Vector and Scalar IMs in Structural Response Estimation: Part I - Hazard Analysis

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A realistic assessment of building economic losses and collapse induced by earthquakes requires monitoring several response measures both story-specific and global. The prediction of such response measures benefits from using multiple ground motion intensity measures (IMs) that are, in general, correlated. To allow the inclusion of multiple IMs in the risk assessment process it is necessary to have a practical tool that computes the vector-valued hazard of all such IMs at the building site. In this paper, Vector-valued Probabilistic Seismic Hazard Analysis (VPSHA) is implemented here as a post processor to scalar PSHA results. A group of candidate scalar and vector IMs based on spectral acceleration values, ratios of spectral acceleration values, and spectral accelerations averaged over a period range are defined and their hazard evaluated. These IMs are used as structural response predictors of 3D models of reinforced concrete buildings described in a companion paper (Kohrangi et al., 2015b).

INTRODUCTION

Performance-based earthquake engineering (PBEE) (Cornell and Krawinkler, 2000) has become commonplace in the industry for assessing response of buildings and other structures subjected to seismic loading. Studies based on PBEE are now routinely used by a variety of stakeholders such as building owners, developers, insurers, lending institution and earthquake

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engineers. For instance, owners of important buildings use it to make critical decisions about buying an appropriate level of earthquake insurance or identifying a retrofitting solution. Engineers use it for designing structural components to withstand forces and control displacements induced by target design ground motions with a margin of safety consistent with well performing, code-compliant structures. Regardless of the specific PBEE application, it is critical that estimates of the likelihood that a structure's response exceeds a given level of severity, ranging from onset of damage to incipient collapse, be as accurate as reasonably possible.

To increase the accuracy of estimating the structure's response, engineers have taken advantage of the computational capabilities of modern computers by developing more realistic two-dimensional (2D) and three-dimensional (3D) numerical models. These computer models are subjected to many different ground motions of different intensities to assess the structure's performance. Statistical techniques are typically used to provide functional relationships between the IMs of the ground motion and response measures that are associated with required levels of performance (e.g., operational, life safety or collapse).

The response of such complex models, however, is better estimated by monitoring multiple response measures, which are often referred to as Engineering Demand Parameters (EDPs). In turn, estimates of the maximum values of these measures are better predicted by a pool of IMs of the ground motion in both horizontal (and sometimes vertical) directions rather than by a single IM. For example, a good predictor of Maximum Inter Story Drift Ratio in the X- direction of a building (MIDRX) may be the spectral acceleration at the first period of vibration, T_{1x}, of the structure in its X-direction, $Sa_x(T_{1x})$; and similarly, $Sa_y(T_{1y})$ is a good predictor for MIDRY, where T_{1y} is the first period of vibration of the structure in its Y-direction. The collapse of a building, however, is more likely to happen when both MIDRX and MIDRY and, therefore, $Sa_x(T_{1x})$ and $Sa_y(T_{1y})$, are large rather than when either one is large. In addition, damage to structural, non-structural components and equipment of a building are better estimated by different EDP types (e.g. peak floor spectral ordinate and maximum inter story drifts), whose estimation is better served by utilizing different appropriate IMs.

If EDPs are estimated via multiple IMs, the long-term risk computations require the convolution of IMs versus EDPs relationships (FEMA-P-58 2012) and, therefore, the knowledge of the joint hazard probability distributions of the (generally correlated) IMs at the building site. The methodology for computing the joint hazard was first introduced in 1998 and was called vector-PSHA (Bazzurro, 1998; Bazzurro and Cornell, 2001 and 2002) or VPSHA for short. A few software programs were developed since for such a purpose (Bazzurro, 1998; Thio, 2003; 2010; 2010) but were limited to a vector of two *IM*s and were not capable of providing the disaggregation of the joint hazard. To avoid the complexity of the joint hazard computation for a vector of IMs, researchers over the years introduced several complex scalar IMs that are combination of multiple IMs (e.g., Fajfar et al., 1990, Cordova et al., 2002, followed by Vamvatsikos and Cornell, 2005, Luco et al., 2005a; Luco and Cornell, 2007, Mehanny, 2009, Bianchini et al., 2010, Bojórquez and Iervolino, 2011). These complex IMs are often more effective in the prediction of EDPs than each single IM that compose them but arguably less effective than considering a vector of those IMs in the response prediction.

To help promoting the use of VPSHA, a methodology was developed and implemented (Bazzurro et al., 2009 and 2010) that allows the computation of the joint hazard using results from any standard scalar PSHA software. This “indirect” approach to VPSHA is more computationally efficient than the original VPSHA “direct” integration method. It also has a major advantage over the direct integration method: it can accommodate a higher number of Random Variables (RVs) without significant loss of joint hazard accuracy.

In this paper, we review the direct and indirect VPSHA methodologies and elaborate on the pros and cons of each. The “indirect” method is then used to compute the VPSHA for a set of IMs in terms of spectral acceleration and average spectral acceleration for a site close to Istanbul based on the scalar PSHA results computed using the software OpenQuake. In the companion paper (Kohrangi et al., 2015), these PSHA and VPSHA results are used to perform a risk-based assessment of three 3D models of reinforced concrete infilled frame buildings of 3-, 5- and 8-stories typical of the European Mediterranean countries.

VECTOR-VALUED PROBABILISTIC SEISMIC HAZARD ANALYSIS (VPSHA)

As mentioned earlier, the original methodology for computing the joint hazard of multiple ground motion IMs (e.g., Peak Ground Acceleration, PGA, and S_a(1.0s)), which are dependent RVs (Bazzurro, 1998; Bazzurro and Cornell, 2001 and 2002), is based on direct integration of the joint probability density function (pdf) of the same IMs at a site caused by each earthquake considered in the analysis. The joint distribution of correlated IMs at a site, which can be modeled as a multivariate Gaussian distribution if the IMs are represented by their natural logarithms (Jayaram and Baker, 2008), is computed separately for each earthquake scenario. The total hazard is obtained by summing the contributions from all scenarios weighted by their occurrence rates. This method contains no approximation besides the implicit numerical accuracy of the integration solver. This so-called "direct method" is considered in this study only to obtain a set of joint hazard results for the many ground motion IMs considered. These results are used as a benchmark to validate the results from the indirect method.

The joint Gaussian pdf conditional on the parameters of the earthquake (i.e., magnitude M, source-to-site distance R, number of standard deviations from the mean GMPE prediction, ε, the rupture mechanism, and the soil conditions) can be computed when ground motion prediction Equations (GMPEs) are available for the IMs involved and with the knowledge of their variance-covariance matrix. Inoue (1990) and, more recently, Baker and Jayaram (2008), Goda and Hong (2008) and Akkar et al. (2014), have empirically derived the correlation structure for spectral accelerations with different periods and different record component orientations. Figure 1(a) shows one example of such empirical correlation structure. In addition, Bradley (2011a, 2011b, 2012a, 2012b) obtained empirical correlations between a few alternative IMs, such as Peak Ground Velocity (PGV), cumulative intensity measures, and ground motion duration. For example, Figure 1(b) shows the contours of the joint pdf for S_a(1.0s) and S_a(0.3s) for a site with Vs_{30}=760 m/s located 7km from a M_w=7.3 event with a strike-slip mechanism as predicted by the GMPE by Boore and Atkinson (2008). According to Baker and Jayaram (2008), the correlation coefficient for S_a(1.0s) and S_a(0.3s) is 0.5735 for this particular case. Although conceptually straightforward, direct integration is numerically challenging, especially when (a) high precision in the tails of the distribution is sought; (b) the number of earthquake scenarios is large, which is usually the case in realistic applications; and (c) the number of IMs exceeds three or four. In fact, to our best knowledge of direct-VPSHA codes in existence, the only software capable of carrying out the computations for more than two RVs is documented in Bazzurro et al. (2010) and all the previous studies are limited to only two RVs (Bazzurro, 1998; Thio, 2003; Gülerce and Abrahamson, 2010). As a consequence, the so-called *direct approach*, due to its complexity and heavy numerical computations, has not been used much so far in the scientific and engineering communities. In the computational efforts in the direct method, one approach would be application of the Monte Carlo simulation. For instance, Bazzurro et al. 2010 used an integration algorithm based on a quasi-Monte Carlo simulation developed by Genz and Bertz, 1999, 2002. Although, these integration techniques seem appealing, still it might not, in any way, alleviate the computational burden of the direct method. To overcome this hurdle, Bazzurro et al. (2009) proposed an alternative approach for the calculation of VPSHA based on processing only the results of available scalar PSHA codes. This is what we called here the *indirect method,* which is discussed in the next section.

Figure 1. (a) The variance-covariance structure of log spectral accelerations at different periods in a random horizontal component of a ground motion record (Jayaram and Baker, 2008); (b) the joint pdf for *Sa*(1.0s) and *Sa*(0.3s) for a given scenario earthquake (adopted from Bazzurro et al., 2010).

INDIRECT APPROACH TO VPSHA

Under the rational of joint normality of log IMs (Jayaram and Baker, 2008), the joint Mean Rate Density, MRD (for definition and details, see Bazzurro and Cornell, 2002) or, similarly, the Mean Annual Rate (MAR) of occurrence of any combination of values of a pool of ground motion IMs could be computed only with the knowledge of the following items (Bazzurro et al., 2009):

1. ***Site-specific seismic hazard curves of the ground motion IMs considered in the vector—*** The vector of ground motion IMs is denoted herein as ***S***. This vector could include, for example, three parameters: the spectral acceleration at two different periods in one of the horizontal directions, and at one period in the orthogonal horizontal direction. These periods could correspond, for example, to the first and second mode of

vibrations of a building in the longitudinal directions and the first mode in the transverse direction. The three hazard curves corresponding to these periods can be obtained with any standard PSHA code.

2. ***The variance-covariance matrix of all the ground motion IMs*****—**Empirical estimates of this variance-covariance matrix are available in the literature as discussed in previous section (see the reference list for some such studies).
3. ***The disaggregation results from scalar PSHA* —**The joint distributions of all the basic variables, **X**, including M, R, ε, the style of faulting, the distance to the top of the co-seismic rupture, and all other variables required by the GMPE of choice that contribute to the joint occurrence of specific values of IMs at the site. This is a straightforward extension of the disaggregation results routinely available from standard scalar PSHA codes.

For brevity, following Bazzurro et al. (2009) the details of the methodology are presented below only for the case of three IMs that, in this specific case, are spectral accelerations. However, this approach, which requires some straightforward matrix algebra, is scalable to a larger number of (RVs) and can include any other ground motion parameters (e.g., ground motion duration, near-source forward-directivity pulse period, Arias intensity and cumulative absolute velocity) if the proper correlation structure and prediction equations are available. For simplicity, in the derivations below the RVs are treated as discrete rather than continuous quantities.

Let $\boldsymbol{S}=[Sa_1;Sa_2;Sa_3]$ denote the vector of RVs for which we seek to obtain the joint hazard expressed by the mean annual rate of occurrence of the three spectral acceleration quantities Sa_1, Sa_2 and Sa_3 in the neighborhood of any combination of three spectral acceleration values a_1, a_2 and a_3, respectively. Mathematically, this is $\text{MAR}[Sa_1;\ Sa_2;\ Sa_3] = \text{MAR}_{Sa1;\ Sa2;\ Sa3}[a_1;\ a_2;\ a_3]$. Note that Sa_1, Sa_2 and Sa_3 represent here the natural logarithm of the spectral accelerations but the logarithm operator has been dropped to avoid lengthy notations. The quantity $\text{MAR}[Sa_1;\ Sa_2;\ Sa_3]$ could, for example, denote the Mean Annual Rate (MAR) of observing at a building site values in the neighborhood of (the natural logarithm of) 1.0g, 1.5g, and 0.8g for the spectral acceleration quantities at the periods of the first and second modes of vibration in the building longitudinal direction and the spectral acceleration at the period of the first mode in the building transverse direction. These spectral acceleration values may be related to the onset of an important structural limit-state determined from a statistical analysis of the response of a structure subjected to many ground motion records. Then, using the theorem of total probability, one can express the following:

$$\mathrm{MAR}[Sa_1;Sa_2;Sa_3] = \mathrm{P}[Sa_1 \,|\, Sa_2;Sa_3] \cdot \mathrm{P}[Sa_2 \,|\, Sa_3] \cdot \mathrm{MAR}[Sa_3]\,, \tag{1}$$

where:

$$\mathrm{P}[Sa_1 \,|\, Sa_2;Sa_3] = \sum_{\mathbf{X}} \mathrm{P}[Sa_1 \,|\, Sa_2;Sa_3;\mathbf{X}] \cdot \mathrm{P}[\mathbf{X} \,|\, Sa_2;Sa_3] \tag{2}$$

Equation (2) represents the conditional distribution of Sa_1, Sa_2 and Sa_3. This term can be numerically computed by conditioning it to the pool of variables **X** in a standard PSHA that appear in the selected GMPE and integrating over all possible values of **X**, as shown on the right hand side of Equation (2). Exploiting the joint log normality of ***S***, for every possible value of **X**, the quantity P[Sa_1; Sa_2; Sa_3] can be computed simply with the knowledge of the variance-covariance matrix of Sa_1, Sa_2 and Sa_3 and the GMPE of choice. Further details on the mathematics are provided below. P[**X** | Sa_2; Sa_3] can be obtained via disaggregation and Bayes theorem as follows:

$$\mathrm{P}[\mathbf{X} \,|\, Sa_2;Sa_3] = \frac{\mathrm{P}[\mathbf{X}, Sa_2 \,|\, Sa_3]}{\sum_{X} \mathrm{P}[Sa_2 \,|\, Sa_3;\mathbf{X}] \cdot \mathrm{P}[\mathbf{X} \,|\, Sa_3]} = \frac{\mathrm{P}[Sa_2 \,|\, Sa_3;\mathbf{X}] \cdot \mathrm{P}[\mathbf{X} \,|\, Sa_3]}{\mathrm{P}[Sa_2 \,|\, Sa_3]} \tag{3}$$

Where P[**X** | Sa_3] can be derived using conventional scalar PSHA disaggregation. P[Sa_2 | Sa_3; **X**], as for the P[Sa_1 | Sa_2; Sa_3; **X**] term above, can be computed with the knowledge of the variance-covariance matrix of Sa_2 and Sa_3 and the adopted GMPE. $\sum_{\mathbf{X}} \mathrm{P}[Sa_2 \,|\, Sa_3;\mathbf{X}] \cdot \mathrm{P}[\mathbf{X} \,|\, Sa_3]$ can be evaluated as explained above. MAR[Sa_3] is the absolute value of the discretized differential of the conventional seismic hazard curve for the scalar quantity Sa_3 at the site. After some simplifications, Equation (1) can be rewritten as follows:

$$\mathrm{MAR}[Sa_1;Sa_2;Sa_3] = \mathrm{MAR}[Sa_3] \cdot \sum_{\mathbf{X}} \mathrm{P}[Sa_1 \,|\, Sa_2;Sa_3;\mathbf{X}].\mathrm{P}[Sa_2 \,|\, Sa_3;\mathbf{X}] \cdot \mathrm{P}[\mathbf{X} \,|\, Sa_3] \tag{4}$$

The two first conditional terms in Equation (4) (i.e., P[Sa_1 | Sa_2; Sa_3; **X**] and P[Sa_2 | Sa_3; **X**] can be evaluated using the multivariate normal distribution theorem. In general, if ***S*** = [Sa_1 , Sa_2, …, Sa_n]^T is the vector of the natural logarithm of the random variables for which the joint hazard is sought, then ***S*** is joint normally distributed with mean, μ, and variance-covariance matrix, **Σ**, i.e., in mathematical terms ***S*** =~N($\boldsymbol{\mu}$, ∑). If ***S*** is partitioned into two vectors, $\boldsymbol{S_1} = [Sa_1\ ;\ Sa_2;\ \ldots,\ Sa_k]^T$ and $\boldsymbol{S_1} = [Sa_{k+1}\ ;\ Sa_{k+2};\ \ldots,\ Sa_n]^T$, where $\boldsymbol{S_2}$ comprises the conditioning variables (in the example above $\boldsymbol{S_1} = [Sa_1]^T$ and $\boldsymbol{S_2} = [Sa_{2,}\ Sa_3\ ]^T$), one can write the following:

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_1 \\ \boldsymbol{S}_2 \end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{bmatrix} \right) \qquad (5)$$

For jointly normal distribution, the conditional mean and variance can be determined as:

$$\boldsymbol{S}_1 \mid \boldsymbol{S}_2 \sim \mathrm{N}(\boldsymbol{\mu}_{1|2}, \boldsymbol{\Sigma}_{1|2}), \qquad (6)$$

$$\boldsymbol{\mu}_{1|2} = \boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1} \cdot (\mathbf{S}_2 - \boldsymbol{\mu}_2); \boldsymbol{\Sigma}_{1|2} = \boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}\boldsymbol{\Sigma}_{21} \qquad (7)$$

Equation (1) can be generalized to *n* variables as follows:

$$\begin{aligned} &\mathrm{MAR}[Sa_1; Sa_2; Sa_3; \ldots; Sa_{n\text{-}1}; Sa_n] = \\ &\sum_X \mathrm{P}[Sa_1 \mid Sa_2; Sa_3; \ldots; Sa_{n\text{-}1}; Sa_n; \mathbf{X}] \cdot \mathrm{P}[Sa_2 \mid Sa_3; \ldots; Sa_{n\text{-}1}; Sa_n; \mathbf{X}] \\ &\cdot \mathrm{P}[Sa_3 \mid Sa_4; \ldots; Sa_{n\text{-}1}; Sa_n; \mathbf{X}] \cdot \mathrm{P}[Sa_{n\text{-}1} \mid Sa_n; \mathbf{X}] \cdot \mathrm{MAR}[Sa_n] \end{aligned} \qquad (8)$$

DIRECT VERSUS INDIRECT APPROACHES

Bazzurro et al. (2010) performed a series of comparison tests between the results obtained by both "direct" and "indirect" VPSHA. That study shows that, while both methods have their respective strengths and weaknesses, the indirect method has several qualities that, arguably, make it superior to the direct integration method. The advantages of the indirect method are:

1) its implementation does not require much modification of already existing scalar PSHA codes;
2) It can compute the joint hazard for a higher number of IMs than the direct method;
3) it is computationally faster than the direct method for two reasons. First, integrating multivariate standard normal distributions with three or more dimensions with very high accuracy is typically an extremely time consuming task. It should be noted that the indirect method has also mathematical challenges, such as matrix inversions, which, however, require considerably lower computation time. Second, in the direct method multi-dimension integration needs to be

repeated for every earthquake considered in the PSHA. In the indirect method, the number of events affects only the total run time of the scalar hazard analyses, which is negligible when compared to the total run time of a comparable joint hazard study.

4) It is easily scalable to higher dimensions of variables;

5) given its recursive nature, when adding the *n*-th dimension, the indirect method can re-use results previously computed for the first *n*-1 dimensions. Conversely, adding an additional dimension in the direct method requires restarting the hazard analysis.

In fairness, the "indirect" method has also some weaknesses such as:

1) it requires larger computer memory space than the direct method;

2) It yields results that are approximate when the number of bins used to discretize the domains of the RVs is limited, a restriction which becomes a necessity in applications with four or more IMs. However, a judicious selection of bins guided by disaggregation results can limit the error in the estimates of the joint and marginal MARs to values typically lower than 3% for the entire range of IMs of engineering significance (Bazzurro et al., 2010).

In light of the considerations above, the "Indirect VPSHA" methodology is applied herein to evaluate the joint hazard of vectors of IMs that contain average spectral accelerations over a period range and ratios of spectral accelerations at different periods. The definition of such IMs and the technicalities needed for their inclusion in the VPSHA framework are presented in the sections below.

AVERAGE SPECTRAL ACCELERATION

The average spectral acceleration, Sa_{avg}, is a complex scalar IM that is defined as the geometric mean of the log spectral accelerations at a set of periods of interest (Cordova et al., 2000; Bianchini et al., 2010). These periods, for example, could be equally spaced in the range from $0.2 \cdot T_1$ to $2 \cdot T_1$, where T_1 is the first-mode elastic period of the structure. This array of periods might cover higher mode response and also the structural period elongation caused by the nonlinear behavior due to the accumulation of damage. Alternatively, perhaps more effectively, Sa_{avg} could be defined as the geometric mean of log spectral accelerations at relevant vibration periods of the structure, such as T_{1x}, T_{1y}, T_{2x}, T_{2y}, $1.5 \cdot T_{1x}$, and $1.5 \cdot T_{1y}$, where x and y refer to the two main orthogonal axes of the buildings and the indices 1 and 2 refer to the first and second vibration modes of the structure in those directions. Mathematically, Sa_{avg} can be defined in the following two equivalent ways:

$$Sa_{avg} = \left[\prod_{i=1}^{n} Sa(T_i) \right]^{1/n}, \tag{9}$$

$$\ln(Sa_{avg}) = \left(\frac{1}{n} \right) \cdot \sum_{i=1}^{n} \ln(Sa(T_i)) \tag{10}$$

Therefore, from Equation (10) it is clear that the mean and variance of $\ln(Sa_{avg})$are:

$$\mu_{\ln(Sa_{avg})} = \left(\frac{1}{n} \right) \cdot \sum_{i=1}^{n} \mu_{\ln(Sa(T_i))}, \tag{11}$$

$$\mathrm{var}(\ln Sa_{avg}) = \left(\frac{1}{n} \right)^2 \cdot \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{\ln Sa(T_i),\ln Sa(T_j)} \cdot \sigma_{\ln Sa(T_i)} \cdot \sigma_{\ln Sa(T_j)} \tag{12}$$

where $\mu_{\ln Sa(Ti)}$ and $\sigma_{\ln Sa(Ti)}$ are the logarithmic mean and standard deviation of spectral accelerations at the *i*-th period obtained from a standard GMPE and $\rho_{\ln Sa(Ti),\ln Sa(Tj)}$ is the correlation coefficient between $\ln Sa(Ti)$ and $\ln Sa(Tj)$. The correlation coefficient of two average spectral acceleration at two orthogonal directions, $Sa_{avgX} = \left[\prod_{i=1}^{n} \left(Sa_x(T_i) \right) \right]^{1/n}$ and $Sa_{avgY} = \left[\prod_{j=1}^{m} \left(Sa_y(T_j) \right) \right]^{1/m}$, could be computed as follows:

$$\rho_{\ln Sa_{avgX},\ln Sa_{avgY}} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} \rho_{\ln Sa_x(T_{ix}),\ln Sa_y(T_{jy})} \cdot \sigma_{\ln Sa_x(T_{ix})} \cdot \sigma_{\ln Say(T_{iy})}}{m \cdot n \cdot \sigma_{\ln Sa_{avgX}} \cdot \sigma_{\ln Sa_{avgY}}} \tag{13}$$

SPECTRAL ACCELERATION RATIO: GMPE AND CORRELATION COEFFICIENTS

The vectors of *IM*s considered here include both spectral accelerations and also ratios of spectral accelerations at different ordinates of the spectrum. Ratios are considered to avoid any negative collinearity effects (e.g., Kutner et al., 2004) due to the presence of high correlation between spectral accelerations at different but closely spaced periods. This operation, however, requires the evaluation of correlation coefficients of ratios of spectral accelerations and spectral accelerations at different periods. Equations (14) and (15), which show such correlation coefficients, were derived based on the hypothesis of joint normality of the distribution of the logarithm of spectral accelerations.

$$\rho_{\ln[Sa(T_1)],\ln\left[\frac{Sa(T_3)}{Sa(T_2)}\right]} = \frac{\rho_{1,3}\cdot\sigma_1\cdot\sigma_3 - \rho_{1,2}\cdot\sigma_1\cdot\sigma_2}{\sigma_{3/2}}, \tag{14}$$

$$\rho_{\ln\left[\frac{Sa(T_2)}{Sa(T_1)}\right],\ln\left[\frac{Sa(T_4)}{Sa(T_3)}\right]} = \frac{\rho_{1,3}\cdot\sigma_1\cdot\sigma_3 + \rho_{2,4}\cdot\sigma_2\cdot\sigma_4 - \rho_{1,4}\cdot\sigma_1\cdot\sigma_4 - \rho_{2,3}\cdot\sigma_2\cdot\sigma_3}{\sigma_{2/1}\cdot\sigma_{4/3}} \tag{15}$$

in which $\rho_{i,j}$ is the correlation coefficient between $Sa(\mathrm{T}_i)$ and $Sa(\mathrm{T}_j)$, $\sigma_{i/j}$ is $\sigma_{\ln\left[Sa(T_i)/Sa(T_j)\right]}$, the dispersion of the spectral acceleration ratio. The mean and variance of this variable can be computed using the following equations based on a preferred GMPE and the corresponding correlation coefficients:

$$\mu_{\ln\left[\frac{Sa(T_i)}{Sa(T_j)}\right]} = \mu_{\ln Sa(T_i)} - \mu_{\ln Sa(T_j)}, \tag{16}$$

$$\sigma^2_{\ln\left[\frac{Sa(T_i)}{Sa(T_j)}\right]} = \sigma^2_{\ln Sa(T_i)} + \sigma^2_{\ln Sa(T_j)} - 2\cdot\rho_{\ln Sa(T_i),\ln Sa(T_j)}\cdot\sigma_{\ln Sa(T_i)}\cdot\sigma_{\ln Sa(T_j)} \tag{17}$$

where $\mu_{\ln Sa(Ti)}$ and $\mu_{\ln Sa(Tj)}$ are the mean logarithm (or, equivalently, the logarithm of the median) of values of $Sa(T_i)$ and $Sa(T_j)$ obtained from the GMPE.

SITE SPECIFIC SEISMIC HAZARD ANALYSIS

The OpenQuake (Monelli et al., 2012) open-source software for seismic hazard and risk assessment, developed by the Global Earthquake Model (GEM) foundation, was used to perform the seismic hazard computations. These computations are based on the area source model and the Fault Source and Background (FSBG) model (black and red lines in

Figure **2**(a), respectively) developed during the SHARE Project (Giardini et al., 2013). The former model assumes a homogeneous distribution of earthquakes in time and space. Area sources are polygons, each one comprising a region of homogeneous seismic activity. The latter model uses fault specific information, most importantly the fault slip rate, to estimate earthquake activity rates. This is different from the area source model, which uses solely the earthquake catalog to estimate the rates of occurrence of earthquakes occurring in a zone. These SHARE models were constructed via an iterative process of collecting, reviewing and updating national and regional models (Giardini et al., 2013). We adopted the GMPE proposed by Boore and Atkinson (2008).

INTENSITY MEASURES TESTED IN THIS STUDY

The group of considered scalar and vector IMs is listed in Table 1. The effectiveness of these IMs in the estimation of building EDPs is compared in the companion paper (Kohrangi et al., 2015) while herein we only address the details of the hazard analysis methodology carried out for each IM. The IMs selected here are different combinations of the predictors most commonly available to engineers, namely the elastic pseudo spectral accelerations at different periods used singularly or jointly for assessing the response of 3D buildings (as opposed to 2D models, as often done). Therefore, other more complicated nonlinear IMs, such as inelastic spectral displacement (Tothong and Cornell, 2007) are not considered here. Still, it is important to note that IMs of practically any complexity can be incorporated in the assessment without needing to rerun the structural analyses. As observed by Vamvatsikos and Cornell (2005), changing the IM is simply an exercise in post processing. On the other hand, the estimation of hazard will need to be repeated using appropriate GMPEs, which are available for all the IMs tested herein, but not necessarily for other less common ones (e.g., the so-called Fajfar Index, I_v defined in Fajfar et al., 1990).

The spectral acceleration at the first modal period of the structure, $Sa(T_1)$, termed Sa_{S1} in Table 1, is the most commonly adopted scalar IM for seismic response assessment of 2D structural models. However, the selection of the value of T_1 might not be obvious for 3D structural models of buildings especially when the first modal periods in the two main horizontal directions are significantly different.

Alternatively, the engineer may decide to carry out the assessment for each direction separately, hence disregarding the interaction between the responses of the building in the two main horizontal axes. This latter approach is often adopted with the understanding that it produces conservative results. In this context, FEMA P-58 (2012) suggests using the spectral acceleration at the average of the period in the two main horizontal orthogonal axes of the building, $\bar{T}=\left(T_{1x}+T_{1y}\right)/2$, termed Sa_{S2}. However, this approach might not be effective for structures with well-separated periods in the two horizontal axes.

Table 1. IMs considered in the response estimation

INTENSITY MEASURE (IM) **	**ABBREVIATION***
SCALAR IMs	
Natural logarithm of arbitrary spectral acceleration at the first modal period $\ln\left[Sa_x(T_{1x})\right]$ or $\ln\left[Sa_y(T_{1y})\right]$.	Sa_{S1}
Natural logarithm of the geometric mean of spectral acceleration at the average period, $\ln\left[Sa_{g.m.}\left(\bar{T}=\left(T_{1x}+T_{1y}\right)/2\right)\right]$.	Sa_{S2}
$\ln\left[Sa_x\left(\alpha_1\cdot T_{1x}\right)\cdot Sa_x\left(T_{1x}\right)\cdot Sa_x\left(\alpha_u\cdot T_{1x}\right)\cdot Sa_y\left(\alpha_1\cdot T_{1y}\right)\cdot Sa_y\left(T_{1y}\right)\cdot Sa_y\left(\alpha_u\cdot T_{1y}\right)\right]$	Sa_{S3}
$\ln\left(\left[\prod_{i=1}^{n}\left(Sa_x\left(T_{xi}\right)\right)\right]^{1/n}\right)\cdot\ln\left(\left[\prod_{j=1}^{m}\left(Sa_y\left(T_{yj}\right)\right)\right]^{1/m}\right),\ \alpha_1\cdot T_1\le T_i\le\alpha_u\cdot T_1, m=n=10$ §	Sa_{S4}
Natural logarithm of the geometric mean of Peak Ground Acceleration, $ln[PGA_{g.m.}]$	PGA
VECTOR IMs	
$\ln\left\{Sa_x(T_{1x}),\frac{Sa_y(T_{1y})}{Sa_x(T_{1x})},\frac{Sa_x(1.5\cdot T_{1x})}{Sa_y(T_{1y})},\frac{Sa_y(1.5\cdot T_{1y})}{Sa_x(1.5\cdot T_{1x})}\right\}$	Sa_{V1}
$\ln\left\{Sa_{g.m.}(\bar{T}),\frac{Sa_{g.m.}(0.5\cdot\bar{T})}{Sa_{g.m.}(\bar{T})},\frac{Sa_{g.m.}(1.5\cdot\bar{T})}{Sa_{g.m.}(0.5\cdot\bar{T})}\right\}$	Sa_{V2}
$\ln\left\{\left[Sa_x\left(\alpha_1\cdot T_{1x}\right)\cdot Sa_x\left(T_{1x}\right)\cdot Sa_x\left(\alpha_u\cdot T_{1x}\right)\right]^{1/3},\left[Sa_y\left(\alpha_1\cdot T_{1y}\right)\cdot Sa_y\left(T_{1y}\right)\cdot Sa_y\left(\alpha_u\cdot T_{1y}\right)\right]^{1/3}\right\}$	Sa_{V3}
$\ln\left\{\left(\left[\prod_{i=1}^{n}\left(Sa_x\left(T_{xi}\right)\right)\right]^{1/n}\right),\left(\left[\prod_{j=1}^{m}\left(Sa_y\left(T_{yj}\right)\right)\right]^{1/m}\right)\right\},\ \alpha_1\cdot T_1\le T_i\le\alpha_u\cdot T_1, m=n=10$	Sa_{V4}

*All the *IM*s are based on natural logarithm transformation. The notation ln is removed from the abbreviations for brevity
** α_1 is equal to 0.8, 0.2 and 0.2 for the 3-, 5- and 8-story, respectively. α_u is equal to 1.5 in all cases.
§ The periods are equally spaced.

In addition, as the structure becomes nonlinear, the structural response is more correlated with spectral acceleration at vibration periods longer than the linear elastic response at T_1. Vamvatsikos and Cornell (2005) and Baker and Cornell (2008) showed also that for tall structures one needs to account for both longer and shorter periods rather than just T_1 to appropriately describe both the inelastic response and the spectral shape (related to higher modes) expressed in terms of Maximum IDR. On the other hand, a desirable IM should be an efficient and sufficient predictor of *multiple response quantities* (i.e. IDRs and peak floor accelerations, PFAs, along the structure's height) rather than performing very well for predicting one EDP type and very poorly for predicting others. An efficient IM provides low dispersion of the predicted response given IM and a sufficient IM offers statistical independence of the response given IM from ground motion characteristics, such as magnitude, distance, etc. Efficiency helps reduce the number of time history analysis for reliable assessment of response, while sufficiency is a *sine qua non* requirement for combining PSHA with structural analysis results. See Luco and Cornell (2007) for more detailed definitions of efficiency and sufficiency. As discussed by Kazantzi and Vamvatsikos (2015) and in the companion paper (Kohrangi et al. 2015), an IM that is effective for predicting both IDR and PFA responses at all story levels should combine spectral accelerations at a wide range of periods bracketing the first mode. To this end, the hazard calculations for several scalar and vector IMs are addressed here.

Sa_{V1} and Sa_{V2} are vectors of $Sa(T_1)$ and the ratio(s) of spectral accelerations at different spectral ordinates and orientations. In Sa_{V1} the focus has been on addressing the IDR response estimation and, therefore, we utilized the arbitrary spectral acceleration component (Sa_{arb}, referred to Sa_x or Sa_y in Table 1) since it can capture the 3D response of both orthogonal directions separately. This IM, however, is expected to be less effective in PFA response estimation since it lacks information about spectral accelerations at periods consistent with higher modes of the structure. Sa_{V2}, on the other hand, is a three-component vector IM based on the geometric mean of spectral acceleration at $\bar{T}_1$ and two periods lower and higher than $\bar{T}_1$. This IM is expected to be appropriate for both IDR and PFA response prediction; however, it might fail in capturing the 3D modeling effect, as explained earlier. Two scalar IMs in the form of average spectral acceleration (Sa_{S3} and Sa_{S4} in Table 1) were also defined using the geometric mean to combine the intensities in two orthogonal directions. Sa_{S3} is constructed with the spectral accelerations at three building-specific spectral ordinates in both directions for a total of six components, whereas Sa_{S4} is defined over ten periods for a total of 20 components. Either of these two IMs is expected to be promising for different applications. Again, since Sa_{S3} and Sa_{s4} combine the two orthogonal excitations with equal weights, they are expected to be less effective for 3D asymmetric structural models whose vibrations may be very different in the two main orthogonal directions. Hence, Sa_{V3} and Sa_{V4} are introduced as the corresponding vector IMs by separating the contribution of each horizontal ground motion component into a two-element vector.

In the range of periods longer than T_1, the value of $T=1.5\cdot T_1$ has been selected as an appropriate upper period limit for all IMs. This was decided based on a preliminary nonlinear response history analysis for the three buildings (see Kohrangi et al., 2015) where $Sa(T=1.5\cdot T_1)$ consistently provides the lowest dispersion in response estimation for all directions. As stressed earlier, in the range of periods lower than T_1, one needs to provide a balance in the efficiency of the same IM in the estimation of both PFA and IDR. It is well known that values of PFA are considerably more influenced by higher modes compared to those of IDR. In other words, adding many short period ordinates to a vector IM, or averaged spectral acceleration scalar IM, may help in PFA prediction only but it may not be as effective for predicting IDR. Opposite considerations hold when adding many spectral ordinates with periods longer than the fundamental one in the predictive vector. Therefore, care should be exercised when selecting the relative weight placed on the short versus the long period ranges for each building. In this study, minimum periods of 0.8, 0.2 and 0.2 of T_1 for the considered 3-, 5- and 8-story buildings, respectively, were observed to provide such balance in the response prediction. The PGA is also considered as a candidate IM here because it is expected to be a valuable predictor for estimating PFA, especially for short and relatively rigid structures or at lower floors of taller buildings, as confirmed in the companion paper (Kohrangi et al., 2015). Finally, as mentioned earlier, to avoid problems caused by multi-collinearity of different predictors in the vector IMs of Sa_{V1} and Sa_{V2}, all spectral accelerations other than the first component of the vector (i.e., $Sa_x(T_{1x})$) are normalized to the previous component in the series.

PSHA AND VPSHA ANALYSIS RESULTS

A site in the south of the Sea of Marmara in Turkey was considered in this study and all earthquake sources within 200 km from it where included in the hazard calculations.

Figure **2**(a) shows the site map along with the considered faults. A reference "stiff or soft rock" soil class with average shear wave velocity over the top 30m (Vs_{30}) equal to 620 m/s was assumed to be present at the site. The minimum magnitude of engineering significance used in the hazard analysis was M_w =4.5. The hazard calculations are based on the GMPE proposed by Boore and Atkinson (2008) that provides GMRotI50 of spectral acceleration (i.e., a median value of the geometric mean over multiple incident angles) rather than the geometric mean of the spectral accelerations of two recorded horizontal components or the spectral acceleration of one arbitrarily chosen component. Baker and Cornell (2006) showed that even though the GMRotI50, the geometric mean ($Sa_{g.m.}$) and the arbitrary component (Sa_{arb}) have statistically similar median values for any given earthquake at any given location, their logarithmic standard deviations are different (the values for Sa_{arb} being higher due to the component-to-component variability). Therefore, one should be careful in consistently applying the same definition of spectral acceleration both in hazard calculations and in the response assessment. In this study, consistent definitions of spectral acceleration variables (arbitrary component or geometric mean) were used by modifying the standard deviation of the applied GMPE, according to the definition of spectral acceleration considered.

Figure 2. Hazard Analysis results: (a) Site map showing the location of fault sources (blue lines), background source model (red lines), the area source model (black lines), and the assumed location of the building (yellow pin), (b) Mean Annual Rate (MAR) of exceedance of Sa at periods of relevance to the 8-story building (Kohrangi et al., 2015) (solid line: $Sa_{g.m.}$, dashed line: Sa_{arb}) and Sa_{avg} made of the same spectral accelerations.

Figure **2**(b) shows the hazard curves related to the 8-story building described in the companion paper for spectral acceleration at four different periods (solid lines for the geometric mean and dashed lines for the arbitrary component) corresponding to T_{1x}=1.30s and T_{1y}=0.44s and periods 1.5 times the first vibration mode of each direction, along with the curve for their average, Sa_{avg}. As mentioned earlier, the VPSHA *indirect* approach was implemented using the PSHA output of OpenQuake. The disaggregation results for finely discretized bins of 0.5 magnitude unit and 2.5 km distance were considered. In the PSHA, the hazard curves for the spectral accelerations were computed for values ranging from 0.0001g to 3.5g with a logarithmic increment of ln(0.2) and the spectral acceleration ratios ranging from 0.01 to 50 with a constant logarithmic increment of ln(1.17). Such fine discretization of spectral acceleration hazard curves was employed as required to achieve sufficiently accurate estimates of the marginal MARs (see Bazzurro et al., 2010). Bazzurro et al., (2010), performed a sensitivity analysis on the effect of bin size on the precision of the method and the interested reader is referred to that study.

The same GMPE (Boore and Atkinson, 2008) and site conditions were adopted for VPSHA for consistency reasons. In a real, complex case problem in which several GMPEs are considered in a logic tree format, the VPSHA indirect computations may also be complicated by the handling of multiple GMPEs and the corresponding proportions, which was avoided here. An additional simplification adopted is the assumption that all the earthquakes were generated by a strike-slip rupture mechanism. This eliminates the need for rupture mechanism bookkeeping when disaggregating the site hazard. The correlation coefficients proposed by Baker and Jayaram (2008) via Equations (13), (14) and (15) were used for the computation of the hazard of complex IMs. Note that, for simplicity, these correlation coefficients were applied to every scenario event, although a recent study (Azarbakht et al., 2014) has shown some dependence of the correlation structure on magnitude and distance.

As an example,

Figure **3**(c) shows VPSHA results for a selected vector case with two components.

Figure **3**(d) displays the M and R disaggregation of the joint hazard at $Sa(T_1{=}0.57\text{s}){=}0.067\text{g}$ and $Sa(1.5{\cdot}T_1)/Sa(T_1){=}1.021$, which are IMs relevant for the 3-story building analyzed in the companion paper (Kohrangi et al., 2015). The code generated in this study is capable of computing the joint hazard for a vector up to 4 components. One simple, but not necessarily sufficient, validation for the vector PSHA is the comparison between the hazard curves obtained using scalar PSHA for each IM in the vector, with the marginal distributions of the joint IM distribution obtained from VPSHA for the same IMs. Such validation was performed for the entire vector computations tested here and good consistency was observed in all cases.

Figure **3** (c) shows one such comparison for the VPSHA case of equaling pairs of $Sa(1.5{\cdot}T_1)/Sa(T_1)$ and $Sa(T_1)$ values (see

Figure **3**(a)). In

Figure **3**(b), the MAR of exceeding for this example is shown.

Figure 3. Hazard Analysis results: (a) MAR of equaling joint values of $Sa_x\left(T_{1x}\right)$ and of $Sa_x\left(1.5{\cdot}T_{1x}\right)/Sa_x\left(T_{1x}\right)$ at $T_{1x}=0.57s$, (b) MAR of exceeding joint values of $Sa_x\left(T_{1x}\right)$ and of $Sa_x\left(1.5{\cdot}T_{1x}\right)/Sa_x\left(T_{1x}\right)$ at $T_{1x}=0.57s$ c) comparison of the MAR of equaling derived from the scalar PSHA and from the marginal of VPSHA; d) disaggregation results for a joint MAR of equaling at a given ground motion intensity level with $Sa_x\left(T_{1x}\right)=0.067g$ and $Sa_x\left(1.5{\cdot}T_{1x}\right)/Sa_x\left(T_{1x}\right)=1.021$ at $T_{1x}=0.57s$.

It should be noted, again, that to achieve a good accuracy of the hazard estimates the domain of all the random variables considered in the VPSHA calculations must be well discretized especially around the region where the probability density function is more concentrated. For instance, the joint MAR of equaling for $Sa_x(T_{1x})$ and $Sa_x(1.5{\cdot}T_{1x})/Sa_x(T_{1x})$ ratio for T_{1x} =0.57s shown in

Figure **3**(a) needs a fine discretization especially in the 0.5 to 1.0 range for the $Sa_x(1.5{\cdot}T_{1x})/Sa_x(T_{1x})$ ratio and of 0.001g to 1.0g for $Sa_x(T_{1x})$. As explained earlier, in this study a constant and rather fine discretization was considered to cover all the ranges appropriately. However, the user can adopt different discretization schemes with respect to the importance of each adopted range, perhaps to reduce the analysis time and to reach the accuracy of interest.

An improvement of this software for carrying out VPSHA compared to previous ones is the ability to compute the contributions to the joint hazard in terms of the *M*, *R*, and, if needed, the rupture mechanism of the causative events. Although not implemented here, the joint hazard disaggregation could also be extended to identify the latitude and longitude of the events, so that the specific faults that control the hazard can be uniquely recognized (Bazzurro and Cornell, 1999). Several refinements of the disaggregation exercise can be carried out to meet the requirements of the users. For example, in a 2D joint hazard case, the disaggregation can be implemented to extract the contributions to the MAR of “equaling” a certain joint IM cell (e.g., *Sa*(0.3s)=0.2g and *Sa*(1.0s)=0.1g), or to the MAR of equaling or exceeding it (e.g., *Sa*(0.3s) ≥ 0.2g and *Sa*(1.0s) ≥ 0.1g). One example of such results is shown in

Figure **3**(d). The VPSHA software developed for this study in MatLab is available at Kohrangi, 2015a.

CONCLUSIONS

Computing the seismic risk of realistic buildings for both loss estimation and collapse assessment requires monitoring building response measures that may include story-specific measures, such as peak inter story drifts and floor response spectra at all stories, and global measures, such as maximum peak inter story drift along the height of the building and residual, post-earthquake lateral displacement. A confident assessment of these response measures requires sophisticated structural and non-structural modeling that is better served by using 3D computer models of the building. Predicting the response of such models in both the main horizontal axis and, in some cases, vertical direction (e.g., for assessing the damage to suspended ceilings) is facilitated by the use of more than one IM of the ground motion in one or more directions and at one or more oscillator periods.

Estimating response measures as a function of different IMs involves statistical and probabilistic techniques that have been already, in large part, developed and fine-tuned. However, which IMs are superior for a practical estimation of *both losses and collapse* of buildings modeled as 3D structures and how to compute the joint hazard of these IMs at the building site is still a very fertile ground for research.

This article and its companion one (Kohrangi et al., 2015) describe the use of more than one IM for assessing building response for both loss and collapse estimation. The present article focuses on defining the IMs that are jointly used as predictors of building response in the companion paper and outlines a method for performing vector-valued PSHA for these IMs. Performing vector-valued PSHA for complex IMs that are derived from common ones (e.g., spectral accelerations at different periods) is not trivial and requires modifying the existing ground motion prediction models and computing the variance-covariance matrix of such IMs.

All these aspects are covered here for the most common practical IMs appearing in the literature namely spectral accelerations, ratios of spectral accelerations and averages of spectral accelerations over different periods and orientations, which are used as predictors of building response both in scalar form and in vector form. More precisely, the scalar IMs considered here are spectral accelerations at first mode period of the structure in each orthogonal main directions of the building, or at the average of the first modal periods in the two orthogonal directions. Another scalar IM used is the averaged spectral acceleration at multiple periods of oscillation that are important for the structures considered. It is emphasized, however, that the methodology described for performing vector-valued PSHA goes beyond the boundaries of these specific applications that use only spectral accelerations. Other less conventional IMs (e.g. *PGV*, *PGD*, Arias Intensity, duration, and Cumulative Absolute Velocity), can be used following the same approach provided that legitimate ground motion prediction models and correlation coefficients for those IMs are available.

For the applications at hand, the conventional scalar PSHA for scalar IMs and the vector-valued PSHA were performed using the software OpenQuake. The vector-valued PSHA were carried out using a methodology that was called the "Indirect" approach since it does not implement the numerical integration of the joint distribution of all the correlated IMs considered, as the "direct" approach does. The "indirect" approach uses the marginal hazard curve for each IM, the disaggregation results from those IMs, and the correlation coefficients for each pair of IMs to obtain the joint hazard. Hence, this method could be considered as a simple post processor of any available scalar PSHA code. This "indirect" method is arguably superior to the "direct" integration approach in many aspects as explained in the body of the paper. However, when applying the "indirect" approach to vector PSHA, care should be exercised in the selection of the bin sizes that discretize the mutli-dimensional domain of the IMs. The bin sizes should be rather small especially in the part of the domain where the highest concentration of probability is concentrated.

The software that post-process scalar PSHA results and that produced the joint hazard estimates used in this study is available at Kohrangi, 2015. As will be discussed in the companion paper (Kohrangi et al., 2015), using vectors of IMs in seismic performance assessment of structures is a very promising avenue. It is hoped that the software for performing vector PSHA made available here will decrease the hurdle that has hindered its use in the past and will enable more complex and accurate seismic response assessment studies of realistic buildings.

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APPENDIX: INCORPORATING MULTIPLE GMPES IN VPSHA

In a complex PSHA, where multiple GMPEs are used, one could proceed either by using a single GMPE, but accepting some level of inaccuracy; or by incorporating all the GMPEs in computing the median and standard deviation of the corresponding IM. In the latter option, the values of the median and the standard deviation needed in equations (A.1) and (A.2) could be approximately obtained using the following equations:

$$\mu_{\ln \mathrm{IM}_i|rup} \approx \sum_k p_k \, \mu_{\ln IM_{i,k}|rup} \, , \tag{A.1}$$

$$\sigma_{\ln \mathrm{IM}_i|\mathrm{rup}} \approx \sqrt{\sum_k p_k \left(\sigma^2_{\ln IM_{i,k}|\mathrm{rup}} + \left(\mu_{\ln IM_{i,k}|\mathrm{rup}} - \mu_{\ln IM_i|\mathrm{rup}} \right)^2 \right)} \, , \tag{A.2}$$

In which:

$\mu_{\ln \mathrm{IM}_i|rup}$, is the logarithmic mean obtained incorporating all the GMPEs of the *i*-th IM in the vector of IMs given a scenario (Magnitude, distance, etc.).

$\mu_{\ln IM_{i,k}|rup}$, is the logarithmic mean obtained from *k*-th GMPE in the logic tree of the *i*-th IM in the vector of IMs given a scenario (Magnitude, distance, etc.).

p_k is the PSHA weight assigned to the *k*-th GMPE in the logic tree.

$\sigma_{\ln \mathrm{IM}_i|\mathrm{rup}}$, is the logarithmic standard deviation obtained by incorporating all the GMPEs of the *i*-th IM in the vector of IMs given a scenario (Magnitude, distance, etc.).

$\sigma_{\ln \mathrm{IM}_{i,k}|\mathrm{rup}}$, is the logarithmic standard deviation obtained from *k*-th GMPE in the logic tree of the *i*-th IM in the vector of IMs given a scenario (Magnitude, distance, etc.).

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