The Hysteretic Energy as a Performance Measure in Analytical Studies

Athanasia K. Kazantzi, [a] **and Dimitrios Vamvatsikos,** [a] **M.EERI**

Hysteretic energy dissipation is often employed as a measure of performance for systems subjected to earthquake excitation. This mainly stems from quasi-static cyclic tests where fuller hysteresis loops (i.e., higher energy absorption) are taken to indicate better performance when comparing systems with similar strength under the same cyclic loading protocol. However, seismic loading offers a different proving ground, where energy absorption is strongly correlated with energy input while the non-stationary loads imply that the beneficial hysteretic effects observed in a cyclic test may never be realized. Given the current state of art in models and methods of Performance-Based Earthquake Engineering, we ask whether earthquake records at a given seismic intensity will cause peak/residual displacements or accelerations that favor models having fuller hysteresis. Using incremental dynamic analysis on story-level oscillators with varying hysteretic characteristics, it is demonstrated that hysteretic energy dissipation does not consistently correlate with seismic performance.

INTRODUCTION

The hysteretic energy absorbed by a structural system undergoing nonlinear deformation is typically recognized as a potentially useful seismic performance indicator (e.g., Park *et al*. 1987; Bojorquez *et al*. 2011). The general intuition is that ground motion records contain certain amounts of energy at each frequency. This energy is imparted upon structures, e.g., in accordance to their eigenfrequencies, and must be dissipated quickly (or efficiently) to ensure low response. Thus, a system composed of members possessing stable hysteretic loops with large hysteretic energy dissipation capacity should be guaranteed a better system performance, implying that there is good correlation between dissipated hysteretic energy and inelastic deformation/acceleration demands. This notion is often founded on observations made in quasi-static cyclic tests, where it seems apparent that between two systems with

[a] School of Civil Engineering, National Technical University of Athens, 9 Heroon Polytechneiou, 157 80, Athens, Greece

similar strength, tested under the same cyclic loading protocol, the one with the higher energy absorption, i.e., “fuller” hysteresis loops, should exhibit superior performance. Thus, dissipated hysteretic energy is a term that has become synonymous to performance, at least conceptually rather than quantitatively, and it is so pervasive as to become common nomenclature of modern seismic codes, for example Eurocode 8 (CEN 2004) or the Japanese seismic code (Akiyama 1985, BSL 2009).

Most tellingly, at the basis of seismic design, the definition of the behavior (reduction) factor q (or R) is often cast (e.g., in EN1998, paragraphs 3.2.2.5, 5.2.2.2) in the sense that the high strength of a linear elastic system having no energy absorption can be substituted by the equally effective dissipating behavior of an elastoplastic system with a base shear strength that is q times lower (at least for moderate and long periods where the equal displacement rule holds). While q is generally quantified on the basis of overstrength and ductility capacity, rather than dissipated energy capacity per se, a physical connection with the latter is often implied. The prevailing concept is that, in order to economize, it is important to compensate for strength by providing sufficient ductility *and energy dissipation*. While there is no question about the need for ductility and its connection to the behavior factor, the role of energy dissipation, a term that is often interchangeably used (e.g., in EN1998) to denote ductility capacity, is still imperfectly understood. There is only limited evidence that links the analytically computed energy capacity with what it is actually measured experimentally and with how the structure performs.

Energy dissipation in analytical studies is typically thought of as a proxy for viscous damping, a concept that was perhaps first introduced by Jacobsen (1960) through the use of equivalent linearization techniques. Such methods provide an estimate of the (average) nonlinear displacement of elastoplastic oscillators by employing an equivalent linear single-degree-of-freedom (SDOF) system characterized by a longer period (estimated at a secant stiffness) and an increased value of viscous damping. Crucially, the increase in damping is often provided as a function of the area under the force-deformation curve of the nonlinear oscillator, a quantity that is well correlated to the quasi-statically dissipated hysteretic energy. It is no wonder then that higher energy dissipation seems to be equivalent to higher damping, ergo better performance. Is this really true though and how much faith can be put in the analytically evaluated energy capacity of a system given the present modeling limitations?

On the contrary, Miranda and Ruiz-Garcia (2002) have shown that using the actual area under the backbone of an elastoplastic system to define equivalent damping yields worse results for maximum displacement estimation compared to other more abstract approaches. A number of recent studies have also explored the effect of the type of cyclic hysteresis on the seismic performance of structural systems. Henceforth the term "hysteresis" is used to refer exclusively to the set of rules that define the cyclic force-displacement behavior of a system, excluding its backbone capacity curve, as obtained under monotonic loading conditions. Thus, two SDOF systems may have the same backbone (e.g., elastic-plastic) but different hysteresis, or may have the same hysteresis (e.g., kinematic hardening) but different backbones. In many cases, it has been observed that either there is no clear correlation between the hysteresis type and the analytically evaluated ductility demands (Rahnama and Krawinkler 1993; Foutch and Shi 1998; Huang and Foutch 2009) or that the hysteresis type becomes important mainly when the system approaches its global collapse state (Ibarra *et al*. 2005). Furthermore, with respect to the widely-used Park and Ang (1985) damage index, which is expressed as a linear function of the dissipated energy and ductility, it was demonstrated by Kappos (1997) that, for well-designed structures, the energy term of the damage index has a small contribution to the index value compared to the ductility term. Given that the hysteretic rules largely decide the amount of energy dissipation, questions may be easily raised. Similarly, when the connection of dissipated energy and performance is extrapolated from quasi-static tests to hysteretic models under non-stationary loads, i.e., actual ground motions, current ideas about the importance of hysteretic energy may not be generalizable. Such results have actually led Priestley (1993) to declare energy as "one of more pervasive myths of earthquake engineering".

Therefore, we want to pose the question of whether hysteretic energy dissipation remains a fundamental quality of system performance when we expand from the physical structure to the structural models. In other words, when comparing two analytical models having similar (or the same) backbone and subject to the same ground motion, we are asking whether the one with the "fuller" hysteresis loops (as evidenced from classic quasi-static cyclic tests) or, more generally, the one dissipating more energy via hysteresis in dynamic loading, is the one having the edge on seismic performance as measured in terms of acceleration and displacement (or ductility). Or, viewed from a different angle, whether among two systems with the same cyclic hysteresis type but differing force-displacement capacity (backbone) curves, the one with the enveloping capacity curve always performs best. While these two questions tackle a very basic issue in hysteretic energy they are, nonetheless, not meant to be all-encompassing. Any issues related to material or member failure/degradation criteria and whether these should be based on dissipated hysteretic energy will not be discussed. Such questions can only be resolved unambiguously by experiments and not via computational studies, like the one that we are going to embark upon. However, it is still important to explore even on a purely computational basis and to the extent the available analytical models can simulate reality, whether energy dissipation is a useful indicator of seismic performance as currently employed in state-of-art Performance-Based Earthquake Engineering. Any relevant conclusions may be considered applicable to cases where fracture energy related catastrophic failure, e.g., due to material fracture by low-cycle fatigue, is not reached. Still, for the vast majority of computational models for component behavior, the present investigation should be quite relevant.

ENERGY BALANCE EQUATION AND HYSTERETIC ENERGY

The definition of dissipated hysteretic energy comes from the classic equation of motion for structural dynamics. For a damped SDOF system subjected to a horizontal ground motion record, this can be written as

$$m\ddot{u} + c\dot{u} + f_s = -m\ddot{u}_g \tag{1}$$

where m is the mass of the system, c is the viscous damping coefficient, f_s is the restoring force, $\ddot{u}_g$ is the ground acceleration and u, $\dot{u}$, $\ddot{u}$ are the relative displacement, velocity and acceleration, respectively, of the mass with respect to the ground. The absorbed energy is evaluated according to the energy balance equation (Uang and Bertero 1990), derived from integrating over time Equation (1), representing the equilibrium of forces, multiplied by the instantaneous displacement $du = \dot{u}dt$:

$$\int m\ddot{u}\dot{u}\,dt + \int c\dot{u}^2 dt + \int f_s\,\dot{u}\,dt = -\int m\ddot{u}_g\dot{u}\,dt \tag{2}$$

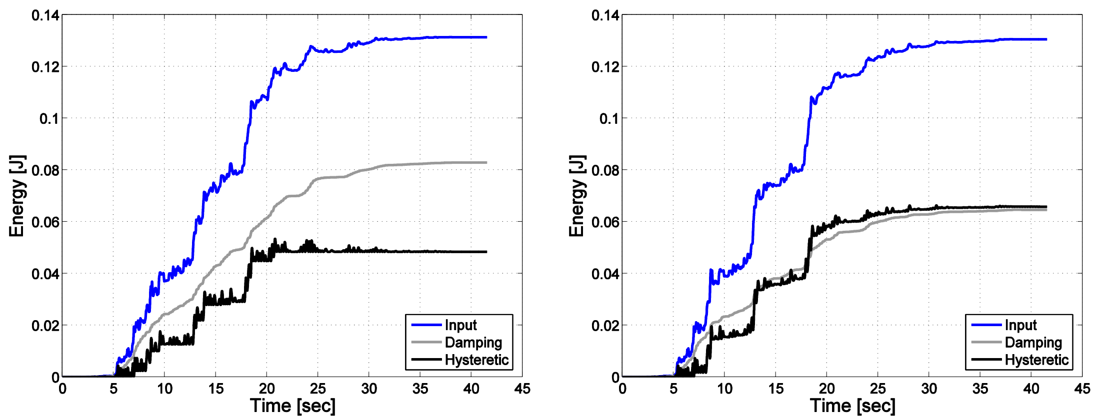
The energy balance equation is valid throughout the duration of the motion. The first term depicts the “relative” kinetic energy of the system, as measured with respect to the ground, representing energy temporarily stored in the kinematics of the system. The second is the damping energy dissipated by viscous damping, and the third is termed the absorbed energy, consisting of the irrecoverable hysteretic energy and the recoverable strain energy. Despite the presence of the recoverable part, the name “absorbed energy” is perfectly valid when integration is carried out until the system comes to rest, where strain energy vanishes. The final term is the relative input energy imparted by the ground motion to the system, as measured relative to the ground, excluding any rigid body translation. Still, if integration is carried out to the time when the system comes to rest this is essentially equivalent to the absolute input energy (Uang and Bertero 1990) rendering the distinction mostly academic. The actual input energy induced to a system during an earthquake event is thus dissipated in its entirety by means of viscous damping and hysteretically absorbed energies.

A simplistic interpretation of the energy equation would maintain that the earthquake contains some given amount of energy that is imparted to the structure and then dissipated through the two mechanisms of damping and hysteresis. On the contrary, the amount of energy input is not fixed for any ground motion record. While it naturally depends on the oscillator period and damping, as Equation (2) reveals it is also dependent on the details of the system force-deformation relationship. The ground motion acceleration is multiplied by the oscillator velocity at each time instant, resulting to either a positive or a negative energy increment. The same is true for the hysteretic energy as well, where the sign of the restoring force f_s and the velocity may become opposite. On the other hand, this is never the case for damping, as it offers an ever-present dissipating action due to the square on the velocity term (Equation (2)). Thus, increasing the damping has the straightforward effect of reducing the seismic demands. Changing the force-deformation characteristics, though, to amplify the area of the cyclic hysteresis loops or the area under the monotonic force-deformation curve, does not necessarily mean an improved performance. Such changes will not only affect the dissipated energy but also the input energy, thus rendering conclusion-drawing a difficult task. In other words, despite the fact that fixing the energy input will directly result in the model with the fuller hysteretic loops behaving better, it is quite probable that the aforementioned energy discrepancies, implied by the equation of motion, will shape things in quite an unpredictable way at a global level.

For example, Figures 1a,b show the energy time histories of the same elastic-perfectly-plastic SDOF oscillator, subjected to a single ground motion record but having two different cyclic hysteresis behaviors. In this case, the full loops of the kinematic strain hardening model happen to cause almost the same energy input as the skinnier loops of a peak-oriented system. Still, the first has lower hysteretic energy dissipation than the latter with the difference being taken up by the damping energy (despite both having 5% viscous damping).

Even so, it is not easy to guess from Figure 1 which of the two cases registered the lowest peak displacement; actually, they were quite similar in that respect.

At a different level, it is equally troublesome to try to derive conclusions regarding system performance based not on dynamically-absorbed energy but on quasi-statically absorbed instead. Such tests are typically performed under a displacement-controlled loading protocol that not only imposes certain displacements but, given the hysteretic model, essentially also prescribes the input energy. By virtue of removing any influence of damping, this also ensures that all the energy will be dissipated via hysteresis only. Clearly this is something that can never happen in dynamic tests therefore any connections would be difficult to justify.



(a) kinematic hardening (b) peak-oriented

Figure 1. Energy time histories of an elastoplastic oscillator with T = 0.5sec and 5% viscous damping for a single ground motion scaled to $2S_{ay}$ (i.e., twice the lowest spectral acceleration to cause yielding) and for two different hysteresis rules.

METHODOLOGY

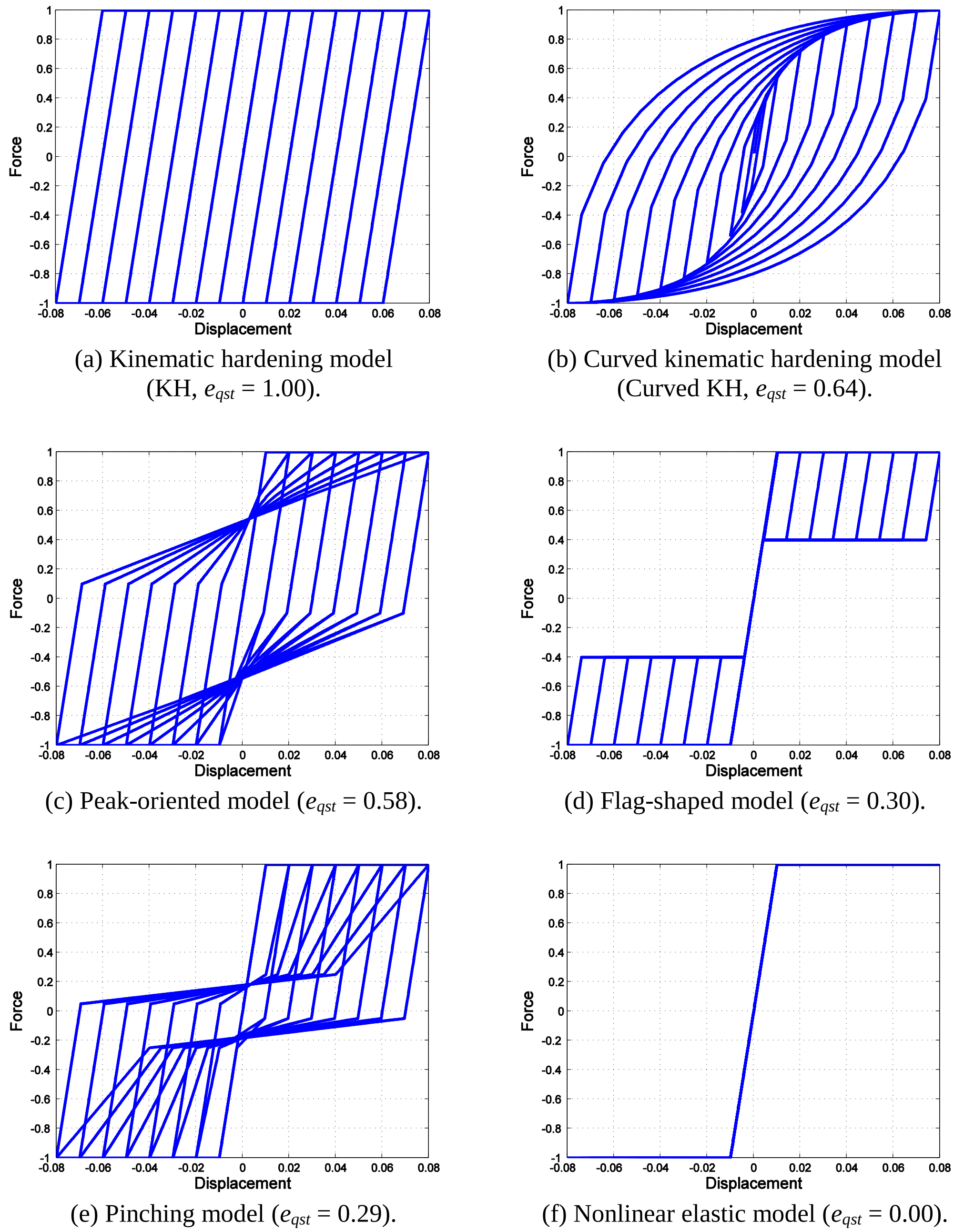
To investigate the correlation between hysteretic energy and seismic performance in computational studies, a number of SDOF systems will be used, each having different force-deformation characteristics. To evaluate their seismic performance Incremental Dynamic Analysis (IDA) is employed (Vamvatsikos and Cornell 2002). A suite of sixty ground motion records were selected from the PEER Strong Motion database from a relatively narrow magnitude and distance bin, having moment magnitude within 6.5 – 6.7 and closest distance to fault rupture ranging from 13 to 32km (PEER 2011). The records can be characterized as ‘ordinary’ in the sense that they do not raise any concerns regarding soft soil or near-source directivity. Their significant durations, as defined by their 5-75% Arias intensities (Bommer and Martinez-Pereira 1999), range within 3.3 to 19.6 sec, as typical for strong crustal earthquakes. Note that as records are scaled the number of the hysteretic cycles that go beyond yielding increases; thus, regardless of actual duration, the system will experience longer times of inelastic deformation through higher intensity. To define IDA curves of seismic intensity versus response, two scalars are now needed: An Intensity Measure (IM) and an Engineering Demand Parameter (EDP) to record the structural response.

As an IM, the 5% damped spectral acceleration at the vibration period of the SDOF systems, $S_a(T)$, was adopted, since it is considered to be efficient especially for SDOF oscillators (Shome *et al*. 1998). Moreover, to allow comparisons between the different models and periods investigated, the elastic spectral acceleration S_a will typically be shown normalized by its value S_{ay} at yield; this will provide the dimensionless ratio $R = S_a/S_{ay}$, which is akin to the strength reduction (or behavior) factor R. Subjecting SDOF systems to a record scaled by a factor of R or maintaining an unscaled record while reducing the yield strength by R is essentially the same thing; thus the motivation for employing this strength ratio. Still, R should be understood herein as indicative of the intensity of the earthquake relative to the yield strength, without necessarily implying any connection with codified values of reduction/behavior factors for actual structures.

Regarding the EDP, displacements, energies and accelerations are all viable candidates. The peak absolute acceleration sustained a_{abs}, or, equivalently, the inelastic spectral acceleration, correlates well with contents' damage. The maximum displacement d_{max} will be employed as a measure for the peak seismic demands. Residual displacements d_{res} will also be recorded as a useful indicator of whether a damaged building should be retrofitted or demolished (e.g., Ruiz-Garcia and Miranda 2006). Finally, the total absorbed hysteretic energy E_{hyst} will serve as the candidate indicator of performance, used to investigate correlations with the established a_{abs}, d_{max} and d_{res}. To allow an accurate estimation of quantities to be measured at the end of the analysis, i.e., residual displacements and total hysteretic energies, the oscillators are allowed to undergo several free vibration cycles at the end of each record until they return to rest.

HYSTERETIC MODELS

We consider a series of single-degree-of-freedom (SDOF) oscillators, all sharing practically the same elastic-plastic force-deformation backbone (allowing for some curved transition in one case) but with varying hysteretic characteristics to depict a wide spectrum of force-deformation behaviors. We shall investigate each system without entering the discussion of whether it is a fully realistic representation of physical systems, i.e., whether there might be other physical processes in action that dissipate energy beyond what is provided by the assumed hysteresis and viscous damping considered. After all, this is how they are employed in the literature to cover a wide range of structural behaviors.



(a) Kinematic hardening model (KH, e_{qst} = 1.00).

(b) Curved kinematic hardening model (Curved KH, e_{qst} = 0.64).

(c) Peak-oriented model (e_{qst} = 0.58).

(d) Flag-shaped model (e_{qst} = 0.30).

(e) Pinching model (e_{qst} = 0.29).

(f) Nonlinear elastic model (e_{qst} = 0.00).

Figure 2. Backbone and hysteretic loops of the considered nonlinear oscillators.

The six systems considered are presented in Figure 2, arranged in order of decreasing quasi-statically absorbed hysteretic energy. At the very top of the group, a classic elastoplastic system with kinematic strain hardening (Figure 2a) was chosen to serve as the benchmark for comparing the performance of the more elaborate systems to follow. Adding a curved transition between the elastic segment and the plastic plateau while maintaining kinematic strain hardening defines the "curved" system of Figure 2b. Cyclic stiffness degradation is introduced to different degrees of severity by the peak-oriented and the pinching systems (Ibarra *et al.* 2005) in Figures 2c and 2e, which are chosen to slightly depart from the norm by having elastic unloading branches that extend somewhat below zero strength. Recent advances in self-centering systems (Christopoulos *et al.* 2002) are represented by the flag-shaped hysteretic loops of Figure 2d. Finally, the nonlinear-elastic oscillator of Figure 2f lies at the opposite extreme compared to the kinematic hardening, having the same backbone but no hysteretic energy dissipation capacity. In each case, a 5% viscous damping was employed and a range of periods, from 0.5 to 2.0sec, was investigated. All analyses were carried out via OpenSees (McKenna *et al.* 2000).

The hysteretic energy dissipation capacity under quasi-static cyclic loads was evaluated on the premise of the energy ratio e_{qst}:

$$e_{qst} = \frac{E_{hyst,i}}{E_{hyst,KH}} \tag{3}$$

where $E_{hyst,i}$ is the hysteretic energy absorbed by model i and $E_{hyst,KH}$ is the energy absorbed by the kinematic hardening (KH) model, when both are subjected to the same cyclic loading protocol. The corresponding e_{qst} ratios are reported in Figure 2 for each system and they vary from one to zero, with one representing the KH model (Figure 2a) and zero associated with the nonlinear elastic model (Figure 2f). It is worth noting that the e_{qst} ratios for the moderately pinching (Figure 2e) and the flag-shaped (Figure 2d) models are almost identical, at least for this example. Note that hysteretic energy is often employed in the literature in a normalized form, e.g., divided by the product of yield strength and displacement (Bojorquez *et al*. 2011). This makes no difference for the energy ratios defined herein as the systems compared have the same (nominal or actual) yield point.

To evaluate the systems' performance under dynamic loads, we also need to assess their behavior in terms of energy, acceleration and displacement. For simplicity, the dynamic responses will be normalized for each system, ground motion record and intensity level considered by the corresponding results of the benchmark KH model. For example, for the peak-oriented system with T = 1.0sec, Figure 3a presents the "spaghetti plot" of 60 IDA curves of d_{max} ratios versus the normalized intensity R. All such curves are naturally coincident in the elastic region, i.e., until R = 1.0. Past the yield point considerable record-to-record variability manifests itself, making the summarization of these results a necessity. This is achieved by estimating at each level of R the x% fractile values, over all records, of the hysteretic energy ratio e_{dyn}, the maximum displacement ratio r_{max} (equivalent to the max ductility ratio as all systems share the same nominal yield displacement), the residual displacement ratio r_{res} and the peak absolute acceleration r_{acc} defined for each system i with respect to the KH hysteresis:

$$e_{dyn,x\%} = \left[\frac{E_{hyst,i}}{E_{hyst,KH}}\right]_{x\%} \quad r_{max,x\%} = \left[\frac{d_{max,i}}{d_{max,KH}}\right]_{x\%} \quad r_{res,x\%} = \left[\frac{d_{res,i}}{d_{res,KH}}\right]_{x\%} \quad r_{acc,x\%} = \left[\frac{a_{abs,i}}{a_{abs,KH}}\right]_{x\%} \tag{4}$$

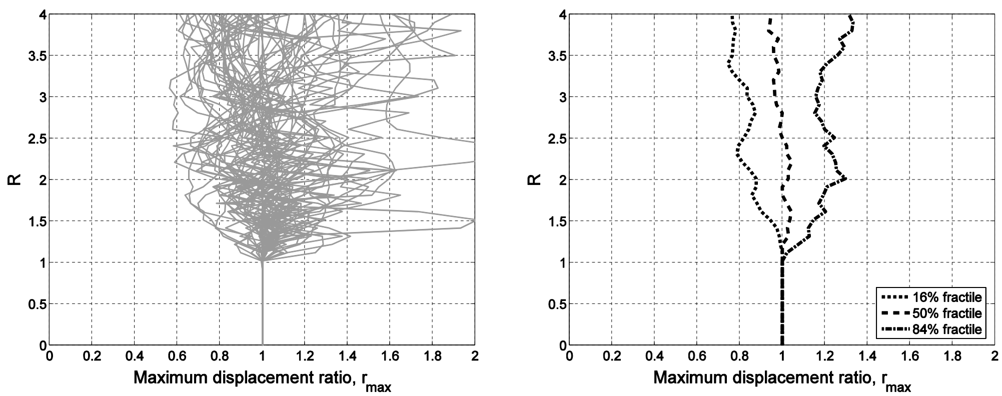


Figure 3. (a) The 60 IDA curves and (b) the estimated fractile values of maximum displacement response ratio, r_{max}, of the peak-oriented over the KH system (T = 1.0sec).

Typical values for x include 50%, i.e., the median as a central value, and 16, 84% to evaluate the associated dispersion. The corresponding results appear in Figure 3b, where the peak-oriented and the KH system are shown to have the same median d_{max} response (i.e., a ratio of practically 1.0) at all levels of intensity, in agreement with past studies (Rahnama and Krawinkler 1993; Vamvatsikos and Cornell 2006). This unit ratio comes with a coefficient of variation of roughly 20% for all R-values greater than 1.0. This essentially means that 84% – 16% = 68% of the records will not generate differences higher than ±20% between the two models. Such differences are about equally probable to favor one or the other model, being symmetric around the median.

PERFORMANCE COMPARISONS BETWEEN THE ANALYTICAL MODELS

The performance of the five hysteretic models has been compared versus the KH system across multiple periods. For brevity, only results at T = 0.5sec and T = 1.0sec will be reported. Results at longer periods are practically the same introducing only marginal differences. A summary of results for T = 1.0sec appears in Table 1 where the six systems are compared at three different levels of intensity, R = 1.5, 2.0 and 4.0, to highlight the difference in response in the near and far post-yield region (note that the responses of the six systems have been evaluated over a range of R values between 0-4.0 despite the fact that the results in Table 1 are presented for only three R levels). First, it becomes apparent that dynamically and statically absorbed energies (as analytically computed) are not well correlated. For example, at R = 1.5, all the systems, except the flag-shaped and nonlinear elastic models, absorb more energy than the KH (in a median sense). The curved system actually absorbs nearly twice the hysteretic energy of the KH model, despite having almost 40% lower quasi-static energy absorption. At the same intensity, the skinny loops of the pinching system absorb only half the energy of the peak-oriented system in quasi-static loading, yet it has almost 30% higher dissipation for dynamic loading. These energy ratios are slowly evened out at higher intensities, all but the two self-centering systems reaching nearly unity at R = 4.0. Actually, by R = 4.0 the KH has slightly overtaken most of the other systems.

Table 1. Summarized comparison of the quasi-statically and dynamically dissipated hysteretic energy versus the displacement and acceleration response for the considered hysteretic models (T = 1.0sec). All quantities are shown as median values of response ratios normalized by the kinematic strain hardening system

Loading	Response	curved KH	peak-oriented	flag-shaped	pinching	nonlinear elastic
Quasi-static	e_{qst}	0.64	0.58	0.30	0.29	0.00
Dynamic *R* = 1.5	$e_{dyn,50\%}$	2.35	1.39	0.79	1.83	~ 0
	$r_{max,50\%}$	0.90	1.04	1.04	1.03	1.17
	$r_{res,50\%}$	0.22	1.34	~ 0	0.76	~ 0
	$r_{acc,50\%}$	0.56	1.00	1.00	1.00	1.02
Dynamic *R* = 2.0	$e_{dyn,50\%}$	1.52	1.22	0.81	1.36	~ 0
	$r_{max,50\%}$	0.92	1.02	1.07	1.04	1.33
	$r_{res,50\%}$	0.18	1.48	~ 0	0.81	~ 0
	$r_{acc,50\%}$	0.64	0.99	1.01	1.00	1.05
Dynamic *R* = 4.0	$e_{dyn,50\%}$	0.96	1.02	0.69	0.86	~ 0
	$r_{max,50\%}$	0.99	0.95	1.14	1.06	1.50
	$r_{res,50\%}$	0.29	1.28	~ 0	0.70	~ 0
	$r_{acc,50\%}$	0.80	0.98	1.03	1.00	1.08

The apparent peculiarity of these analytical results can be explained by looking into the details of the hysteretic behavior. For example, the handicap of the KH system at low intensities is a consequence of its purely elastic unloading-reloading behavior. The KH model can only dissipate energy when it deforms along the yield plateau. Thus, the ground motion acceleration spikes that are not strong enough to cause plastification in either direction are instead wasted in non-dissipative unloading/reloading. They cause this energy to be temporarily stored as recoverable strain energy. This is the cause of the many jagged peaks above the five plateaus representing the successive nonlinear excursions of the KH system registered for R = 2 and T = 1.0sec in Figure 4a. On the contrary, the pinching model, subjected to the same ground motion record, displays hysteretic absorption even when cycling below the plastic plateau, thus steadily dissipating energy (rather than temporarily storing it). The difference is better understood by inspecting the corresponding force-displacement hysteretic responses in Figure 4b: Where the KH system spends most of its time in elastic unloading-reloading, the pinching one displays a distinctive rhomboid dissipative region that largely accounts for the observed difference in absorbed energies. Still, for this particular scaled ground motion, the pinching system, despite its superior energy dissipation capacity exhibits a higher peak deformation (Figure 4b) compared to the KH system.

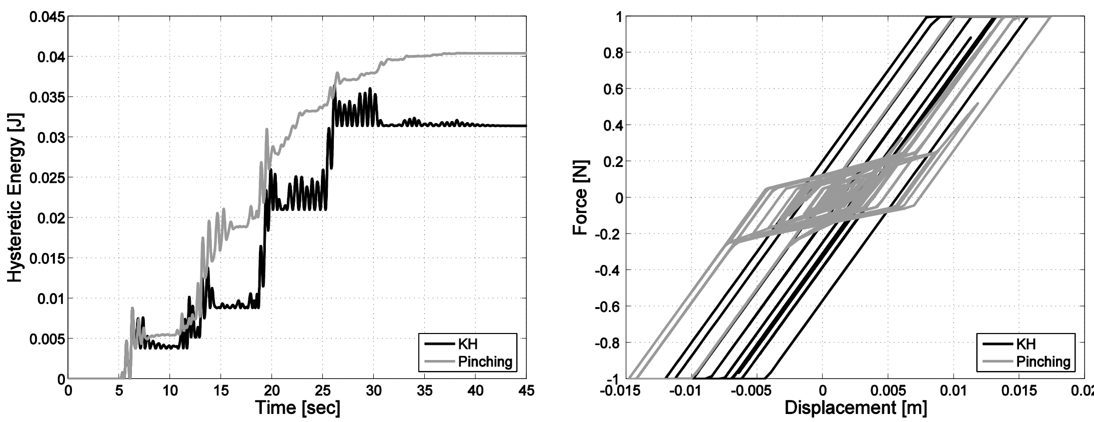


Figure 4. (a) Hysteretic energy time histories and (b) force-displacement hysteretic loops for kinematic hardening and pinching models (T = 1.0sec), for a single record scaled to R = 2.0.

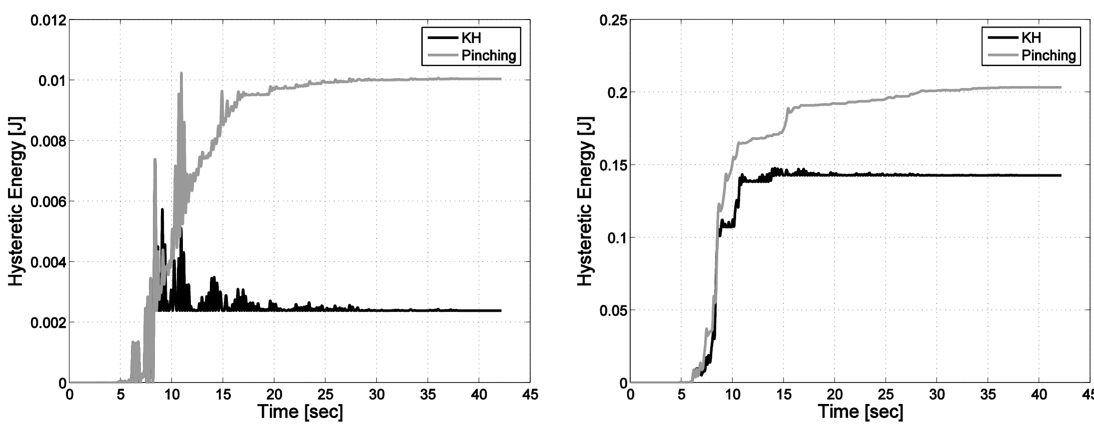
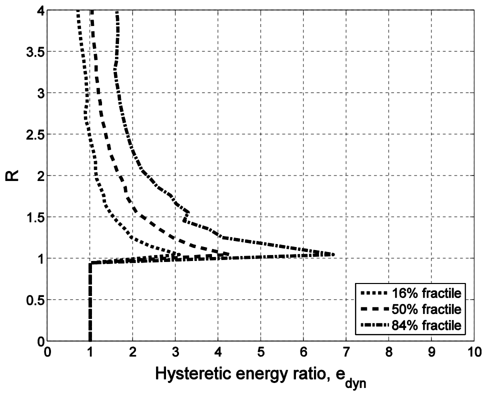


Figure 5. Hysteretic energy time histories for kinematic hardening and pinching hysteretic behavior (T = 0.5sec), for a single record scaled to (a) R = 1.2 and (b) R = 3.0.

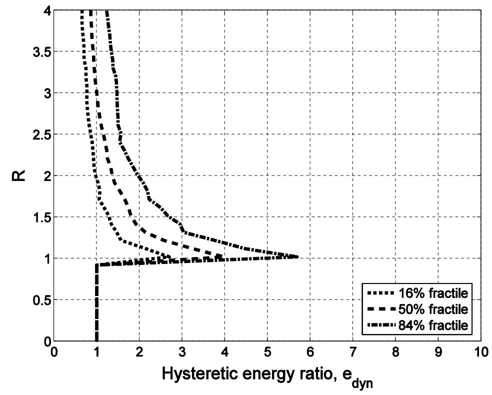
At lower intensity levels, this difference is further accentuated, as shown for example in Figure 5a. Therein, at R = 1.2, the KH system manages just a single nonlinear excursion and only one third of the energy dissipation of the pinching oscillator. When scaling to higher levels (or equivalently reducing the oscillator yield strength), then more complete load reversals happen, increasing the KH energy absorption (Figure 5b). Eventually, the fuller loops of the KH will assert themselves in terms of energy dissipation at higher intensities, as shown, e.g., by the shape of the fractile IDA results in Figures 6a and 6d, and should provide it with the advantage suggested by quasi-static experiments.

Interestingly, though, Table 1 shows that no such "advantage" ever appears in the computational studies of the examined hysteretic models in terms of maximum or residual displacement. At all levels of intensity, the KH, peak-oriented, pinching and curved KH models share practically the same d_{max} response. This trend remains consistent for practically all R-values, as shown for the pinching model in Figures 6b and 6e. Furthermore, the pinching and curved KH models clearly have superior performance in terms of residual displacement compared to the KH at all levels of intensity. For example, Figures 6c and 6f shows the relatively good behavior of the pinching system that sustains residual displacements in the order of 70-100% of the KH, at least in the median; individual records show r_{res} ratios to be widely distributed in [0.3,4].

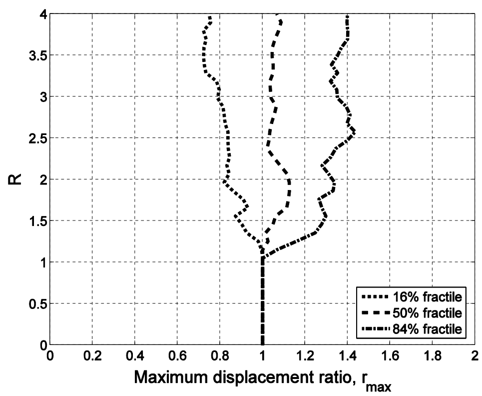
The peak absolute acceleration response is also insensitive to energy dissipation. All modeled systems display the same demand for any given intensity with the notable exception of the curved KH oscillator, a fact also observed by Wiebe and Christopoulos (2010). The absolute acceleration is the sum of the relative and the ground accelerations, thus it is essentially bounded by the sum of their peak values. For systems sharing the same elastoplastic backbone, the peak relative acceleration for any R > 1 is equal to the yield strength divided by the mass, while the peak ground acceleration (PGA) is R times the unscaled PGA of the ground motion. The peak absolute acceleration will be less than or equal to their sum, its actual value largely dependent on timing. On a median sense this turns out to be practically the same for all systems. The only glaring exception is the curved KH oscillator whose backbone lies consistently below the elastoplastic for all but the highest intensities. It is no wonder then that its peak acceleration response is significantly lower than the KH response for R = 1.5 and steadily catches up to it as the intensity increases. The nonlinear elastic system also shows a statistically significant deviation from the KH system results, but at a 5-8% difference this is still not of practical importance.



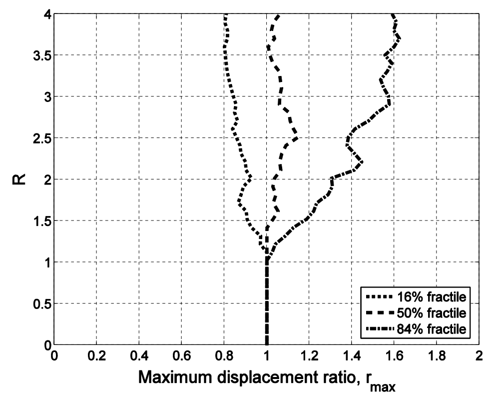
(a) T = 0.5sec, total hysteretic energy ratio, e_{dyn}



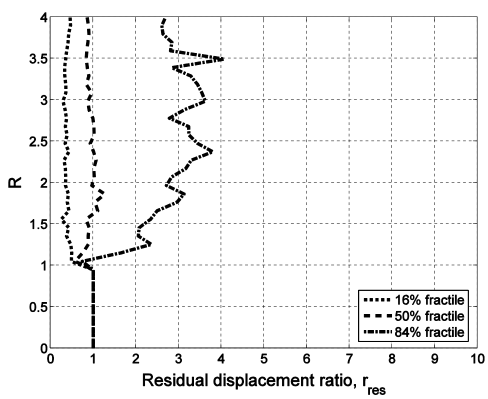
(d) T = 1.0sec, total hysteretic energy ratio, e_{dyn}



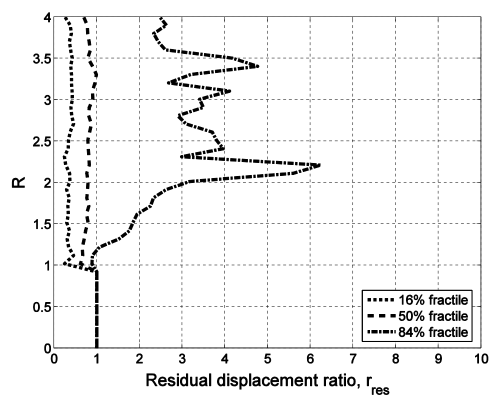
(b) T = 0.5sec, max displacement ratio, r_{max}



(e) T = 1.0sec, max displacement ratio, r_{max}



(c) T = 0.5sec, residual displacement ratio, r_{res}



(f) T = 1.0sec, residual displacement ratio, r_{res}

Figure 6. The 16, 50, 84% fractiles of the ratio of total dissipated hysteretic energy and of maximum and residual displacement for the pinching over the KH model for T = 0.5sec (left) and T = 1.0sec (right).

It is worth noting that the peak-oriented model was found to sustain higher residual displacement demands compared to the KH across the entire intensity range examined in Table 1. This may seem to go against the results reported by other researchers (Ruiz-Garcia and Miranda 2006; Christopoulos *et al*. 2003) that favor similar rather than unequal demands. Actually, the observed difference is a direct consequence of the details of the hysteresis adopted. As pointed out earlier, in our case the elastic unloading branch does not terminate at zero strength but extends lower. If, instead, the unloading stiffness change appears at zero strength, we get a slightly lower e_{qst} = 0.55 and a residual displacement response that practically equals the KH results. More than anything else this finding clearly shows that minor details of the hysteretic modeling may significantly affect the residual deformation demands in a way that energy dissipation cannot effectively capture, at least not when evaluated by the considered (and widely adopted) hysteretic models and analytical tools.

Providing additional evidence, the flag-shaped system, which in a median sense dissipates 20–30% less energy than the KH model, shows only marginally higher peak displacements (Table 1). Even so, the wide dispersions in the dissipated energy and displacement ratios appearing in Figure 7 suggest that there are several records that combine both lower dissipation and lower peak displacement compared to the KH system. At the extreme end, the non-dissipating nonlinear elastic system responds with increasingly higher d_{max} as R increases, reaching a 40-50% higher median at R = 4.0. Interestingly, Figures 8a and 8b show that roughly 10–20% of the accelerograms favor the nonlinear elastic model at both short and moderate periods, showing lower or equal d_{max}. On the other hand, residual displacement results thoroughly vindicate these two self-centering systems. Together with the curved KH system they display the lowest values of r_{res}, aided by their unloading behavior that, as shown in Figure 2, tends to relieve some or all of the maximum displacement when returning to rest. Such results have prompted Christopoulos *et al*. (2002) to suggest that a flag-shaped system of similar strength and period can be found to achieve equal or smaller displacement demands to those of the kinematic hardening system, despite the fact that the hysteretic energy absorption is lower for the former.

It is apparent for the whole range of the considered cases that there is no correlation between the computationally evaluated e_{qst} and e_{dyn} ratios. Thus, observations under cyclic loading regarding the hysteretic behavior of a system may not be stable indicators of its dynamic energy dissipation. Furthermore, any superior hysteretic energy dissipation performance does not seem to be reflected in the analytically evaluated maximum or residual displacement demands. Thus, when it comes to computations involving such systems as the ones studied, it is not the area of the loops themselves that seems to matter but rather the finer details of the hysteretic rules that are employed in each case.

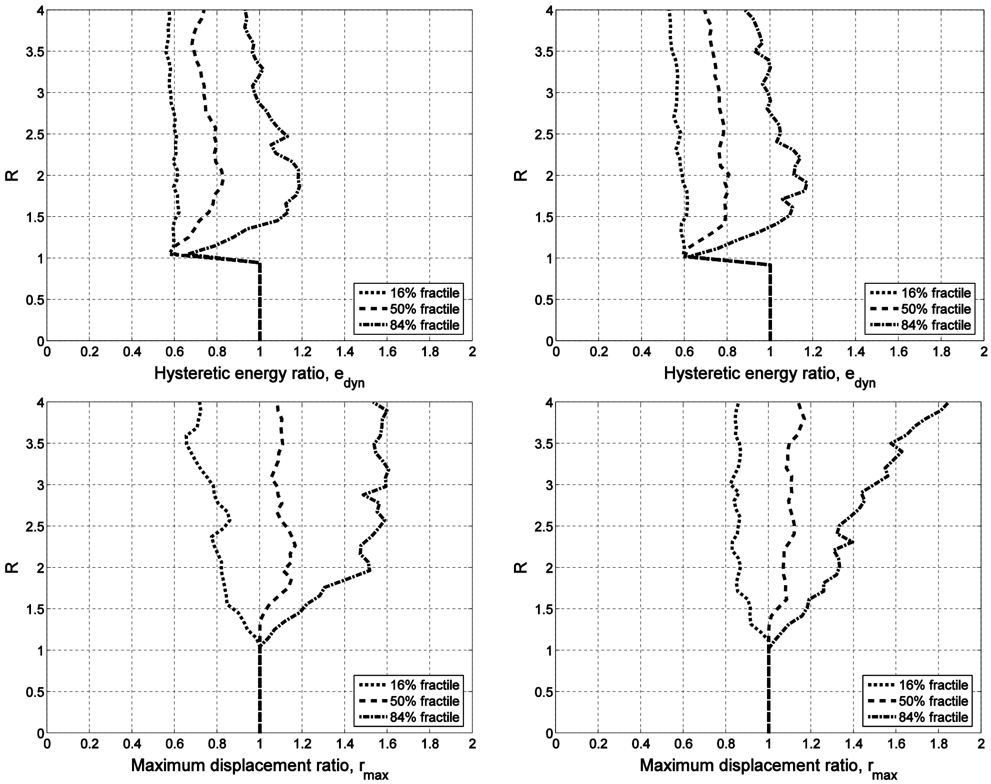


Figure 7. The 16, 50, 84% fractiles of the ratio of total dissipated hysteretic energy and of maximum displacement for the flag-shaped over the KH model for T = 0.5sec (left) and T = 1.0sec (right).

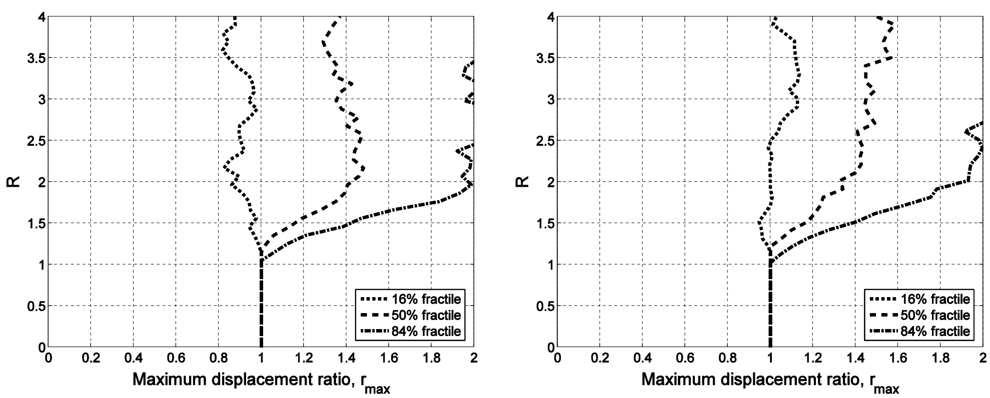


Figure 8. Maximum displacement ratio $r_{max,x\%}$ of nonlinear elastic over the kinematic hardening model for two characteristic periods: (a) T = 0.5sec and (b) T = 1.0sec.

Additional SDOFs were also examined, employing various hysteresis rules, different damping, and elastic-hardening or elastic-negative backbones that incorporate in-cycle strength degradation. In all cases, the results are in agreement with our earlier conclusions. One of the most characteristic cases is illustrated in Figure 9 for the modified Ibarra-Medina-Krawinkler deterioration model (Lignos and Krawinkler 2009), which directly employs the dissipated hysteretic energy to estimate the cyclic deterioration of strength and stiffness. For the two different levels of cyclic deterioration considered (Figures 9a and 9b), the peak

(Figure 9c) and residual (Figure 9d) displacements are found to be very similar to each other. Still, a combination of longer duration ground motions, shorter period systems and more aggressive cyclic deterioration parameters may indeed make a difference here (e.g., see Ibarra and Krawinkler 2005). Otherwise, for the entire spectrum of the hysteretic models examined, little correlation of the hysteretic energy absorption and the computed seismic deformation demands was found.

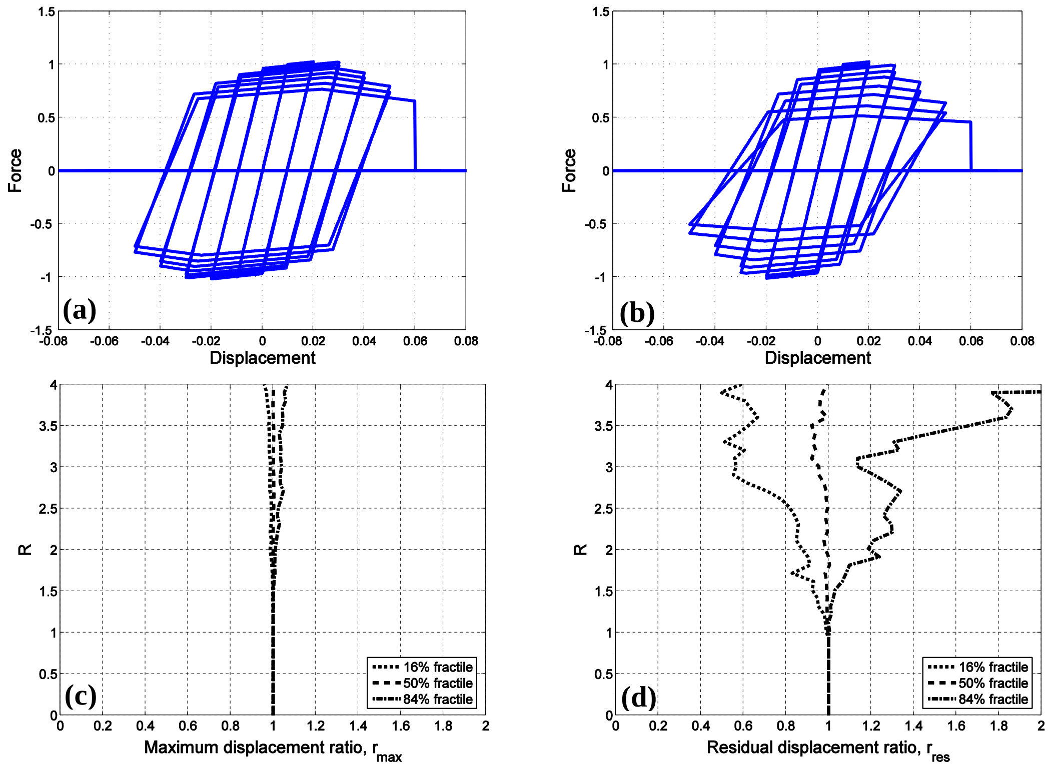


Figure 9. The modified IMK model for (a) low, (b) high cyclic deterioration. Little difference between the two is shown by the 16, 50, 84% fractiles of the ratio of maximum (c) and residual (d) displacement for the high over the low deterioration at T = 1.0sec.

AREA UNDER THE CAPACITY CURVE AND PERFORMANCE

Having shown that the correlation between the computed dissipated energy and seismic performance is not always credible, it becomes important to investigate the use of other proxies for energy. One such popular concept is the (implicit) classification of systems according to the area enveloped under the monotonic force-deformation capacity curve. Essentially, this can be viewed as a substitute for hysteretic energy that disregards cyclic unloading-reloading rules to deliver an index to approximate the seismic performance of a system based on purely static monotonic loading. The motivation is obvious as no hysteretic modeling is needed. The question now concerns the effect of the capacity curve itself, if the hysteretic rule remains the same, contrary to our previous tests where all the oscillators shared the same backbone but different hysteresis. In effect, could the notion of the "area under the curve" (as equivalent to energy) become a useful indicator of performance? Such ideas have already entered seismic codes, at least implicitly, e.g., regarding the definition of an equivalent SDOF system for nonlinear static procedures. For example, Eurocode 8 (CEN 2004) stipulates that the equivalent elastoplastic SDOF system should enclose the same area under the capacity curve (from zero up to the target displacement) as the MDOF system studied. In effect, this suggests that these two systems would show the same (or similar) seismic performance, at least at the level implied by the target displacement.

In reality, the "area under the curve" concept may sometimes serve as a useful general indicator of the combined effect of strength and ductility. For example, Figure 10a shows three different elastic-hardening-negative pinching systems with different negative stiffness segments, where indeed a larger area under the curve means improved collapse capacity and lower displacement demands. Nevertheless, in the authors' opinion, the complexity of a capacity curve does not always lend itself to simplified descriptions via the enclosed area. Recent studies (Ibarra and Krawinkler 2005; Vamvatsikos and Cornell 2006) have already cast serious doubts on the usefulness of such an index to predict the seismic performance, offering detailed observations on how the characteristics of the static pushover curve affect structural response. Actually, the SPO2IDA tool (Vamvatsikos and Cornell 2006) offers a unique perspective of the relationship between the capacity curve and the maximum displacement by instantly providing an estimate of the IDA fractile curves for very complex oscillators with peak-oriented or pinching hysteresis. An example appears in Figure 10b where moderate period oscillators with a matching negative stiffness segment have nearly identical median d_{max} response, despite the large differences in strength and enclosed area under the capacity curve. Figure 11 provides further evidence for MDOF structures (Vamvatsikos and Fragiadakis 2010) where a seemingly significant advantage or disadvantage in pushover peak strength (Figure 11a) never translates to any appreciable difference in maximum interstory drift demand (Figure 11b).

Furthermore, De Luca *et al*. (2012) have carefully studied the use of "equal area" concepts to derive equivalent backbone curves for static pushover analysis under the same hysteresis. They have shown that procedures such as the one suggested by Eurocode 8 do not always result to an equally performing equivalent SDOF system. In addition they have presented many cases where the curvature in the capacity curve may count for more than the area it encloses. For example, the piecewise linear backbone appearing in Figure 12 may almost always envelop the curved backbone, yet it displays up to 20% lower S_a capacity and nearly 40% higher d_{max} demands in the post-peak range, surprisingly showing a significantly subpar performance. Such observations have proven that seismic performance is a more complex creature than often believed, and for sure it cannot be judged easily by any current definition of seismic energy or “area under the capacity curve” ideas.

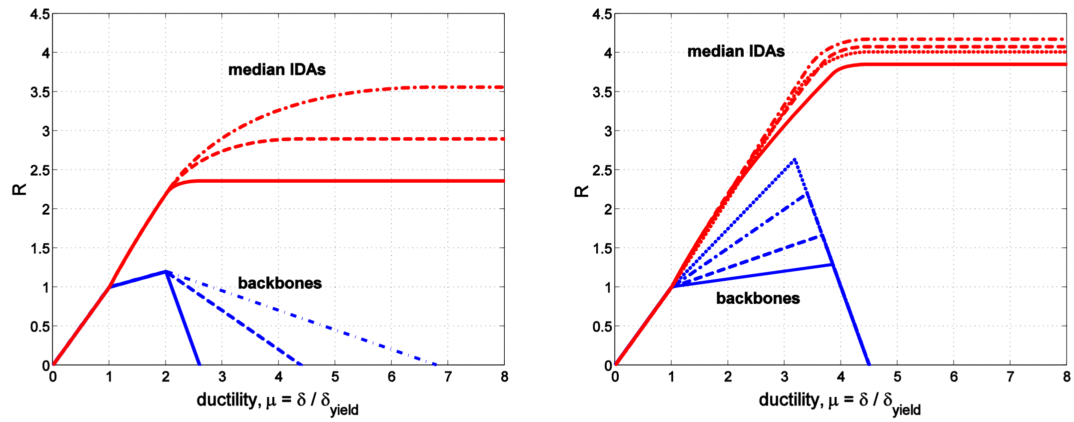


Figure 10. The median IDAs produced by SPO2IDA for four different SDOF systems having the same period and (a) coincident elastic-hardening or (b) coincident negative stiffness segments. The area under the curve correlates well with collapse capacity and median displacement demand in the first case but not in the second (Vamvatsikos and Cornell 2006).

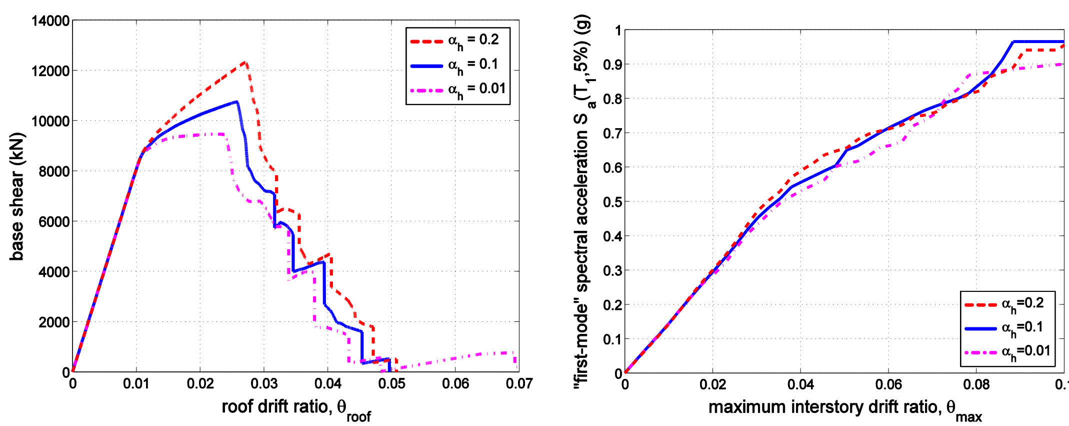


Figure 11. Three different realizations of the same building using different values for the hardening stiffness ratio a_h of the beam plastic hinges. Despite the obvious differences in the SPO curves (a), the median IDA (b) dynamic response is indistinguishable (Vamvatsikos and Fragiadakis 2010).

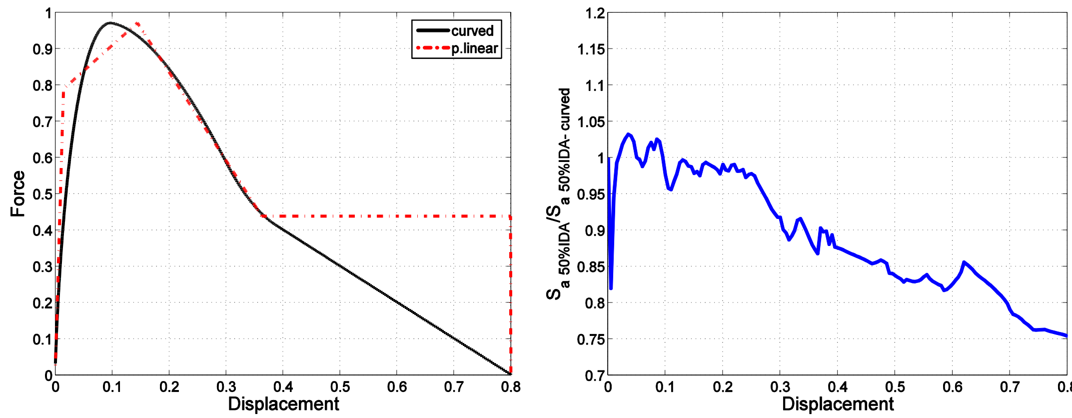


Figure 12. Two SDOF systems with the same period and hysteresis. The one having a piecewise linear capacity curve envelops the curvilinear one (a), but shows inferior performance, i.e., lower median S_a capacities for a given displacement (b) for practically all intensity levels (De Luca *et al.* 2012).

ROAD PATH

Given the abovementioned findings, one can conclude that we should be careful of how we interpret the results of quasi-static tests for assessing the seismic mitigation capabilities of any existing or novel seismic device/system. Hence, one may not use the area confined by the hysteretic loops or the amount of pinching as a proving ground for the performance assessment of a hysteretic energy dissipating element. Instead its performance should be judged based on other force-deformation (or moment-rotation) characteristics, such as cyclic and in-cycle degradation, ductility and residual deformations. Undoubtedly, hysteretic elements do enhance the seismic performance of existing buildings since they limit structural deformations during low-to-moderate earthquakes while they also act as sacrificial ductile elements during severe seismic events that move plastic zone formation away from the existing dissipation areas (often non-ductile) of the structural frame (Benavent-Climent, 2011). Nevertheless, it seems their ductility, strength and self-centering characteristics rather than their hysteretic energy dissipation performance to be the most influential parameters for the overall system performance (e.g. Kam *et al*. 2007).

For assessing the seismic performance of structural components several local indices exist in the literature (see Kappos 1997). We suggest future exploitations on this area to focus more on deformation and/or strength criteria rather than on the hysteretic energy dissipation. A potential candidate index might be one that combines a deformation-based index, defined as the ratio of the max deformation of the system in question over the max deformation of the kinematic hardening (elastoplastic) reference system ($d_{max,system}$ /$d_{max,KH}$), having the same period, damping and for the same record, with another deformation index that accounts for the residual drifts, defined as the amplitude of the residual deformation over the maximum deformation both of which refer to the system in question ($d_{res,system}$ /$d_{max,system}$). Depending on the user needs a weighted combination of the above indices might yield optimum results, but some calibration is required to determine the weighting factors. Even though dissipated energy was shown to be an inconsistent seismic performance indicator when considering systems with widely different hysteretic characteristics, it might still be calibrated to serve as an indicator for the seismic performance of any particular single system. In such a case, to link the analytically evaluated dissipated energy with the system's seismic performance, existing archival datasets from tests on similar component typologies can be leveraged to correlate the energy dissipated with specific damage states. Obviously, such an energy-based index would need to be recalibrated to be applicable to different component hysteretic characteristics.

CONCLUDING DISCUSSION

What has been shown in detail, by means of a purely computational work, is that estimates of hysteretic energy are not well correlated to acceleration or displacement demands across different hysteresis types. On the contrary, the numerically evaluated structural response is largely dependent on the details of the monotonic capacity curve and the force-displacement hysteresis in ways that cannot be described by energy dissipation in any of its tested forms. Force-deformation characteristics, such as the shape and curvature of the backbone and whether it shows self-centering behavior or not, have a more pronounced impact on the response of structural models. Hence, the computed hysteretic energy capacity may or may not be critical depending on the system hysteretic characteristics. That is not to say that useful results cannot be gained based on the dissipated energy, as many authors have already shown, but rather that the energy dissipation cannot be understood as a sole quantity. When confining oneself to a single class of capacity curve shapes and a specific hysteretic behavior, energy can become a proxy for ductility, strength or other system characteristics (either in isolation or in combination), thus allowing the calibration of customized indices that can relate to seismic performance. It is doubtful, though, whether such relationships would survive the shift to a different backbone or hysteresis without recalibration.

Therefore, our primary objection to the unconscious use of energy dissipation as a measure for seismic performance in analytical studies stems from the fact that, to the extent that the currently available models can simulate reality, energy might not be the best possible choice. In the bygone era of elastoplastic systems with kinematic strain hardening, it was a robust quantity that could convey strength and ductility information in a single scalar value. With the advent of complex backbones and hysteretic rules, the use of energy as a discriminating index can be outright misleading. Most importantly, it completely misrepresents the capabilities of the new promising class of rocking and self-centering systems that may have decreased energy dissipation but show comparable, if not superior, performance to other, highly dissipative systems. A long record of publications has shown that hysteretic energy still has its uses but, in our opinion, we should either employ appropriately calibrated hysteretic models that directly incorporate the effect of energy on structural response and damage or be very careful when trying to correlate estimates of dissipated energy with seismic performance.

REFERENCES

Akiyama, H., 1985. Earthquake-resistant limit state design for buildings, University of Tokyo Press, Tokyo.

Benavent-Climent, A., 2011. An energy-based method for seismic retrofit of existing frames using hysteretic dampers, *Soil Dynamics and Earthquake Engineering* **31**(10), 1385–1396.

Bojorquez, E., Teran-Gilmore, A., Ruiz, S. E., and Reyes-Salazar, A., 2011. Evaluation of structural reliability of steel frames: Interstory drift versus plastic hysteretic energy, *Earthquake Spectra* **27**(3), 661–682.

Bommer, J. J., and Martinez-Pereira, A., 1999. The effective duration of earthquake strong motion, *Journal of Earthquake Engineering* **3** (2), 127–172.

BSL, 2009. *The Building Standard Law of Japan*, The Building Center of Japan, Tokyo.

CEN, 2004. *Eurocode 8: Design of structures for earthquake resistance—Part 1: General rules, seismic actions and rules for buildings*, European Committee for Standardization, Brussels.

Christopoulos, C., Filiatrault, A., and Folz, B., 2002. Seismic response of self-centering hysteretic SDOF systems, *Earthquake Engineering and Structural Dynamics* **31**, 1131–1150.

Christopoulos, C., Pampanin, S., and Priestley, M. J. N., 2003. Performance-based seismic response of frame structures including residual deformations. Part I: Single-degree of freedom systems, *Journal of Earthquake Engineering* **7**(1), 97–118.

De Luca, F., Vamvatsikos, D., and Iervolino, I., 2012. Near-optimal piecewise linear fits of static pushover capacity curves for equivalent SDOF analysis, *Earthquake Engineering and Structural Dynamics*, **42**(4): 523–543.

Foutch, D. A., and Shi, S., 1998. Effects of hysteresis type on the seismic response of buildings, in *Proceedings, 6th U.S. National Conference on Earthquake Engineering*, EERI, Seattle, WA.

Huang, Z., and Foutch, D. A., 2009. Effect of hysteresis type on drift limit for global collapse of moment frame structures under seismic loads, *Journal of Earthquake Engineering* **13**, 1363–2469.

Ibarra, L. F., and Krawinkler, H., 2005. *Global Collapse of Frame Structures under Seismic Excitations, Report No. 152,* The John A. Blume Earthquake Engineering Center, Stanford University, Stanford, CA.

Ibarra, L. F., Medina, R. A., and Krawinkler, H., 2005. Hysteretic models that incorporate strength and stiffness deterioration, *Earthquake Engineering and Structural Dynamics* **34**, 1489–1511.

Jacobsen, L. S., 1960. Damping in composite structures, in *Proceedings, 2nd World Conference on Earthquake Engineering*, Tokyo and Kyoto, Japan, 1029–1044.

Kam, W.Y., Pampanin, S., Carr, A.J., and Palermo, A., 2007. Advanced flag-shape systems for high seismic performance including near-fault effects. *Proceedings, 2007 NZSEE Conference*, Palmerston North, New Zealand.

Kappos, A.J., 1997. Seismic damage indices for RC buildings: evaluation of concepts and procedures, *Progress in Structural Engineering and Materials*, **1**(1): 78–87.

Lignos, D. G., and Krawinkler, H., 2009. *Sidesway collapse of deteriorating structural systems under seismic excitations. Rep. No. TB 172*, The John A. Blume Earthquake Engineering Center, Stanford University, Stanford, CA.

McKenna, F., Fenves, G., Jeremic, B., and Scott, M., 2000. Open system for earthquake engineering simulation. <http://opensees.berkeley.edu>, (Oct. 9, 2011).

Miranda, E., and Ruiz-Garcia, J., 2002. Evaluation of approximate methods to estimate maximum inelastic displacement demands, *Earthquake Engineering and Structural Dynamics* **31**, 539–560.

Pacific Earthquake Engineering Research Center, 2011. PEER Strong motion database. <http://peer.berkeley.edu/peer_ground_motion_database>, (Mar. 10, 2011).

Park, Y.J., and Ang, A.H., 1985. Mechanistic seismic damage model for reinforced concrete, *Journal of Structural Engineering*, **111**(4), 722–739.

Park, Y.J., Ang, A.H., and Wen, Y.K., 1987. Damage-limiting aseismic design of buildings, *Earthquake Spectra* **3**(1), 1–26.

Priestley, M.J.N., 1993. Myths and fallacies in earthquake engineering – Conflicts between design and reality, *Bulletin of the New Zealand National Society for Earthquake*, 26(3), 329–341.

Rahnama, M., and Krawinkler, H., 1993. *Effects of soft soil and hysteretic model on seismic demands, Report No 108*, John A. Blume Earthquake Engineering Center, Stanford University, Stanford, CA.

Ruiz-Garcia, J., and Miranda, E., 2006. Evaluation of residual drift demands in regular multi-storey frames for performance-based seismic assessment, *Earthquake Engineering and Structural Dynamics* **35**(13), 1609–1629.

Shome, N., Cornell, C. A., Bazzurro, P., and Carballo, J. E., 1998. Earthquakes, records and nonlinear responses, *Earthquake Spectra* **14**(3), 469–500.

Uang, C. M., and Bertero, V. V., 1990. Evaluation of seismic energy in structures, *Earthquake Engineering and Structural Dynamics* **19**(1), 77–90.

Vamvatsikos, D., and Cornell, C. A., 2002. Incremental dynamic analysis, *Earthquake Engineering and Structural Dynamics* **31**(3), 491–514.

Vamvatsikos, D., and Cornell, C. A., 2006. Direct estimation of the seismic demand and capacity of oscillators with multi-linear static pushovers through Incremental Dynamic Analysis, *Earthquake Engineering and Structural Dynamics* **35**(9), 1097–1117.

Vamvatsikos, D., and Fragiadakis, M., 2010. Incremental dynamic analysis for estimating seismic performance uncertainty and sensitivity, *Earthquake Engineering and Structural Dynamics* **39**(2), 141–163.

Wiebe, L., and Christopoulos, C., 2010. Characterizing acceleration spikes due to stiffness changes in nonlinear systems, *Earthquake Engineering and Structural Dynamics* **39**(14), 1653–1670.