denotes the cumulative sum of the modes.

	Mode No.	Period (s)	PMRX (%)	PMRY (%)	PMRRZ (%)	ΣPMRX (%)	ΣPMRY (%)	ΣPMRRZ (%)
3 story	1	0.66	5.78	50.21	26.40	5.78	50.21	26.40
	2	0.57	77.16	8.28	0.03	82.94	58.49	26.43
	3	0.43	2.60	23.43	60.37	85.54	81.91	86.80
5 story	1	0.67	0.00	80.70	0.00	0.00	80.70	0.00
	2	0.46	71.28	0.00	10.45	71.28	80.70	10.45
	3	0.42	10.07	0.00	72.54	81.35	80.70	82.99
8 story	1	1.30	99.04	0.00	0.03	99.04	0.00	0.03
	2	0.46	0.02	18.48	73.00	99.07	18.48	73.03
	3	0.44	0.04	68.43	18.66	99.10	86.91	91.69

Figure 2. Pushover curves for three buildings with and without infill panels (Solid line: Y axis, Dotted line: X axis)

GROUND MOTION DATABASE

Nonlinear Dynamic Analysis is used to obtain the distribution of structural response, as expressed in terms of engineering demand parameters (e.g., MIDR and PFA), for different IM levels. Given the approximations included in the first of the four-step PEER approach, where the complexity of ground motions is represented most often by a single IM, the ground motion record selection may often play a key role in ensuring accuracy in the estimation of the responses. Loosely speaking, the ground motions selected should appropriately reflect the distribution of seismological characteristics not accounted for by an insufficient IM at the given site. Given the limitations in the existing databases of real accelerograms, any available method for record selection is imperfect. In this study we used the selection method (Jayaram, et al., 2011; and Lin et al., 2013a) based on the Conditional Spectrum, CS, to assemble three sets of input ground motion records. In this methodology, a suite of ground motions is selected and scaled such that each resulting suite collectively matches the entire conditional distribution of spectral accelerations given the IM value as represented by the CS. This way, both the mean and variance of the record set spectra are consistent with the seismic hazard at the site.

For this study, 10-12 intensity levels of spectral acceleration, $Sa(\bar{T})$, were used for the 3- and 8-story buildings and 12 levels for the 5-story building. Numerically they range from 0.037g to 2.46g and they were selected to cover the entire range of response from elastic to severely inelastic. Each stripe consists of both horizontal components of 20 accelerogram pairs in which the geometric mean of the spectra of the two components is used to match the corresponding anchoring point of the CS at the period $\bar{T}$. The GMPE of Boore & Atkinson (2008) and mean magnitude, distance and epsilon ($\bar{M}$, $\bar{R}$ and $\bar{\varepsilon}$) obtained from disaggregation results for each intensity level were used for computing the CS. The conditioning period ($\bar{T}$) in all cases is the average of the first mode building vibration periods in X and Y directions, $\bar{T} = \left(T_{1x} + T_{1y}\right)/2$, as proposed by FEMA P-58 for period-specific scalar IM*s* case to be used for assessing the response of 3D buildings.

Table 2. Summary of the mean M, R and ε values obtained via disaggregation for the selected levels of $Sa(\bar{T})$ for the three considered buildings.

Intensity Level	$Sa(\bar{T})$ [g]	3- story ($\bar{T} = 0.62s$)			5- story ($\bar{T} = 0.57s$)			8- story ($\bar{T} = 0.87s$)		
		$\bar{R}$	$\bar{M}$	$\bar{\varepsilon}$	$\bar{R}$	$\bar{M}$	$\bar{\varepsilon}$	$\bar{R}$	$\bar{M}$	$\bar{\varepsilon}$
IML1	0.04	68.7	5.7	0.7	69.3	5.6	0.7	66.3	5.9	0.8
IML2	0.07	57.0	5.9	0.9	57.5	5.9	0.9	54.0	6.2	0.9
IML3	0.12	43.4	6.2	0.9	44.0	6.2	0.9	39.7	6.5	1.0
IML4	0.22	31.0	6.5	1.1	31.5	6.4	1.1	28.6	6.7	1.2
IML5	0.33	25.4	6.6	1.3	25.7	6.6	1.3	23.7	6.7	1.6
IML6	0.50	21.2	6.7	1.7	21.5	6.7	1.6	19.8	6.8	1.9
IML7	0.61	19.4	6.7	1.8	19.6	6.7	1.7	17.7	6.8	2.2
IML8	0.74	17.5	6.7	2.1	17.7	6.7	2.0	15.6	6.8	2.2
IML9	0.90	15.3	6.7	2.2	15.7	6.7	2.2	12.2	6.7	2.2
IML10	1.35	6.3	6.5	2.1	7.7	6.5	2.1	2.6	6.4	2.4
IML11	2.01	-	-	-	2.6	6.4	2.4	-	-	-
IML12	2.46	-	-	-	2.5	6.7	2.5	-	-	-

Figure 3. (a) The mean conditional spectra (CMS) at 12 intensity measure levels and the selected ground motion suites used as input to the nonlinear dynamic analysis of the 5-story building, (b) "Hazard consistency" verification of the selected ground motion records for the 8-story building based on the conditioning period $\bar{T} = 0.87s$ for CS matching (dashed line: Direct hazard curve from PSHA, solid line: Hazard curve from the selected records).

Since the same GMPE was applied both here and in the PSHA calculations, the results of the record selection could be considered accurate. However, some approximations might be introduced by neglecting the different causal earthquake scenarios that may not be adequately captured by the average M, R and ε values considered in the process (see Lin et al., 2013, for more details). The records in this database can be classified into two groups: ordinary far field records and pulse-like records. Based on the method proposed by Shahi (2013), in this database 9 records for the 3-story building, 8 records for the 5-story building, and 22 records for the 8-story building are classified as pulse like.

As an example, Figure 3(a) shows the Mean of Conditional Spectra (CMS) at 12 intensity levels as well as the geometric mean of the spectra of the two components of all 240 (selected and scaled) individual records used as input to the nonlinear dynamic analysis of the 5-story building. The approach proposed by Lin et al. (2013b) was used to verify the hazard consistency of the selected records for the three buildings. Figure 3(b) compares the direct hazard curves of the site with the rate of exceedance of $Sa(\overline{T}_1)$, $Sa(0.5 \cdot \overline{T}_1)$ and $Sa(1.5 \cdot \overline{T}_1)$ in the record data set selected for the 8-story building. The consistency of the selected records with the hazard curves at different structural ordinates of the spectrum is acceptable. Although omitted here, this consistency was also verified for the record sets selected for the 3- and the 5-story buildings.

NONLINEAR DYNAMIC ANALYSIS, INTENSITY MEASURES AND ENGINEERING DEMAND PARAMETERS

Nonlinear Dynamic Analysis was performed for the risk-based assessment of the three buildings using the CS-based records. It is emphasized that the difference between what is done here and the well-known Incremental Dynamic Analysis (IDA) approach (Vamvatsikos and Cornell, 2002) is that this study uses different records for the lower, middle and higher stripes, whereas in IDA the same records are incrementally scaled up until collapse of the structure is reached. Thus, the present study does not depend as much on the quality of the IM to achieve reliable results.

Structural and non-structural deformation-sensitive damage is typically correlated to the peak (in time) inter story drift ratios (IDRX and IDRY) at each story. As a single indicator of global collapse, the respective maxima over height, MIDRX and MIDRY, may also be employed. For simplicity, the directionless square root of the sum of the squares (SRSS) of the corresponding X and Y EDPs in the two directions, termed IDR for individual stories or MIDR for the whole building, is sometimes used instead. In this study, as torsion may be an issue, such values are averaged over the four corners of the building's rectangular plan. To assess the acceleration-sensitive losses of nonstructural components and contents, the absolute peak floor acceleration is also employed. This is also taken as the peak (in time) of the SRSS of the floor accelerations in the two main orthogonal directions at the middle point of the floor slab. Note that the IDRs in X or Y direction are more suitable for monitoring the response of single components according to their orientation and, therefore, we chose to show the results of IDR for two orthogonal directions, separately. For PFA, however, for which the direction has less significance, we present the SRSS results instead. On the other hand, the use of directionless SRSS values may understate the magnitude of change observed in a specific direction, somewhat softening the perceived impact of 3D ground motion excitation and the improvement brought on by some of the more specialized IMs tested.

The global response of the buildings in terms of (directionless) IDR and PFA is shown in is shown in Figure 4. The thin gray lines represent the maximum floor response of individual analyses while the thick blue lines identify their median at different IM levels. Two collapse criteria were considered. The first is the global side-sway collapse that we equated to non- convergence of the analysis after large lateral displacements were reached. In addition, we considered a local collapse criterion that can be associated to the loss of load bearing capacity of the non-ductile columns (Aslani and Miranda, 2005). This was set at an *IDR* value of 0.04, on average.

Figure 4. Structural response of for the 8-story building obtained from nonlinear dynamic analysis carried out for the CS-based selected records. Each thick blue line is the median of the responses for each stripe analysis.

The set of scalar and vector IM candidates considered are listed in Table 3. A detailed summary of the tested IMs and the criteria employed for selecting them are presented in companion paper (Kohrangi et al., 2015b).

Table 3. IMs considered in the response estimation

INTENSITY MEASURE (IM) **	**ABBREVIATION***
SCALAR IMs	
Natural logarithm of arbitrary spectral acceleration at the first modal period $\ln\left[Sa_x(T_{1x})\right]$ or $\ln\left[Sa_y(T_{1y})\right]$.	Sa_{S1}
Natural logarithm of the geometric mean of spectral acceleration at the average period, $\ln\left[Sa_{g.m.}\left(\overline{T}=\left(T_{1x}+T_{1y}\right)/2\right)\right]$.	Sa_{S2}
$\ln\left[Sa_x\left(\alpha_1\cdot T_{1x}\right)\cdot Sa_x\left(T_{1x}\right)\cdot Sa_x\left(\alpha_u\cdot T_{1x}\right)\cdot Sa_y\left(\alpha_1\cdot T_{1y}\right)\cdot Sa_y\left(T_{1y}\right)\cdot Sa_y\left(\alpha_u\cdot T_{1y}\right)\right]$	Sa_{S3}
$\ln\left(\left[\prod_{i=1}^{n}\left(Sa_x\left(T_{xi}\right)\right)\right]^{1/n}\right)\cdot\ln\left(\left[\prod_{j=1}^{m}\left(Sa_y\left(T_{yj}\right)\right)\right]^{1/\mathrm{m}}\right),\ \alpha_1\cdot T_1\le T_i\le\alpha_u\cdot T_1, m=n=10$ §	Sa_{S4}
Natural logarithm of the geometric mean of Peak Ground Acceleration, $ln[PGA_{g.m.}]$	PGA
VECTOR IMs	
$\ln\left\{Sa_x(T_{1x}),\frac{Sa_y(T_{1y})}{Sa_x(T_{1x})},\frac{Sa_x(1.5\cdot T_{1x})}{Sa_y(T_{1y})},\frac{Sa_y(1.5\cdot T_{1y})}{Sa_x(1.5\cdot T_{1x})}\right\}$	Sa_{V1}
$\ln\left\{Sa_{g.m.}(\overline{T}),\frac{Sa_{g.m.}(0.5\cdot\overline{T})}{Sa_{g.m.}(\overline{T})},\frac{Sa_{g.m.}(1.5\cdot\overline{T})}{Sa_{g.m.}(0.5\cdot\overline{T})}\right\}$	Sa_{V2}
$\ln\left\{\left[Sa_x\left(\alpha_1\cdot T_{1x}\right)\cdot Sa_x\left(T_{1x}\right)\cdot Sa_x\left(\alpha_u\cdot T_{1x}\right)\right]^{1/3},\left[Sa_y\left(\alpha_1\cdot T_{1y}\right)\cdot Sa_y\left(T_{1y}\right)\cdot Sa_y\left(\alpha_u\cdot T_{1y}\right)\right]^{1/3}\right\}$	Sa_{V3}
$\ln\left\{\left(\left[\prod_{i=1}^{n}\left(Sa_x\left(T_{xi}\right)\right)\right]^{1/n}\right),\left(\left[\prod_{j=1}^{m}\left(Sa_y\left(T_{yj}\right)\right)\right]^{1/\mathrm{m}}\right)\right\},\ \alpha_1\cdot T_1\le T_i\le\alpha_u\cdot T_1, m=n=10$	Sa_{V4}

*All the *IMs* are based on natural logarithm transformation. The notation ln is removed from the abbreviations for brevity
** α_1 is equal to 0.8, 0.2 and 0.2 for the 3-, 5- and 8-story, respectively. α_u is equal to 1.5 in all cases.
§ The periods are equally spaced.

RESPONSE ESTIMATION AND *IM* EFFICIENCY ASSESSMENT

The selection of an appropriate IM is driven mainly by its efficiency and sufficiency (Luco and Cornell, 2007). An efficient IM is one which, when used as predictor of an EDP, results in a relatively small variability in the EDP given the value of the IM. Sufficiency, on the other hand, reflects the independency of the distribution of EDP given IM from other ground motion characteristics, such as magnitude of the causative earthquake, distance of site from the source, epsilon of the record, etc. Higher efficiency results in the reduction in the necessary number of records needed to obtain a reliable estimate of the EDP/IM. Higher sufficiency results in decreased (or non-existent) bias. Due to the careful record selection approach adopted, we shall assume that sufficiency is achieved, at least with respect to spectral shape, and we will only concentrate on efficiency. As such, the efficiency of the examined IMs is gauged by the variance of the residuals of the linear regression analysis of the EDP as a function of the IM. For vector IMs, each element IM_i of the vector was employed separately as a predictor. The regression models adopted for response prediction and efficiency checking appear in Equation (1) and Equation (2), these having linear and complete quadratic IM terms, respectively. It should be noted that for scalar IMs the additional quadratic terms introduced in Equation (2) are not as useful, thus only Equation (1) is used.

$$\ln\left(EDP\right) = b_0 + \sum_{i=1}^{n}\left[b_i \cdot \ln\left(IM_i\right)\right] + \varepsilon_r \cdot \sigma_{\ln EDP|IM} \quad (1)$$

$$\begin{aligned}\ln\left(EDP\right) = b_0 &+ \sum_{i=1}^{n}\left[b_i \cdot \ln\left(IM_i\right)\right] + \frac{1}{2}\cdot\sum_{i=1}^{n}\sum_{j=1}^{n}\left[b_{ij} \cdot \ln\left(IM_i\right)\cdot \ln\left(IM_j\right)\right] \\ &+ \frac{1}{2}\cdot\sum_{i=1}^{n}\sum_{j=1}^{n}\left[b_{ii} \cdot \ln\left(IM_i\right)\right]^2 + \varepsilon_r \cdot \sigma_{\ln EDP|IM}\end{aligned} \quad (2)$$

In these equations the b_i values are the regression parameters; $\sigma_{\ln EDP|IM}$ is the standard error of estimation, and ε_r is the standardized error term. The advantage of the linear regression method is that it provides a well-developed theory regarding model selection and confidence intervals for regression coefficients, but there are a few disadvantages as well. Firstly, this approach assumes homoscedasticity, namely a single standard deviation of the error for the entire data range, whereas it is shown in earlier studies (e.g., Modica and Stafford, 2014, to name one of the most recent references) that structural response in terms of MIDR is indeed heteroscedastic, which in this case means that the response variance increases with increasing IM values. Secondly, the regression model extrapolates in the data range that is not covered well by the observed data points. For example, in the case of a vector of IMs, for certain values of T_1 it is rare to have real records with a low $Sa(T_1)$ value and a high $Sa(1.5\cdot T_1)/Sa(T_1)$ value from a real ground motion. Luckily, this issue is not a very serious one because the mean rate of occurrence of such unlikely pairs of spectral acceleration values in the joint hazard is so small as to render irrelevant the perhaps inaccurate extrapolations of the model. Thirdly, the significance of each component of the vector of IMs might be different across the range of the data. For example, for low levels of $Sa(T_1)$ when the structure behaves mainly linearly, $Sa(1.5 \cdot T_1)$ is less effective in predicting the IDR response, whereas for high values of $Sa(T_1)$, when the structure is highly nonlinear, $Sa(1.5 \cdot T_1)$ has a significant predictive power on the nonlinear IDR response. This implies that IM interaction terms should be included in the multiple linear regression model (e.g., Baker, 2007) when used for the vector IM cases. Modica and Stafford (2014), in fact, used a quadratic functional form consisting not only of interaction terms, $\ln(IM_i) \cdot \ln(IM_j)$, but also of quadratic ones, $\left[\ln(IM_i)\right]^2$, when assembling the prediction model for estimating EDP*s*.

An alternative method for response estimation is the non-parametric running median (or in general running quantile) approach. In this method, the median of a moving window of the data is computed and the standard deviation is obtained using 84th and the 16th percentiles of the residuals with the assumption of the normal or lognormal distribution of the data (Vamvatsikos and Cornell, 2005). The primary advantages of this method are that it provides a standard deviation that varies across the data range, which is more faithful to the data, and that it does not need a parametric assumption for the error term. In addition, because it uses a quantile of the data (e.g., the 50% quantile for median), it can deal with collapse data points and non-collapse data points at the same time. Although appealing, this method may work well for scalar IMs but it becomes impractical as the number of components of the vector IM increases (e.g. more than 2) since the data points tend to be sparse in a multi-dimensional space. Hence, the need for fitting a model becomes unavoidable.

IM EFFICIENCY FOR INTER STORY DRIFT RATIOS

Figure **5** and

Figure **6** compare the response dispersion profiles expressed in terms of IDRX, IDRY and PFA for the three buildings computed using different IMs. Note that, as explained earlier, the linear regression analysis of the response data points provides single dispersion at all IM levels. As such, in these figures the dispersion is not related to a specific IM level. The lowest dispersion of IDR/IM in the lower stories and in almost all cases and for both directions is provided by the vector Sa_{V1}. The decrease in dispersion of IDRX, IDRY given Sa_{V1} compared to the simplest scalars Sa_{S1} and Sa_{S2} is significant, whereas it is negligible when compared against vectors Sa_{V3} and Sa_{V4} for the 3-story building and moderate for the 5- and 8-story buildings. This could be explained by the fact that, the averaged spectral accelerations used as components of Sa_{V3} and Sa_{V4} indiscriminately combine Sa values at multiple periods, thus introducing a slight disadvantage for the taller structures.

Figure 5. Comparison of the dispersion $\sigma_{\ln EDP|IM}$ profiles of IDRX, IDRY using different IMs for the 3-, 5- and 8-story buildings (note: Single dispersion is estimated for all IM levels)

For instance, in an average Sa, generally indicated as Sa_{avg} henceforth, incorporating only two spectral accelerations at T_1 and $1.5\cdot T_1$, one record with $Sa(T_1)$=0.8g and $Sa(1.5\cdot T_1)$=0.4g will provide the same value of Sa_{avg} of a record with the values switched (i.e. $Sa(T_1)$=0.4g and $Sa(1.5\cdot T_1)$=0.8g), even though the response of the structure to these two records will be different. Therefore, a vector with separated spectral accelerations such as Sa_{V1} is expected to show a better performance in this case, as can be observed from the results here. The only saving grace of such indiscriminate averaging is the relative scarcity of one of the two equal- Sa_{avg} pairs (as discussed earlier). In addition, given the concentration of the nonlinearity in the lower floors for our case-studies, the IDR response of these floors is highly correlated with the spectral accelerations at elongated periods and less with the ones lower than T_1. That is why, even if Sa_{V1} does not reflect the spectral accelerations at the low range periods, it remains the best predictor of the response in the lower floors. In upper floors, on the other hand, where the higher mode effects become more important, Sa_{V1} slightly loses its effectiveness due to its lack of spectral accelerations at lower periods whereas Sa_{V3} and Sa_{V4} which contain such terms, become superior. Sa_{V2} being a vector IM, is similarly more effective compared to its corresponding scalar, Sa_{S2}. On the other hand, it is less efficient compared to other vector IMs used here, something that may be attributed to its use of the geometric mean Sa at each period rather than of the arbitrary component S_a.

Among the scalar IMs, Sa_{S3} and Sa_{S4} are superior to Sa_{S1} and Sa_{S2}. Sa_{S2} shows specifically a poor performance for the 8-story building with widely different periods in the two orthogonal directions (1.3s and 0.44s for X and Y directions, respectively). The difference in the dispersion estimated by Sa_{S3} and Sa_{S4} is very small which suggests the superiority of Sa_{S3} to Sa_{S4} because it is a simpler application of Sa_{avg} with only 6-components compared to 20 components of Sa_{S4}. It should be highlighted that there can be cases where the more complex Sa_{S4} becomes a better solution. For instance, for structures with multiple important higher modes, providing more weight to spectral accelerations at periods lower than T_1, essentially helps to improve the efficiency of this IM for such structures. The periods used in Sa_{S4} and Sa_{V4} are equally spaced between the minimum and maximum periods in this study, providing almost equal number of periods lower and higher than T_1, and consequently giving the same weight to the nonlinear elongated and linear higher mode response. On the other hand, using equally-spaced periods in a logarithmic scale will give more weight to periods lower than T_1. Such an IM could be more effective for structures with relatively significant importance of higher modes or for the estimation of PFA as discussed in the next section.

IM EFFICIENCY FOR PEAK FLOOR ACCELERATION

PGA is a superior predictor of PFA at the ground and maybe some additional lower floors than any other IM tested here, but it becomes progressively less effective with height. This could be simply explained by an example of a single-story single-degree-of-freedom (SDOF) system in which the PFA at the ground floor matches the PGA, while by definition it matches $Sa(T_1)$ at the level of the roof. This is why PGA will always be the best predictor for the ground floor. Recall that the PFA results shown here are the SRSS of the values for the two orthogonal directions of the building. This is the reason why a small dispersion of 10% is observed at the ground floor, since the PGA used as IM is based on the geometric mean (rather than the SRSS) of the two record components regardless of the times when the peaks occur. $Sa(T_1)$, on the other hand, is one of the worst IMs at the ground floor and its efficiency progressively improves with height. This might suggest the idea of using a vector IM including both PGA and $Sa(T_1)$, to cover the efficiency at the lower and upper floors for PFA prediction. However, since PGA is not a good predictor of IDRs for any but the shortest-period buildings, such a vector IM will not be globally effective, unless, for example the estimation of losses at the lowest floors is deemed to be the most significant contributor to losses.

Another interesting point is that PFA is more closely related to the seismic forces applied to the structure than deflections. When the structural ductility after yielding increases and the stiffness reduces, the seismic forces stabilize and do not increase appreciably, akin to an isolation effect. Therefore, the PFA values in most analyses are observed at a time or IM level where the structure is still in the linear elastic regime or close to it. This fact is even more emphasized for RC infill frames, such as those analyzed here, in which the stiffness of the structure is initially high due to the presence of the infill panels but decreases abruptly after they fail. This can explain the vector Sa_{V1} not providing considerable improvements compared to the corresponding scalar Sa_{S1}, since Sa_{V1} is more appropriate for nonlinear response prediction as was shown in the previous section. Scalars Sa_{S3}, Sa_{S4} and their corresponding vector IMs of Sa_{V3}, Sa_{V4} are fairly efficient IMs for PFA prediction and perform favorably well all along the height and for all the buildings tested here.

Figure 6. Comparison of the dispersion, $\sigma_{\ln EDP|IM}$, profiles of PFA using different IMs for the 3-, 5- and 8-story buildings (note: uniform dispersion is estimated for all IM levels)

As explained earlier, these IMs could become even more effective in PFA prediction by giving more weight to the spectral acceleration at periods lower than T_1. However, by doing so, they will lose some of their efficiency in IDR response estimation. This fact could be seen well expressed by the slight improvement of Sa_{S4} and Sa_{V4} compared to Sa_{S3} and Sa_{V3}, as they contain spectral acceleration at several more periods lower than T_1. Having said that, it should also be emphasized that for the vector IMs in case of Sa_{V3}, Sa_{V4} there is no traceable improvement compared to their counterpart scalar IMs of Sa_{S3}, Sa_{S4}. The reason is related to the fact that PFA here is an SRSS of the values at two main orthogonal directions of the structure, therefore, separating the excitation for X and Y directions apparently does not help to improve their efficiency. This fact can also explain the effectiveness of Sa_{V2} in the lower floors. This IM contains the geometric mean of spectral acceleration at $0.2\cdot\overline{T}_1$, which is very close to the geometric mean of PGA, the top IM for PFA at the ground floor.

RISK ASSESSMENT

Following Shome and Cornell, 1999, the rate of exceeding different values of an *EDP*, $\lambda(EDP > edp)$, can be computed using the conditional complementary cumulative distribution function of *EDP|IM* for the non-collapsed data, $\mathrm{P}(EDP > edp \mid \mathrm{NC}, IM)$, and the probability of collapse given IM, $P_{col|IM}$, along with the rate of occurrence of the scalar or vector IM of interest, $\lambda(IM)$, formally:

$$\lambda(EDP > edp) = \int \left[\mathrm{P}\left(EDP > edp \mid \mathrm{NC}, IM\right)\cdot\left(1 - P_{col|IM}\right) + P_{col|IM} \right]\cdot\left|d\,\lambda IM\right| \tag{3}$$

Logistic regression (Kutner et al., 2004) was used to compute the probability of collapse for each IM level while linear regression (Equation (1)) was used to model $P\left(EDP > edp \mid \mathrm{NC}, IM\right)$. As an example of the results obtained,

Figure **7** shows the response hazard curves for MIDRX and for maximum PFA all along the height for the 5-story building. The results for all other EDPs are similar. In this particular example, MIDRX values in the order of 3 to 4% are associated with collapse occurrence estimates of mean annual rates. The latter vary by an order of magnitude among the different IMs, that is from 2×10^{-5} (when Sa_{S4} was used as response predictor) to 2×10^{-4} (for Sa_{S1}). In theory, though, since even the lowly Sa_{S1} is riding on the back of careful CS-based record selection, there is no obvious argument that would lead us to prefer one estimate over another. Still, the remarkable consistency in the estimates provided by the vector IMs does lend some credence to the notion that they probably represent a more accurate estimate. Until further research provides a more concrete answer, we cannot assign bias to any of these bonafide estimates: We are bound to uniformly treat the entirety of the variance shown in Figure 7 as a product of epistemic uncertainty.

Figure 7. Comparison of response hazard curves obtained using different scalar IMs and a vector of IMs for the 5-story building: a) MIDRX, b) maximum PFA along the building height.

The fidelity of the linear regression analysis was checked for various IMs based on the confidence intervals of regression coefficients and their corresponding p-values. All the parameters used in the regression analysis were thus shown to be significant with few exceptions. For instance, for the 8-story building the elongated period in Y direction, $Sa_y\left(\alpha_u \cdot T_{1y}\right)/Sa_x\left(\alpha_u \cdot T_{1x}\right)$, was shown to be insignificant in predicting MIDRX when Sa_{V1} is the IM. It should be noted that using spectral accelerations instead of the ratios of spectral accelerations in Sa_{V1} and Sa_{V2} would have led to wider confidence intervals for many of these coefficients making them less effective in response estimation. In addition, the results of the EDP hazard curves using regression with linear and quadratic terms showed some small differences in the low ranges of IDR, differences that are due to the effect of the interaction terms as explained previously.

DISCUSSION

Based on the data produced in this study and by looking at the results, only part of which is shown here, the following observations can be made:

- In general, one cannot claim with certainty which of the approaches applied provides the most accurate risk-based assessment and consequently is the most reliable method to be used for loss estimation; however, it is legitimate to expect that, given the lower dispersions suggested by some of the scalar or vector IMs used here, those IMs could be considered to be better options.

- We could say that among the applied scalar IMs, the ones based on Sa_{avg} are preferred. However, for asymmetric buildings or buildings with well-separated periods into the two orthogonal directions, like the 3- and 8-story buildings herein, other vector IMs, such as $\{Sa_{avgX}, Sa_{avgY}\}$, could provide a better solution. Such a vector IM consisting of two components is easier to compute and more practical to handle (e.g., in data fitting and programming) compared to three or four element vectors.
- A multi-element vector IM, such as Sa_{V1} can better discern the contribution of separate spectral ordinates, thus it should be a more effective IM compared to the simpler vectors or scalars based on the average of the same or similar ordinates, such as Sa_{S3}, Sa_{S4}, Sa_{V3} and Sa_{V4}. However, the addition of further spectral ordinates is easier to handle with scalar or two-component vector IMs using averaging, rather than adding too many elements in a vector. Computing a scalar hazard curve for spectral averages would not grow appreciably more complex with the number of spectral ordinates, while computing a joint hazard for a vector IM with more than 5 components, although theoretically doable, is practically cumbersome, computationally intensive, and prone to numerical inaccuracies. In addition, the " curse of dimensionality" will haunt model fitting in multiple dimensions via Equations (1) and (2), as the inherent scarcity of data in multi-element vector spaces will eventually defeat any attempt to properly represent the IM-EDP relationship with the required detail.
- As a corollary, the higher efficiency of vector IMs, contrarily to what it implies for scalars (Luco and Cornell, 2007), should not necessarily mean that fewer records could be used in nonlinear dynamic analysis for achieving the same accuracy in the EDP estimates. The complexity of fitting a regression model using more than two predictors suggests always using a reasonably high number of analyses and records in order to provide reliable results. It can be stated here, however, that the smaller dispersion of EDPs given vector IMs is more likely to produce response estimates

and, in turn, risk estimates that are not biased compared to those achieved using IMs that show a higher level of dispersion.

- Among the IMs tested, the response hazard curves based on different vector IMs are more consistent, showing only a small variation among them, whereas the scalar IMs produce less consistent results that more widely vary around their mean. While the best scalar candidates can get close to the performance of the vectors, some very bad scalar choices are obviously available that will lead the assessment to erroneous results. The MIDRX, MIDRY and maximum PFA (along the height) hazard curves of the individual (dotted lines) and the mean of all (solid line) are shown in
- Figure **8** in the left panels. In the right panels the Coefficient of Variation (C.o.V) of the MAR of response exceedance for all three buildings is presented. Clearly, the closer the building is to collapse, the more uncertain the result.

Figure 8. Left panels: Response hazard curves (Dotted lines: individual IM results; solid lines: mean), Right panels: Coefficient of Variation (C.o.V) of the response exceeding rate for three building cases tested here (blue = 3-story, red = 5-story, green = 8-story).

CONCLUSIONS

An ideal IM for 3D structures should be efficient in response prediction at any story within the building at both linear and nonlinear states of the structure. For a linear SDOF system or for a linear first mode dominated building, $Sa(T_1)$ is an appropriate IM. However, as the structure becomes nonlinear, the spectral acceleration at longer periods is needed. For MDOF systems, such as the ones tested herein, the effect of higher modes and spectral shape on the response becomes important. In addition, for a 3D structural model, with coupled response in two orthogonal directions, this IM should contain separated information about the excitations in both directions. Moreover, such an ideal IM should have fairly balanced predictive potential for different structural response types such as IDR and PFA and work well all along the height of the building. As a scalar IM, average spectral acceleration is shown to be an appropriate IM for response prediction of both PFA and IDR. However, we observed here that its efficiency is relatively lower for 3-D asymmetric buildings or buildings with well separated periods in two main orthogonal directions. As such, a superior approach is offered that considers the average spectral acceleration of two orthogonal directions in a two-component vector IM. Such an IM, at least for the examples considered here, can enhance all of the advantages mentioned earlier for 3D buildings in terms of PFA and IDR. The use of a vector IM, however, comes at a price since vector hazard estimation needs to be performed rather than the routine scalar PSHA for carrying out long-term response hazard or loss calculations. This vector IM route is more accessible if one uses the indirect method to vector hazard analysis, discussed in the companion paper (Kohrangi et al. 2015), rather than its original formulation.

ACKNOWLEDGEMENTS

We would like to thank Dr. Marco Pagani for his precious help in carrying out the PSHA analysis for this study. We hereby acknowledge that the building data used was in part provided by Prof. Fabrizio Mollaioli, Prof. Rui Pinho and Romain Sousa. Partial support was provided to the third author by the European Research Executive Agency via Marie Curie grant PCIG09-GA-2011-293855.

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