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A View of Seismic Robustness Based on Uncertainty

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ABSTRACT: A simplified view of robustness and redundancy is presented for the seismic assessment and design of structures. It is argued that the topological simplicity of the seismic load, i.e., its highly correlated nature of application for practically every component in all but the ultra-long structures, means that simpler formulations can be devised compared to blast, wave or wind hazards. In that sense, robustness may be considered to express the influence of structural uncertainties on the seismic performance of the structure, in essence showing the available margin of safety subject to material variability. Quantitatively, a pertinent robustness/redundancy index is defined as the ratio of two different estimates of the mean annual frequency (MAF) of exceeding a limit-state of interest, such as global collapse: On the denominator lies the MAF estimate that incorporates all sources of variability while on the nominator is the estimate that neglects structural uncertainties. It is shown that a simple closed form solution is available that directly relates robustness to the dispersion of response due to model parameter uncertainty. As an example, a steel frame where only beams are allowed to yield is shown to be more robust compared to another version where columns become the sacrificial element.

Structural robustness and redundancy are highly desirable structural properties that are understood to be related to structural safety, especially against collapse. Qualitatively, one may think of robustness as the property of the system that prevents isolated local failures from progressing unchecked to become extensive or even global. According to EN1990, robustness in design is enforced by mandating that "a structure should be designed and executed in such a way that it will not be damaged to an extent disproportionate to the original cause". Quantitatively, this is a difficult concept to capture, as it pertains to the level and magnitude of loading, structural typology, member properties and even the consequences of failure (e.g. loss, time-to-repair etc.).

Various robustness indices have been proposed at times, each carrying its own combination of factors that contribute to increased structural safety. In general, the focus has been on assessing either the consequences or the safety associated with a locally damaged structure. For example, Baker et al (2008), suggest separating the consequences (e.g. losses) due damage into (a) the direct losses of the damaged components and (b) the indirect ones due to cascading failures of other components or loss of functionality. Then the ratio of direct over the total losses defines a robustness index. Another approach sees the comparison of the global safety of the damaged structure versus the intact, where a wealth of different indices is available to supply quantification; see for example Sorensen (2011), Mondal and Tesfamariam (2013), Starossek and Haberland (2011) and references therein for a more comprehensive review.

Redundancy is a concept quite similar to robustness. For example, Frangopol and Curley (1987) define it as the absence of critical components whose failure would cause global collapse. It is also generally taken to be associated to the presence of alternate load paths within the structure, and thus the presence of multiple elements to resist a given load. It has also received a fair share of attention, especially in seismic engineering, due to the inclusion of an explicit redundancy factor in US codes (ICBO 1997) that favors some structural configurations over others, an issue that has received considerable criticism (Wen and Song, 2002, Liao et al 2007). In the end, as elaborately argued by Kanno and Ben-Heim (2011), redundancy and robustness can be thought as two faces of the same coin, conveying the ability of the system to transfer loads with safety in face of the uncertainty in the structure itself and its environment.

1. CONCEPTUAL DEVELOPMENT

Existing approaches to determine robustness and redundancy need to be comprehensive, and thus often undeniably complex, mainly due to the need to examine the potential failure of many different components due to localized loads. In this particular case, though, the focus is on structures under seismic loads. This is a highly specific situation where topology is not necessarily important. It is quite different in comparison to, e.g., a lifeline network under seismic loads. There, having multiple alternate routes connecting a given node often confers an advantage, as each may experience different loading levels due to the spatial variability of the ground motion. It would also be different for the blast assessment of a building where the highly localized effects of an explosion become important. Then, having a large number of widely-spaced structural elements working in unison improves the survivability. Topology becomes important for such cases because the damaging loads have a very distinct spatial distribution. A single structure subject to an earthquake, instead, generally experiences the same (or a highly correlated) ground motion in all of its components (with the probable exception of long bridges). Based on the above, it can be reliably stipulated that for any single structure we can define a seismic robustness index that does not need to account for topology per se.

Then, structural robustness becomes a simpler issue of quantifying how the safety margin of capacity versus demand is influenced by the uncertainty arising from the structure itself. In that sense, robustness is not far from the notion of a reliability index that pertains to the influence of epistemic and aleatory uncertainties inherent to the structure, rather than the method of analysis or the seismic hazard. Following this mode of thinking, a simple definition for a seismic robustness index is proposed to characterize the "resistance" of structural systems to uncertainty. Specifically, the uncertainty robustness index (URI) is defined as the ratio of two estimates of the mean annual frequency (MAF) of exceeding a limit-sate of interest. At the nominator is the MAF that includes only (aleatory) record-to-record randomness, λ_R, while the denominator is the MAF estimate that also incorporates (aleatory or epistemic) structural uncertainties, λ_{RU}:

$$I_r = \frac{\lambda_R}{\lambda_{RU}} \qquad (1)$$

Obviously $0 < I_r \leq 1$. Equality to 1.0 is only theoretically possible, if there is no uncertainty in the structure. I_r is not just a generic index with no connection to the structural performance. Its inverse, $1/I_r$, is actually the factor by which the MAF is increased due to the presence of uncertainties in the structure, thus making it a very practical quantity with obvious consequences for seismic safety.

Alternatively, we can also represent I_r in terms of the familiar format of "probability of exceedance p in t years", as in "10% in 50years" being typical for a Life-Safety criterion. Then, $\lambda = -\ln(1 - p)/t \approx p/t$ for low values of λ. Thus, I_r can be related to the probabilities p_{RU} and p_U of exceeding the limit-state of interest in a period of t years with and without uncertainty, respectively:

$$I_r = \frac{\lambda_R}{\lambda_{RU}} \approx \frac{p_R/t}{p_{RU}/t} = \frac{p_R}{p_{RU}} \qquad (2)$$

2. SAC/FEMA APPROXIMATION

A more intuitive and conceptually simpler expression for the URI can be achieved by using the SAC/FEMA approximations to estimate the MAF. Estimating the probability of violating a certain performance objective or limit-state (i.e. demand > capacity) requires knowledge of the seismic hazard at the site, which is represented by the hazard curve, $H(s)$. $H(s)$ is obtained from a probabilistic seismic hazard analysis, and is parameterized by the seismic intensity measure IM = s, typically selected to be the spectral acceleration at the first mode period, $S_a(T_1)$. Scalar representations of demand, D, and capacity, C, of the structure, may be expressed in terms of an engineering demand parameter (EDP). Then, the conditional failure probability, also known as the fragility, $P(C < D|s)$, conveys the probability of limit-state exceedance for a given intensity level. By convolving the fragility with the seismic hazard, $H(s)$, the MAF of limit-state exceedance λ can be estimated as (Jalayer 2003):

$$\lambda = \int_0^{+\infty} P(C < D \mid s)\,\left|\mathrm{d}H(s)\right| \tag{3}$$

To simplify the above integration, an approximate closed-form solution was derived by Cornell et al. (2002), introducing the following assumptions:

1. Within a range of s of interest, the (mean) seismic hazard curve is approximated using a local power law fit:

   $$H(s) \approx k_0\, s^{-k_1} = k_0 \exp(-k_1 \ln s) \tag{4}$$

   where k_0 and k_1, the intercept and slope of the log-hazard curve, are positive real numbers.

2. The limit state capacity EDP-value (e.g. the maximum interstory drift that when exceeded signals violation of the limit-state), follows a lognormal distribution with median $\hat{\theta}_c$ and dispersion $\beta_{\theta c}$.
3. The EDP demand at a specific value of the IM = s is assumed to have a lognormal distribution (generally accurate for displacement and ductility quantities away from global collapse, NIST 2010) with a constant dispersion of $\beta_{\theta d}$ and a conditional median given by:

   $$\hat{\theta}(s) \approx a\, s^b \tag{5}$$

   where a and b are positive real numbers.

Under these conditions, a simple closed form approximation to the MAF of exceeding the limit state ($C < D$) is possible:

$$\lambda_R = H\left[\left(\frac{\hat{\theta}_c}{a}\right)^{\frac{1}{b}}\right] \exp\left(\frac{k_1^2}{2b^2}\beta_\theta^2\right) \tag{6}$$

where $\beta_\theta=\sqrt{(\beta_{\theta c}^2 + \beta_{\theta d}^2)}$ is the total aleatory variability in demand and capacity. If the total demand and capacity uncertainty $\beta_{U\theta}$ is also added, and assuming there is no effect on the median capacity, the overall increased MAF becomes:

$$\lambda_{RU} = H\left[\left(\frac{\hat{\theta}_c}{a}\right)^{\frac{1}{b}}\right] \exp\left(\frac{k_1^2}{2b^2}(\beta_\theta^2 + \beta_{U\theta}^2)\right) \tag{7}$$

If capacity is expressed instead in terms of the required IM value to exceed the limit-state, then, assuming it is lognormal with median $\hat{s}_c$ and aleatory dispersion β_{Sc}, the MAF is estimated as:

$$\lambda_R = H(\hat{s}_c) \exp\left(\frac{k_1^2}{2}\beta_{Sc}^2\right) \tag{8}$$

If uncertainty dispersion β_{USc} is included, the increased MAF becomes:

$$\lambda_{RU} = H(\hat{s}_c) \exp\left(\frac{k_1^2}{2}(\beta_{Sc}^2 + \beta_{USc}^2)\right) \tag{9}$$

Based on the above formulations, a simple closed-form expression for the estimation of I_r is now within reach:

$$I_r = \exp\left(-\frac{k_1^2}{2b^2}\beta_{U\theta}^2\right) = \exp\left(-\frac{k_1^2}{2}\beta_{USc}^2\right) \tag{10}$$

In other words, the URI is directly related to the slope (or, equivalently, the severity) of the hazard curve and the EDP or the IM dispersion due to model parameter uncertainties. In the case where structural uncertainties also influence the median capacity, rather than just the dispersion, Eq. (10) is not an adequate approximation. For some systems, and especially close to collapse, we may observe a reduction from an S_a-capacity of $\hat{s}_{cR}$ to $\hat{s}_{cRU}$. In that case, for example, the second form of Eq. (10) becomes:

$$I_r = \left(\frac{\hat{s}_{cR}}{\hat{s}_{cRU}}\right)^{-k_1} \exp\left(-\frac{k_1^2}{2}\beta_{USc}^2\right) \qquad (11)$$

Furthermore, I_r can find practical use in everyday earthquake engineering calculations. Its use in assessment is obvious, as it is can be employed as a direct divisor to correct a collapse MAF that is estimated without taking into account uncertainties. For typical design applications, it can also be adapted to rationally modify the strength reduction factor R (or behavior factor q) to account for uncertainty robustness. Using the IM-based format of Eq. (9) and the hazard approximation of Eq. (4):

$$\begin{aligned}\lambda_{RU} &= \frac{H(\hat{s}_c)}{I_r}\exp\left(\frac{k_1^2}{2}\beta_{Sc}^2\right) \\ &= k_0\left(\hat{s}_c \cdot I_r^{1/k_1}\right)^{-k_1}\exp\left(\frac{k_1^2}{2}\beta_{Sc}^2\right)\end{aligned} \qquad (12)$$

Let us assume now that the employed q or R factors, e.g. as obtained through a FEMA-P695 (2009) approach, can deal with the effect of ground motion variability, as implicitly (albeit inaccurately) suggested in all seismic codes. Then, one need only multiply the spectral acceleration capacity by the factor

$$R_r = I_r^{1/k_1} \qquad (13)$$

to account for robustness and the influence of structural uncertainties. For a design application, the desired safety is achieved by multiplying the strength reduction R by the correction factor R_r, thus effectively reducing the former to account for uncertainty. In the following pages, we shall illustrate the use and implications of such factors with a simple example.

3. APPLICATION EXAMPLE

The structure selected is a nine-story steel moment-resisting frame with a single-story basement that has been designed for Los Angeles, following the 1997 National Earthquake Hazard Reduction Program (NEHRP) provisions (Foutch and Yun 2002). A centerline model with nonlinear beam-column connections was formed. It allows for plastic hinge formation at beam and column ends (Fig. 1), with the option to deactivate either beam or column hinging selectively to enable or deter the formation of story mechanisms. Thus, the same parametric model can represent a realistic capacity-designed building where plasticity can develop in both beams and columns (termed BCP), only in the beams (BP), or only in the columns (CP). In this case, as shown by static pushover analysis in Fig. 3, the BCP and BP models behave identically, thus leading us to adopt the latter as simpler representative of a ductile capacity-designed frame. This is to be contrasted to its "equivalent" version of a brittle model that allows only column-plasticity (CP). In the latter case, we expect the appearance of story mechanisms that are a well-known bane of robustness in earthquake engineering.

The plastic hinges are modeled as rotational springs with a quadrilinear moment-rotation backbone with six controlling parameters (Fig. 2) that is symmetric for positive and negative rotations and employs a moderately pinching hysteresis without cyclic degradation. The plastic hinge parameters are modeled separately for each member and a spatial correlation model is adopted that incorporates perfect correlation of all plastic hinge parameters of the same type in the structure. The distribution of the parameters themselves is also chosen to be indicative of low construction quality and, thus, relatively well dispersed. For more details on the structural and probabilistic modeling, see Vamvatsikos (2014).

Rayleigh damping of 2% has been applied to the first and second modes. The structural model also includes P-Δ effects, while the internal gravity frames have been directly incorporated. The fundamental mode of the reference frame is at a period of T_1 = 2.35 s and accounts for approximately 84% of the total mass. To perform Incremental Dynamic Analysis (IDA), a suite of 60 ground-motion records has been selected, comprising both horizontal components from 30 recordings. They are all characterized by relatively large magnitudes (6.5–6.9) and moderate distances (15–35 km), while they were all recorded on firm soil and bear no marks of directivity.

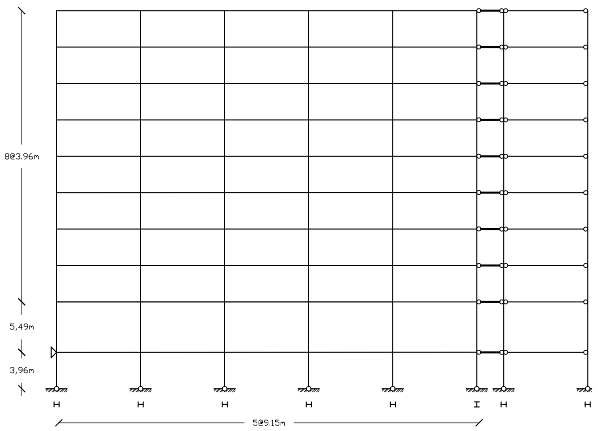

Figure 1: Structural model of the 9-story steel frame, including 5 moment-resisting bays and a single gravity-framing bay on the right instead of the typical leaning column.

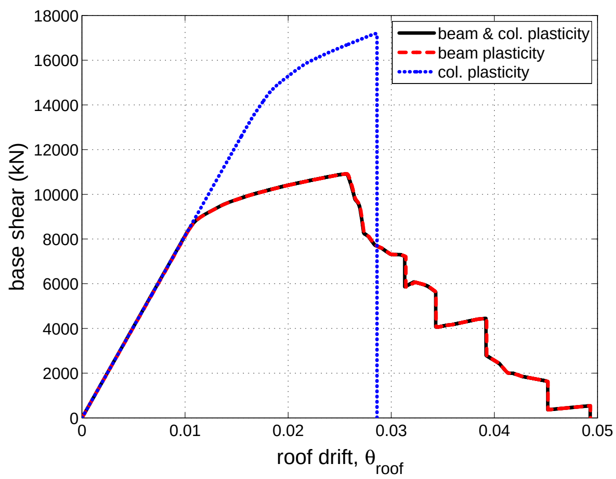

Figure 2: First-mode pushover curves for the BCP, BP, and CP mean parameter models; note that ignoring column plasticity for the BP model does not reduce its accuracy.

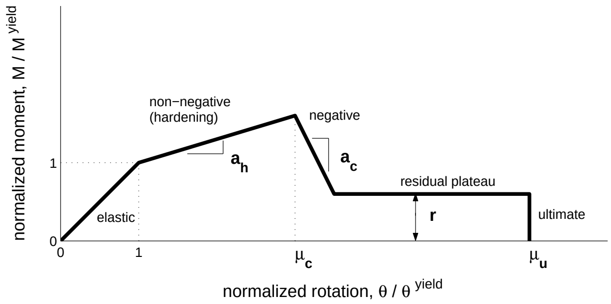

Figure 3: The moment-rotation backbone used for beam and column point hinges shown with its six controlling parameters.

There are several approaches to evaluate the influence of parameter uncertainties, e.g. Liel et al (2008), Dolsek (2008), Vamvatsikos and Fragiadakis (2010). For the present case, a Latin Hypercube Sampling (LHS) algorithm based on IDA will be used (Vamvatsikos 2014). It employs LHS improvements suggested by Tong (2006), Sallabery et al (2008), Schotanus and Franchin (2004) and Nielson et al (2007) and is suitable for the assessment of complex probabilistic models: It achieves significant computational savings by (a) increasing the sample size incrementally and (b) sampling both the structural model and the ground motion records to perform a single-record IDA study per model instance.

The results appear in Figures 4-7, showing the influence of the structural uncertainties on the estimated distribution of demand and capacity. The maximum interstory drift θ_{max} is used as the engineering demand parameter, while the first mode spectral acceleration $S_a(T_1)$ is the intensity measure. Figures 4-5 show the median IDA curve of θ_{max} given the intensity for the two structures. Figures 6-7 present the dispersion of the S_a-capacity, i.e. the dispersion of the S_a values that are required to reach any given level of drift response. For example, to estimate the fragility parameters for a limit-state defined by a given value of interstory drift (and assuming no additional uncertainty exists), one may simply read off the median and dispersion of the associated lognormal fragility function from the corresponding figures of each model.

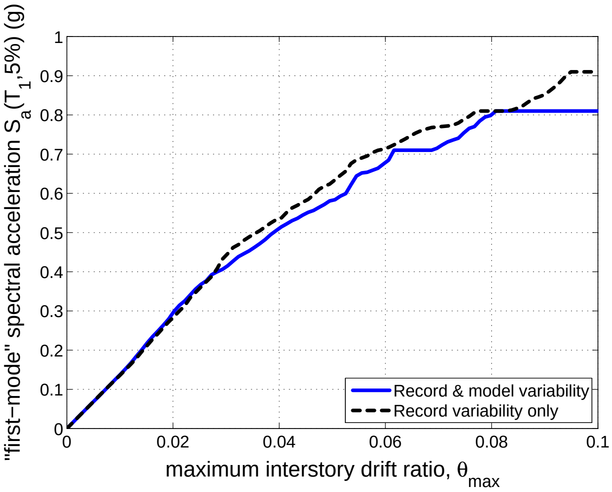

Figure 4: The median IDA curves for the BP model. Plastic hinge uncertainty slightly lowers the median curve, increasing demand and reducing capacity.

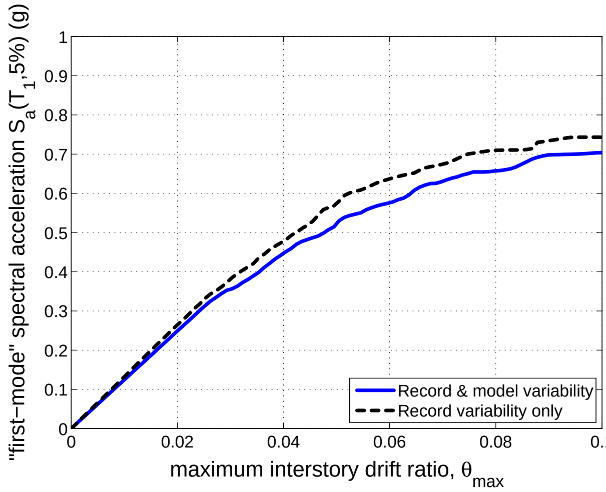

Figure 5: The median IDA curves for the CP model. Plastic hinge uncertainty has a small detrimental effect on the central values of response or capacity.

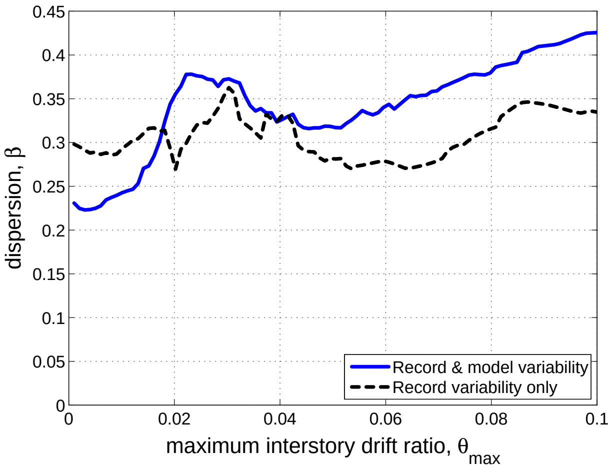

Figure 6: S_a capacity dispersion for the BP model, showing a moderate increase due to parameter uncertainties for drifts higher than 5%.

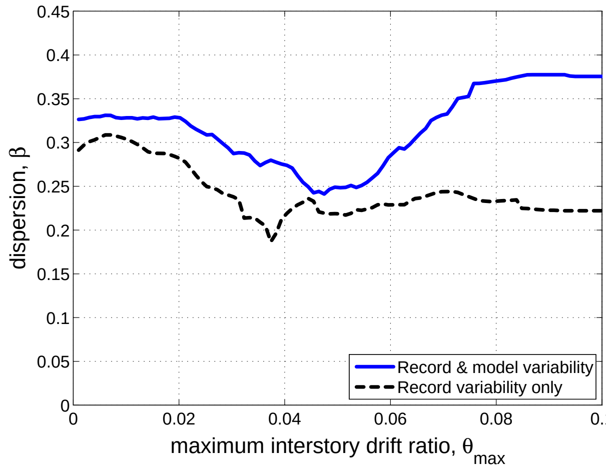

Figure 7: S_a capacity dispersion for the CP model, showing a significant increase due to parameter uncertainties for drifts higher than 4.5%.

It is important to note that the median response of both models in Figures 4-5 is somewhat influenced by the structural uncertainty, showing an increase in the estimated value of θ_{max} with increasing intensity. Actually, at a value of θ_{max} =10% where the structure reaches the highest intensity it can sustain, indicative of global collapse, parameter uncertainties reduce the $S_a(T_1)$ collapse capacity by about 5–10% for both models. Still, this difference is not statistically significant when using 30 ground motion records, thus it will be disregarded in the discussions to follow. In effect, Eq. (10) will be used to derive all URI related quantities.

Let us estimate then the URI for the obvious limit-state of collapse, defining the latter as the exceedance of a maximum interstory drift of 10%. Obviously, lower drift values can be selected with small influence on the reported results. From Figure 5, the BP model shows a total dispersion of β_{RU} = 0.43, and β_R = 0.34. Thus, $\beta_U = \sqrt{(\beta_{RU}^2 - \beta_R^2)}$ = 0.26. Similarly, for the CP structure, β_{RU} = 0.38, β_R = 0.22, thus

β_U = 0.31. Despite this small apparent difference in the dispersion due to model uncertainty (0.31 versus 0.26) between the CP and the BP models, the results in terms of robustness are quite distinctive. Table 1 shows the URI results for three different levels of hazard slope severity. Lower k values are generally associated with sites of lower seismicity. Thus, k_1 = 2 would be indicative of East USA or North/Central Europe, while k_1 = 4 corresponds to West USA and the more seismically active regions of the Mediterranean. Obviously, the robustness index drops for both structures as the seismicity increases. This change is more pronounced, though, for the CP model. This may be declared nearly robust for East USA, having I_r = 0.83, practically the same as a capacity-designed BP frame, but it is severely problematic for West USA, where I_r = 0.46. In the first case, structural uncertainties will increase the MAF of collapse by a factor of 1/0.83 = 1.20, while for the higher seismicity site we expect an increase by 2.17 instead. Current engineering intuition and seismic guidelines seem to reflect such results. Strict rules for avoiding story mechanisms are in effect mainly for the highly seismic parts of Europe and USA, recognizing the inherent non-robustness caused by allowing plasticity in the columns.

Modest reductions to R-factors would be needed to take account of these effects. The needed R_r correction factors range from 0.93–0.87, i.e., a 7–13% reduction for a capacity-designed frame, to 0.91–0.83, or 9–17% reduction for the less ductile option.

Table 1: Collapse URI values for the beam-plasticity (BP) and the column-plasticity (CP) models. Increasing hazard slope (or severity) reduces system robustness to collapse.

hazard	$k_1 = 2$	$k_1 = 3$	$k_1 = 4$
I_r, BP model	0.87	0.73	0.57
I_r, CP model	0.83	0.65	0.46
R_r, BP model	0.93	0.90	0.87
R_r, CP model	0.91	0.87	0.83

4. CONCLUSIONS

The uncertainty robustness index is based on the principle of measuring the effect of structural uncertainties on the seismic performance of the system. Given the non-localized nature of seismic loads, it is a simple yet practical parameter that can reveal the susceptibility of structures to uncertain structural properties. It should be kept in mind that such effects are often dwarfed by the variability in ground motion that tends to overwhelm seismic performance estimates. Still, the latter can never be controlled by the engineer; it simply has to be accepted for what it is. On the other hand, the structural configuration, system design concept and associated construction quality control are subject to engineering decisions. Therefore, supplying a proper index that can help place alternative design concepts within a proper hierarchy can go a long way towards helping engineers understand the consequences of their choices.

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