

# Optimal multi-objective seismic design of a highway bridge by selective use of nonlinear static and dynamic analyses

D. Vamvatsikos

*Department of Civil & Environmental Engineering, University of Cyprus, Nicosia, Cyprus*

**ABSTRACT:** A methodology is introduced for the optimal multi-criteria performance-based seismic design of highway bridges based on evolutionary algorithms by selectively employing nonlinear static and dynamic analysis to emulate full Incremental Dynamic Analysis (IDA). IDA is a novel analysis method that can thoroughly estimate the seismic performance of structures using multiple nonlinear timehistory analyses. Simplified methods based on the static pushover (SPO2IDA) are now able to provide a simpler, albeit less accurate alternative. They are thus employed within the optimization framework to rapidly evaluate the potential of candidate designs to enter the Pareto front in each generation. IDA is only applied to a small, elite set that is selected for inclusion. The final result is a sophisticated tool for optimal design that achieves a compromise between the speed of static procedures and the accuracy of dynamic methods to generate Pareto-optimal designs for bridges under seismic loading.

## 1 INTRODUCTION

Design optimization has recently attracted large interest, especially regarding the use of genetic algorithms and evolutionary strategies to solve the difficult problem of structural design under seismic loads for conflicting objectives (e.g. see Frangopol et al 1985, Beck et al 1999, Papadrakakis et al 2002, Liu et al 2003). Designers are interested to know the trade-offs between initial construction cost and safety or initial cost and life-cycle cost.

In practically every case the use of multi-criteria genetic algorithms or evolutionary optimization techniques (Beyer 2001) has proven to be extremely robust and well-suited to solve design problems compared to the conventional gradient-based algorithms. However, the slow convergence rate of such methods and the requirement for many functional evaluations before the optimum is reached has led to the use of simpler and often less accurate methods to analyze the alternate designs under seismic loads. For example, in Papadrakakis et al (2002) an elastic response spectrum modal analysis is used, while Liu et al (2003) prefer a static pushover methodology using a bilinear single-degree-of-freedom representation of the structure.

While such methodologies offer unmatched computational advantages, especially for preliminary design, they can be highly inaccurate away from the early inelastic region, especially close to dynamic instability where the structure is about to collapse.

On the other hand the recent emergence of powerful analysis techniques represents another possible tradeoff. For example Incremental Dynamic Analysis (IDA, Vamvatsikos & Cornell 2002), offers superb accuracy at the cost of several nonlinear dynamic analyses under multiply-scaled ground motions. Still, its use on optimization problems (Vamvatsikos & Papadimitriou 2005) has shown that the computational cost can be indeed staggering, often beyond the capacity of the average engineering office. To rectify this issue we aim to balance the resource-intensive IDA with the simpler SPO2IDA technique (Vamvatsikos & Cornell 2005) based on the static pushover to achieve optimal designs with excellent accuracy at a reduced cost.

## 2 PROBLEM SETUP

The design problem is formulated in a multi-objective context that allows the simultaneous minimization of the multiple objectives, eliminating the need for using arbitrary weighting factors to weigh the relative importance of each objective. For conflicting objectives there is no single optimal solution, but rather a set of alternative solutions which are optimal in the sense that no other solutions in the search space are superior to them when all objectives are considered. Such alternative solutions, in our case trading-off the cost and safety of the bridge, are known in multi-objective optimization as Pareto

optimal solutions. The set of Pareto solutions can be obtained using Evolutionary Algorithms (Beyer 2001) well-suited to solve multi-objective optimization problems (Fonseca & Fleming 1995, Srinivas & Deb 1994). Casting the problem in a format suitable for multi-criteria optimization means defining the objectives and the constraints to be used. Therefore we aim to (a) define meaningful performance levels, or limit-states, for the bridge and (b) define constraints and objectives based directly on the limit-states and the cost.

Regarding the bridge performance levels, there does not seem to be a consensus yet on what limits to use for performance-based design (Yashinsky & Karshenas 2003). Probably the most relevant guidelines come from the NCHRP Project 12-49 (ATC/MCEER 2001) where two levels of performance are defined for an ordinary bridge, namely an “immediate service level” for an earthquake with 50% in 75 years occurrence, where the bridge is required to sustain minimum damage, and a “significant disruption level” for a 3% in 75 years earthquake, where the bridge may be usable after shoring but could very well be replaced later. Similar two-level criteria are also suggested by the Caltrans Seismic Design Criteria (2002), while three-level criteria are suggested for railway bridges (AREMA 2002).

The adopted approach in defining the limit-states stems from the draft ideas presented by the Pacific Earthquake Engineering Research Center in cooperation with Caltrans (Porter 2002). Therein it is suggested to use four performance levels: Immediately Operational (IO) where no action needs to be taken, Operational (O) where the bridge may have to be closed for a few days and some repairs may be needed, Life Safe (LS) when lateral capacity has been impaired, the bridge has to be closed for an extended time and serious repair work and shoring is needed and finally Collapse Prevention (CP), where the bridge needs to be closed and may later be repaired or replaced, whichever is cheaper. These correspond to distinct limit-states but they are difficult to define using structural response variables. As will be discussed in a later section, we have defined the limit-states using reasonable limits for the maximum pier column drift and the maximum bearing displacement and shear strain. These choices are not restrictive and only represent an engineering decision that may be revised easily.

Taking advantage of the collapse prediction capabilities inherent in IDA we have defined the Global Instability (GI) limit-state. It occurs when the deck falls off the abutment seat or pier seat, or when the pier has reached dynamic instability due to excessive loading, whichever occurs first. This event obviously has a higher (or at most equal) return period than the CP limit-state and when it occurs the bridge has collapsed and needs to be replaced.

When defining the constraints and objectives for the design, it makes sense to choose from these five limit-states plus some measure of the bridge cost. The main idea is that the bridge must satisfy some basic acceptability criteria for the constrained limit-states but the designer will seek possible optimal tradeoffs regarding the unconstrained performance levels versus the cost. Thus, different selections of what to constrain and what to optimize may correspond to very different design schemes. If we are willing to spend money for a bridge that will not close as often, but has in general some fixed MAF of facing serious repairs, means constraining O, LS, CP, GI and optimizing for cost and IO. If on the other hand we are satisfied with some standard limits for frequent small-scale repairs but wish to trade money for the collapse safety of the bridge, then we need to constrain IO, O, LS, CP and optimize for cost and GI.

The latter case was the choice for the example; therefore, we will constrain the IO, O, LS and CP limit-states by directly setting occurrence rates that are deemed reasonable for this ordinary bridge. Specifically, the IO level was set at 75% in 50 years maximum, the O level at 50% in 50 years, the LS level at 10% in 50 years and the CP level at 2% in 50 years. The corresponding minimum allowed return periods are 36, 72, 475 and 2475 years respectively. On the other hand, we will let the optimization pursue the optimal tradeoff for the GI limit-state and the material construction cost. The cost is defined as the combined material cost of the bearings, the concrete and the steel, using typical prices adopted by designers in Greece.

Formally, if we let  $C$  be the material cost,  $\lambda_L$  be the mean annual frequency of exceeding limit-state “L”,  $\tau_L = 1/\lambda_L$  the associated return period and  $\theta$  the parameter vector, we have posed the following optimization problem:

$$\text{minimize } C(\theta), \lambda_{GI}(\theta),$$

$$\text{where } \theta \in \Theta \quad (1)$$

$$\text{subject to } \begin{cases} \tau_{IO} > 36 \text{ yrs} \\ \tau_O > 72 \text{ yrs} \\ \tau_{LS} > 475 \text{ yrs} \\ \tau_{CP} > 2475 \text{ yrs} \end{cases}$$

where  $\Theta$  is the parameter space.

The optimization may further be constrained due to restrictions imposed to the elements of the parameter vector  $\theta$ . The feasible parameter space  $\Theta$  is usually confined in a hypercube by specifying lower and upper limits on each parameter. These limits depend on physical constraints, information about the physical characteristics of the system and modeling experience, and they are going to be discussed in

conjunction with the bridge description in the following section.

### 3 HIGHWAY BRIDGE MODEL

As the testbed for our methodology we will employ a typical highway bridge, shown in Fig. 1, to be constructed in a high seismicity region in Greece. It is a reinforced concrete structure that has a total length of 68m and width of 13.30m. It carries two lanes of traffic on a reinforced concrete superstructure made of four precast, pretensioned beams connected by a cast-in-place concrete deck slab (Fig. 2). There is one pier made of a single hollow rectangular reinforced concrete column (Fig. 3) that is 28m tall. The superstructure is made up of two identical spans supported on elastomeric bearings both on the pier cap and on the seat-type abutments. There is one bearing under each of the four beams on either side of each span and there are also 30cm wide stoppers in the longitudinal direction to control displacement. They allow a maximum displacement of 0.5m before the deck hits them, and a total of 0.8m of displacement, if broken, before the deck slides off the pier cap or the abutment seat.

The parameters of the problem that have been selected for optimization define the stiffness and strength of the bearings and of the pier column. For the (square) bearings these are the width  $b$  and elastomer height  $h$ , both for the abutment bearings ( $b_a$  and  $h_a$ ) and the pier bearings ( $b_p$ ,  $h_p$ ). For the pier column we have included the longitudinal width (along the bridge axis) of the hollow rectangular pier section,  $b_c$ , and the longitudinal (height-wise) reinforcement ratio  $\rho_c$ . Thus, the parameter vector for the optimization problem defined in Eq. 1 is:

$$\boldsymbol{\theta} = (b_a, h_a, b_p, h_p, \rho_c, b_c) \quad (2)$$

These six parameters are allowed to vary freely within some reasonable engineering limits. Specifically, we set the minimum limits for the bearing parameters according to some minimum designer requirements while the maxima were set by taking into account the catalogue of sizes typically manufactured. Thus we allowed the width of the bearings  $b_a$  and  $b_p$  to vary within 0.65m and 1.0m and the total rubber height  $h_a$  and  $h_p$  to have values within 8cm and 25cm. For the column width we set a minimum according to code requirements and expert opinion, so we let it vary between 2m and 4m. Finally, the code-supplied limits were used for the reinforcement, i.e., the allowable range is 1%-4%.

In summary, this is a simple, first-mode dominated structure that can nevertheless become a difficult optimization problem, amenable to treatment by the proposed methodology.

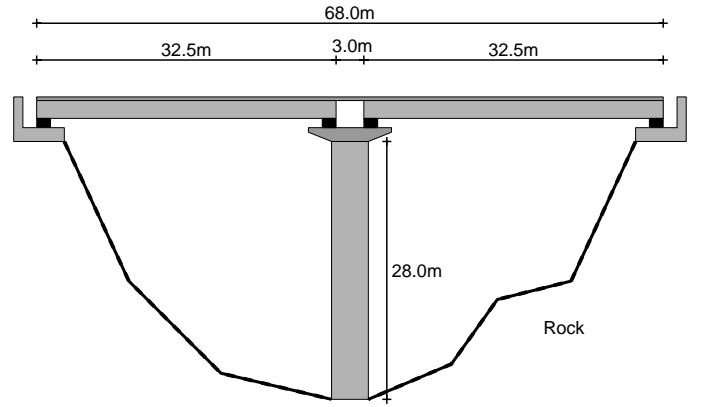


Figure 1. The bridge to be designed.

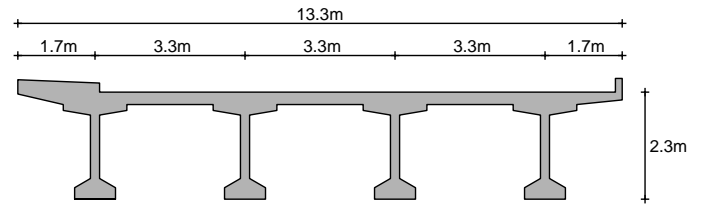


Figure 2. Section of the bridge deck, comprised of four precast, pretensioned I-beams and a cast-in-place deck slab.

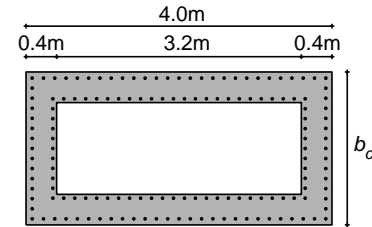


Figure 3. The hollow rectangular section for the pier column: 4m wide in the transverse direction,  $b_c$  width in the longitudinal direction and an all around concrete thickness of 0.40m.

### 4 PERFORMANCE EVALUATION

Incremental Dynamic Analysis (IDA) is an emerging analysis method that offers thorough seismic demand and capacity prediction capability (Vamvatsikos & Cornell, 2002). It involves performing a series of nonlinear dynamic analyses under a multiply scaled suite of ground motion records, selecting proper Engineering Demand Parameters (EDPs) to characterize the structural response and an Intensity Measure (IM), e.g. the 5% damped first-mode spectral acceleration,  $S_a(T_1, 5\%)$ , to represent the seismic intensity, and then generating curves of EDP versus IM for each record. On such IDA curves the appropriate limit-states can be defined by setting appropriate limits on the EDPs and the probabilistic distribution of their capacities can be estimated. Such results combined with probabilistic seismic hazard analysis (Vamvatsikos & Cornell 2002) allow the estimation of mean annual frequencies (MAFs) of exceeding the limit-states. These are exactly what is needed to characterize performance and measure the ability of any candidate design to withstand rare or more frequent seismic threats. IDA has been applied both to structures (Vamvatsikos &

Cornell 2002) and bridges (Mackie & Stojadinovic 2002) and it has been shown to provide excellent prediction capabilities, forming a reliable tool for assessing each alternate design.

Here we are going to explain in detail the elements needed to run IDA, i.e., the structural model and the record suite. Then we will explain how the limit-states are defined and finally we will present the application of IDA for the bridge model for a given selection of the design parameters. In essence, we will take the reader briefly through all the steps needed to perform the performance evaluation of a single design case.

#### 4.1 Structural model

The OpenSEES platform (McKenna et al 2000) was used to form a realistic model of the bridge in 2D. The connecting element between the deck and the piers incorporates the behavior of the elastomeric bearing, including the degradation and eventual breaking of the rubber at high strains, the existing gap between bearings and stoppers and the finite strength of the latter. A detailed fiber model was used for the reinforced concrete section of the pier column while a first-order treatment of geometric nonlinearities (P- $\Delta$  effects) was included.

#### 4.2 Record suite

The number of records selected directly impacts the size of the problem but also the accuracy of the results. Since we have a first-mode dominated, medium period structure ( $T_1 = 0.9 - 1.3$ s), we chose to use ten records as a reasonable compromise. These were selected from a relatively narrow magnitude and distance bin, having moment magnitude within 6.5 – 6.7 and closest distance to fault rupture 18 – 38km (Table 1). They have all been recorded on firm soil and bear no marks of near-fault directivity.

Table 1. The suite of records used for IDA

Event Station	$\phi^1$ deg	Soil <sup>2</sup>	M <sup>3</sup>	R <sup>4</sup> km	PGA G
Superstition Hills, 1987					
Plaster City	135	C,D	6.7	21.0	0.19
Brawley	225	C,D	6.7	18.2	0.16
San Fernando, 1971					
LA Hollywood Sto Lot	180	C,D	6.6	21.2	0.17
Imperial Valley, 1979					
Chihuahua	012	C,D	6.5	28.7	0.27
Plaster City	135	C,D	6.5	31.7	0.06
Compuertas	285	C,D	6.5	32.6	0.15
Northridge, 1994					
Leona Valley #2	090	C,-	6.7	37.7	0.06
Lake Hughes #1	000	C,C	6.7	36.3	0.09
LA Hollywood Sto FF	360	C,D	6.7	25.5	0.36
LA Baldwin Hills	090	B,B	6.7	31.3	0.24

<sup>1</sup> Component

<sup>2</sup> USGS, Geomatrix soil class

<sup>3</sup> Moment magnitude

<sup>4</sup> Closest distance to fault rupture

#### 4.3 Defining the limit-states

There are two elements of the bridge that can sustain damage from earthquakes: The bearings and the pier. Therefore, any limit-state definition needs to take into account limiting values for both of them. In specific for the bearing we need to keep track of the maximum absolute shear strain,  $\gamma_{\max}$ , and the maximum absolute displacement normalized by the bearing width,  $\delta_{\max}$ . According to code guidelines, the rubber may fracture at high values of  $\gamma_{\max}$  necessitating the replacement of bearings, while  $\delta_{\max}$  provides us with a rule-of-thumb to determine the lateral resistance degradation of the bearing under a given axial load. Additionally we have to take into account the “positive” bearing displacement  $d_{\max}^+$ , defined as the displacement away from the bearing seat. This measures exactly the separation between the deck and the abutment or the pier; if it exceeds 0.8m it will result in the deck falling off. Regarding the column we only need to track the peak drift  $\theta_{\max}$ , which has been shown to correlated well with column damage and is frequently used in this role (e.g., Mackie & Stojadinovic 2002). In total we have four different EDPs, all of which can play a part in deciding the violation of each limit-state.

Specifically, IO is set to occur when  $\theta_{\max} = 1\%$  (initiation of cover spalling) or  $\delta_{\max} = 0.33$  or  $\gamma_{\max} = 120\%$  (mild bearing damage), whichever occurs first. O is similarly defined at  $\theta_{\max} = 2\%$  (large visible cracks),  $\delta_{\max} = 0.5$  or  $\gamma_{\max} = 180\%$  (serious bearing damage), LS occurs when  $\theta_{\max} = 3\%$  (degraded column capacity) or  $\delta_{\max} = 0.75$  and finally CP appears at  $\theta_{\max} = 5\%$  (serious degradation of column capacity) or  $\delta_{\max} = 1.0$  (bearing has moved beyond its footprint). The GI limit-state will appear only if one of the two failure modes happens: Either  $d_{\max}^+ = 0.8$ m, i.e., the deck falls off the pier or the abutment seat due to excessive displacement and breaking of the stopper (strong column, weak bearings), or  $\theta_{\max} = +\infty$ , i.e., dynamic instability appears in the columns (weak column, strong bearings). In the vast majority of cases though, only the first event happens, the second appearing only in few, if any, of the possible designs.

#### 4.4 Analysis via IDA and SPO2IDA

Performing IDA for each record involves multiple nonlinear dynamic analyses under records that have been scaled to cover the entire range of bridge behavior, from elasticity to final collapse. After each analysis the four EDPs are recovered and plotted to generate the corresponding IDA curve of each record.

As an example we will show the results for a candidate design having  $b_c = 3.0$ m pier width,  $\rho_c = 2.0\%$  reinforcement ratio,  $b_a = 0.7$  m and  $h_a = 0.12$  m for the abutment bearings and  $b_p = 0.8$  m,  $h_p = 0.17$  m for the pier bearings. This

case has a first mode period  $T_1 = 1.2\text{s}$  and a material cost  $C = 106,573$ . By performing seven analyses per record and interpolating the results we get ten IDA curves for each EDP. Fig. 4 shows the IDA curves for  $d_{\max}^+$ . When these reach 0.8m, a flatline occurs, as the deck has fallen off and the bridge is now considered to have collapsed. These flatlines set the maximum IM limit that the design can withstand and they are the same (in IM terms) for all EDPs. In Fig. 5 are the curves for  $\delta_{\max}$  which actually look quite similar to the curves of  $d_{\max}^+$ , which means that the absolute and the “positive” value are well correlated in this case. Finally, in Fig. 6 we can see the results for the maximum drift  $\theta_{\max}$ . In all cases there is considerable dispersion in the results while the column and bearing EDPs seem well correlated, something to be expected for this rather simple system.

By applying the EDP-based definitions of the limit-states on the IDA curves we can easily estimate the limit-state capacities in IM-terms for each record. Then, using the Pacific Earthquake Engineering Research Center framing equation (Cornell & Krawinkler 2000) plus an appropriate hazard curve for  $S_a(T_1, 5\%)$  (Fig. 7) we can integrate to get MAFs of exceeding each limit-state (Vamvatsikos & Cornell 2004).

While the PEER framework is general enough to allow inclusion of epistemic uncertainties, we will neglect all material and model associated uncertainties, only retaining the uncertainty incorporated into the hazard curve (Cornell et al. 2002). Having said that, if we perform the necessary numerical integration we come up with the following return periods  $\tau_{IO} = 32$ ,  $\tau_O = 93$ ,  $\tau_{LS} = 951$ ,  $\tau_{CP} = 1690$  and  $\tau_{GI} = 3580$ . Obviously the IO and CP constraints are violated, hence this case will be rejected by the algorithm. The total computing time on a Pentium-IV processor was only 8 minutes.

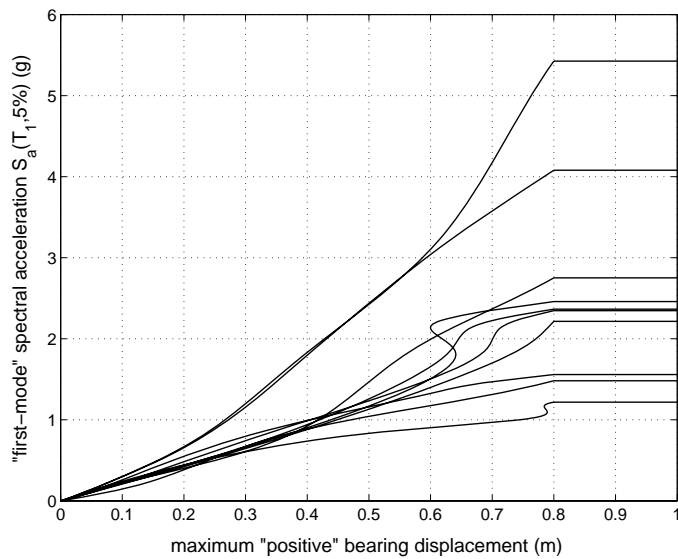


Figure 4. Example IDA curves for the “positive” bearing displacement  $d_{\max}^+$ , i.e. the maximum separation of the deck and the pier or abutment.

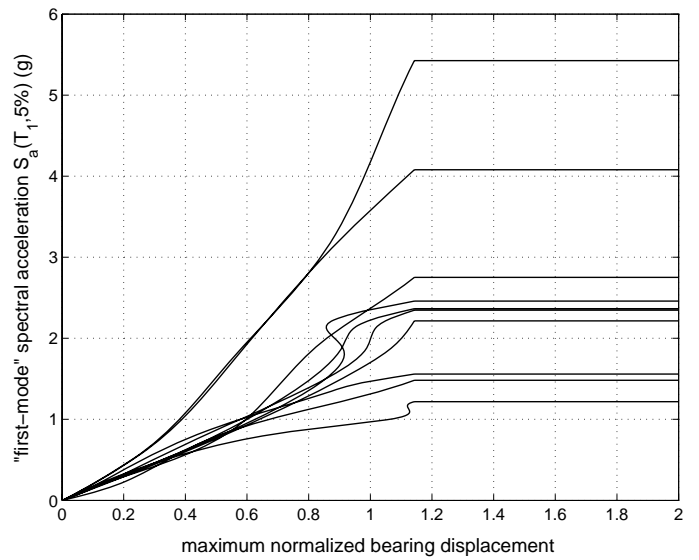


Figure 5. Example IDA curves for maximum bearing displacement normalized by the bearing width,  $\delta_{\max}$ .

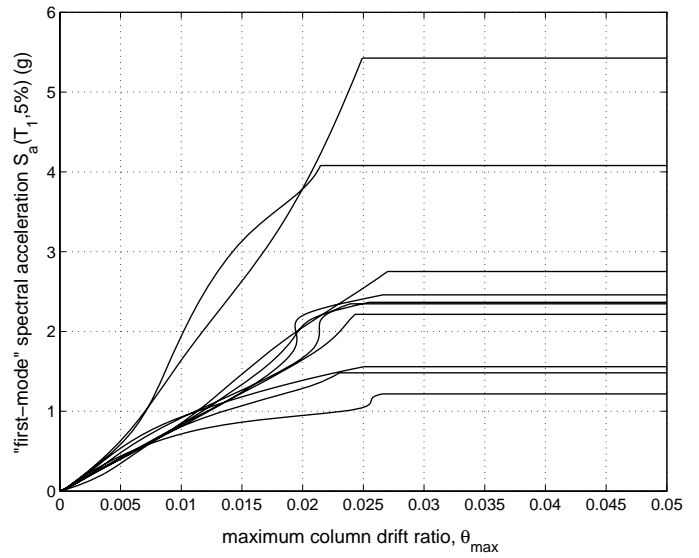


Figure 6. Example IDA curves for a sample design for column drift  $\theta_{\max}$ .

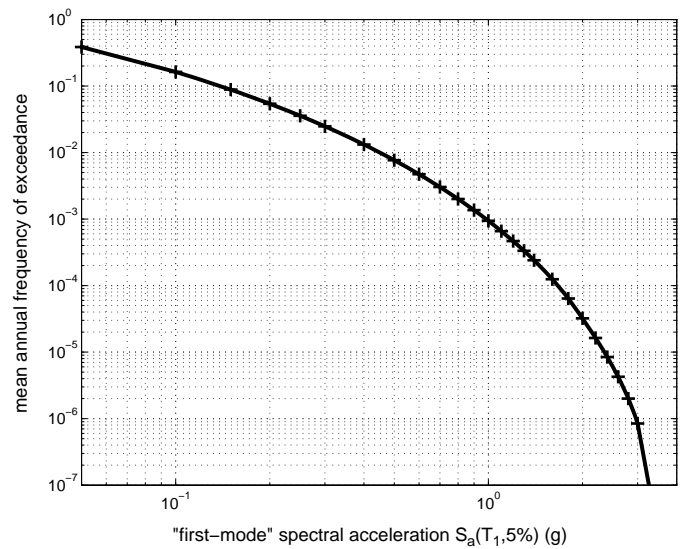


Figure 7. Typical hazard curve for a high seismicity area (Los Angeles, Van Nuys) for  $T=1.2\text{s}$ , corresponding to the bridge example case.

Similar results can be achieved by replacing IDA with SPO2IDA evaluation (Vamvatsikos & Cornell 2006). SPO2IDA is a software tool that was based on the study of numerous SDOF systems having a wide range of periods, moderately pinching hysteresis and 5% viscous damping, while they feature backbones ranging from simple bilinear to complex quadrilinear with an elastic, a hardening and a negative-stiffness segment plus a final residual plateau that terminates with a drop to zero strength as shown in Figure 8. It provides an approximate estimate of the performance of virtually any oscillator without having to perform the costly analyses, and almost instantaneously recreates the fractile IDAs in normalized coordinates of the strength reduction factor  $R$  versus the ductility  $\mu$ . SPO2IDA is in essence a powerful  $R-\mu-T$  relationship that will provide not only central values (mean or median) but also the dispersion, due to record-to-record aleatory randomness, using a multilinear approximation of the static push-over.

The SPO2IDA tool has been extended to first-mode dominated MDOF structures (Vamvatsikos & Cornell 2005), enabling an accurate estimation of the fractile IDA curves even close to collapse without needing any nonlinear dynamic analysis. In addition it has been shown to only slightly increase the error in our estimation, resulting to an accuracy that can be compared to that of an actual IDA using ten ground motion records. Thus, it can render IDA quite effortless, sacrificing only a small part of its accuracy to achieve excellent computational efficiency.

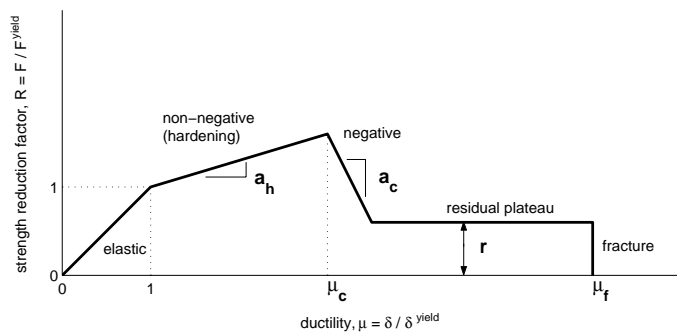


Figure 8. The quadrilinear oscillator backbone of SPO2IDA

## 5 OPTIMIZATION VIA IDA AND SPO2IDA

Evolutionary algorithms are well-suited to perform the multi-objective optimization. They process a set of promising solutions simultaneously and therefore are capable of generating multiple points along the Pareto front. These algorithms are based on an arbitrarily initialized population of search points in the parameter space, which by means of selection, mutation, and recombination evolves towards better and better regions in the search space. We chose to use

the Nondominated Sorting Genetic Algorithm II (NSGA-II, Deb et al. 2001) that has proven to be a robust and widely used method.

The NSGA-II algorithm uses a number of features specific to multi-objective optimization algorithms (Fonseca & Fleming 1995) for finding the multiple Pareto optimal solutions in parallel. It is an elitist algorithm, i.e. it preserves the best candidates from the previous generation in order to maintain a fast convergence rate. It uses non-dominated sorting (Srinivas & Deb 1994) to assess fitness and select individuals. Finally, it maintains diversity in the Pareto front by assigning a crowding distance and eliminating values that cluster at a section of the total front.

The constraints (Eq. 1) are imposed according to the handling scheme proposed by Deb (2000): When comparing two solutions, feasible designs are always considered fitter than infeasible ones, while from two infeasible solutions we select the one with the least constraint violation. This efficiently guides the optimization process to find the feasible space and search for solutions within it without discarding the useful “slightly infeasible” designs that can help us reach the Pareto front rapidly.

Originally, in each generation of the evolutionary algorithm, IDA would be used for each new offspring (i.e. candidate bridge design) to evaluate its seismic performance (Vamvatsikos & Papadimitriou 2005). Since this is a relatively small problem (only two objectives and six parameters) the IDA-based algorithm easily converged to an acceptable Pareto set (Fig. 9), but it still took about 12 days on a Pentium-IV class processor.

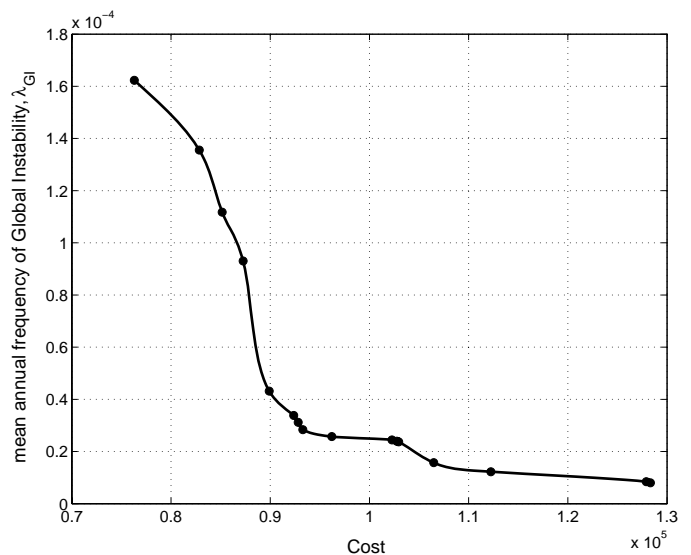


Figure 9. The Pareto front in the objectives' space.

To accelerate the above procedure we intend to perform SPO2IDA on all offsprings to quickly discard the weaker ones, and analyze the remaining few via IDA. There are several possible implementations of the above concept, each one with its own pros and

cons. At present we propose to perform this for NSGA-II with the following method:

- 1 Evaluate the performance of all  $N$  offsprings via SPO2IDA.
- 2 Select the new parent population from the  $N$  parents +  $N$  offsprings.
- 3 If any of the offsprings enters the new parent population, re-evaluate its fitness via IDA and go to step 2, else finish the selection.

Typically, this reduces the number of individuals evaluated via IDA to only 2-10% of the total, depending on how close our current solution is to the stagnation point, beyond which the algorithm cannot easily improve the solution. This in turn results to a similar reduction in the time needed to run the optimization, achieving almost one order of magnitude savings in computational load.

To achieve a fair comparison of pure IDA with the proposed IDA + SPO2IDA method, the same problem was run with an identical initial random seed. Convergence to the practically same front (Fig. 9) was achieved at only 8% of the original time.

## 6 CONCLUSIONS

The multi-criteria optimization procedure is shown to be a powerful method for performance-based seismic design. Using IDA as the tool for performance evaluation lends more accuracy and credibility to the analysis sub-process at the cost of significant computational load. Still, it remains an attractive option as it allows the direct application of constraints on the performance of a structure and provides a method for defining them that is well suited to current guidelines.

This procedure can be significantly accelerated by selectively using the static pushover and SPO2IDA to approximate the results of IDA. This allows an efficient handling of the candidate designs produced at each generation of the evolutionary algorithm, thus enabling the use of the resource-intensive IDA only on those individuals that enter the Pareto-front. The end result is an accurate procedure to perform multi-objective optimal design that can be applied at a reasonable cost.

## REFERENCES

- AREMA 2002. American Railway Engineering and Maintenance-of-Way Association manual. AREMA, Landover, MD.
- Applied Technology Council, Multi-Disciplinary Center for Earthquake Engineering Research (ATC/MCEER) 2001. Recommended LRFD guidelines for the seismic design of highway bridges, Parts I and II. ATC report NCHRP Project 12-49 and MCEER technical report MCEER-02-SP01.
- Beck J.L., Chan E., Irfanoglu A. & Papadimitriou C. 1999. Multi-criteria optimal structural design under uncertainty. *Earthquake Engineering and Structural Dynamics* 28(7): 741–761.
- Beyer H.G. 2001. *The Theory of Evolution Strategies*; SpringerVerlag, Berlin.
- Caltrans 2002. *Seismic design criteria (Version 1.2)*, California Department of Transportation, Sacramento, CA.
- Cornell, C.A. & Krawinkler, H. 2000. Progress and challenges in seismic performance assessment, *PEER Center News* 3(2), <http://peer.berkeley.edu/news/2000spring/index.html>
- Cornell, C.A., Jalayer, F., Hamburger, R.O. & Foutch, D.A. 2002. The probabilistic basis for the 2000 SAC/FEMA steel moment frame guidelines, *ASCE Journal of Structural Engineering* 128(4): 526–533.
- Deb K., Pratap A., Agarwal S. & Meyarivan T. 2000. A fast and elitist multi-objective genetic algorithm: NSGA-II. *KanGAL Report No. 200001*. Indian Institute of Technology, Kanpur, India.
- Fonseca C.M. & Fleming P.J. 1995. An overview of evolutionary algorithms in multiobjective optimization. *Evolutionary Computation* 13(1): 1–16.
- FEMA (2000). Prestandard and Commentary for the Seismic Rehabilitation of Buildings. *Report No. FEMA-356*, Federal Emergency Management Agency, Washington DC.
- Frangopol D.M. 1985. Multicriteria reliability-based optimum design. *Structural Safety* 3(1): 23–28.
- Liu M., Burns S.A. & Wen Y.K. 2003. Optimal seismic design of steel frame buildings based on life cycle cost considerations. *Earthquake Engineering and Structural Dynamics* 32: 1313–1332.
- Mackie K. & Stojadinovic B. 2002. Relation between probabilistic seismic demand analysis and incremental dynamic analysis. *7<sup>th</sup> US National Conference on Earthquake Engineering*, Boston, MA.
- McKenna F. & Fenves G.L., Jeremic B. & Scott M.H. 2000. Open system for earthquake engineering simulation (<http://opensees.berkeley.edu>).
- Papadrakakis M., Lagaros N.D. & Plevris V. 2002. Multi-objective optimization of skeletal structures under static and seismic loading conditions. *Engineering Optimization* 34: 645–669.
- Porter K. 2002. Draft Bridge Decision Variables. *Draft Report on I-880 PEER Testbed*. Pacific Earthquake Engineering Research Center.
- Srinivas N. & Deb K. 1994. Multiobjective optimization using nondominated sorting in genetic algorithms. *Evolutionary Computation* 2(3): 221–248.
- Vamvatsikos D. & Cornell C.A. 2002. Incremental Dynamic Analysis. *Earthquake Engineering and Structural Dynamics* 31(3): 491–514.
- Vamvatsikos D. & Cornell C.A. 2004. Applied Incremental Dynamic Analysis. *Earthquake Spectra* 20(2): 523–553.
- Vamvatsikos D. & Cornell C.A. 2005. Direct estimation of the seismic demand and capacity of MDOF systems through Incremental Dynamic Analysis of an SDOF Approximation. *ASCE Journal of Structural Engineering* 131(4): 589–599.
- Vamvatsikos D. & Cornell C.A. 2006. Direct estimation of the seismic demand and capacity of oscillators with multi-linear static pushovers through Incremental Dynamic Analysis. *Earthquake Engineering and Structural Dynamics* 35(9): 1097–1117.
- Vamvatsikos D. & Papadimitriou C. 2005. Optimal multi-objective design of a highway bridge under seismic loading through Incremental Dynamic Analysis. *Proceedings of the 9<sup>th</sup> International Conference on Structural Safety and Reliability (ICOSSAR)*, Rome, Italy.
- Yashinsky M. & Karshenas M.J. 2003. *Fundamentals of Seismic Protection for Bridges*. EERI No. MNO-9, Earthquake Engineering Research Institute, Oakland, CA.